Image processing

References

- http://szeliski.org/Book/
- http://www.cs.cornell.edu/courses/cs5670/2019sp/lectures/lectures.html
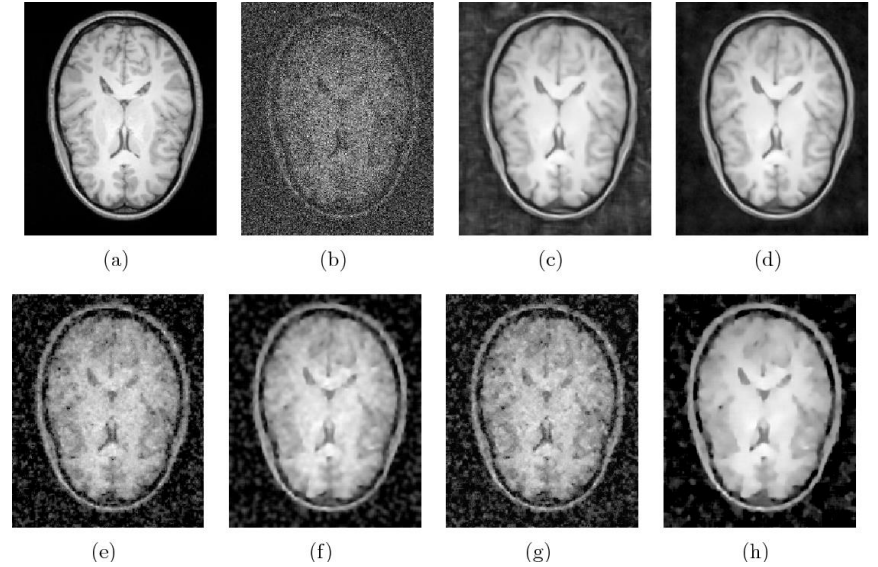- http://www.cs.cmu.edu/~16385/

Some motivation

Art
(Photoshop image correction)

Medicine
(MRI denoising)
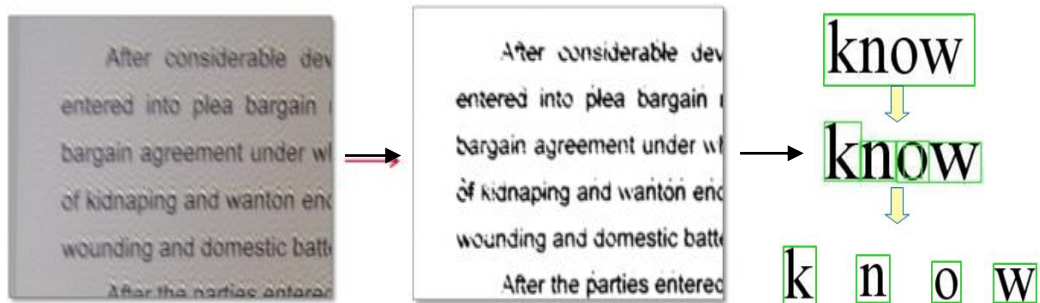
Robotics
(OCR – optical character recognition)

Agriculture
(color ripeness detection)

contents

- **Image representation**
- Pixel-wise operations
- Noise and filtering
- Frequency representation
- Morphology operators
- Connected components

Image representation

- We cane think of image as a 3d matrix of discrete RGB values.
- The values mark the intensity of each color channel and are usually of type uint8 = $\{0, \ldots, 255\}$.
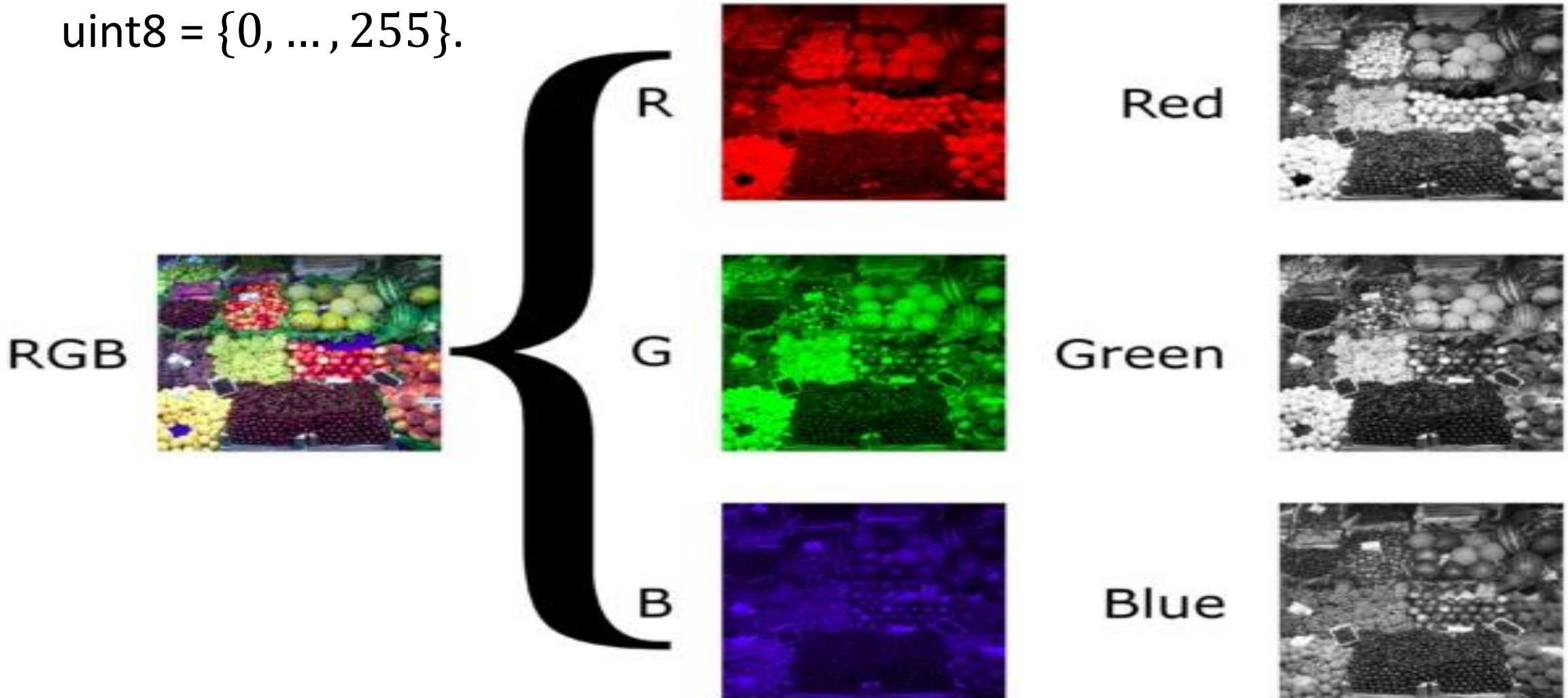
Image representation
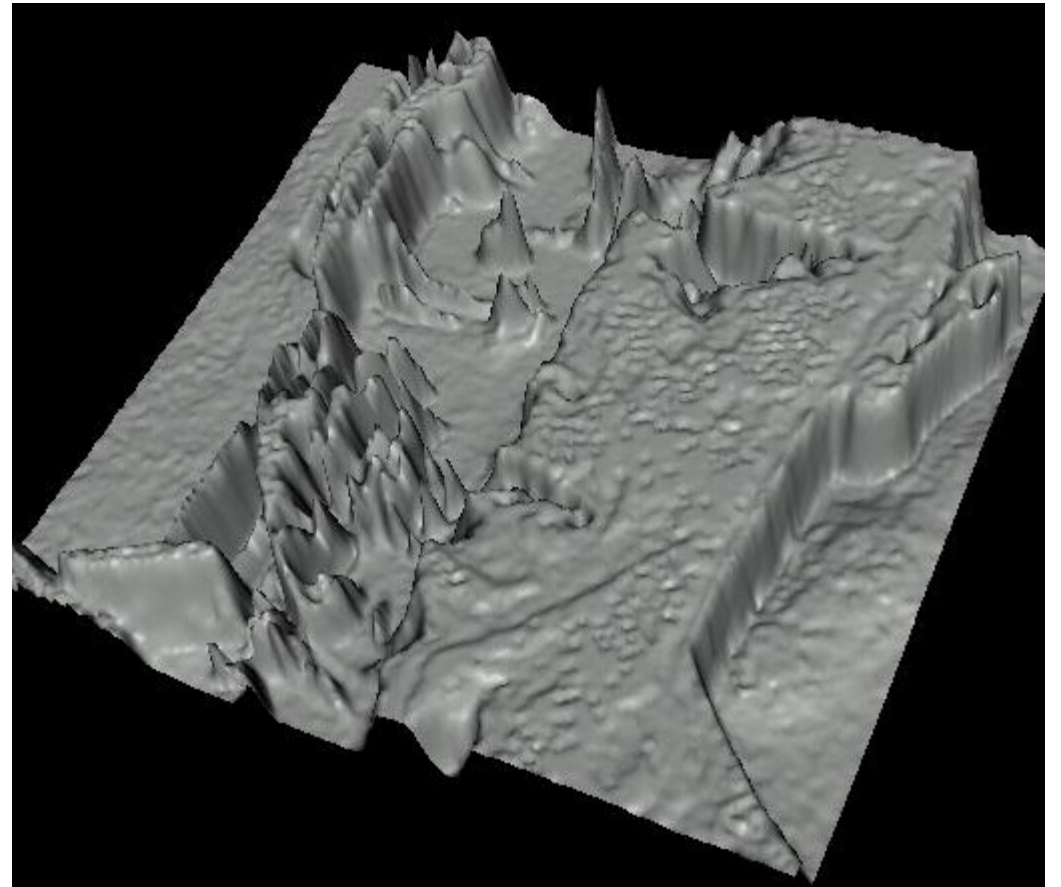
- We can also think of an image as a function $f(x, y)$.

