

Soundness and completeness theorem:

The set S of clauses is inconsistent $\Leftrightarrow S \mid \text{Res} \square$

The backtracking algorithm stops in two cases:

1) The empty clause was derived and the conclusion will be that S is inconsistent

2) for the top clause all the possible side clauses were used but the empty set was not derived, then we conclude that the set S is consistent

U is a theorem $\Leftrightarrow \text{CNF}(\neg U) \mid \text{Res} \square$

Linear resolution is a refutation method because it model the "reductio ad absurdum" and it uses the negation of the formula to prove something.

$$U = (t \rightarrow \neg s \vee r) \rightarrow (t \rightarrow \neg s \wedge r)$$

$$\neg U = \neg((t \rightarrow \neg s \vee r) \rightarrow (t \rightarrow \neg s \wedge r))$$

We change $X \rightarrow Y$ with $\neg X \vee Y$

$$\equiv \neg((\neg t \vee (\neg s \vee r)) \rightarrow (\neg t \vee (\neg s \wedge r)))$$

$$\equiv \neg(\neg(\neg t \vee (\neg s \vee r)) \vee (\neg t \vee (\neg s \wedge r)))$$

We apply De Morgan's laws

$$\equiv (\neg t \vee (\neg s \vee x)) \wedge \neg(\neg t \vee (\neg s \wedge x))$$

Besides already at hand
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$$\equiv (\neg t \vee (\neg s \vee x)) \wedge (t \wedge (s \vee \neg x))$$

$$\equiv (\neg t \vee \neg s \vee x) \wedge t \wedge (s \vee \neg x)$$

$$S = \{\neg t \vee \neg s \vee x, t, s \vee \neg x\}$$

$$C_1 = \neg t \vee \neg s \vee x$$

$$C_2 = t$$

$$C_3 = s \vee \neg x$$

We use C_1 as a top clause

I. C_2 is used as a side ^{clause} for C_1

$$C_4 = \text{Res}_t(C_1, C_2) = \neg s \vee x$$

$$C_5 = \text{Res}_s(C_4, C_3) = \neg x \vee x \equiv T (\text{tautology})$$

process blocked

II. We use C_3 as a side clause for C_1

$$C_6 = \text{Res}_s(C_1, C_3) = \neg t \vee x \vee \neg x \equiv T (\text{tautology})$$

The search process is blocked

A complete linear derivation process search was performed

But \Box was not derived $\Rightarrow S$ is a consistent set \Rightarrow

$\Rightarrow U$ is not a theorem