Camera calibration

References

- http://szeliski.org/Book/
- http://www.cs.cornell.edu/courses/cs5670/2019sp/lectures/lectures.html
- http://www.cs.cmu.edu/~16385/

- Camera intrinsics
- Camera extinsics
- Radial distortion

What is camera calibration

- **Geometric camera calibration**, also referred to as **camera resectioning**, estimates the parameters of a lens, image sensor, position and view direction of a camera.
- You can use these parameters to correct for lens distortion, measure the size of an object in world units, or determine the location of the camera in the scene. These tasks are used in applications such as machine vision to detect and measure objects. They are also used in robotics, for navigation systems, and 3-D scene reconstruction.
- [from: https://www.mathworks.com/help/vision/ug/camera-calibration.html]

Starting from the end

- The camera matrix is a full transformation from 3D objects in the scene to a 2D image with the specific camera parameters:

$$P = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$$P \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \mapsto \begin{bmatrix} \frac{u}{w} \\ \frac{v}{w} \end{bmatrix}$$

- How many DOFs are we looking for?

Starting from the end

- The camera matrix is a full transformation from 3D objects in the scene to a 2D image with the specific camera parameters:

$$P = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$$P \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \mapsto \begin{bmatrix} \frac{u}{w} \\ \frac{v}{w} \end{bmatrix}$$

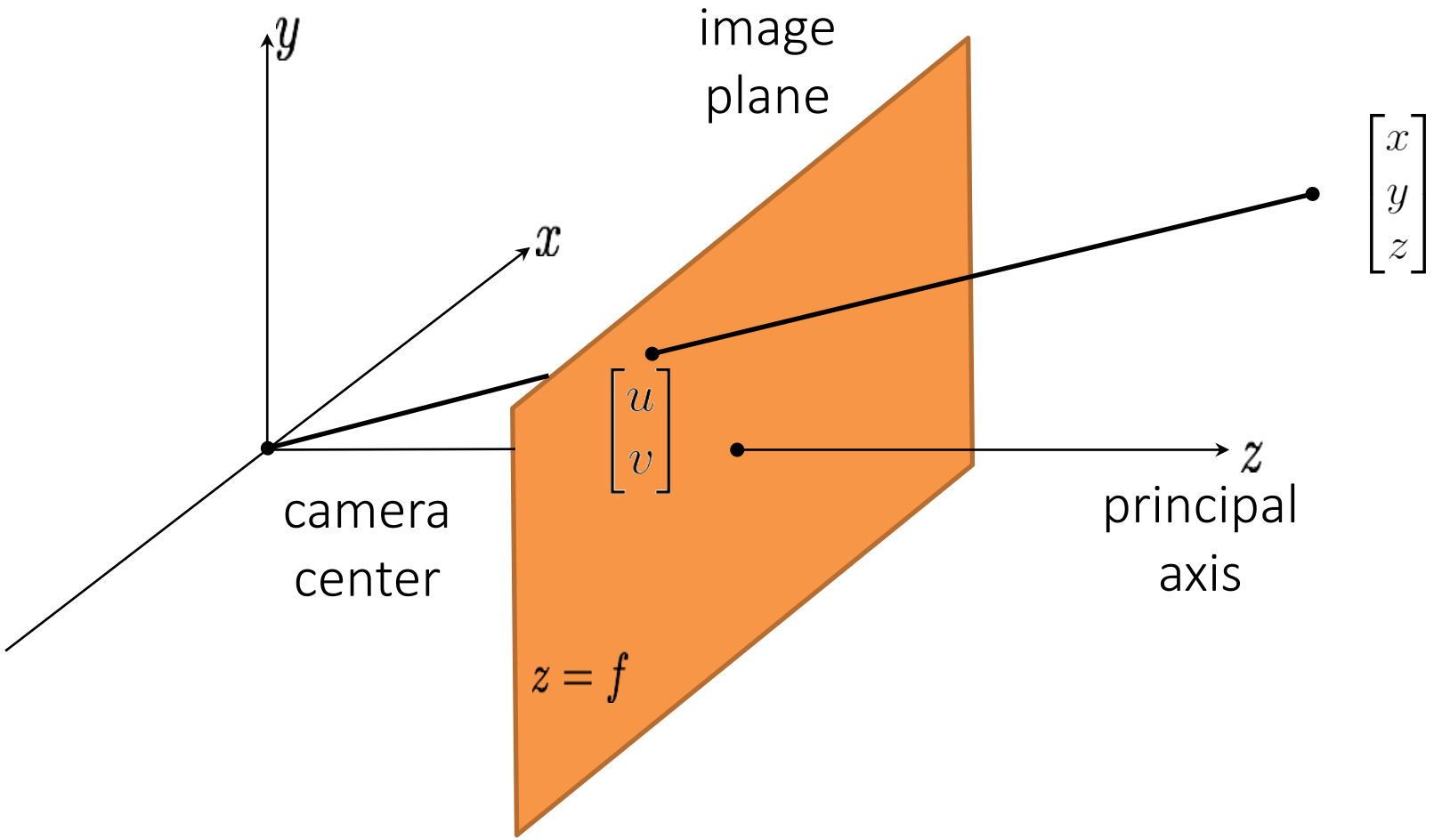
- How many DOFs are we looking for?
  - 11, because in homogenous coordinates the answer is always correct up to a scale.

