2D cases: Line Integral Data

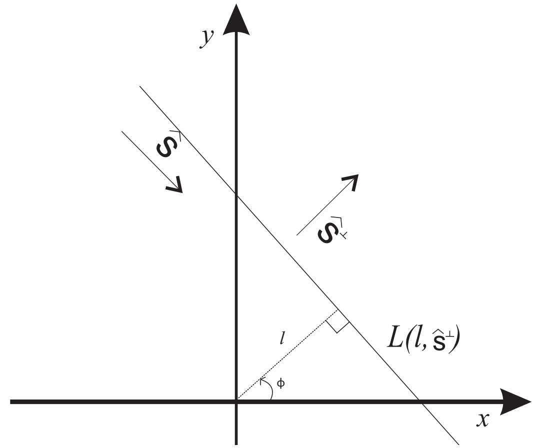
Two types of line integral measurements are used to reconstruct the field $\mathrm{e}(\mathrm{x})$.

Longitudinal:

$$I_L^{\|}(l, s^{\perp}) = \int_L \hat{\mathrm{s}} \cdot \mathrm{e}(\mathrm{x})\, d\ell(\mathrm{x}) = \mathcal{R}_\Omega\{\hat{\mathrm{s}} \cdot \mathrm{e}\},$$

Transverse:

$$I_L^{\perp}(l, s^{\perp}) = \int_L \hat{\mathrm{s}}^{\perp} \cdot \mathrm{e}(\mathrm{x})\, d\ell(\mathrm{x}) = \mathcal{R}_\Omega\{\hat{\mathrm{s}}^{\perp} \cdot \mathrm{e}\}.$$



$\hat{\mathrm{s}}$ is the unit vector in the direction of line L

$\hat{\mathrm{s}}^{\perp}$ is the unit vector that is orthogonal to the line L

Line: $L(l, \hat{\mathrm{s}}^{\perp}) := \{\mathrm{x} = (x, y) \in \Omega : \mathrm{x} \cdot \hat{\mathrm{s}}^{\perp} = l\}$.

$\mathcal{R}_\Omega\{.\}$ denotes the Radon transform of e restricted on Ω

$$\mathcal{R}_\Omega\{\hat{\mathrm{s}}_\theta \cdot \mathrm{e}\}(l, \hat{\mathrm{s}}^{\perp}) = \int_L \hat{\mathrm{s}}_\theta \cdot \mathrm{e}(\mathrm{x})\, d\ell(\mathrm{x}) = \int_\Omega \hat{\mathrm{s}}_\theta \cdot \mathrm{e}(\mathrm{x}) \delta(l - \mathrm{x} \cdot \hat{\mathrm{s}}^{\perp})\, d\mathrm{x}.$$

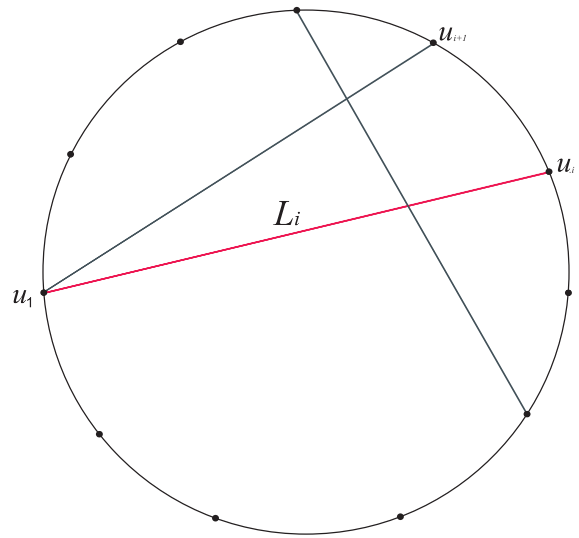
$\hat{\mathrm{s}}_\theta$ is an arbitrary unit length vector (e.g. $\hat{\mathrm{s}}^{\perp}$).

Longitudinal and Transverse Integrals of the Electric Field

The electric field is expressed as $\mathrm{e}(\mathrm{x}) = -\nabla u \quad \text{in} \quad \Omega.$
Given a set of potential measurement $\{u(\mathrm{x}_1), \ldots, u(\mathrm{x}_n)\}$,
the longitudinal integral can be directly measured, i.e.

$$I^{\parallel}_{L_i} = \int_{L_i} \hat{\mathrm{s}} \cdot \mathrm{e}(\mathrm{x})\; d\ell(\mathrm{x}) = \int_{L_i)} \hat{\mathrm{s}} \cdot (-\nabla u)\; d\ell(\mathrm{x}) = u(\mathrm{x}_1) - u(\mathrm{x}_i) = V_i.$$

The longitudinal integral is associated solely with the harmonic component of the field.



The transverse integral is related with the field inside the domain.

$$I^{\perp}_{L_i} = \int_{L_i} \hat{\mathrm{s}}^{\perp} \cdot \mathrm{e}(\mathrm{x})\; d\ell(\mathrm{x}) = \int_{L_i} \frac{\partial u}{\partial y} s_x - \frac{\partial u}{\partial x} s_y \; d\ell(\mathrm{x}).$$

where $\hat{\mathrm{s}}^{\perp} = (-s_y, s_x)$.

Algebraic Reconstruction Technique

Reconstruction result if it were possible to measure both line integrals: