Maastricht University

Multi Agent Systems
Group 6

Strategic Voting

Authors
Diez Pérez María - i6268086
Gianzina Alexandra - i6294354
Osnabrugge Niels - i6252882
Santing Lukas - i6298143
Simon Eliott - i6184618
Vico Gianluca - i6183186

April 13, 2022

Contents

Abstract

Individuals who are given the opportunity to vote, depending on the circumstances and the content they are voting for, may vote honestly or may vote tactically. The aim of this project is to see whether a strategic voter can change the outcome of the voting situation and increase his happiness levels. So, we implemented a Tactical Voting Analyst, which, given an honest voting situation, computes the outcome and calculates the individual happiness and the overall happiness of the voters. We conducted different experiments to see the sensitivity of each scheme and tested whether voter happiness depends on the definition of the happiness function and whether different definitions of happiness can bring different levels of happiness. We came to the conclusion that certain functions of happiness can lead to happier voters. In addition, we experimented with different ranges of voters. We started some experiments with a small number of voters and then also increased the number of voters to 100. We found that it is possible for a voter to find a tactical strategy even when he is among 99 voters. Finally, we did some experiments on multiple voters who can cooperate and agree on a tactical vote, and we concluded that voters can do it, but depending on the scheme they may have more or less tactical voting options.

1 Happiness and Risk

1.1 Happiness

The happiness function we propose is based on how similar an individuals vote is compared to the overall vote. This is based on the fact that humans want to be acknowledged, and therefore also want their vote to be acknowledged. This creates the happiness function as can be seen below in 1. *Points* is the calculated points a candidate got based on a ranking and a voting scheme.

$$H_i = \sum_{c \in C} \left(\frac{P(c) \times (C - R(c,i))}{\sum_{t \in C} P(t)} \right) \tag{1}$$

where P = Points, R = Ranking, and C = Candidates.

An important aspect of this happiness function is the fact that you not only care who wins, but also care who becomes second, third etc. This means that two people who voted on the winner can have quite a different happiness score. We consider this to be an advantage; for instance, what if the person you hate becomes close second, and therefore still has quite some power?

An example voting situation can be seen in the table below. Using the 'Vote for two' voting scheme, we would get the following happiness for Voter 1:

$$H_1 = \frac{1}{8} * 1 + \frac{3}{8} * 3 + \frac{2}{8} * 2 + \frac{2}{8} * 0 = \frac{14}{8}$$

A	B	C	D
2	2	4	3
3	4	1	2
1	1	3	1
4	2	2	3

1.2 Risk

For the risk, we consider two different measures: how many voters have at least one tactical vote, and how many of each individual voter's total possible votes are tactical votes.

If we assume that the voters are rational and able to find a tactical vote, then a tactical vote occurs. The risk, in this case, is given by the ratio between the voters with at least one tactical vote and number of voters. For examples, if there are 3 votes with 2, 1 and 0 tactical votes each, then the risk is $\frac{2}{3}$. If the tactical votes for the voters are respectively 10, 1 and 0, then the risk is still $\frac{2}{3}$, because the number of voter that can vote tactically is the same.

However, the voters might not have perfect information about the other voters' preference, or they might not be fully rational. Therefore, they might not be able to find all the tactical votes or select the best option. In this case, the risk depends on the number of individual alternative votes that are tactical votes.

Then, the risk can be defined as follows:

$$risk = \frac{\sum_{i \in Voters} count\_tv(i)}{\sum_{i \in Voters} count\_av(i)} \tag{2}$$

In, Eq. (2), $count\_tv(\cdot)$ counts the number of possible tactical votes for the given voter, while $count\_av(\cdot)$ counts the number of alternative possible votes. If we exclude bullet voting, $count\_av(i)$ is equal to $|Options|! - 1$ for every agent i. For instance, 2 voters with respectively 1 and 2 possible tactical votes and 5 possible alternative preferences give a risk of 30% of tactical votes. With the first definition of risk, it would have been 100%.

The downside of this risk definition is that it needs to compute all the permutations of preferences. Give m voters, n options and a coalition of size c ($c = 1$ for the basic TVA), there are $n!$ alternative preferences for each voter, and every time we need to compute the happiness for the voters.

The happiness of a group of voters can be computed in $O(cn)$ time. The preferences of a coalition require $O((n!)^c)$ steps, and there are $O(\frac{m!}{c!(m-c)!})$ coalitions. Therefore, the time complexity for the risk is $O(\frac{m!}{c!(m-c)!}(n!)^c cn)$, which does not scale well when the number of options n increases.