contents

- Image representation
- **Pixel-wise operations**
- Noise and filtering
- Frequency representation
- Morphology operators
- Connected components

Pixel-wise operators

- Pixel-wise operators, or point operators, are defined as such that each output pixel's value depends on only the corresponding input pixel value.

Pixel-wise operators

original

x

darken

lower contrast

Gamma compression

invert

lighten

raise contrast (?)

Gamma expansion

Pixel-wise operators

original

x

darken

lower contrast

Gamma compression

invert

$255 - x$

lighten

raise contrast (?)

Gamma expansion

Pixel-wise operators

original

x

darken

$x - 128$

lower contrast

Gamma compression

invert

$255 - x$

lighten

raise contrast (?)

Gamma expansion

Pixel-wise operators

original

x

darken

$x - 128$

lower contrast

Gamma compression

invert

$255 - x$

lighten

$x + 128$

raise contrast (?)

Gamma expansion

Pixel-wise operators

original

$$x$$

darken

$$x - 128$$

lower contrast

$$\frac{x}{2}$$

Gamma compression

invert

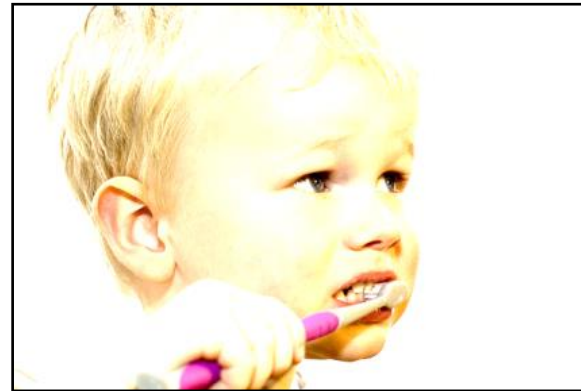$$255 - x$$

lighten

$$x + 128$$

raise contrast (?)

Gamma expansion

Pixel-wise operators

original

darken

lower contrast

Gamma compression

x

$x - 128$

$\dfrac{x}{2}$

invert

lighten

raise contrast (?)

Gamma expansion

$255 - x$

$x + 128$

$x \times 2$

Contrast

- **Contrast** in visual perception is the difference in appearance of two or more parts of a field seen.

- The human visual system is more sensitive to contrast than absolute luminance;

- **Contrast ratio**, or **dynamic range**, is the is the ratio between the largest and smallest values of the image or :

$$CR = \frac{V_{max}}{V_{min}}$$

Pixel-wise operators

original

darken

lower contrast

Gamma compression

$$x$$

$$x - 128$$

$$\frac{x}{2}$$

$$\left(\frac{x}{255}\right)^{1/3} \times 255$$

invert

lighten

raise contrast (?)

Gamma expansion

$$255 - x$$

$$x + 128$$

$$x \times 2$$

Pixel-wise operators

original

$$x$$

darken

$$x - 128$$

lower contrast

$$\frac{x}{2}$$

Gamma compression

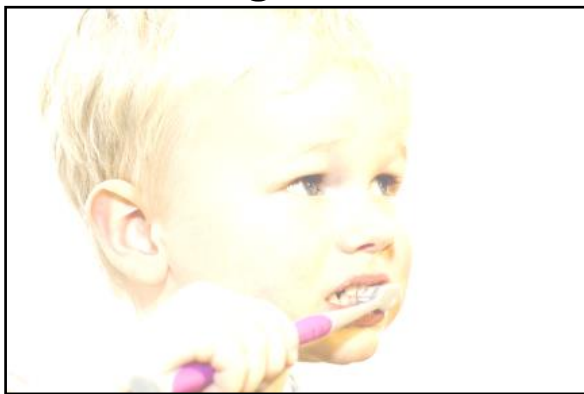

$$\left(\frac{x}{255}\right)^{1/3} \times 255$$

invert

$$255 - x$$

lighten

$$x + 128$$

raise contrast (?)
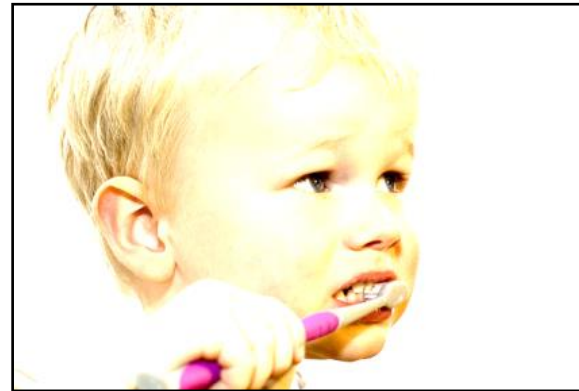
$$x \times 2$$

Gamma expansion

$$\left(\frac{x}{255}\right)^{2} \times 255$$

Gamma correction

- Originally, Due to non-linearities in the old CRT televisions, intensities was seen different then they are.

