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In order to prove if the conclusion is derivable from the hypotheses I am going to use the Deduction in first-order logic theorem.

Let $U_1, U_2, ..., U_n$ be first order formulas, and V ~~(C)~~ the conclusion. V is deducible from $U_1 U_2 ..., U_n$ if there exists a sequence of formulas $(f_1, f_2, ..., f_m)$ such that $f_m = V$ and we have:

$\forall i \in \{1, ..., m\}$

a) $f_i \in A_{pl}$ (axiom of predicate logic)

b) ~~($f_i \in$)~~ f_i is one of the hypotheses

c) We can derive ~~another formula from two already exis~~ f_i from two formulas which exist in the sequence using modus ponens.

d) We can obtain f_i using the universal generalitation on an existing formula in the sequence.

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H_1: Anyone who plays tennis or soccer is healthy.

H_2: Anyone who plays chess is smart

H_3: All CS students play chess

H_4: Samuel is a CS student and plays tennis

C: Samuel is smart and healthy

D - is the domain: the universe of people

Samuel - constant $\in$ D

Predicate symbols:

unary:

$CS : D \to \{T, F\}$, $CS(x) = T$ if x is a CS student

$chess : D \to \{T, F\}$, $chess(x) = T$ if x plays chess

$tennis : D \to \{T, F\}$, $tennis(x) = T$ if x plays tennis

$soccer : D \to \{T, F\}$, $soccer(x) = T$ if x plays soccer

$healthy : D \to \{T, F\}$, $healthy(x) = T$ if x is healthy

$smart : D \to \{T, F\}$, $smart(x) = T$ if x is smart

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$H_1: (\forall x)((tennis(x) \vee soccer(x)) \to healthy(x))$

$H_2: (\forall x)(chess(x) \to smart(x))$

$H_3: (\forall x)(CS(x) \to chess(x))$

$H_4: CS(Samuel) \wedge tennis(Samuel)$

$C: Smart(Samuel) \wedge healthy(Samuel)$

$H_4 \vdash_{simplification} CS(Samuel) = f_5$

$U \wedge V \vdash_{simplif} U$

$U \wedge V \vdash_{simplif} V$

$H_3 \vdash_{univ.-inst} CS(Samuel) \to chess(Samuel) = f_6$

universal
instantiation

$(\forall x)\, U(x) \vdash_{univ.-inst} U(t)$

t - term (variable or constant of the domain)

$f_5, f_6 \vdash_{mp} chess(Samuel) = f_7$

modus
ponens

$V, V \to U \vdash_{mp} U$

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$H_2 \vdash_{univ\_inst}$ clean(Samuel) -> smart(Samuel) = f_8

$f_4, f_8 \vdash_{mp}$ smart(Samuel) = f_9

$H_4 \vdash_{simplification}$ tennis(Samuel) = f_{10}

$H_1 \vdash_{univ\_inst}$ (tennis(Samuel) v soccer(Samuel)) -> healthy(Samuel) = f_{11}

$f_{10}, f_{11} \vdash_{mp}$ healthy(Samuel) = f_{12}

$f_9, f_{12} \vdash_{conjunction}$ smart(Samuel) $\wedge$ healthy(Samuel) = C

(H_1, H_2, H_3, H_4, $f_5, f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_{12}$) is the proof of C. Therefore Samuel is smart and healthy.