Moreover, we do not distinguish tactical voting by compromising or by burying, but we allow a mix of both strategies.

The first definition of risk is indicated throughout the report as **Boolean Risk** (or **Bool risk** for short), while the second one is indicated as **Risk**.

2 Experiments and Results

The experiments conducted using the TVA can be found below. In order to make the results clearer, we use bar charts to represent both the number of tactical votes per voter, as well as the happiness of each voter before and after applying it for the different vote schemes.

2.1 Basic TVA

2.1.1 Different happiness functions

We start off the experiments by trying out three different happiness functions. As will be explained, the first is the happiness function we settled on for the rest of our experiments, as described in equation 1 above in section 1.1. Equations 3 and 4 described below are alternate descriptions of happiness that we explored for the purposes of experimentation.

Equation 3 considers only the top candidate (the candidate who was voted by each voter to be in the first position) as relevant for voter happiness. Equation 4 defines happiness as the weighted total distance of the personal vote against the final outcome, per candidate.

$$H_i = \frac{P(TP(i)}{\sum_{t \in C} P(t)} \tag{3}$$

$$H_i = (\frac{\sum_{c \in C} |R(c) - R(c,i)| \times (C - R(c,i))}{C!}) \tag{4}$$

In this experiment, we tested the three different happiness functions on the same voting situation with 10 voters; A to J, and 4 candidates; 0 to 3, as follows:

A	B	C	D	E	F	G	H	I	J
0	0	2	3	0	1	3	1	0	1
1	1	0	1	3	3	2	3	3	2
2	2	3	2	1	0	0	2	1	0
3	3	1	0	2	2	1	0	2	3

Let's start the comparison of these three functions by comparing the original happiness of each candidate using the Vote-For-One voting scheme.

As expected, different ways of measuring happiness, results in different levels of happiness per voter for the same voting situation and voting scheme. Interestingly, the first and third results are the most similar, and are also the only two that take the full list of votes into account, albeit in different ways. Each other voting scheme similarly shows differences, but is not presented here in the interest of space.

Let's also look at the difference that these happiness functions make on the number of tactical votes each voter has. Again, there is more similarity between the results from functions 1 and 4.

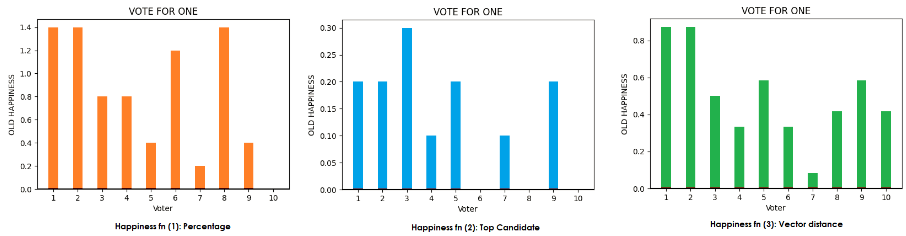


Figure 1: Original happiness for each voter for the different happiness functions with the Vote-For-One voting scheme

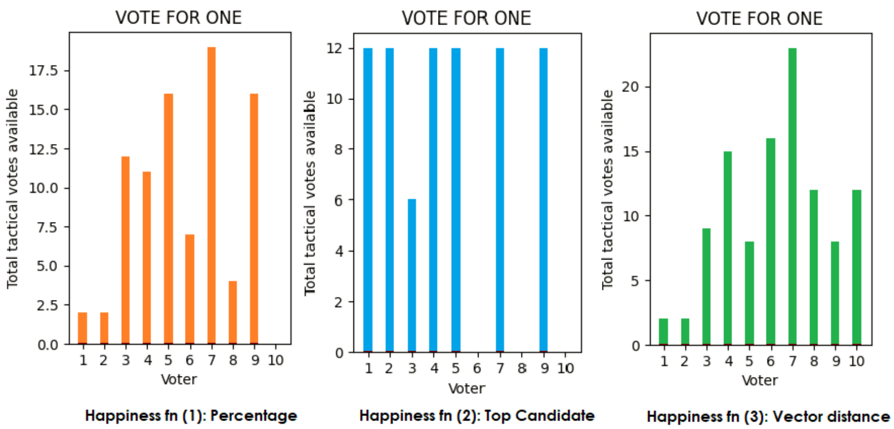


Figure 2: Number of tactical votes for each voter for the different happiness functions with the Vote-For-One voting scheme

An interesting observation is that equation 3 has a smaller variance in happiness than the other two. This can be seen as a bad thing because in reality this is rarely the case. In the end, the decision was taken to continue our other experiments using the 1 happiness function. We think this definition captures the effect on each individual ranking better than using vector distances 4. After all, the margin in which someone won or lost should play an effect in someone's happiness.