Sample Input to Monitor — Graph of Input: RAMP

Output from Monitor — Graph of Output: $L = V \wedge 2.5$

Υ=2

Υ=1 (original)

Υ=1/2

Υ=1/3

Υ=1/4
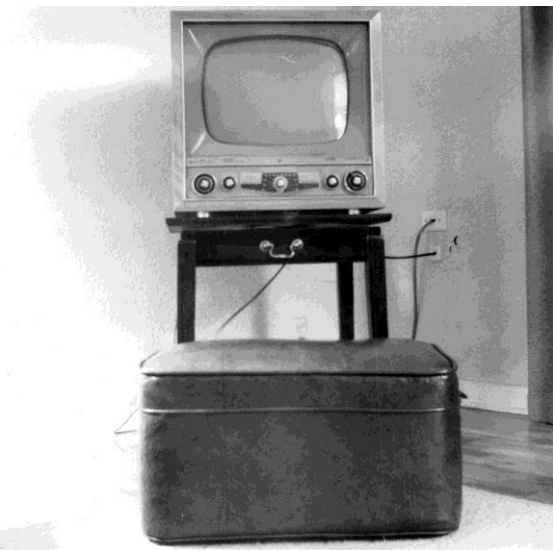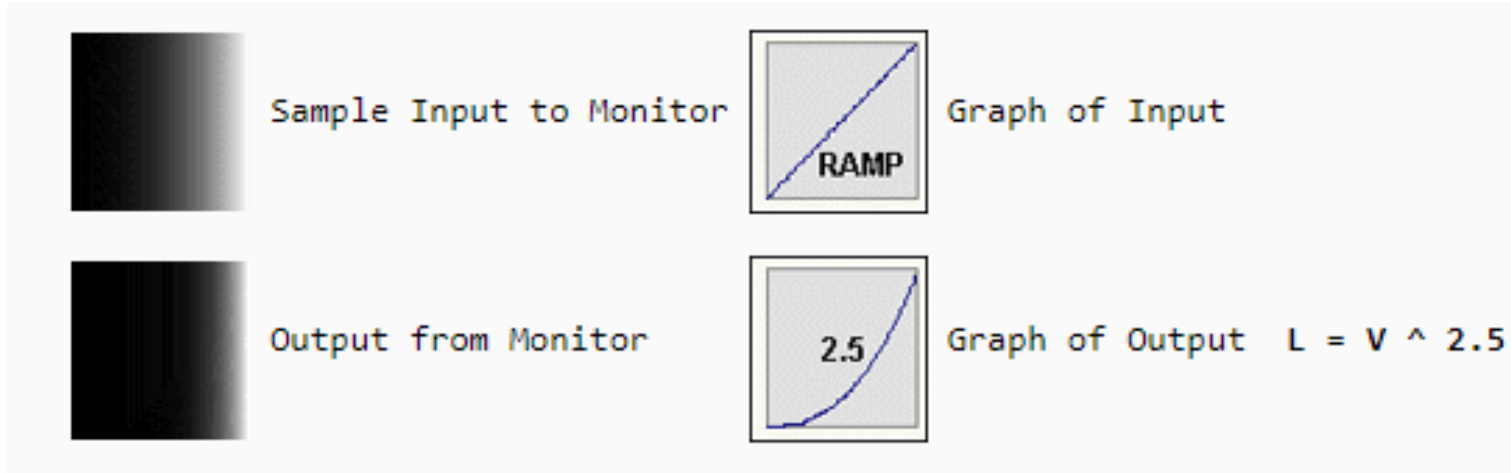
Gamma correction

- To correct this non linear transformation, gamma correction was done:

$$V_{out} = V_{in}^{\gamma}$$

- This is, of course, also applicable for image enhancements.
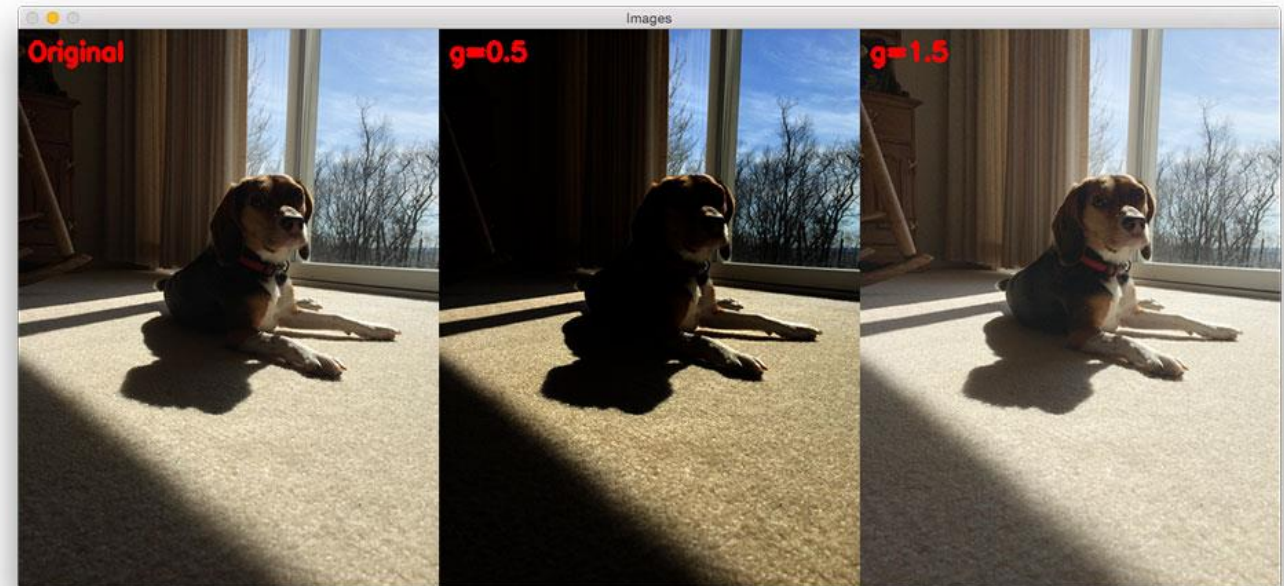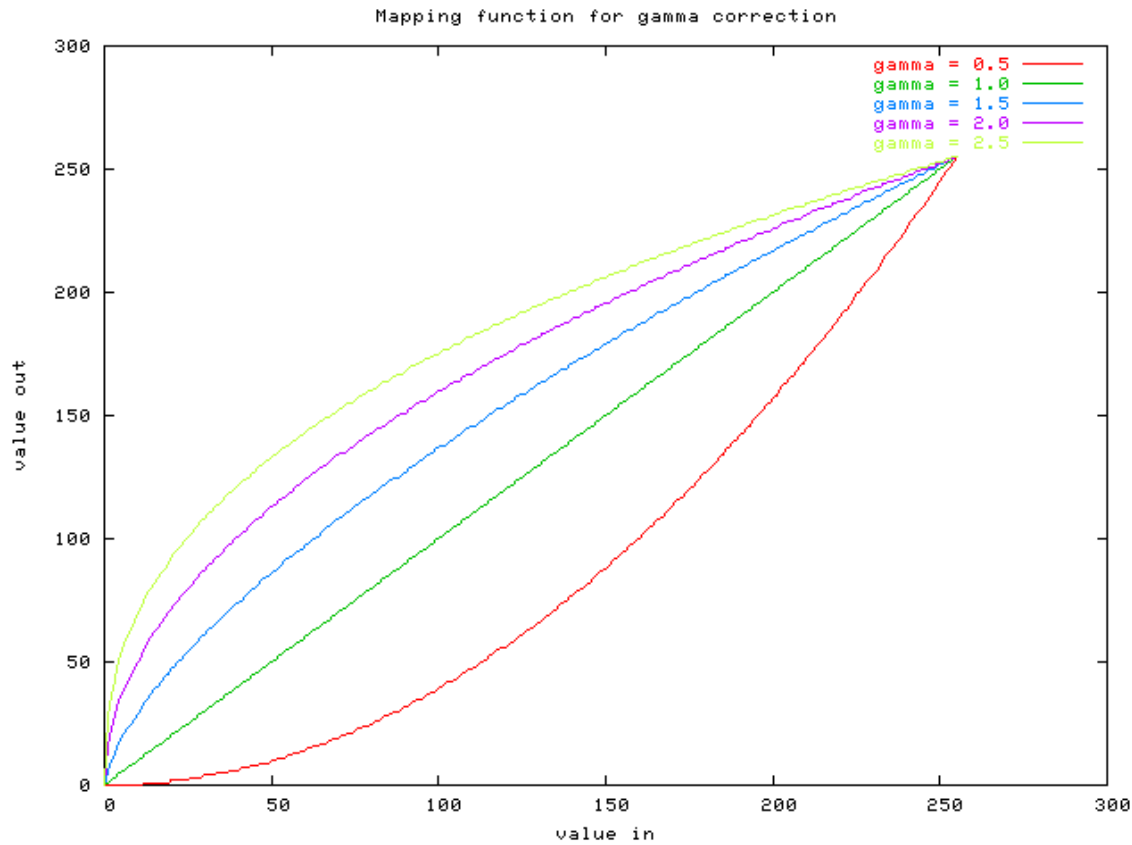
contents

- Image representation
- Pixel-wise operations
- **Noise and filtering**
- Frequency representation
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Gaussian Noise

- Gaussian noise is an additive noise that can appear in images due to the system electrical circuitry.

- This noise is independent of signal strength and independent at each pixel (IID).

$$p(z) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}$$

Gaussian Noise σ=0.01

Gaussian Noise σ=0.05

Gaussian Noise σ=0.1

SNR=34.0206

SNR=19.1825

SNR=13.7121

Salt & Pepper noise

- Noise that can be caused by analog-to-digital converter errors, bit errors in transmission, etc.

- This noise is **not** additive to the signal strength (a replacement of original value with noise value).

- This noise is independent of signal strength and independent at each pixel.

And some more noise

- **Shot noise** - caused by statistical quantum fluctuations, that is, variation in the number of photons sensed at a given exposure level in the darker parts of an image (where there are just few photons that enter each pixel "bin"). Modeled as Poisson noise.
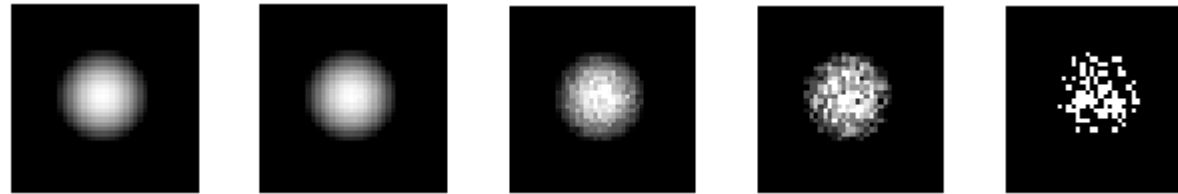
- **Quantization noise** – caused by quantizing the pixels of a sensed image to several discrete levels (analog to digital conversion).
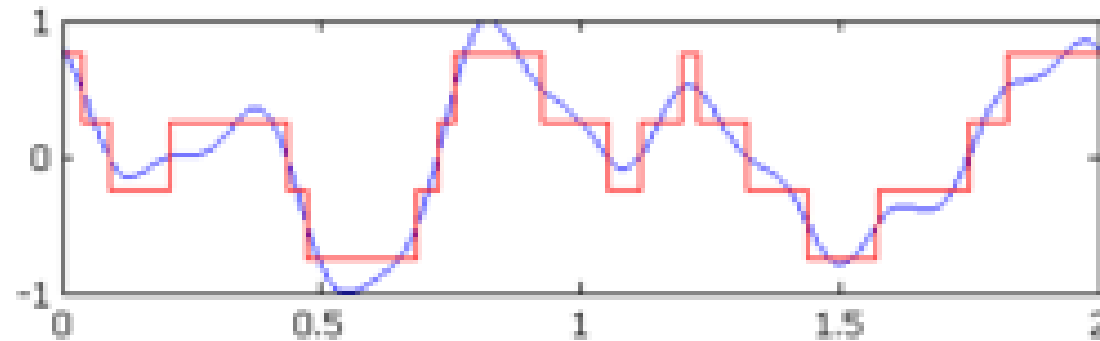
Noise reduction with LTI filters

- **linear filtering operators**, which involve weighted combinations of pixels in small neighborhood.

- The combination is determined by the filter's *kernel*.

- The same kernel is *shifted* to all pixel locations so that all pixels use the same linear combination of their neighbors.

- That's why it's called **linear shift-invariance** (LTI) filter.