Starting from the end

$$P_{3X4} = K_{3X3}[I|0]_{3X4}\Pi_{4X4}$$

Recap: perspective projection

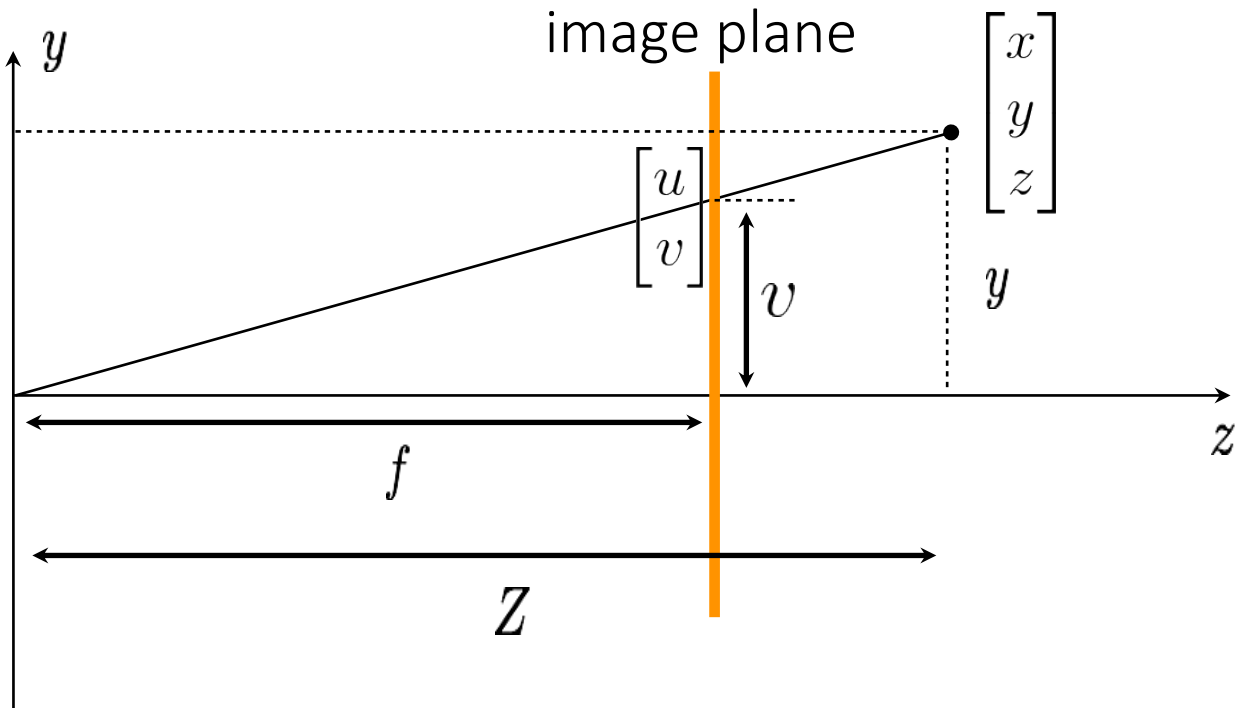
- **Perspective projection** (also known as **perspective transformation**) is a linear projection where three dimensional objects are projected on the image plane.



Recap: perspective projection

- Using triangle proportions (Thales' theorem) we can easily conclude that:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \mapsto \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} f\frac{x}{z} \\ f\frac{y}{z} \end{bmatrix}$$



Recap: perspective projection

- Let's use the homogeneous coordinates:

$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} fx \\ fy \\ z \end{bmatrix} \mapsto \begin{bmatrix} f\frac{x}{z} \\ f\frac{y}{z} \end{bmatrix}$$

  - Units of $[m]$

Recap: perspective projection

- Let's split into 2 matrices and use 3D->2D homogenous coordinates:

$$\begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} fx \\ fy \\ z \end{bmatrix} \mapsto \begin{bmatrix} f\frac{x}{z} \\ f\frac{y}{z} \end{bmatrix}$$

Intrinsic camera matrix

Projection matrix

Intrinsic camera matrix

- The intrinsic matrix $\boldsymbol{K}$ contains 5 intrinsic parameters. These parameters encompass
  - Scaled x & y focal length, image sensor format, and principal point.

