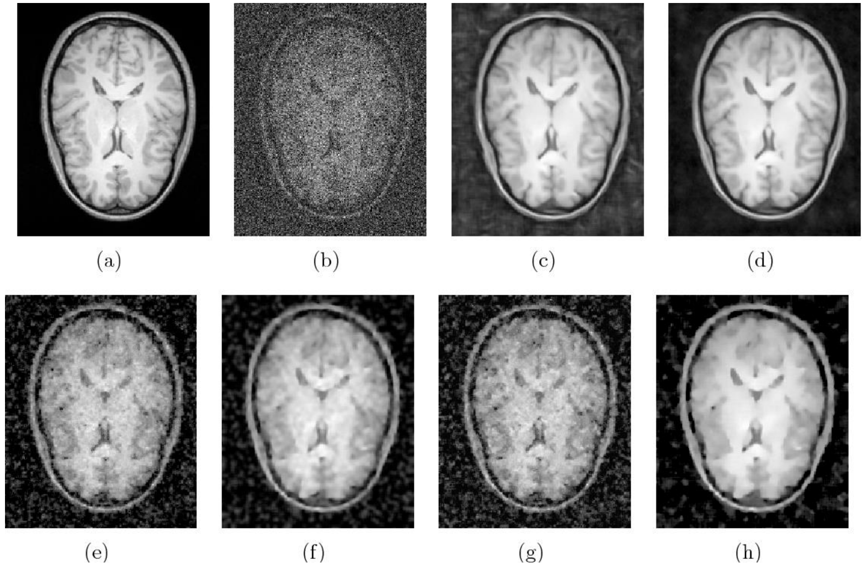
Image processing

References

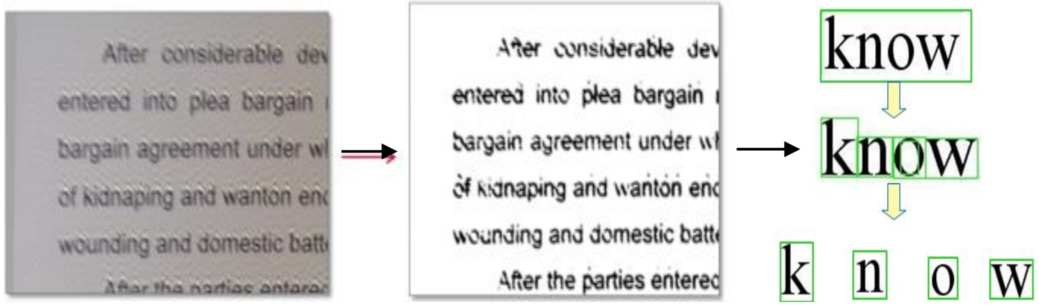
- http://szeliski.org/Book/
- http://www.cs.cornell.edu/courses/cs5670/2019sp/lectures/lectures.html
- http://www.cs.cmu.edu/~16385/

Some motivation

Art
(Photoshop image correction)



Medicine
(MRI denoising)



Robotics
(OCR – optical character recognition)

Agriculture
(color ripeness detection)

contents

- **Image representation**
- Pixel-wise operations
- Noise and filtering
- Frequency representation
- Morphology operators
- Connected components

Image representation

- We cane think of image as a 3d matrix of discrete RGB values.
- The values mark the intensity of each color channel and are usually of type uint8 = $\{0, \ldots, 255\}$.

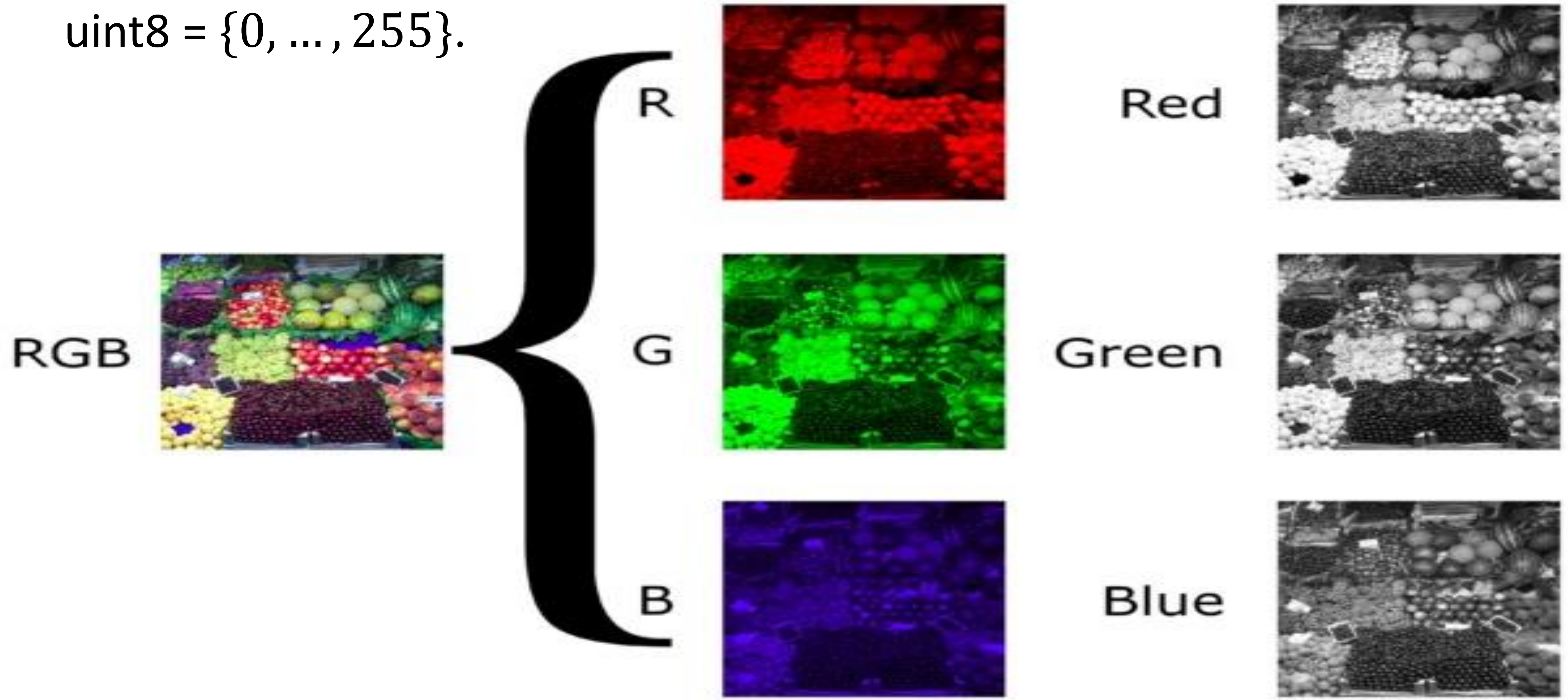
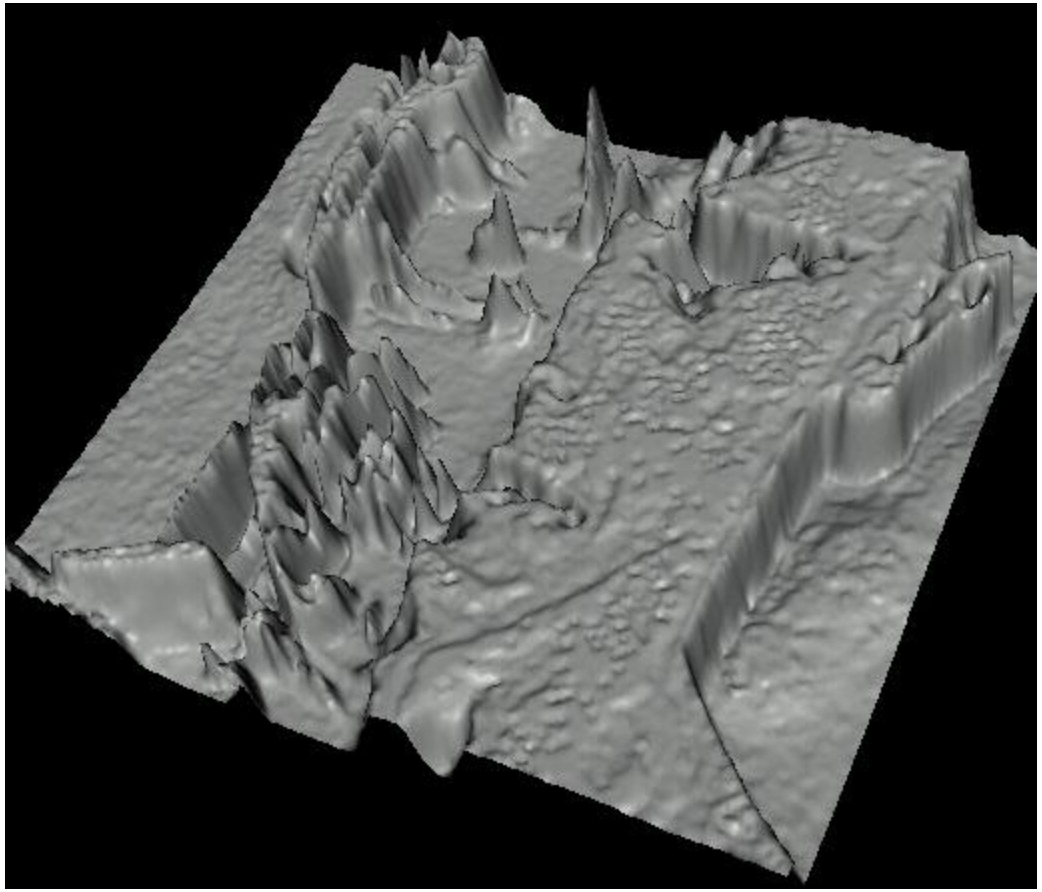