Convolution

- Works on LTI filters.
- Let h be the image, f be the kernel (of size 2k+1 x 2k+1), and g be the output image:

$$g(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} h[u,v] f[i-u, j-v]$$

  – **Note:** by definition, this operator flips the kernel both horizontally and vertically.
- This operation is called **convolution operator** and is more compactly noted as:

$$g = h * f$$

Example: mean filter

- The kernel is:

$$\frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

- Replaces pixel with local average.

- Has smoothing (blurring) effect

- The kernel can be in any other size as well (see .ipynb).

Run the filter

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

Note that we assume that the kernel
coordinates are centered.
Here the kernel is symmetric
horizontally and vertically, so the
flipping is not noticeable.

Run the filter

Run the filter

kernel

$$\frac{1}{9}$$

1	1	1
1	1	1
1	1	1

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0								

shift-invariant: as the pixel shifts, so does the kernel

Run the filter

kernel

$$\frac{1}{9}$$

1	1	1
1	1	1
1	1	1

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

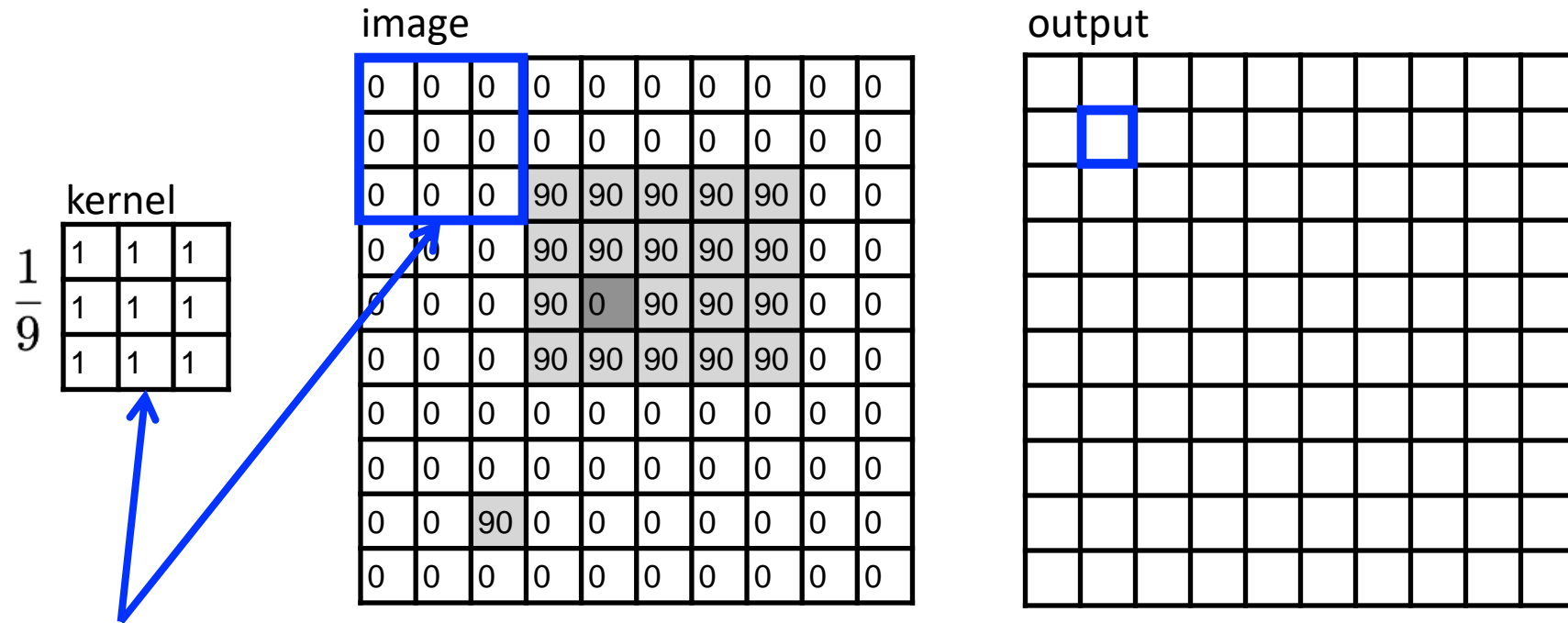|									
	0	10							

Run the filter

$g[\cdot,\cdot]$

kernel

$$\frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

image
$f[\cdot,\cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output
$h[\cdot,\cdot]$

	0	10	20						

$$h[m,n] = \sum_{k,l} g[k,l]f[m+k,n+l]$$

output   filter   image (signal)

Run the filter

kernel

$$\frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10	20	30					

Run the filter

kernel

$$\frac{1}{9}$$

1	1	1
1	1	1
1	1	1

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10	20	30	30				

Run the filter

... and the result is

kernel

$$\frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	0	10	20	30	30	30	20	10	
	10	10	10	10	0	0	0	0	
	10	10	10	10	0	0	0	0	

Gaussian filter

- Another kind of blur filter.

- this filter can be controlled by its size and STD.

$$h(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2+y^2}{2\sigma^2}}$$
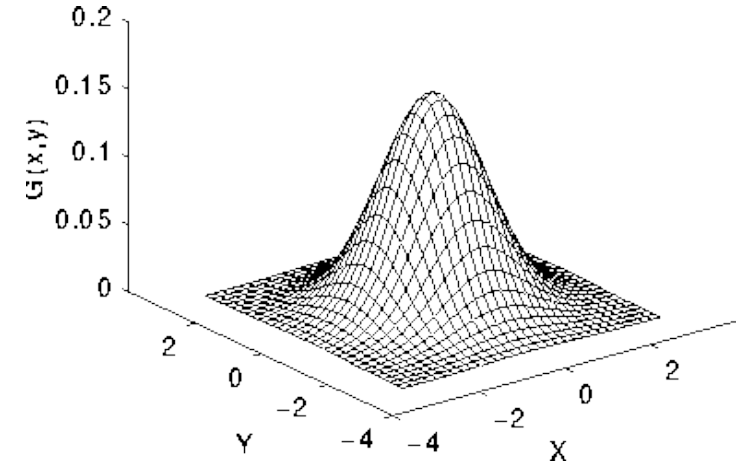
- The kernel is discretized to bins according to the wanted kernel size.

- Isn't Gaussian function infinite?

   – Most often, the kernel cuts out the remaining lower bins (usually at 2-3 σ).

- **Both Gaussian and mean filters are good against Gaussian noise, but not effective against S&P noise.**

Median filter

- Takes the median value from the given neighbors (and hence can also be considered as a "blurring effect").
- For example:
  - The median of [1, 0, 100] is 1.
- **Median filter is good against salt and pepper noise, and against Gaussian noise (but not as effective).**
- **Median filter is also more computationally expansive.**
- This filter is not LTI because it's not linear on its weights.