Intrinsic calibration matrix

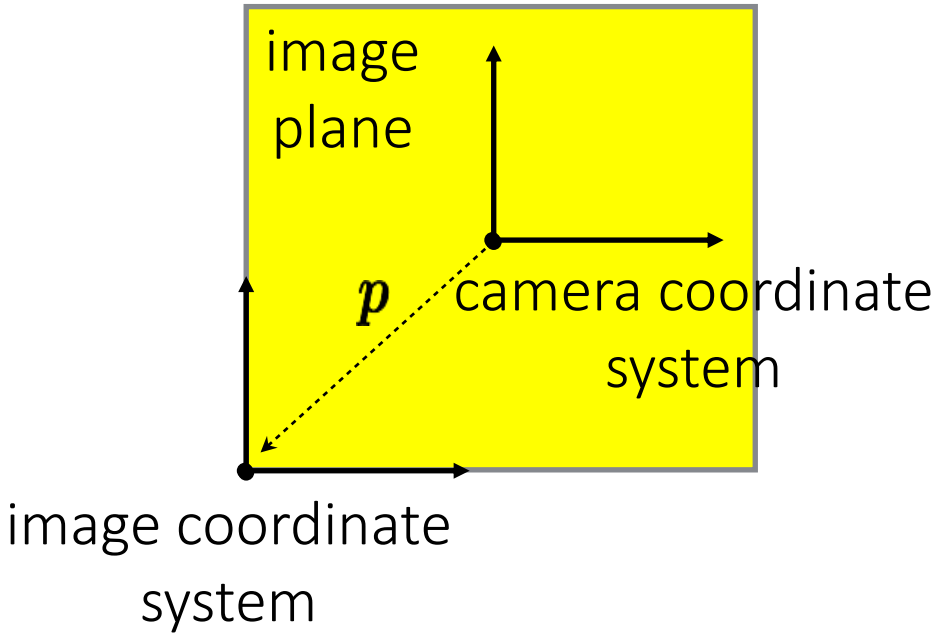
- Transforming to units of pixels in image space:
  - Pixel size in x dimension is m_x and Pixel size in y dimension is m_y.

$$f_x = \frac{f}{m_x} \;\&\; f_y = \frac{f}{m_y}$$

$$\begin{bmatrix} f_x & 0 & 0 \\ 0 & f_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} f_x x \\ f_y y \\ z \end{bmatrix} \overset{hom.\,coo.}{=} \begin{bmatrix} f_x \frac{x}{z} \\ f_y \frac{y}{z} \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

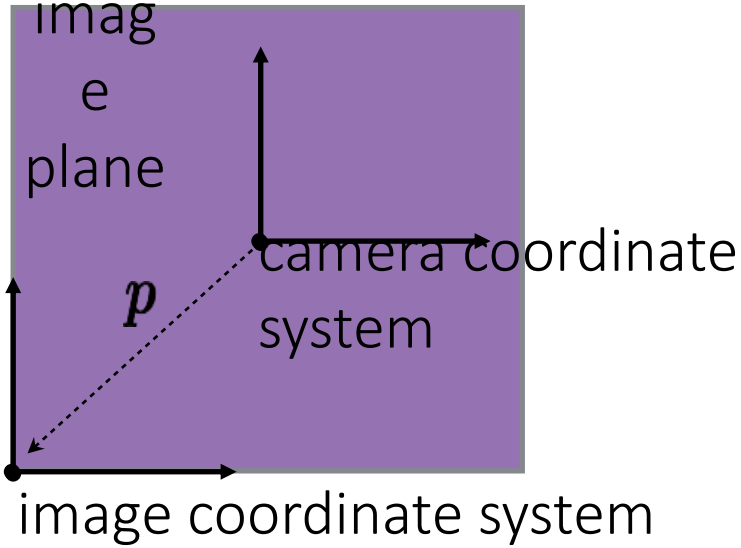
Generalizing the camera matrix

In particular, the camera origin and image origin may be different:



Generalizing the camera matrix

In particular, the camera origin and image origin may be different:



shift vector transforming camera origin to image origin

How does the camera matrix change?

$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

- http://www.vision.caltech.edu/bouguetj/calib_doc/htmls/example.html
- https://docs.opencv.org/2.4/doc/tutorials/calib3d/camera_calibration/camera_calibration.html
- https://webcourse.cs.technion.ac.il/236873/Winter2017-2018/ho/WCFiles/Camera%20Calibration.pdf
- http://ksimek.github.io/2012/08/13/introduction/