Image representation

- We can also think of an image as a function $f(x, y)$.



contents

- Image representation
- **Pixel-wise operations**
- Noise and filtering
- Frequency representation
- Morphology operators
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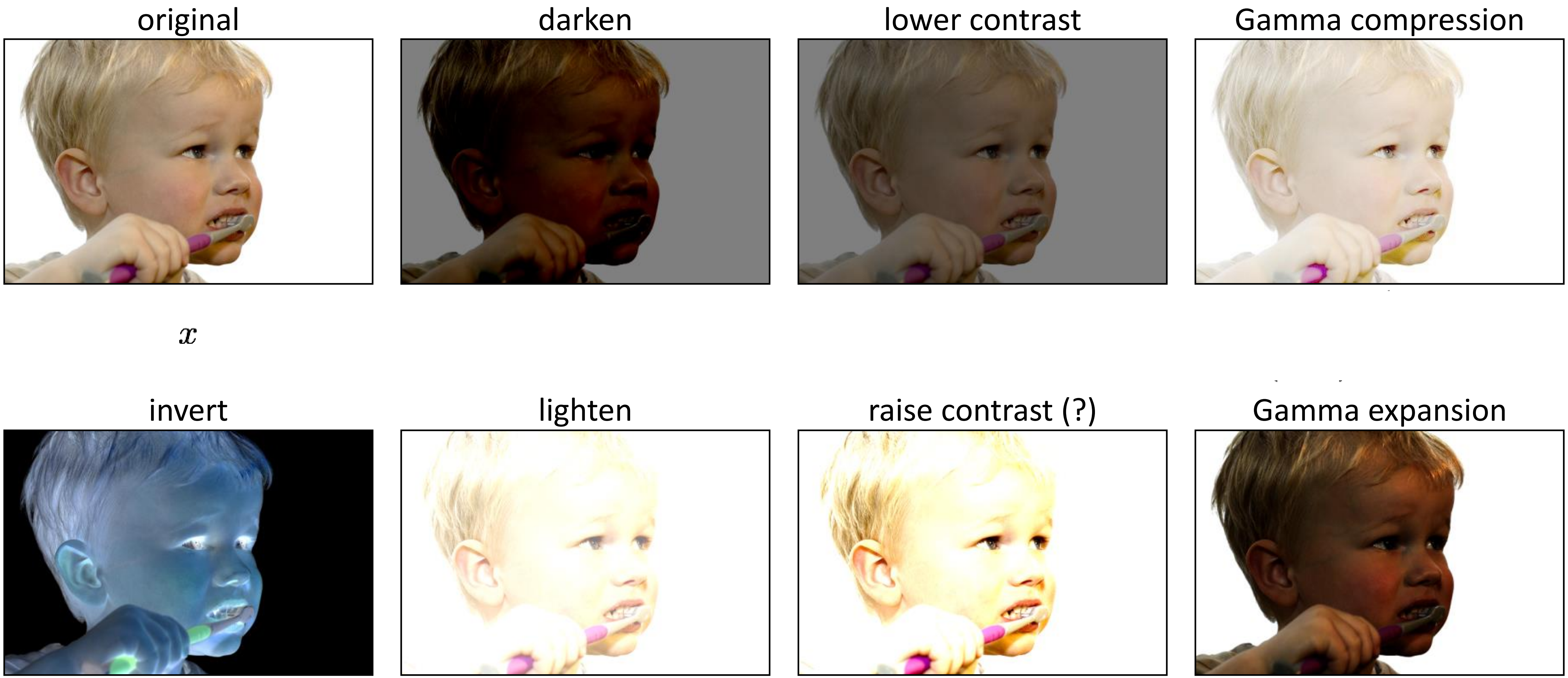
Pixel-wise operators

- Pixel-wise operators, or point operators, are defined as such that each output pixel’s value depends on only the corresponding input pixel value.

Pixel-wise operators



Pixel-wise operators

original

x

darken

lower contrast

Gamma compression

invert

$255 - x$

lighten

raise contrast (?)

Gamma expansion

Pixel-wise operators

original

x

darken

$x - 128$

lower contrast

Gamma compression

invert

$255 - x$

lighten

raise contrast (?)

Gamma expansion

Pixel-wise operators

original

x

darken

$x - 128$

lower contrast

Gamma compression

invert

$255 - x$

lighten

$x + 128$

raise contrast (?)

Gamma expansion

Pixel-wise operators

original

$$x$$

darken

$$x - 128$$

lower contrast

$$\frac{x}{2}$$

Gamma compression

invert

$$255 - x$$

lighten

$$x + 128$$

raise contrast (?)

Gamma expansion

Pixel-wise operators

original

x

darken

$x - 128$

lower contrast

$\frac{x}{2}$

Gamma compression

invert

$255 - x$

lighten

$x + 128$

raise contrast (?)

$x \times 2$

Gamma expansion

Contrast

- **Contrast** in visual perception is the difference in appearance of two or more parts of a field seen.
- The human visual system is more sensitive to contrast than absolute luminance;
- **Contrast ratio**, or **dynamic range**, is the is the ratio between the largest and smallest values of the image or :

$$CR = \frac{V_{max}}{V_{min}}$$

Pixel-wise operators

original

$$x$$

darken

$$x - 128$$

lower contrast

$$\frac{x}{2}$$

Gamma compression

$$\left(\frac{x}{255}\right)^{1/3} \times 255$$

invert

$$255 - x$$

lighten

$$x + 128$$

raise contrast (?)

$$x \times 2$$

Gamma expansion

Pixel-wise operators

original

$$x$$

darken

$$x - 128$$

lower contrast

$$\frac{x}{2}$$

Gamma compression

$$\left(\frac{x}{255}\right)^{1/3} \times 255$$

invert

$$255 - x$$

lighten

$$x + 128$$

raise contrast (?)