MEDIAN FILTER
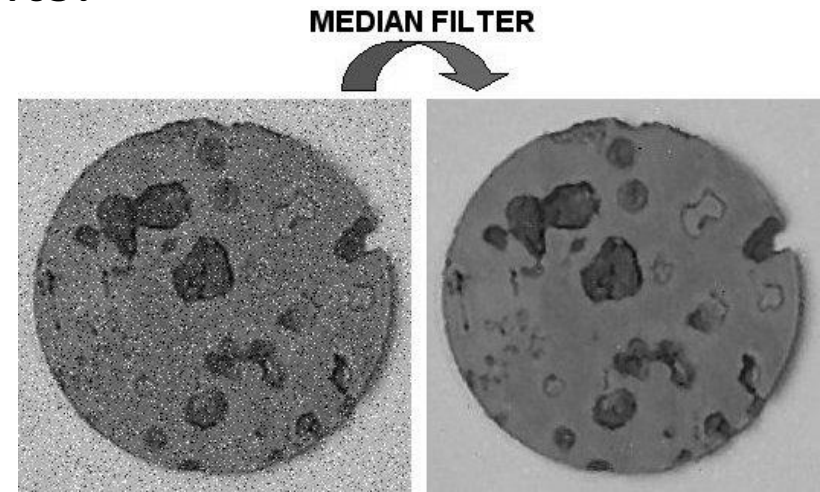
Example in .ipynb

contents

- Image representation
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FFT of an image

- Frequency analysis is not in the scope of the course and is not needed for the rest of the course, but here it can give some intuition.

- In audio signals (or any other 1D signals) Lower frequencies change less over time than higher frequencies.

  - In images, the change is represented in change in distance, so images that changes slowly from pixel to pixel has more lower frequencies then others.

- Natural images are mainly built from low frequencies.

Input Image   Magnitude Spectrum

Low freqs

High freqs
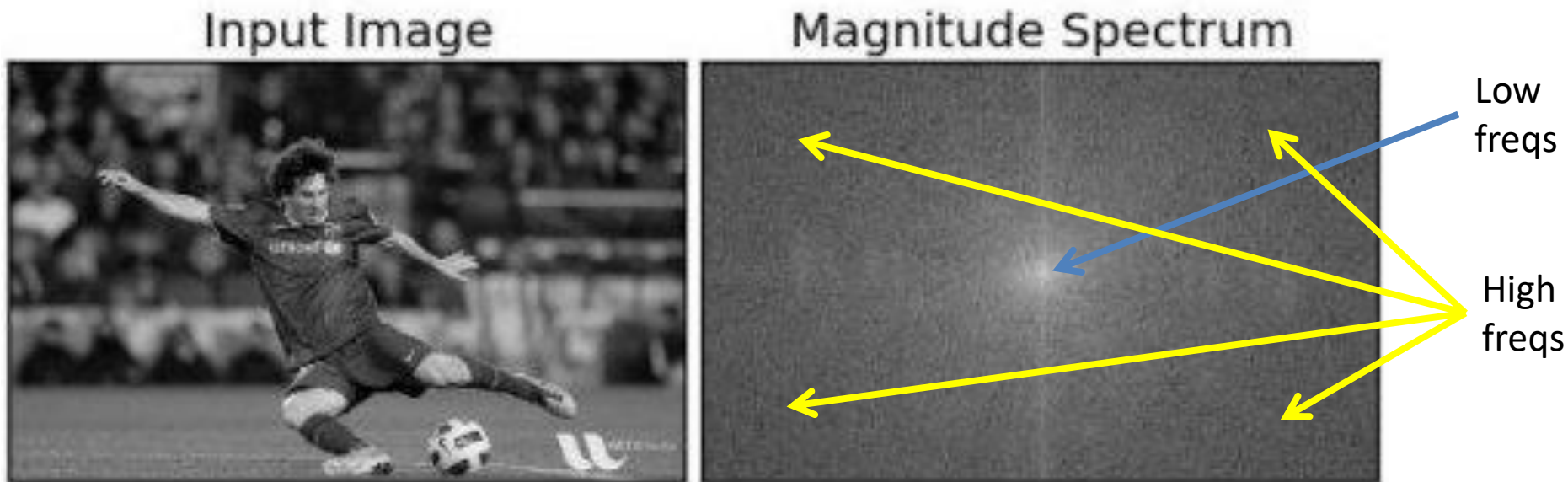
Convolution in frequency domain

- Recall: in time (space) domain:

$$g(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} h[u,v]f[i-u, j-v]$$

$$g = h * f$$

- In frequency domain- simple multiplication:

$$G = H \cdot F$$

Low-pass filters

- Both mean and Gaussian filters are considered low-pass filters because in the frequency domain, they have higher values in the lower frequencies- and when multiplied with frequency spectrums, the high frequencies get smaller.

