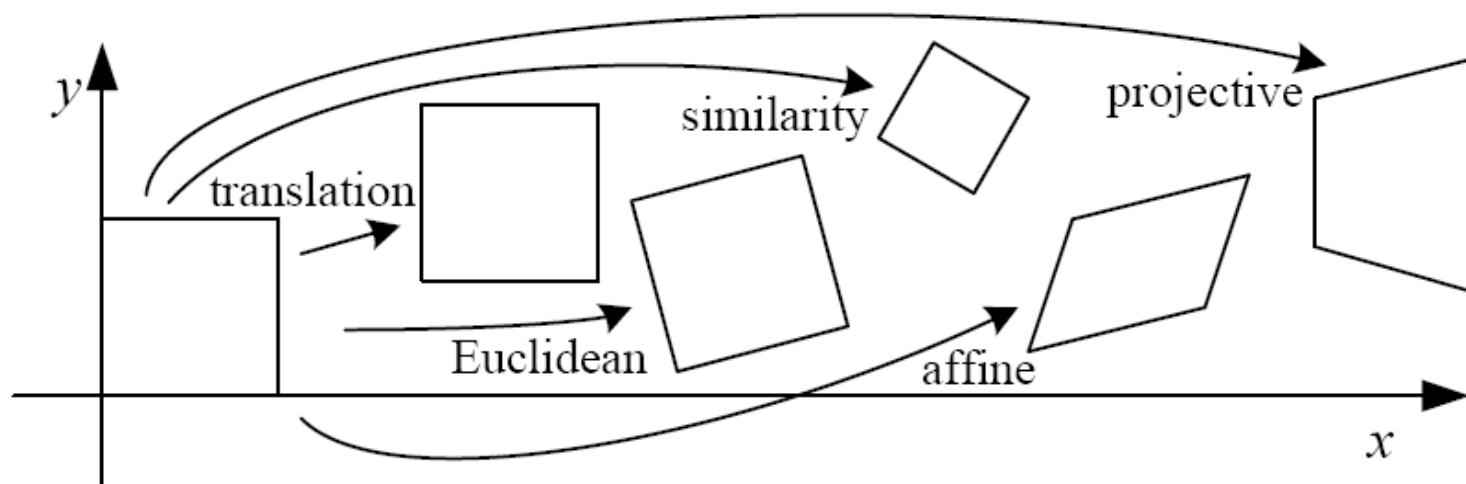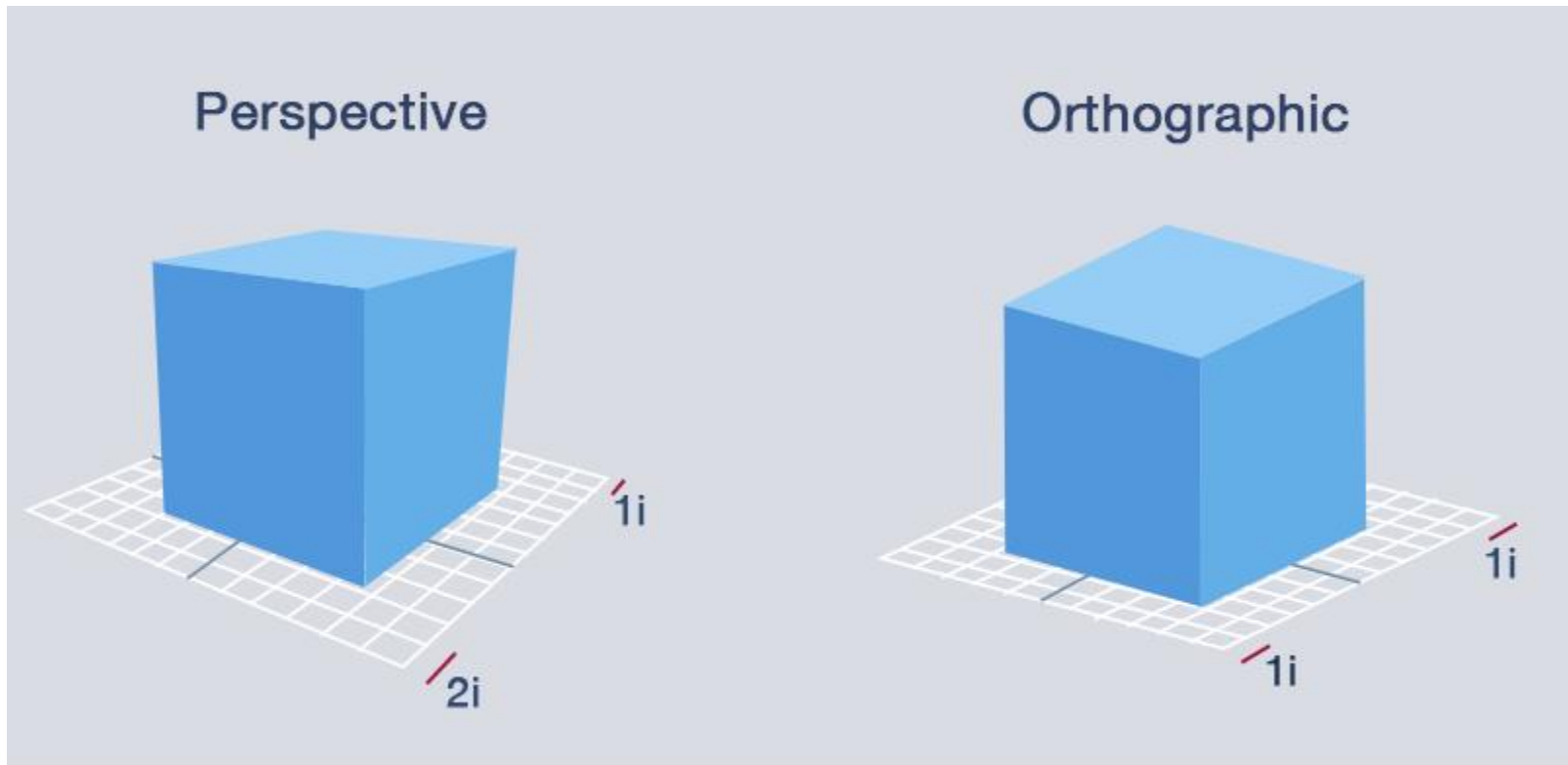