LOGIC CIRCUITS

Individual homework
Exercise 4

Bledea Mihaela-Alexandra

VEITCH-KARNAUGH DIAGRAMS METHOD

- **The cells in a Veitch/Karnaugh diagram are arranged so that there is only a single variable change between neighbor (adjacent cells).**
- **Adjacency (graphical neighborhood) relation is defined by a single variable change.**
- **The diagrams are circular, meaning that the first row/column and the last row/column are neighbors.**
- **Two minterms belonging to two neighbor cells factorize => *simple factorization*.**
- **The result of a *k-factorization* is a monom which contains the common variables of all 2^k neighbor minterms (cells).**
- **The maximal monoms are obtained from the diagram (Veitch/Karnaugh) by applying factorizations as follows: for a Boolean function of *n* variables we try first *n-factorization*, we continue with *n-1 factorizations*, ... , *simple factorizations* and *0-factorizations* (isolated minterms).**

Veitch-Karnaugh Diagrams Method

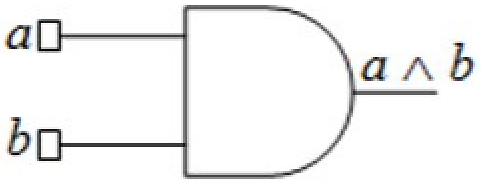
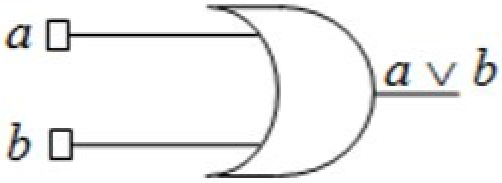
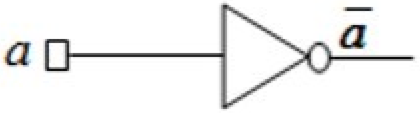
- Factorization process => the set of *maximal monoms* $M(f)$.
- From the set of maximal monoms the *central monoms* are selected => $C(f)$
- The *central monoms* are the maximal monoms which cannot be covered by the disjunction of all the other maximal monoms.
- Identification of the *central monoms:*

  if the group of the minterms covered by a maximal monom contains at least one cell (minterm) circled once, then the maximal monom is a central one and it belongs to all the simplified forms of the function.
- The groups of the minterms corresponding to the central monoms are shaded in the diagram.
- The minterms from the function's expression which are unshaded, are not covered by the central monoms and they will be covered in all the possible ways using a minimum number of unused maximal monoms and with a minimum number of overlaps, resulting *all the simplified forms* of the initial function.

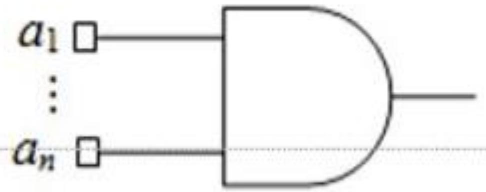
Logic Circuits

- Binary Boolean algebra provides a concise way to express the operation of a logic circuit formed by a combination of logic gates so that the output can be determined for various combinations of input values.
- The basic elements used to build electronic circuits are called basic gates and they implement one of the Boolean operations: $\wedge$, $\vee$, $\bar{\ }$.
- A gate takes the values from its input wires and combines them with the appropriate Boolean operation to produce the label on its output wire
- A Boolean function models the functionality of a logic circuit.
  The variables used in the function's expression are represented by the inputs of the circuit. Each wire in the circuit represents some part of the expression. The final output of the whole circuit represents the function's expression.

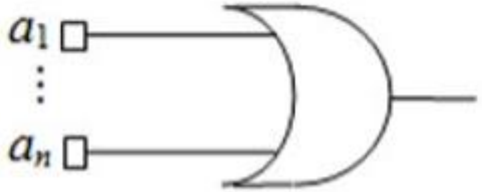
Basic gates – IEEE standards

gate	symbol for the gate	Boolean operation, symbols used
AND		**conjunction** $a \wedge b$, $a \cap b$, $a * b$, ab
OR		**disjunction** $a \vee b$ $a \cup b$, $a + b$
NOT		**negation** $\neg a$, $\overline{a}$

The AND and OR gates can be generalized to have more input variables:



$a_1 \wedge a_2 \wedge \ldots \wedge a_n$ or $a_1 a_2 \ldots a_n$



$a_1 \vee a_2 \vee \ldots \vee a_n$

Exercise 4.

Write a Boolean function of 4 variables given by its table of values, simplify it and draw the logic circuits corresponding to all its simplified forms.