- When image is left only with the lower frequencies, the rapidly changes parts of the image (e.g.: edges, noise) are smoothen.
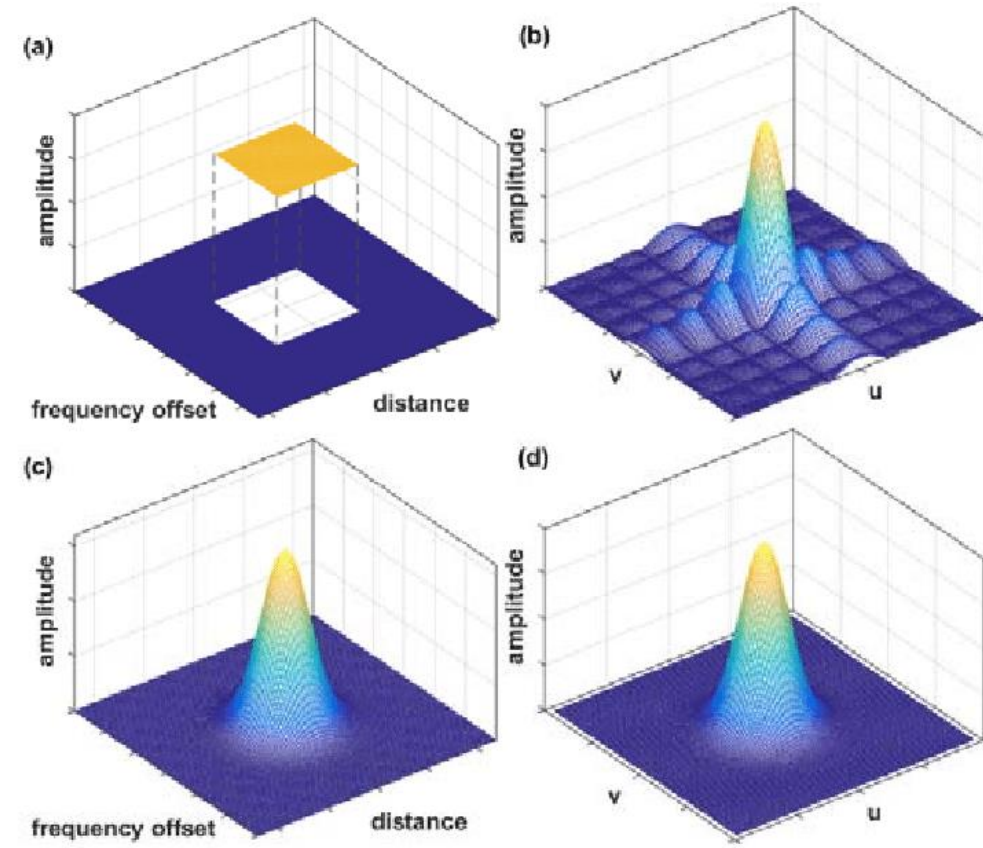
LP example
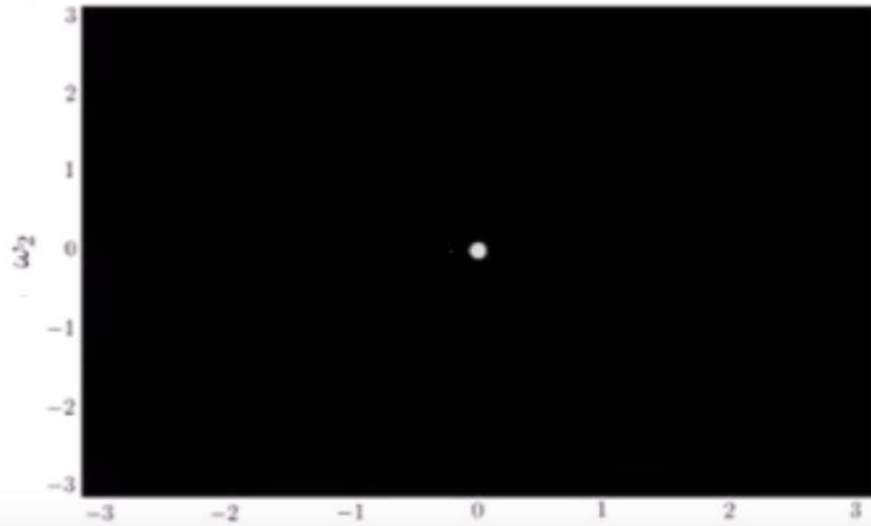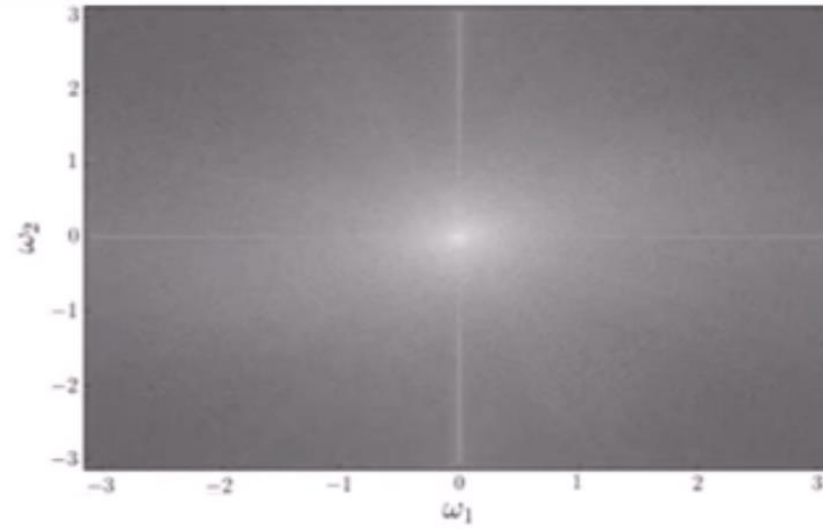
FFT of gaussian noise

- Since gaussian noise (AWGN) is distributed along all frequencies, LP filter reduce this kind of noise significantly.

Input Image   Magnitude Spectrum
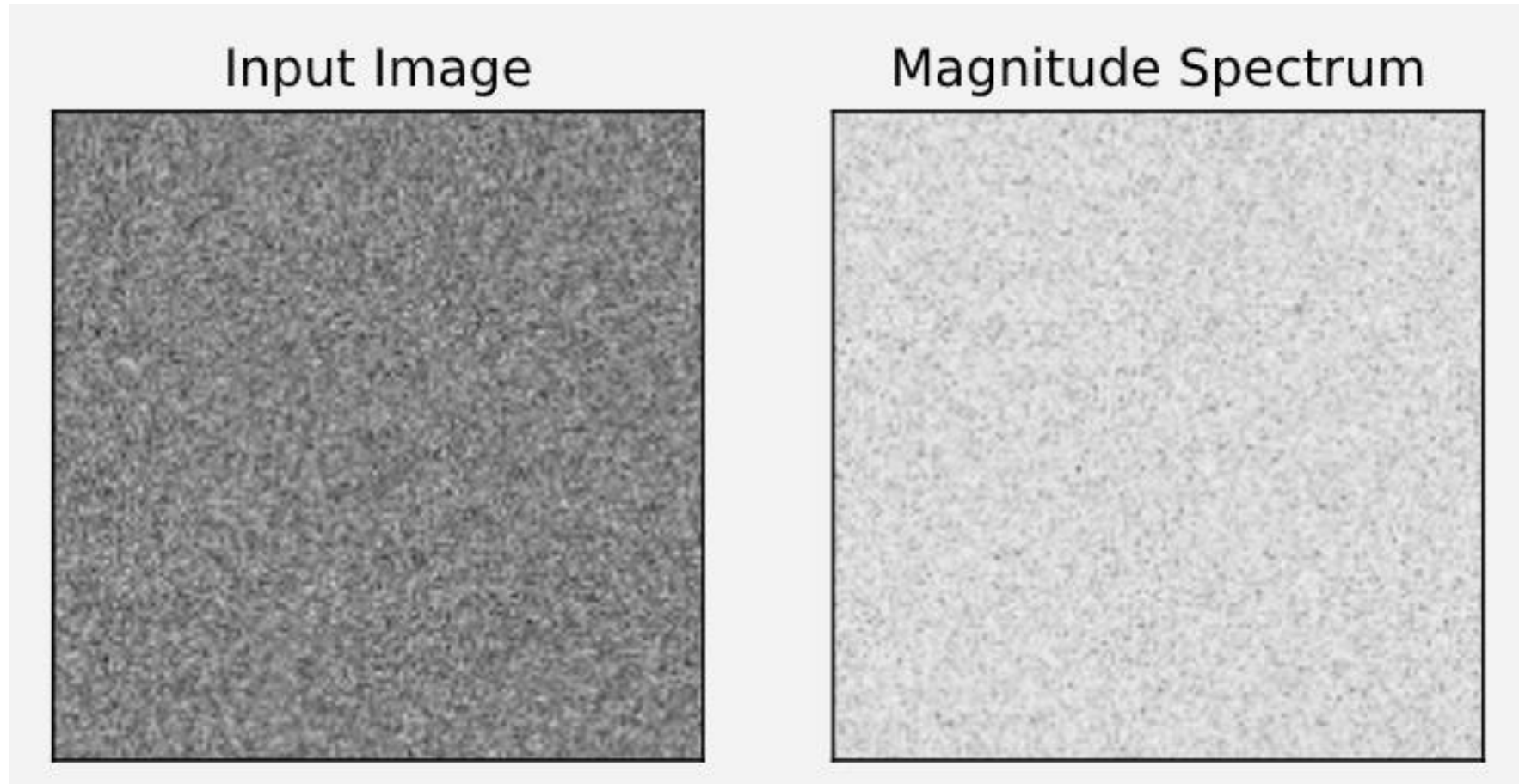
Mean vs. Gaussian filter