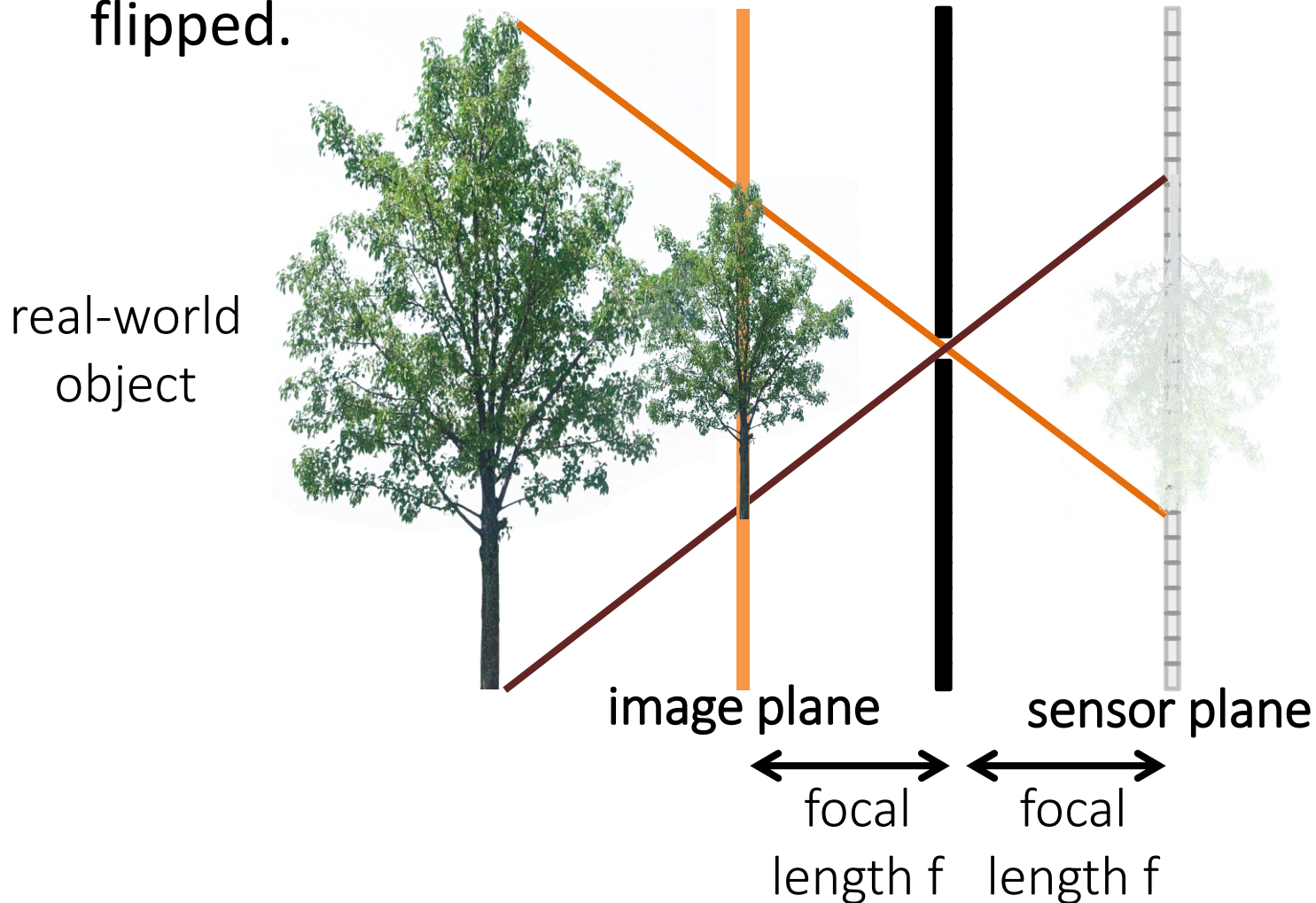# Image transformation