2.1.2 5 voters and 3 candidates

We continue the experiments using the initial happiness as described in section 1.1. The first experiment that we ran is with 5 voters and 3 candidates. In Figure [3] we can observe that each scheme might have a voter who does not have a tactical vote available. Also, the number of tactical votes seems to have the same distribution for each scheme. Thus, we can conclude that the scheme we choose does not have a different sensitivity to strategic voting, as the influence that each scheme has seems to be the same. We also notice that the average of tactical votes for scheme *vote for one* is slightly higher, which means that it could be easier to find a strategic vote that maximizes the total happiness.

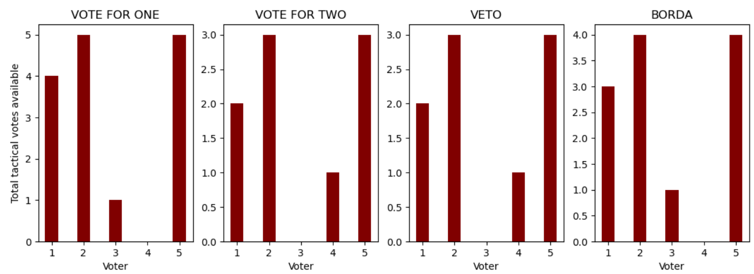


Figure 3: Total tactical votes for 5 voters and 3 candidates

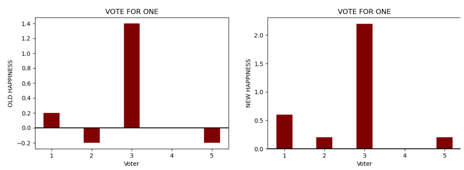
Figure [4] represents the happiness obtained per voter. The old happiness shows the happiness of each voter without any tactical strategy, while the new happiness shows the happiness of each voter after applying the strategic voting. We would like to underline the fact that if a voter is applying a tactical vote for himself, then this will not immediately result in the increase of the happiness of the other voters.
The individual happiness of each voter differs in the chosen strategy. For example, in *vote for one* [4a], only one voter has a high level of happiness (> 2) while the others do not get even the half. Using the others strategies the level of happiness is more similar, for instance, to *Veto* [4c], the range is between 1 and 1.4.

The number of tactical votes is not relevant to the increase of the happiness, because a voter only needs one tactical vote to improve it. Our Tactical Voting Agent is looking for any tactical strategy that can increase the individual happiness of each voter, but chooses a random one that increases his happiness that may not be optimal.

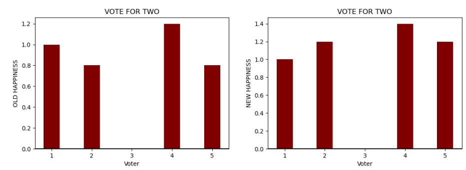
Tables [1] show the total risk and the average risk per scheme, as described in section 1.2.

2.1.3 20 voters and 3 candidates

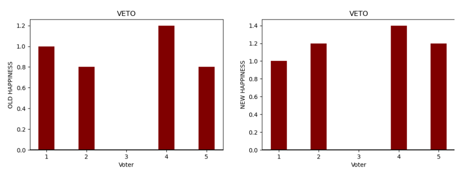
Now we are going to analyse what happens when we have a higher number of voters with an odd number of candidates. This scenario has 20 voters and 3 candidates, the following tactical voters image [5], shows that there is one voter who does not have any tactical vote for all the strategies. We also can see that *vote for one* has 4 voters with 5 tactical votes, while *vote for two, veto and borda* have 6. On the other hand, when we use *borda*, almost half of the voters have only 1 tactical vote.



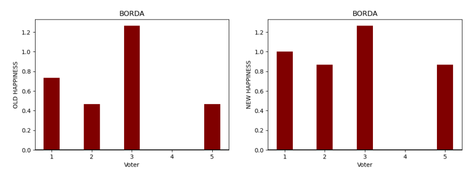
(a) Vote for one



(b) Vote for two



(c) Veto



(d) Borda

Figure 4: Happiness for 5 voters and 3 candidates

	Total Risk	Average Risk
Vote for one	15	0.6
Vote for two	9	0.36
Veto	9	0.36
Borda	12	0.48