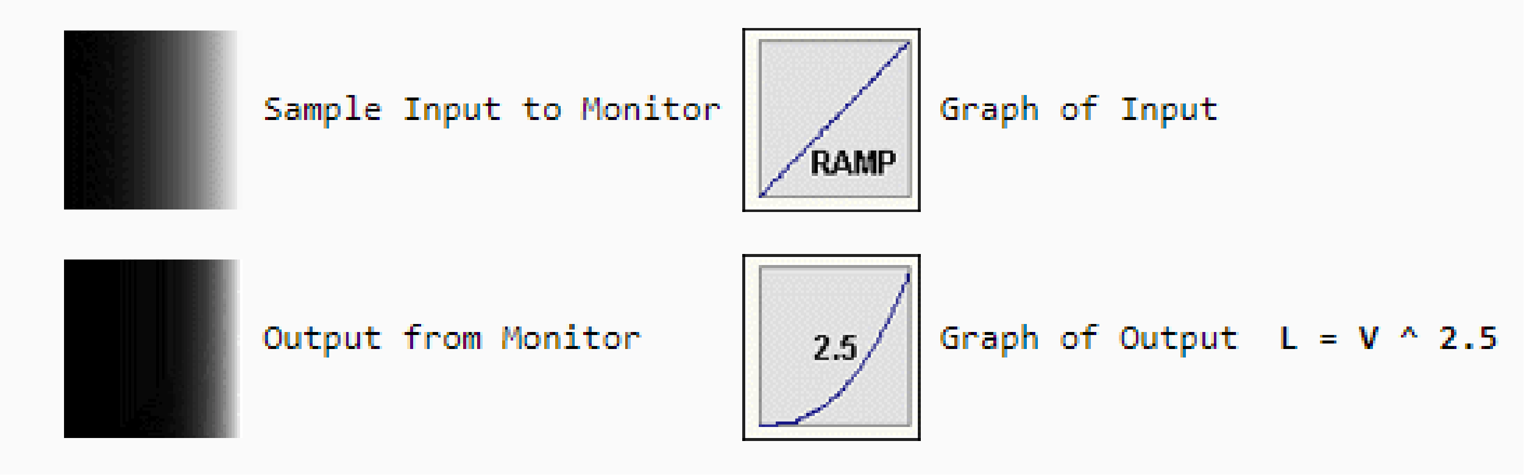
$$x \times 2$$

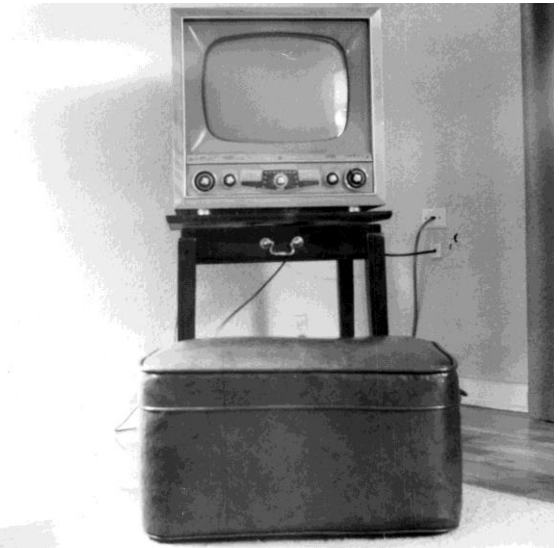
Gamma expansion

$$\left(\frac{x}{255}\right)^{2} \times 255$$

Gamma correction

- Originally, Due to non-linearities in the old CRT televisions, intensities was seen different then they are.