- 3d->2d trans

3D projection

- **3D projection** is any method of mapping three-dimensional points to a two-dimensional plane.
- Two types of projections are **orthographic** and **perspective**.
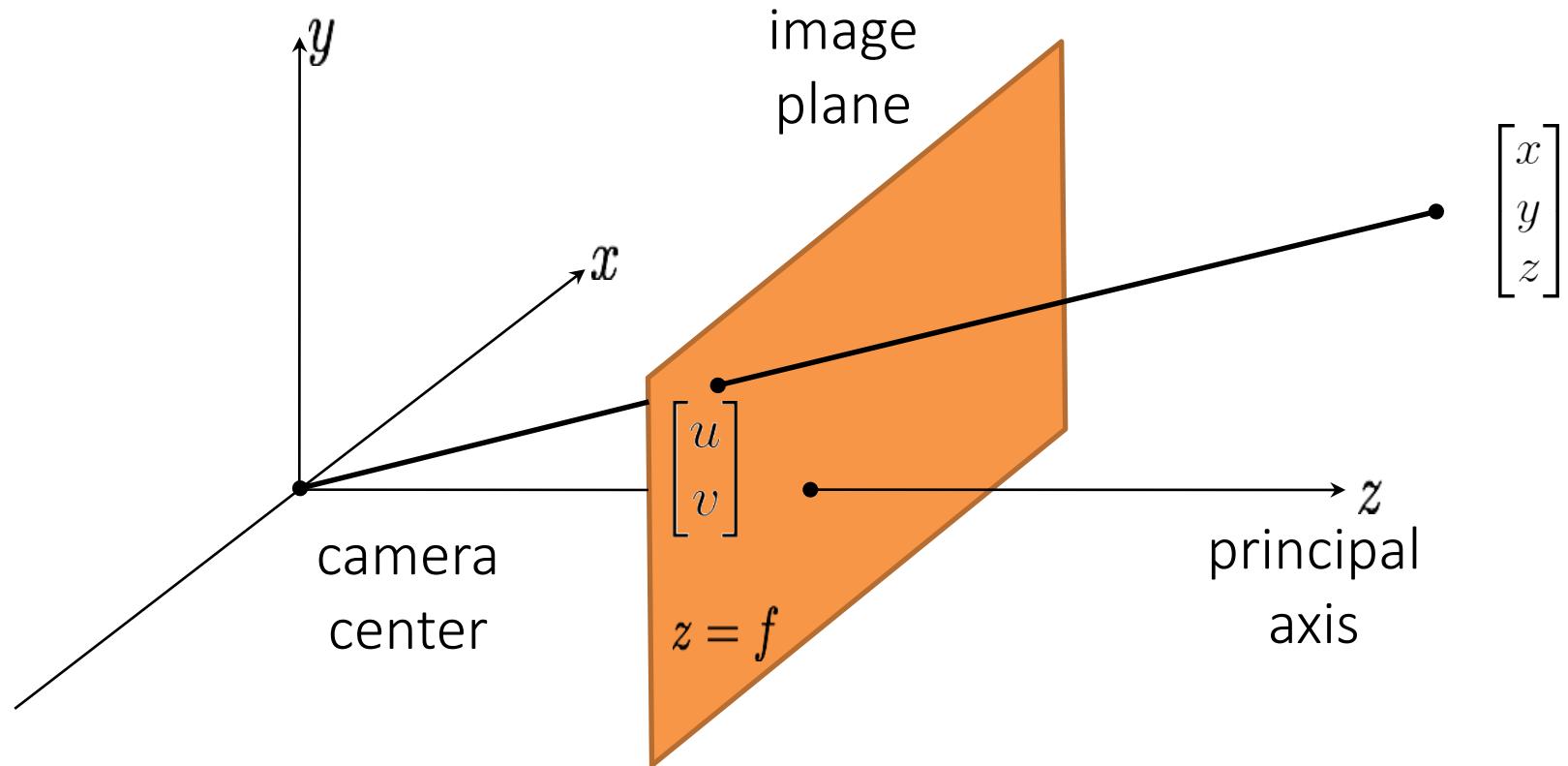