Table 1: Total Risk and Average Risk for 5 voters and 3 candidates

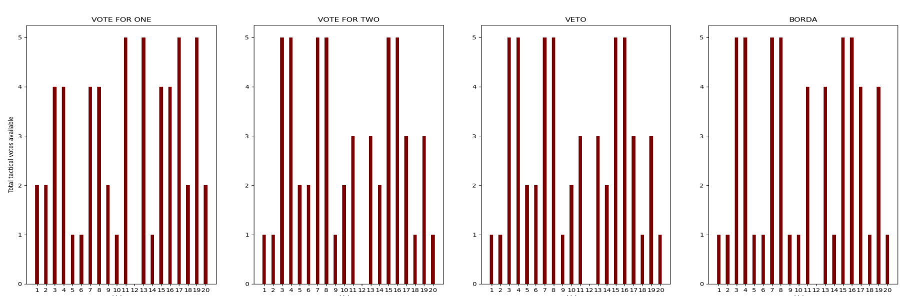


Figure 5: Total tactical votes for 20 voters and 3 candidates

If we compare the tactical votes that each voter has with each strategy, we notice that the voters with more tactical votes are less happy, while the voters with only one tactical vote are the happiest. Let's focus on *borda* graph [6], for instance, in the old happiness bar chart we can derive that the voters 1, 2, 5, 6, 9, 10, 13, 18 and 20 have the higher level of happiness, almost 1.2, and these are the same voters that have only one tactical vote. Even though they have the same number of tactical votes and the same happiness when we apply the tactical votes, we can see that the happiness of these voters increases but not at the same rate. This result is something we expected because the Tactical Voting Agent always takes into account the rise of the happiness of each voter after the application of a strategic voting. The fact that a voter may have more options to choose from does not mean that, even if he applies the best option, this will result in the highest individual happiness compared to other voters. It is very likely that another voter will have only one option that will lead to the highest possible level of individual happiness.

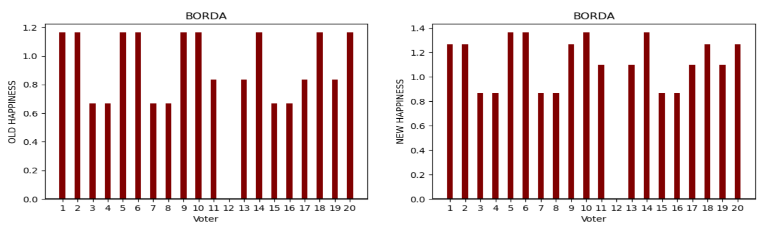


Figure 6: Borda

2.1.4 100 voters and 4 candidates

Another experiment we want to look at is what happens when we increase the number of voters to 100, and we choose an even number of candidates. We observe that according to Figure [7] each scheme has different distribution to the tactical votes available. We will examine voters that have less than 10 tactical votes. The scheme *vote for one* has 56 voters, the Scheme *vote for two* has 53 voters, the scheme *veto* has 63 voters and scheme *borda* has 50 voters. The conclusion that we draw from this data is that *borda* and *vote for two* are a little more flexible to tactical voting because half of the voters have more than 10 options for tactical voting.

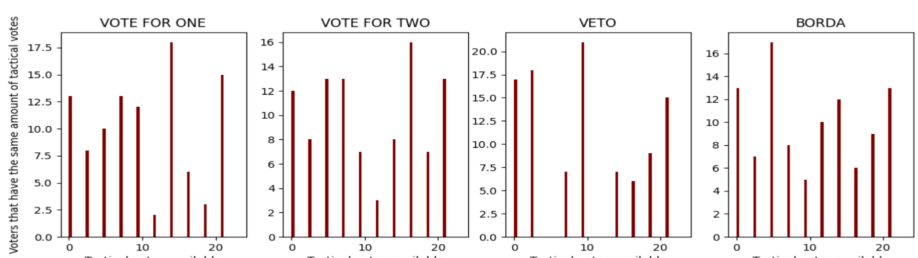
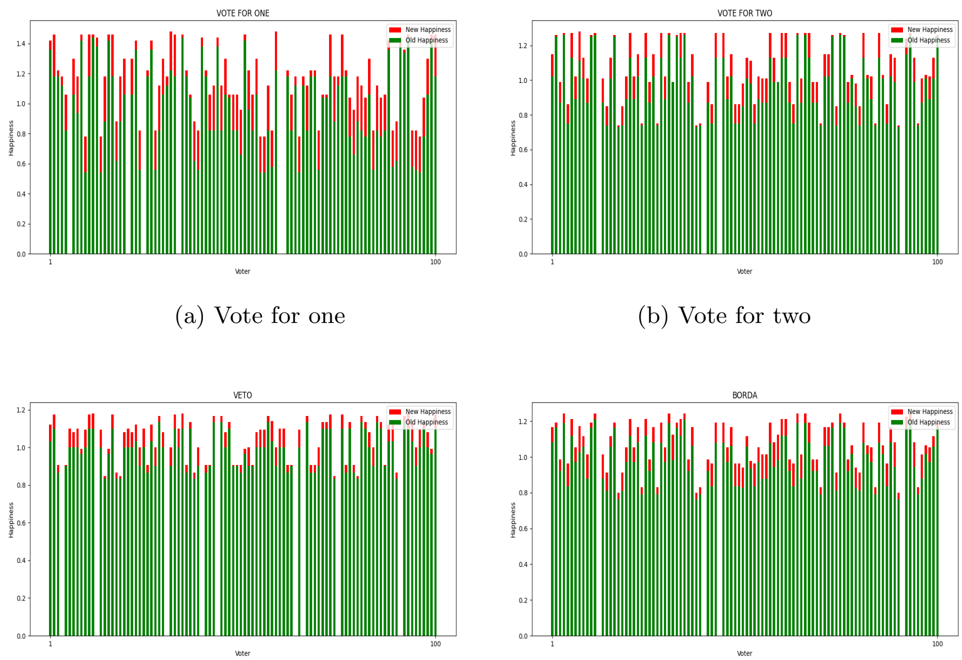