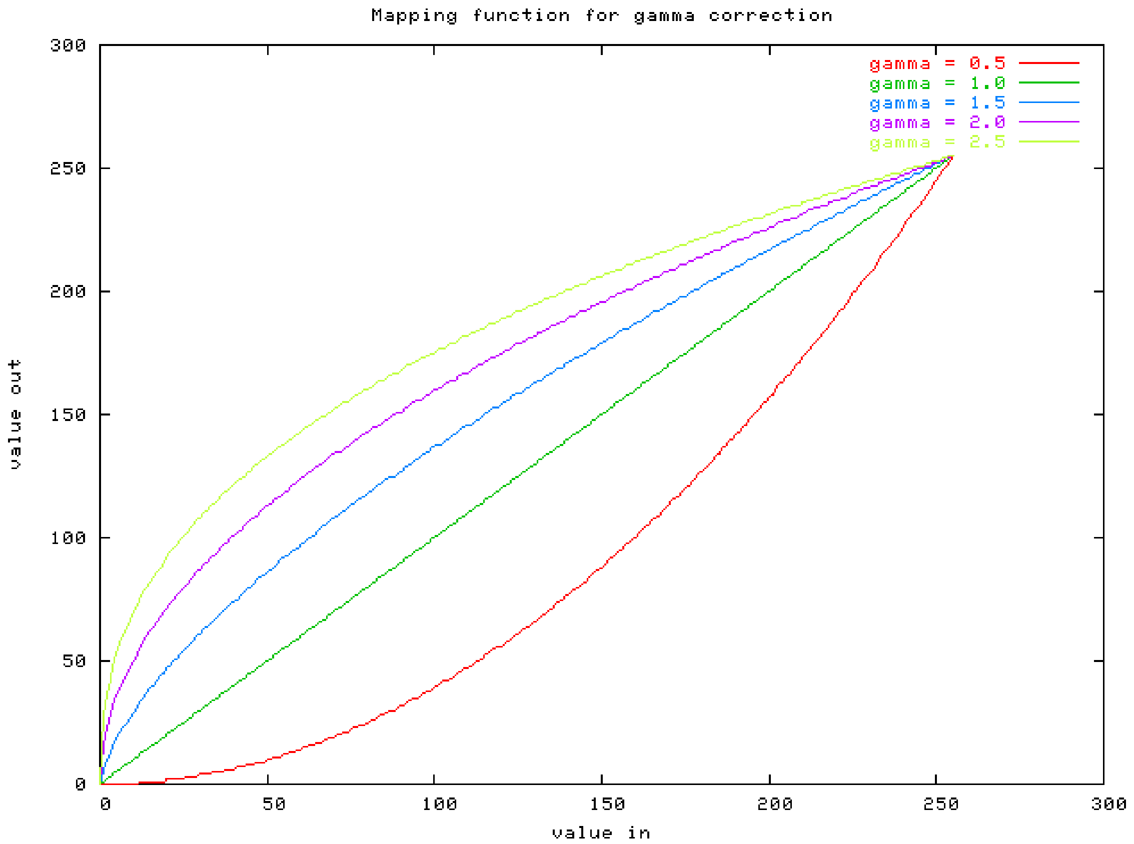


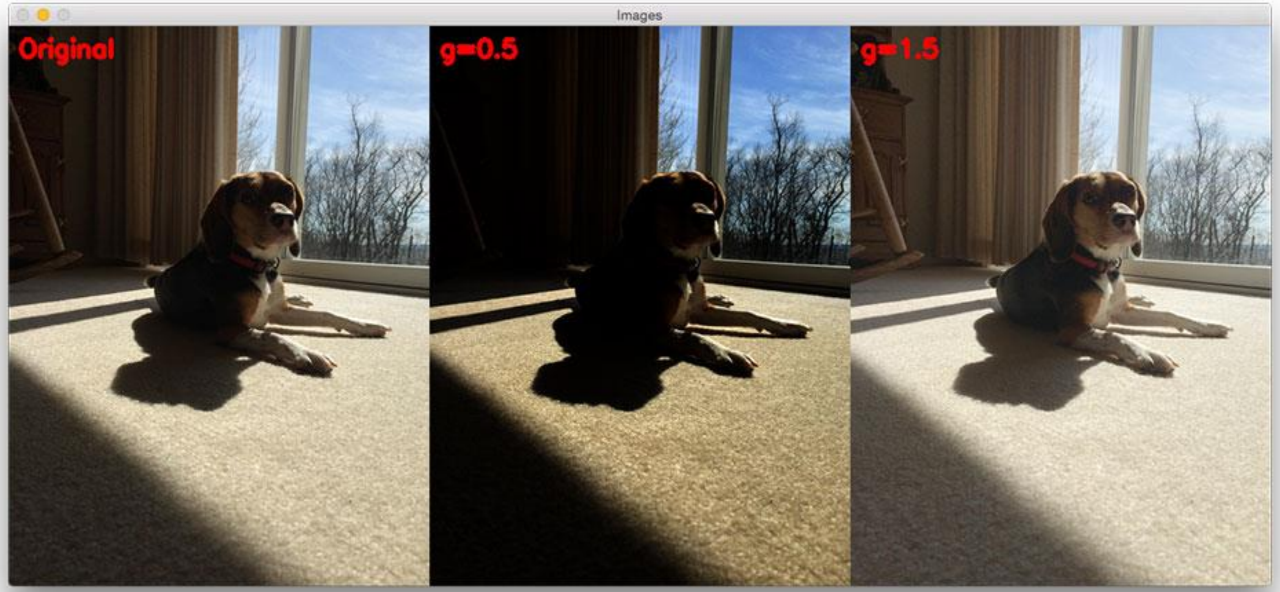
Gamma correction

- To correct this non linear transformation, gamma correction was done:

$$V_{out} = V_{in}^{\gamma}$$

- This is, of course, also applicable for image enhancements.





contents

Gaussian Noise

- Gaussian noise is an additive noise that can appear in images due to the system electrical circuitry.
- This noise is independent of signal strength and independent at each pixel (IID).

$$p(z) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}$$

Gaussian Noise σ=0.01

SNR=34.0206

Gaussian Noise σ=0.05

SNR=19.1825

Gaussian Noise σ=0.1

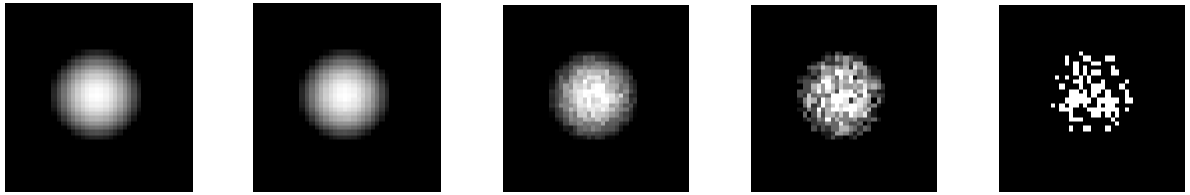
SNR=13.7121

Salt & Pepper noise

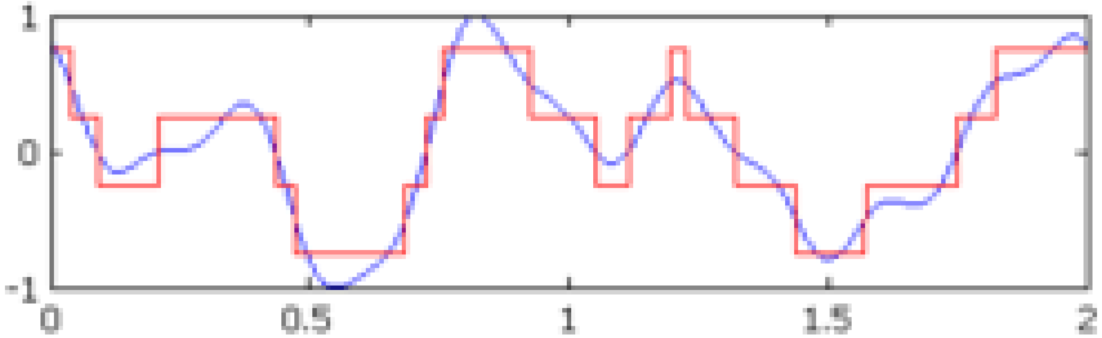
- Noise that can be caused by analog-to-digital converter errors, bit errors in transmission, etc.
- This noise is **not** additive to the signal strength (a replacement of original value with noise value).
- This noise is independent of signal strength and independent at each pixel.

And some more noise

- **Shot noise** - caused by statistical quantum fluctuations, that is, variation in the number of photons sensed at a given exposure level in the darker parts of an image (where there are just few photons that enter each pixel “bin”). Modeled as Poisson noise.



- **Quantization noise** – caused by quantizing the pixels of a sensed image to several discrete levels (analog to digital conversion).



Noise reduction with LTI filters

- **linear filtering operators**, which involve weighted combinations of pixels in small neighborhood.
- The combination is determined by the filter's *kernel*.
- The same kernel is *shifted* to all pixel locations so that all pixels use the same linear combination of their neighbors.
- That's why it's called **linear shift-invariance** (LTI) filter.

Convolution

- Works on LTI filters.
- Let h be the image, f be the kernel (of size 2*k*+1 x 2*k*+1), and g be the output image:

$$g(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} h[u,v] f[i-u, j-v]$$

  – **Note:** by definition, this operator flips the kernel both horizontally and vertically.
- This operation is called **convolution operator** and is more compactly noted as:

$$g = h * f$$

Example: mean filter

- The kernel is: $$\frac{1}{9}\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline\end{array}$$
- Replaces pixel with local average.
- Has smoothing (blurring) effect
- The kernel can be in any other size as well (see .ipynb).

Run the filter

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

Note that we assume that the kernel coordinates are centered. Here the kernel is symmetric horizontally and vertically, so the flipping is not noticeable.

Run the filter

kernel

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

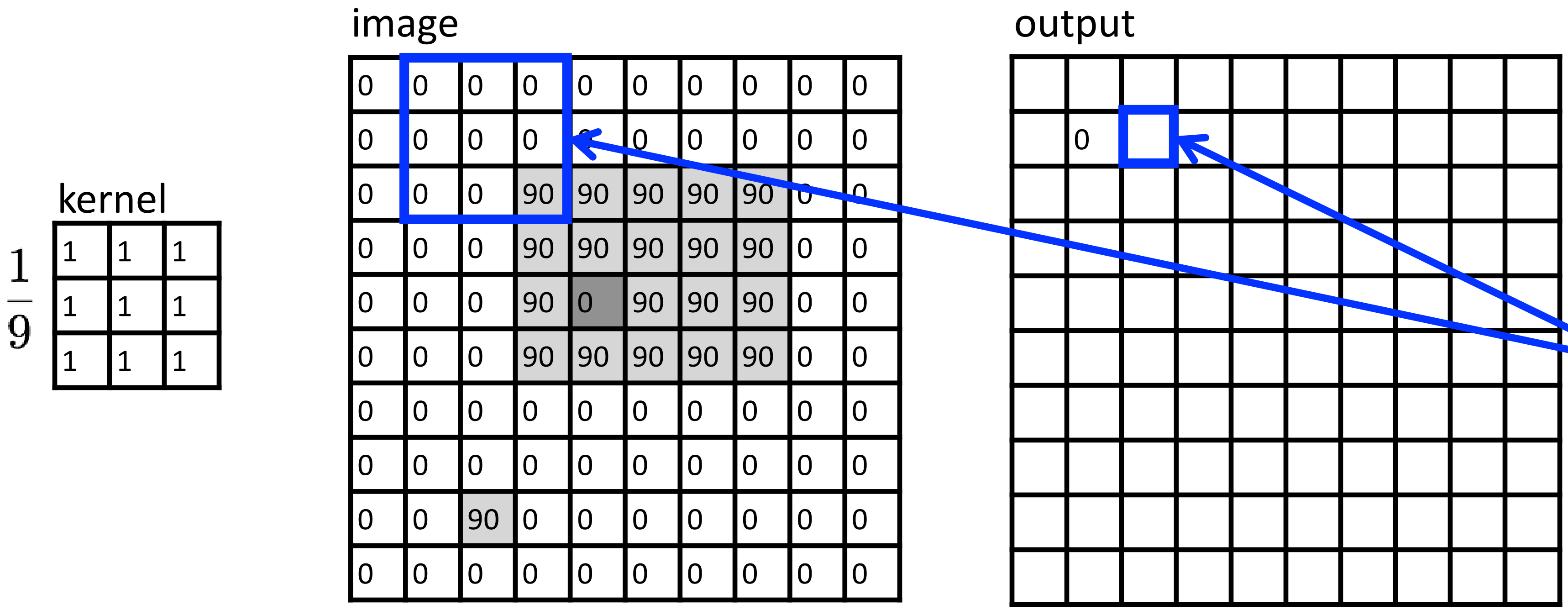
image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0								