The disjunctive canonical form is:

$$f(x_1, x_2, x_3, x_4) = \overline{x}_1\overline{x}_2\overline{x}_3\overline{x}_4 \vee \overline{x}_1\overline{x}_2\overline{x}_3 x_4 \vee \overline{x}_1\overline{x}_2 x_3\overline{x}_4 \vee \overline{x}_1 x_2\overline{x}_3\overline{x}_4 \vee$$

$$\vee\ \overline{x}_1 x_2\overline{x}_3 x_4 \vee \overline{x}_1 x_2 x_3\overline{x}_4 \vee x_1\overline{x}_2 x_3\overline{x}_4 \vee x_1 x_2\overline{x}_3 x_4 \vee x_1 x_2 x_3\overline{x}_4$$

$$\vee\ x_1 x_2 x_3 x_4$$

X_1	X_2	X_3	X_4	$F(X_1, X_2, X_3, X_4)$
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

The disjunctive canonical form is:

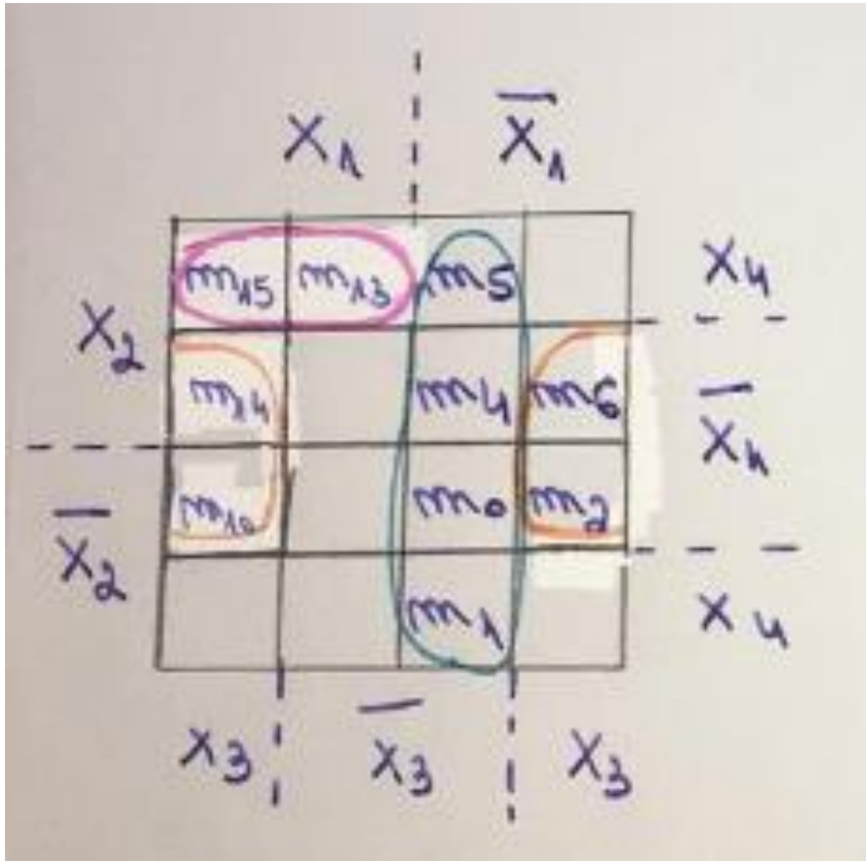
$$f(x_1,x_2,x_3,x_4)=\overline{x}_1\overline{x}_2\overline{x}_3\overline{x}_4 \vee \overline{x}_1\overline{x}_2\overline{x}_3 x_4 \vee \overline{x}_1 \overline{x}_2 x_3 \overline{x}_4 \vee \overline{x}_1 x_2 \overline{x}_3 \overline{x}_4 \vee$$

$$\vee\ \overline{x}_1 x_2 \overline{x}_3 x_4 \vee \overline{x}_1 x_2 x_3 \overline{x}_4 \vee x_1 \overline{x}_2 x_3 \overline{x}_4 \vee x_1 x_2 \overline{x}_3 x_4 \vee x_1 x_2 x_3 \overline{x}_4$$

$$\vee\ x_1 x_2 x_3 x_4$$

$$f(x_1,x_2,x_3,x_4) = m_0 \vee m_1 \vee m_2 \vee m_4 \vee m_5 \vee m_6 \vee m_{10} \vee m_{13} \vee m_{14} \vee m_{15}$$

We use Veitch Diagram Method for finding the function's simplified forms.



$$max_1 = m_0 \vee m_1 \vee m_4 \vee m_5 = \overline{x_1}\,\overline{x_3}$$

$$max_2 = m_2 \vee m_6 \vee m_{10} \vee m_{14} = x_3\,\overline{x_4}$$

$$max_3 = m_{13} \vee m_{15} = x_1 x_2 x_4$$

The set of maximal monoms → $M(f) = \{max_1, max_2, max_3\}$

The set of central monoms → $C(f) = \{max_1, max_2, max_3\}$

$M(f) = C(f)$ => We have one simplified form for f

The simplified form of f:

$$f^S(x_1, x_2, x_3, x_4) = \overline{x_1}\,\overline{x_3} \vee \overline{x_3 x_4} \vee x_1 x_2 x_4$$

$$y(x_1, x_2, x_3, x_4) = \bar{x}_1 \bar{x}_3 \vee x_3 \bar{x}_4 \vee x_1 x_2 x_4$$