Figure 7: Voters that have the same amount of tactical votes

Below, in Figure [8] we demonstrate for each scheme in green the old happiness of each voter and in red the new happiness of each voter after the application of the tactical voting. Each tactical vote applies to the voter individually, resulting in the increase of his happiness. We should take into account that if a voter is applying a tactical vote for himself, then this will not immediately result in the increase of the happiness of the other voters. So, what the following graph does is to print each voter's happiness individually after the application of his strategic voting. We notice that the scheme *vote for one*, according to the graph [8a], has the largest deviation from the original happiness, possibly resulting in much more satisfied voters if they choose to apply strategic voting.



(a) Vote for one

(b) Vote for two

(c) Veto

(d) Borda

Figure 8: New and Old Happiness for 100 voters and 4 candidates

2.2 Advanced TVA

2.2.1 Bullet voting

In this section, we investigate the effect of bullet voting by considering the number of tactical votes, both when this kind of voting is allowed and when it is not allowed.

The voting situation has 10 voters and 4 options. The exact same situation is used for the case with bullet voting and for the case without.

Scheme	Risk - bullet	Risk	Bool Risk - bullet	Bool Risk
Vote for one	0.470	0.469	0.8	0.8
Vote for two	0.400	0.400	1	1
Veto	0.414	0.413	0.8	0.8
Borda	0.4	0.395	0.9	0.9

Table 2: Comparison of the risk when the voters can use bullet voting. Both risk definitions are used.

Table 2 shows the results of the simulation. A decrease of the risk means that no bullet voting increases the happiness of a voter. We can see that the risk is always decreasing with the exception of the scheme *vote for two*. In this case, the risk is the same, meaning that some bullet voting can increase the happiness.

Moreover, for all 4 schemes, if a voter cannot do tactical voting, then bullet voting does not give this voter any advantage.

2.2.2 Voter collusion

In this experiment, we explore what happens when multiple voters can cooperate and agree on a tactical vote.

As before, we generate all the preferences of the voters, but now we combine all the preferences of the voters in the same coalition.

The voter situation for this experiment consists of 5 voters, 3 options, where the voters forms coalitions of size 2. Bullet voting is not allowed. Voters in a coalition perform tactical voting only when this strictly increases the happiness of all the member of the coalition. However, we do not consider whether a tactical vote is optimal for all the members.

Note that since there are only 3 options, the schemes vote for 2 and veto give the same results.

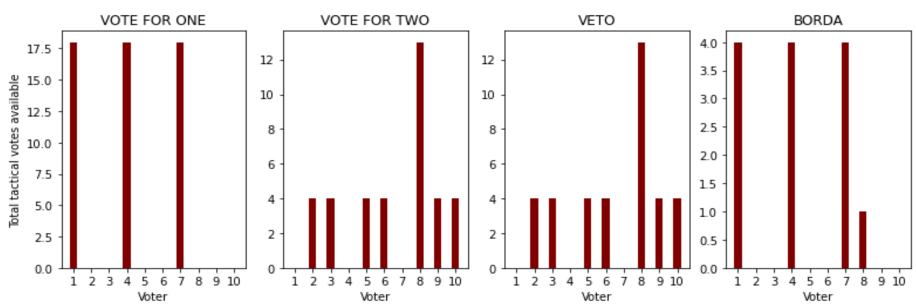


Figure 9: The graph shows on the x-axis the coalition and the y-axis the number of tactical votes that a coalition can perform. All 4 schemes are considered.