Run the filter



shift-invariant:
as the pixel
shifts, so does
the kernel

Run the filter

kernel

$$\frac{1}{9}$$

1	1	1
1	1	1
1	1	1

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10							

Run the filter

$g[\cdot,\cdot]$

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $f[\cdot,\cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $h[\cdot,\cdot]$

	0	10	20						

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

output filter image (signal)

Run the filter

kernel

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10	20	30					

Run the filter

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10	20	30	30				

Run the filter

kernel

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10	20	30	30	30			

… and the result is

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

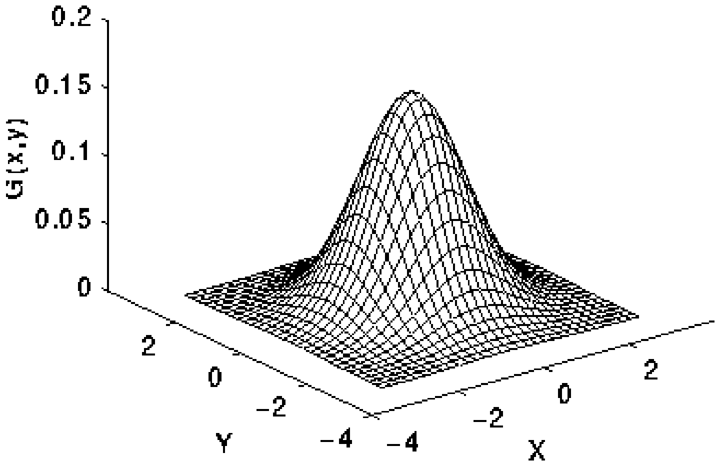
image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	0	10	20	30	30	30	20	10	
	10	10	10	10	0	0	0	0	
	10	10	10	10	0	0	0	0	

Gaussian filter



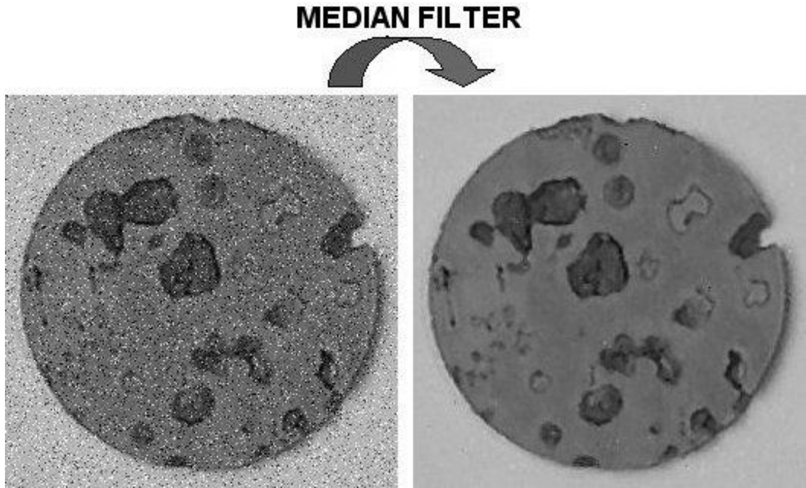
- Another kind of blur filter.
- this filter can be controlled by its size and STD.

$$h(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

- The kernel is discretized to bins according to the wanted kernel size.
- Isn’t Gaussian function infinite?
  – Most often, the kernel cuts out the remaining lower bins (usually at 2-3 σ).
- **Both Gaussian and mean filters are good against Gaussian noise, but not effective against S&P noise.**

Median filter

- Takes the median value from the given neighbors (and hence can also be considered as a “blurring effect”).
- For example:
  - The median of [1, 0, 100] is 1.
- **Median filter is good against salt and pepper noise, and against Gaussian noise (but not as effective).**
- **Median filter is also more computationally expansive.**
- This filter is not LTI because it’s not linear on its weights.



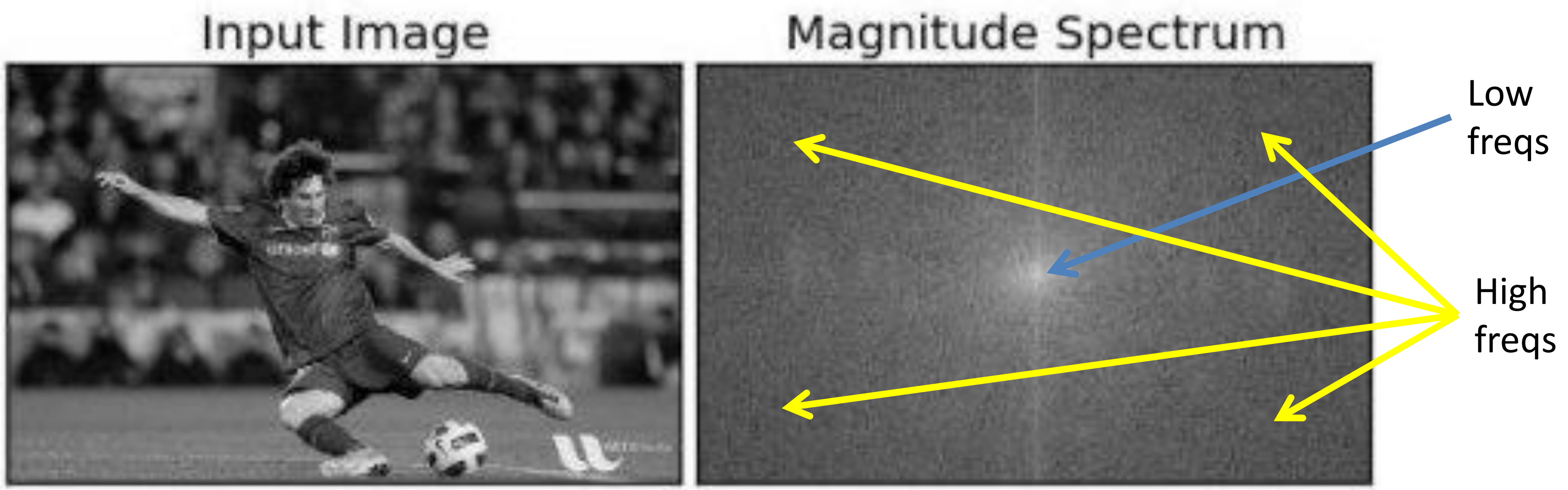
Example in .ipynb

contents

- Image representation
- Pixel-wise operations
- Noise and filtering
- **Frequency representation**
- Morphology operators
- Connected components

FFT of an image

- Frequency analysis is not in the scope of the course and is not needed for the rest of the course, but here it can give some intuition.
- In audio signals (or any other 1D signals) Lower frequencies change less over time than higher frequencies.
  - In images, the change is represented in change in distance, so images that changes slowly from pixel to pixel has more lower frequencies then others.
- Natural images are mainly built from low frequencies.