Perspective

Orthographic

Perspective transformation

- When dealing with imaged 3D scenes, we tend to use the **image plane** rather than the sensor plane which is flipped.

real-world
object

image plane

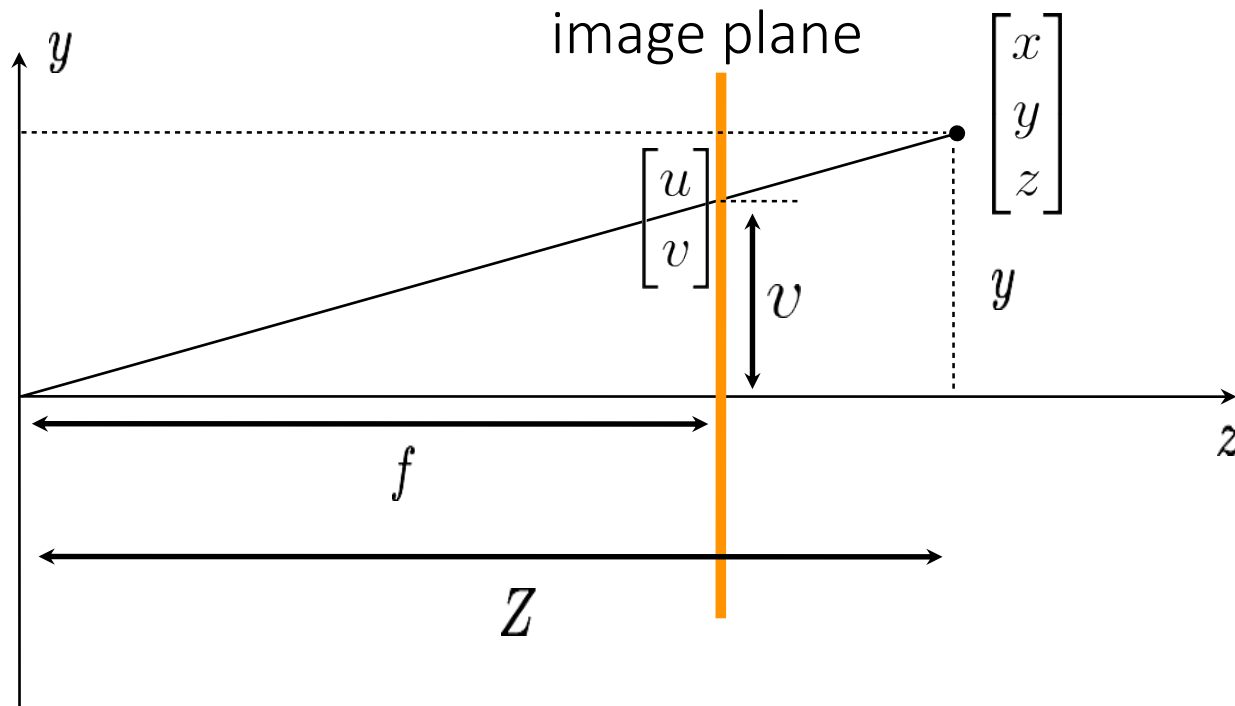sensor plane

focal
length f

focal
length f

Perspective transformation

- **Perspective projection** is a linear projection where three dimensional objects are projected on the image plane.
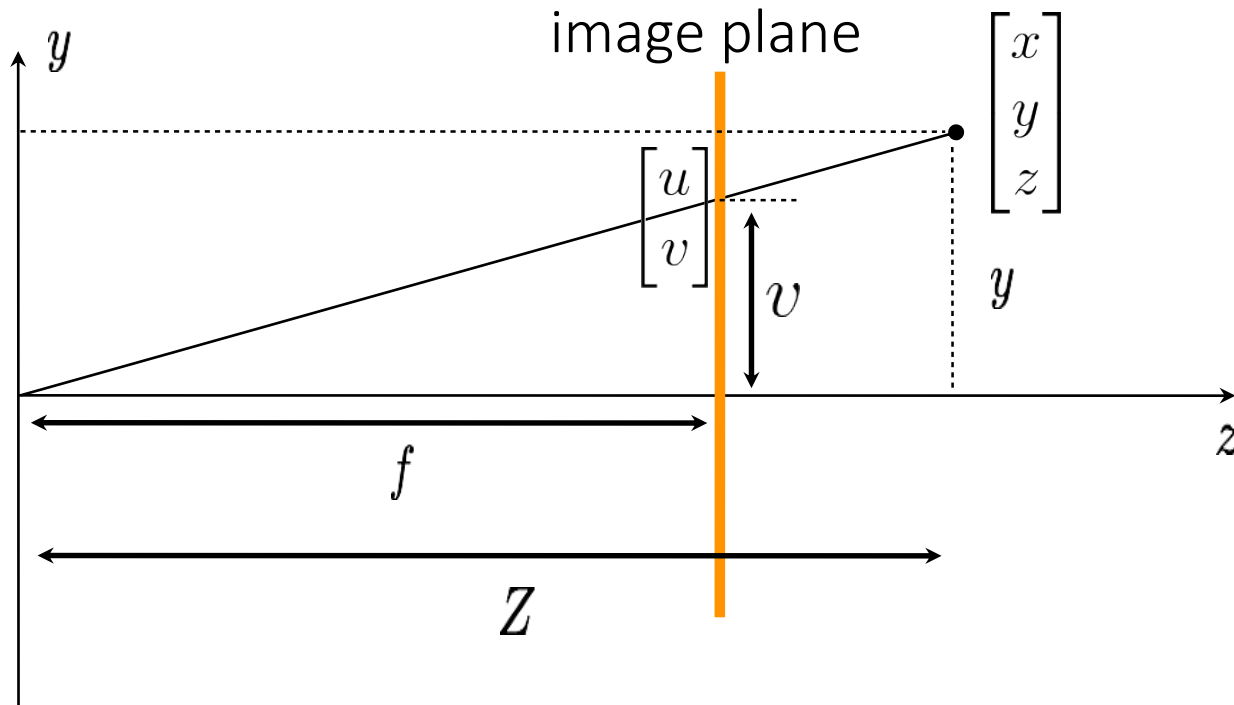
Perspective transformation

- What is the relationship between y & v?

Perspective transformation

- Using triangle proportions (Thales' theorem) we can easily conclude that:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \mapsto \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} f\frac{x}{z} \\ f\frac{y}{z} \end{bmatrix}$$

image plane

Perspective transformation

- Let's use the homogeneous coordinates:

$$
\begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} fx \\ fy \\ z \end{bmatrix} \overset{hom.\,coo.}{=} \begin{bmatrix} f\frac{x}{z} \\ f\frac{y}{z} \\ 1 \end{bmatrix}
$$

  - Units of $[m]$

Perspective transformation

- Transforming to units of pixels in image space:
  - Pixel size in x dimension is m_x and the same for m_y.

$$f_x = \frac{f}{m_x} \ \& \ f_y = \frac{f}{m_y}$$

$$\begin{bmatrix} f_x & 0 & 0 \\ 0 & f_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} f_x x \\ f_y y \\ z \end{bmatrix} \overset{hom.\ coo.}{=} \begin{bmatrix} f_x \frac{x}{z} \\ f_y \frac{y}{z} \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

Street art