Scheme	Risk	Bool Risk
Vote for one	0.216	0.3
Vote for two	0.148	0.7
Veto	0.148	0.7
Borda	0.052	0.4

Table 3: Risk of tactical voting for the different voting schemes and coalitions of size 2.

From Figure 9, it is possible to observe that with the scheme vote for 1, few coalitions have many ways to perform tactical voting. This corresponds to a high risk but a low Boolean risk in Table 3.

With the schemes *vote for two* and *veto*, each coalition has less tactical votes, but more coalitions can do tactical voting. Therefore, the risk is high.

With *borda*, there are few coalitions that can perform tactical voting and, in general, they have fewer ways to do this. Then, *borda* has a lower risk of tactical voting if we allow voter collusion, at least in this voting situation.

3 Discussion and Conclusion

In this section, we highlight and conclude the main insights from the Basic and Advanced TVA experiments.

When it comes to the difference in number of tactical votes per voting scheme, there seems to be very little apparent change. Mild changes do appear, but generally the number of tactical votes are very similar per scheme, and it is the same voters who usually have the most votes across each strategy. There further are relatively few "drastic" changes when it comes to happiness before and after tactical voting. While changes do certainly occur, the same few voters with high happiness tend to also have a higher happiness after tactical voting. This makes sense, because our happiness function takes into account the margin in which their preferences are winning. It rarely occurs that the results of a voting change in an extreme way.

One expectation of ours was that there would be a link between the number of tactical votes available to an individual voter, and the size of the rise in happiness after tactically voting. The reasoning behind this expectation is that if there are more tactical voting options, then the chance of one of those options leading to greater happiness would be higher than the case with fewer options. Surprisingly, this is not reflected in our results, and while there is a rise in happiness after tactical voting as expected, the size of this increase does not seem to correlate with the number of tactical votes available. In fact, in some cases lower number of tactical voting options lead to a relatively larger increase in happiness.

An observation that is clear from the results of our experiments is that there seems to be no direct correlation between the number of tactical votes the voting strategy and the relative number of votes. We often see that the same relative number of votes are seen throughout different voting strategies, for the same voting situation.

Another interesting result from our experiments shows that there may be a link between the number of candidates in a voting situation and a smaller total increase of happiness after tactical voting. One possible reason for this may be that when there are more candidates, the range of possible changes is much larger, and thus a situation leading to greater happiness is unlikely to be reached by a single tactical vote.

An important observation from the three possible definitions of happiness, observed in experiment 2.1.1 is that equations for happiness that take all of the votes into account show a negative correlation between original happiness and number of tactical votes, an observation that feels intuitively logical, as a high happiness with an outcome should intuitively result in fewer ways to "influence" the results to your preference.

Throughout the TVA basic experiments, we noticed that the rise of the individual and the overall level of happiness does not depend on the number of voters we have. By this we mean that even if a voter who wants to apply strategic voting belongs to the minority of voters, as he may be 1 in 100 voters, he will find an alternative voting strategy that will increase his happiness. It does not matter whether he is among other 100 voters or if he is among 5 other voters, the outcome can still change. It would be interesting to expand the experiments and add more voters and find out if one voter among 10.000.000 voters can change the outcome and increase his individual and overall level of happiness.

In the advanced TVA experiments, we can observe that bullet voting does not seem to give any relevant advantage to the voters. For all the voting schemes, the voters are able to vote tactically also without bullet voting.

Schemes such as vote for two or veto seem to suffer more from voter collusion. Borda is the scheme that has relatively few coalitions able to do tactical voting and each coalition has few ways to achieve this.

The main limitation of this work is that the voting situations are generated randomly, but the experiments are performed on a single sample. Therefore, the results highly depend on the specific voting situation. In order to avoid this issue, the number of experiments would have to increase dramatically, to allow for multiple experiments per voting situation and analysis across these multiple experiments, resulting in a more general conclusion for each voting situation.

Another limitation exists within the calculation of tactical votes. We only calculate how much a voter's happiness can possibly increase when it alone would perform a tactical vote. This however does not take into account how other voter's tactical votes affect would affect my happiness. This could be explored in future works to get a better understanding of the happiness variance that tactical voting gives.

Lastly, within our performed experiments, we assume perfect knowledge. Because we have not implemented counter-strategic voting nor voter collusion, this is not a big problem. However, should these two other points be addressed in future work, we would highly recommend performing some experiments without perfect knowledge. After all, this could result in some interesting results like a reduction in tactical voting risk, and therefore a more stable voting.