Convolution in frequency domain

- Recall: in time (space) domain:

$$g(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} h[u,v] f[i-u, j-v]$$

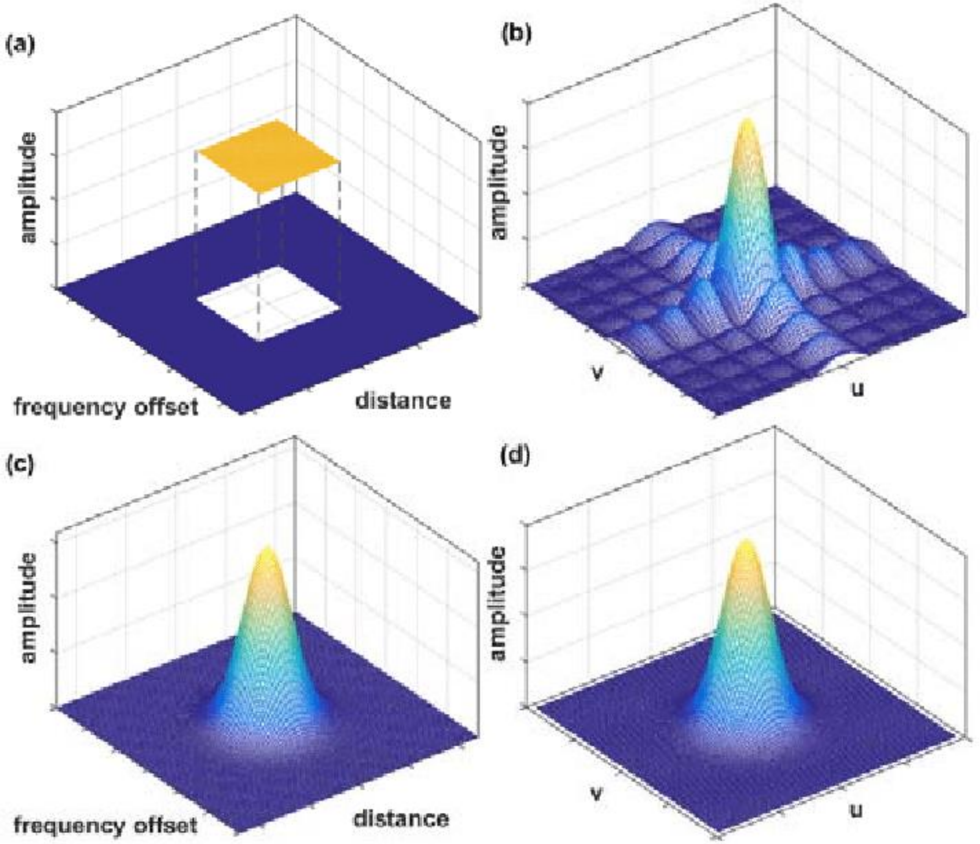
$$g = h * f$$

- In frequency domain- simple multiplication:

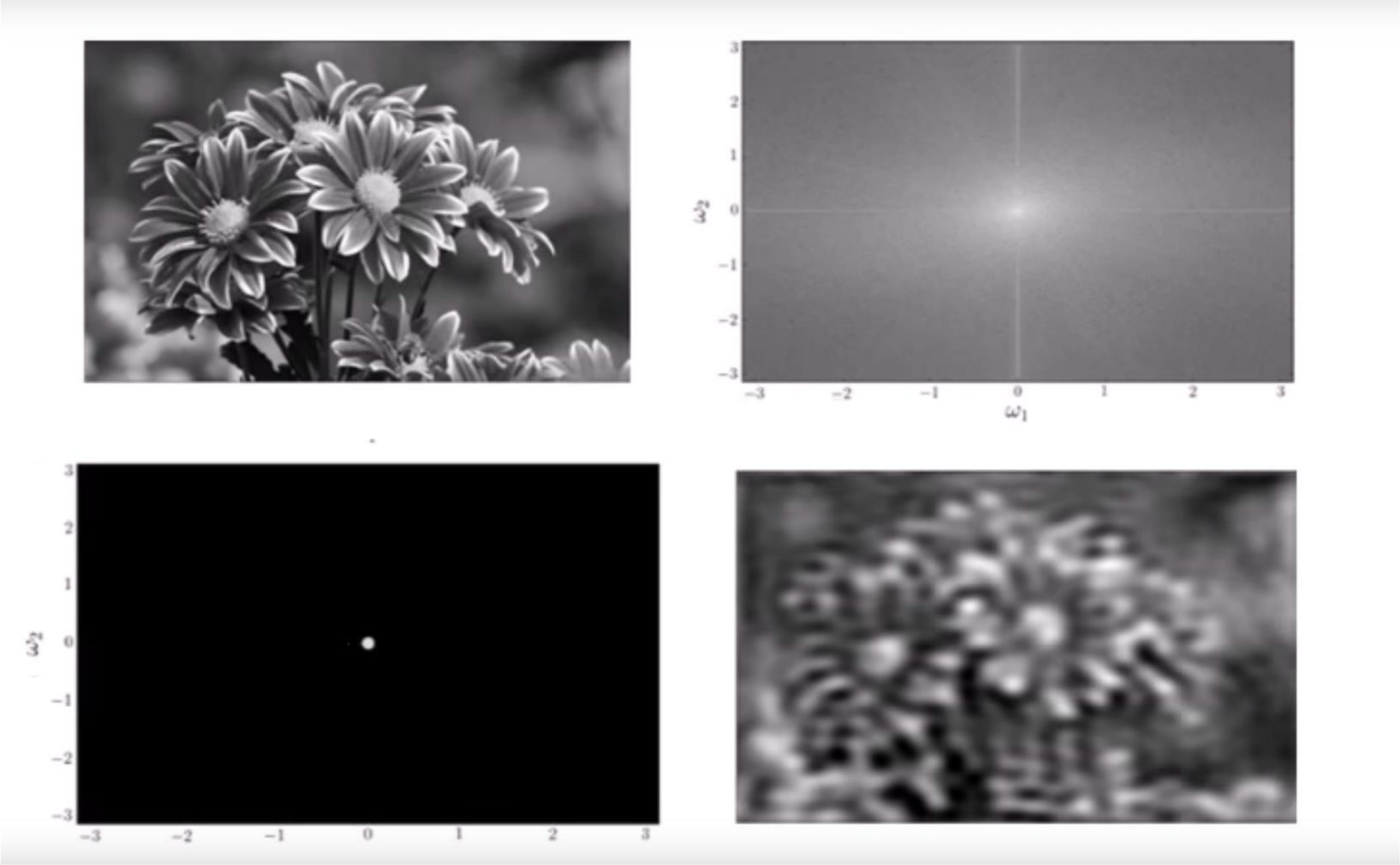
$$G = H \cdot F$$

Low-pass filters

- Both mean and Gaussian filters are considered low-pass filters because in the frequency domain, they have higher values in the lower frequencies- and when multiplied with frequency spectrums, the high frequencies get smaller.
- When image is left only with the lower frequencies, the rapidly changes parts of the image (e.g.: edges, noise) are smoothen.

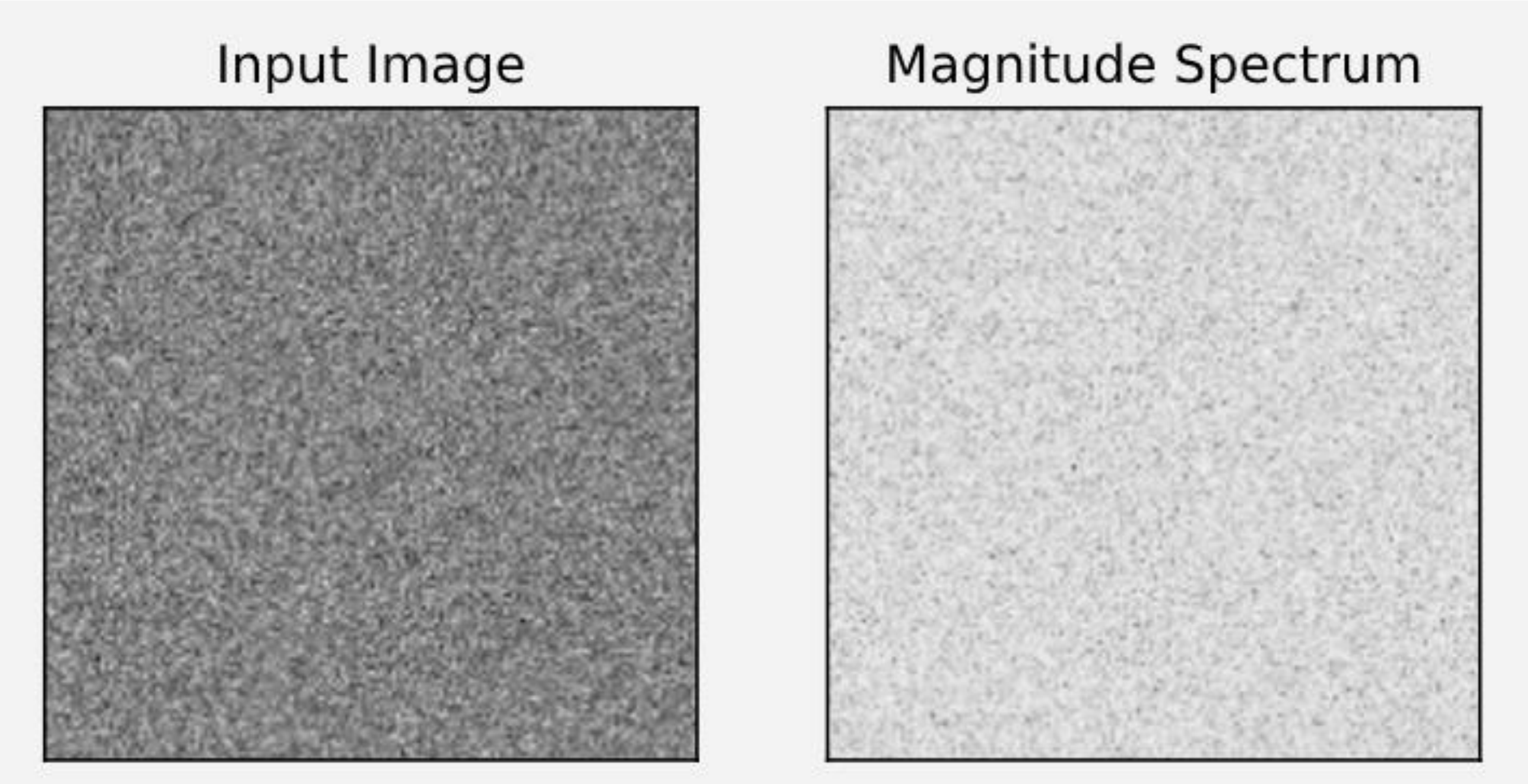


LP example



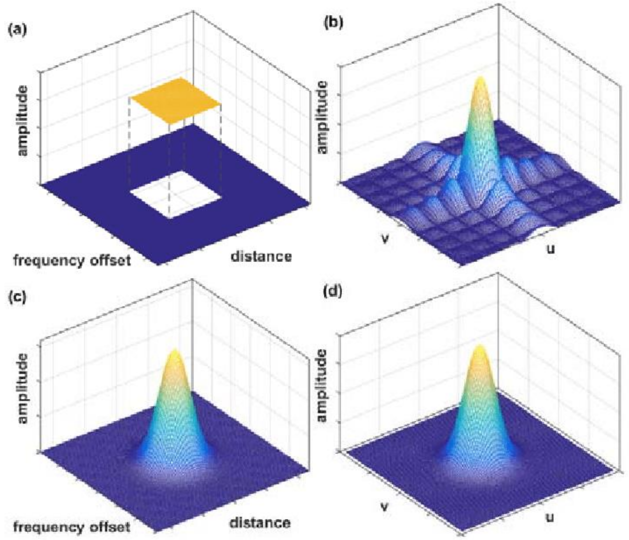
FFT of gaussian noise

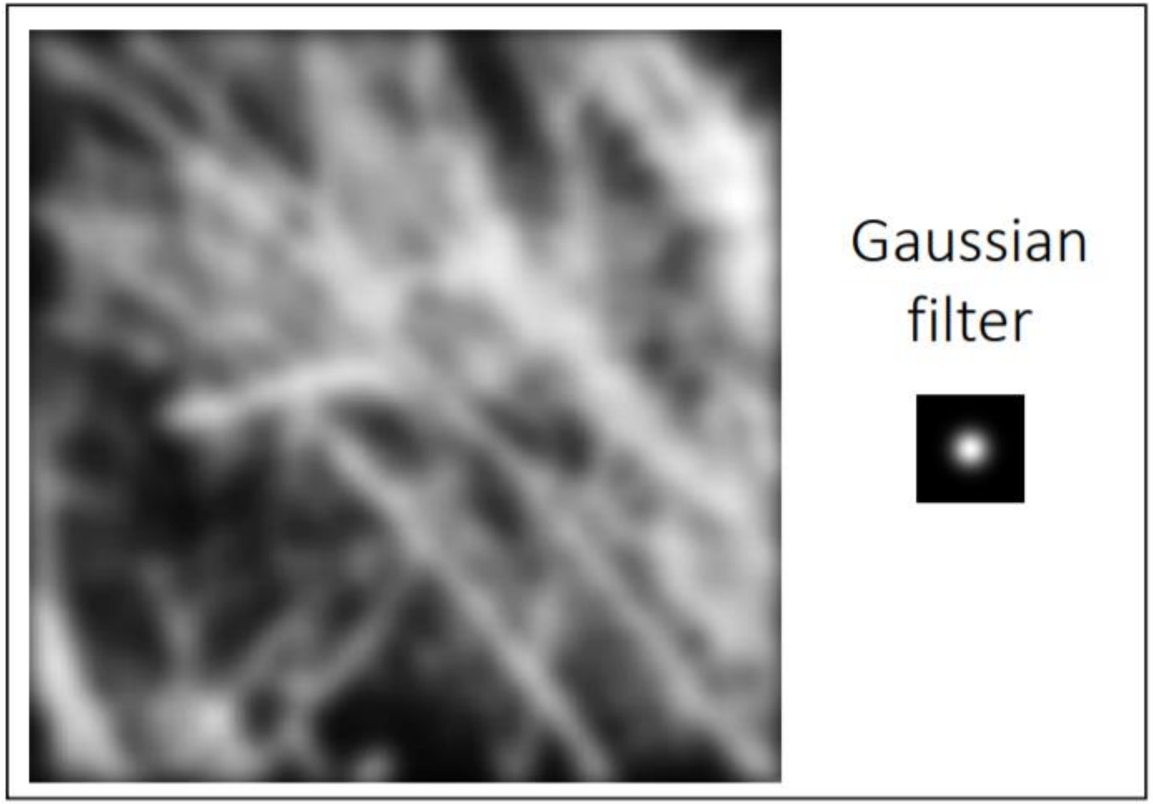
- Since gaussian noise (AWGN) is distributed along all frequencies, LP filter reduce this kind of noise significantly.