4 Literature

5 Who Did What

- Happiness function: Lukas, Maria, Niels
- Risk definition: Gianluca
- Voting situation: Niels
- Voting schemes: Eliott
- Experiments and output graphs: Alexandra, Lukas, Maria
- Implementation and experiments of Advanced TVA: Gianluca
- Report: Everyone

6 Implementation Details

In order to carry out with the TVA Assignment we are using the Python Programming language.

6.1 Basic TVA

6.1.1 Voting Scheme

To implement the basic TVA, we first define all the voting schemes that are required according to the assignment description. The following voting schemes are supported:

- Vote for one: $[1, 0, 0, ...]$
- Vote for two: $[1, 1, 0, 0, ...]$
- Veto: $[1, 1, ..., 1, 0]$
- Borda: $[n - 1, n - 2, ..., 1, 0]$, where n is the number of options

6.1.2 Voting Situation and Outcome

Next, we create the Voting Situation Class. In this Class, a vote is simulated by creating a random matrix where each column represents the preference of each voter. Each entry contains a candidate number, the index of a candidate corresponds to the preference of a voter. The lower the index, the larger the preference.

For instance, with the following matrix, the first voter prefers candidate 1 over candidate 2 while the second voter prefers candidate 2 over candidate 1. Both voters prefer candidate 3 the least.

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 3 \end{bmatrix} \tag{5}$$

A number of votes is given to each candidate depending on the voting scheme. The candidate with the most votes wins. The votes are stored in an array sorted by the options (e.g., the first element is the number of votes of the option 0).

We compute the Outcome, by calling the calculatevote function of the Voting Situation Class for the selected scheme, as shown below.

Listing 1: Example of how to get the outcome

```
    original_outcome = self.situation.calculatevote(scheme_type)
```

6.1.3 Individual Happiness and Overall Happiness

Furthermore, we compute the individual Happiness of the voters (get_voter_happiness function) and the overall Happiness (get_happiness function), given the Voting Situation and the Outcome.

Listing 2: Implementation of the happiness function

```
def get_voter_happiness(self, voter_p):
    weight = np.arange(len(voter_p), -len(voter_p), -2)
    happiness = 0
    total_votes = np.sum(self.election_vector)
    for i, v in enumerate(voter_p):
        happiness += (self.election_vector[v - 1] / total_votes)
                    * (weight[i])

    return happiness
```

Listing 3: Implementation of the overall happiness function

```
def get_happiness(self, voting_matrix):
self.happiness = 0
for voter in range(voting_matrix.shape[1]):
```

```
    self.individual_happiness[voter] =
                    self.get_voter_happiness(voting_matrix[:,voter
                        ↪ ])
self.happiness = np.sum(self.individual_happiness)
```

6.1.4 Tactical Voting Risk

In the following function, we compute the risk of tactical voting whether there is coalition or not. We get the risk for coalitions (or non coalition, depends on the case), and count how many tactical votes each voter has, with this information we calculate the average risk and Boolean risk.

Listing 4: Computing the risk of tactical voting

```
def compute_risk(self):
    # key: scheme, values: (data, risks, avg_risk, avg_bool_risk
        ↪ )
    results = {}
            if self._coalition == 1: # simple tv, 1 voter
            for scheme in VotingScheme:
                res, risk = self._compute_risk_no_coalitions(
                    ↪ scheme)
                tmp = np.array(risk)
                avg_risk = np.sum(tmp) / (self.
                    ↪ alternative_votings * self.voters)
                avg_bool_risk = np.sum(tmp > 0) / self.voters
                results[scheme.name] = (res, risk, avg_risk,
                    ↪ avg_bool_risk)
        else: # advance tv, coalition of voters
            for scheme in VotingScheme:
                res, risk = self._compute_risk_coalitions(scheme)
                tmp = np.array(risk)
                avg_risk = np.sum(tmp) / (
                    self.alternative_votings * math.comb(self.
                        ↪ voters, self._coalition)
                )
                avg_bool_risk = np.sum(tmp > 0) / math.comb(
                    self.voters, self._coalition
                )
                results[scheme.name] = (res, risk, avg_risk,
                    ↪ avg_bool_risk)

    return results
```

6.2 Advanced TVA

The Advanced TVA extends the Basic TVA by allowing bullet voting and coalitions of a fixed size of voters. The voting schemes, voting situations and happiness function remain the same.

The coalitions are obtained by computing all combinations of voters. We generate a set of all possible preference for each voter in the coalition. Then, we compute the Cartesian product of these sets, and we test if these new preferences increase the happiness of all the members of the coalition. If this happens, then we have found a tactical vote.

With bullet voting, we allow a voter to give only a preference. So, instead of a preference like $\begin{bmatrix}1\\2\\0\end{bmatrix}$, we can handle preferences like $\begin{bmatrix}1\\-1\\-1\end{bmatrix}$, where -1 is used as filler value, and it is ignored when computing the outcome of the voting. The bullet preferences are added to the set of possible preferences for each voter. Therefore, now each voter has $n! + n - 1$ possible preferences, where n is the number of candidates. This kind of preferences is taken into account only when searching for tactical voting and not when we generate the initial voting situation.

Listing 5: Code for generating bullet voting.

```
def _get_bullet_votings(self):
    # for each option i create (i, -1, -1, ...)
    bullets = [None for i in range(self.options)]
    for i in range(self.options):
        p = [-1 for i in range(self.options)]
        p[0] = i
        bullets[i] = tuple(p)
    return bullets
```

7 Output Examples

7.1 Voting Situation 1

This voting situation involves 100 voters, and 4 candidates.

7.1.1 Vote For One

Candidate	1	2	3	4
Score	25	30	26	19

In this case, $\sum_i (score_i)$ = number of voters, since each voter votes for exactly one single candidate.

Overall happiness: 99.54%

```
RISK FOR SCHEME:  VOTE_FOR_ONE
Individual risk list:  [5, 17, 22, 6, 8, 17, 6, 6,
Total risk:  1270
Average risk:  0.5521739130434783
```

7.1.2 Vote for Two

Candidate	1	2	3	4
Score	49	60	49	42

In this case, $\sum_i(score_i)$ = 2∗number of voters, since each voter votes for exactly two candidates.

Overall happiness: 99.66%

```
RISK FOR SCHEME:  VOTE_FOR_TWO
Individual risk list:  [15, 23, 21, 1, 11, 7, 16,
Total risk:  1290
Average risk:  0.5608695652173913
```

7.1.3 Veto Voting

Candidate	1	2	3	4
Score	79	76	71	74

In this case, $\sum_i(score_i)$ = 3∗number of voters, since each voter votes for exactly the number of candidates − 1.

Overall happiness: 99.52%

```
RISK FOR SCHEME:  VETO
Individual risk list:  [11, 23, 22, 2, 8, 11,
Total risk:  1282
Average risk:  0.5573913043478261
```

7.1.4 Borda

Candidate	1	2	3	4
Score	170	143	135	152

Overall happiness: 99.91%

```
RISK FOR SCHEME:  BORDA
Individual risk list:  [12, 23, 22, 2, 9, 11, 13,
Total risk:  1302
Average risk:  0.5660869565217391
```

7.2 Voting Situation 2

Output with 50 voters, and 6 candidates:

7.2.1 Vote For One

Candidate	1	2	3	4	5	6
Score	4	10	11	11	7	7

In this case, $\sum_i(score_i)$ = number of voters, since each voter votes for exactly one single candidate.

Overall happiness: 49.52%

```
RISK FOR SCHEME:  VOTE_FOR_ONE
Individual risk list:  [258, 127, 301, 337, 314, 112, 528,
Total risk:  17624
Average risk:  0.49023643949930457
```

7.2.2 Vote for Two

Candidate	1	2	3	4	5	6
Score	17	18	17	22	15	11

In this case, $\sum_i(score_i)$ = 2$*$number of voters, since each voter votes for exactly two candidates.

Overall happiness: 50.04%

```
RISK FOR SCHEME:  VOTE_FOR_TWO
Individual risk list:  [396, 223, 98, 380, 204, 242,
Total risk:  19117
Average risk:  0.5317663421418637
```

7.2.3 Veto Voting

Candidate	1	2	3	4	5	6
Score	44	44	40	40	40	42

In this case, $\sum_i(score_i)$ = 3$*$number of voters, since each voter votes for exactly the number of candidates $-$ 1.

Overall happiness: 50.30%

```
RISK FOR SCHEME:  VETO
Individual risk list:  [593, 491, 606, 446, 686,
Total risk:  19696
Average risk:  0.5478720445062587
```

7.2.4 Borda

Candidate	1	2	3	4	5	6
Score	124	132	128	130	120	116

Overall happiness: 50.15%

```
RISK FOR SCHEME:  BORDA
Individual risk list:  [579, 184, 360, 360, 447,
Total risk:  19644
Average risk:  0.5464255910987482
```