Mean vs. Gaussian filter

- Since Mean filter has higher values in the higher frequencies, edge artifacts sometimes remains.





contents

- Image representation
- Pixel-wise operations
- Noise and filtering
- Frequency representation
- **Morphology operators**
- Connected components

Morphology

- Handy tool whenever needed to clean up binary images.
- Each morphology operator is constructed as such:
    1. Select some kind of structure element (binary kernel)

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Morphology

- Handy tool whenever needed to clean up binary images.
- Each morphology operator is constructed as such:
  1. Select some kind of structure element (binary kernel)
  2. Convolve with input binary image $g = f * s$

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Morphology

- Handy tool whenever needed to clean up binary images.
- Each morphology operator is constructed as such:
  1. Select some kind of structure element (binary kernel)
  2. Convolve with input binary image $g = f * s$
  3. Threshold the output

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\theta_{TH}(x,t) = \begin{cases} 1 & if \quad x \geq t, \\ 0 & else \end{cases}$$

Morphology

- Handy tool whenever needed to clean up binary images.
- Each morphology operator is constructed as such:
  1. Select some kind of structure element (binary kernel)
  2. Convolve with input binary image $g = f * s$
  3. Threshold the output

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\theta_{TH}(x,t) = \begin{cases} 1 & if \quad x \geq t, \\ 0 & else \end{cases}$$

- Overall morphologic operation should look like so:

$$k = \theta_{TH}(f * s, t)$$

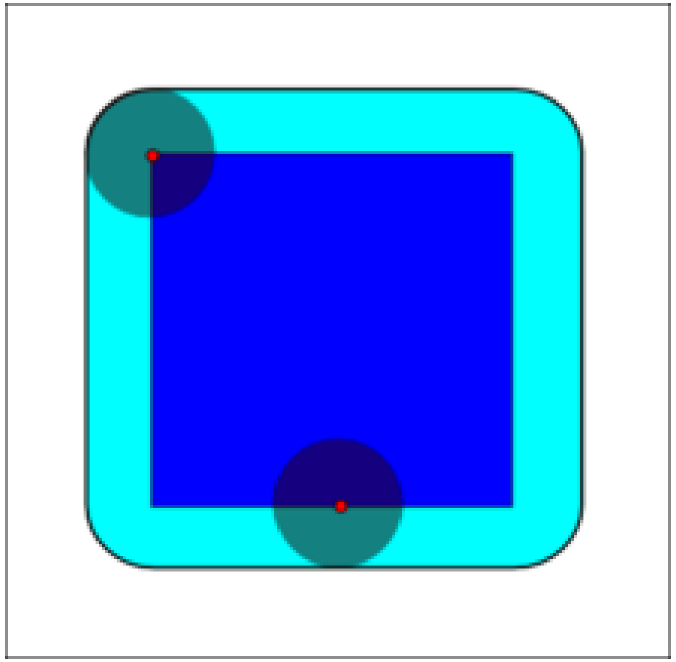
Dilation

1. Select some kind of structure element (binary kernel)
2. Convolve with input binary image $g = f * s$
3. Threshold the output

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\theta_{TH}(x,t) = \begin{cases} 1 & if \quad x \geq t, \\ 0 & else \end{cases}$$

- Dilation: $t = 1$
  - Skinny lines will get thicker



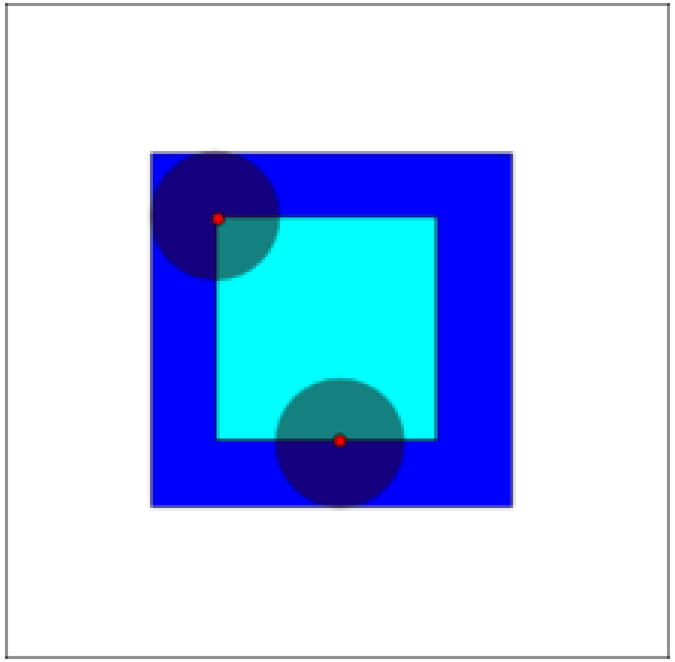
Erosion

1. Select some kind of structure element (binary kernel)
2. Convolve with input binary image $g = f * s$
3. Threshold the output

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

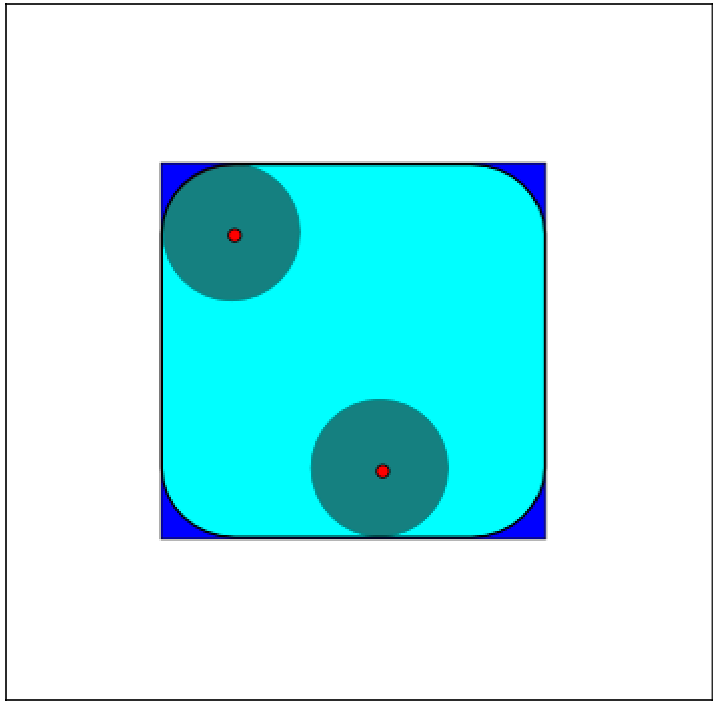
$$\theta_{TH}(x,t) = \begin{cases} 1 & if \quad x \geq t, \\ 0 & else \end{cases}$$

- Erosion: $t = sum(s)$
  - Thicker lines will get skinny

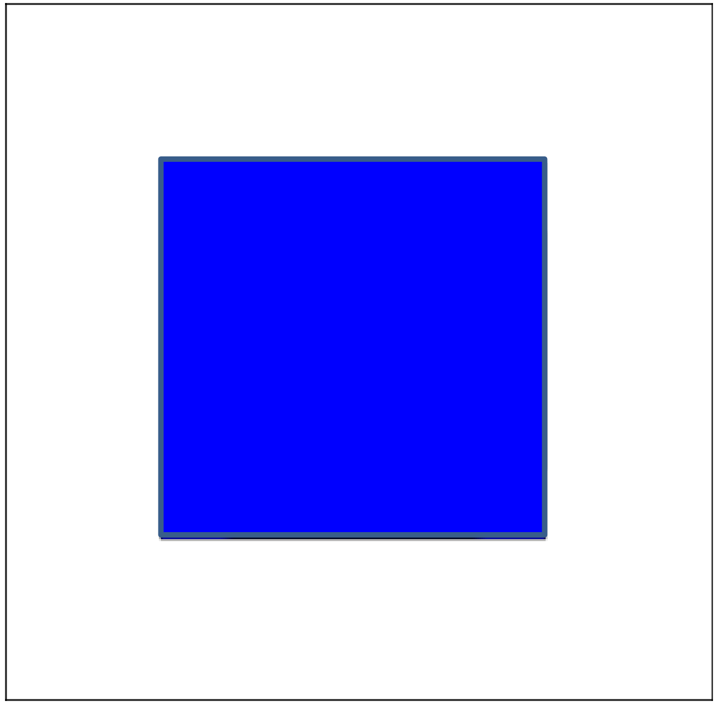


Opening

- Erosion followed by dilation.
  - The effect is of rounding off sharp edges.



Closing



- Dilation followed by erosion.
  - The effect is of closing of narrow gaps and holes.

contents

- Image representation
- Pixel-wise operations
- Noise and filtering
- Frequency representation
- Morphology operators
- **Connected components**

Connected components

- Defined as regions of adjacent pixels that have the same value.
- Commonly used with binary images to find stand alone objects.
  - e.g.: letters in a document.