- Since Mean filter has higher values in the higher frequencies, edge artifacts sometimes remains.
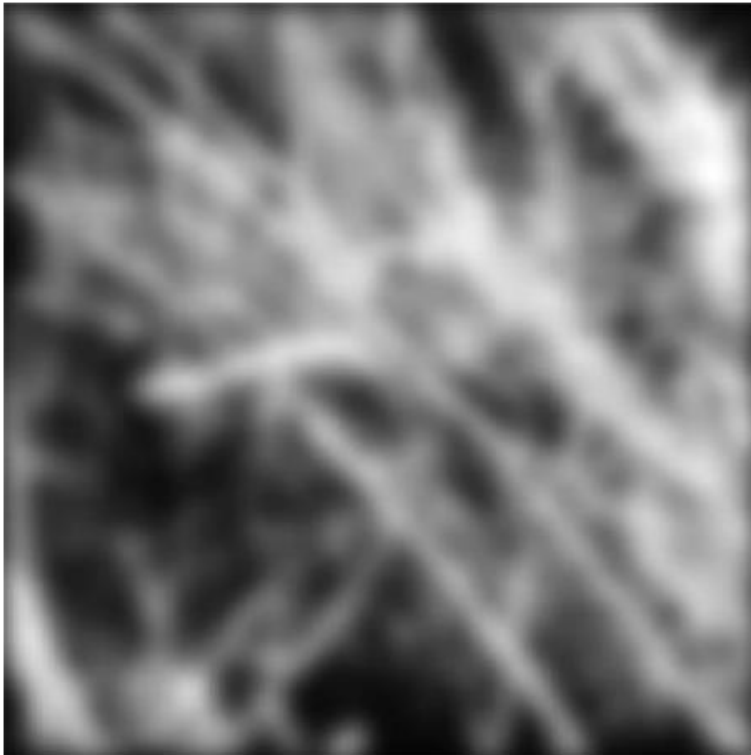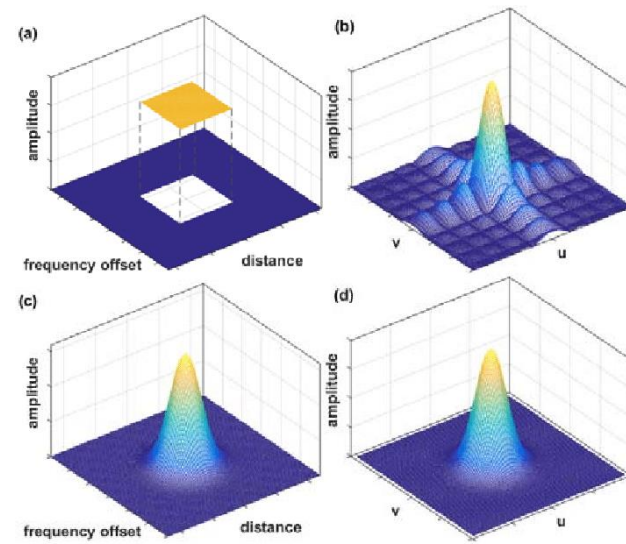
Gaussian filter
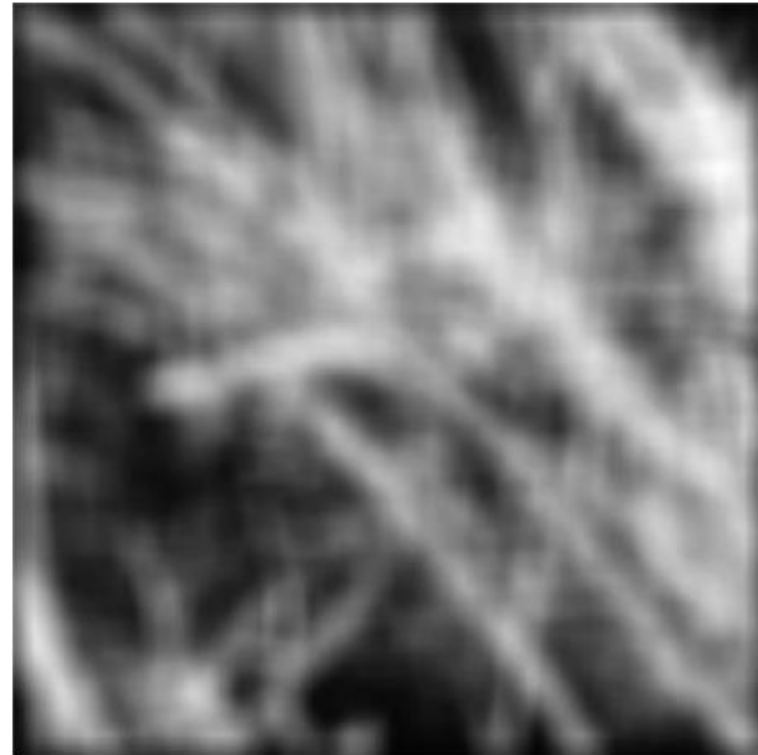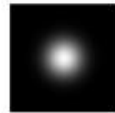
Box filter

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Morphology

- Handy tool whenever needed to clean up binary images.
- Each morphology operator is constructed as such:
  1. Select some kind of structure element (binary kernel)

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Morphology

- Handy tool whenever needed to clean up binary images.
- Each morphology operator is constructed as such:
  1. Select some kind of structure element (binary kernel)
  2. Convolve with input binary image $g = f * s$

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Morphology

- Handy tool whenever needed to clean up binary images.
- Each morphology operator is constructed as such:
  1. Select some kind of structure element (binary kernel)
  2. Convolve with input binary image $g = f * s$
  3. Threshold the output

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\theta_{TH}(x, t) = \begin{cases} 1 & if \quad x \geq t, \\ 0 & else \end{cases}$$

Morphology

- Handy tool whenever needed to clean up binary images.
- Each morphology operator is constructed as such:

  1. Select some kind of structure element (binary kernel)
  2. Convolve with input binary image $g = f * s$
  3. Threshold the output

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\theta_{TH}(x, t) = \begin{cases} 1 & if \quad x \geq t, \\ 0 & else \end{cases}$$

- Overall morphologic operation should look like so:

$$k = \theta_{TH}(f * s, t)$$

Dilation

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$
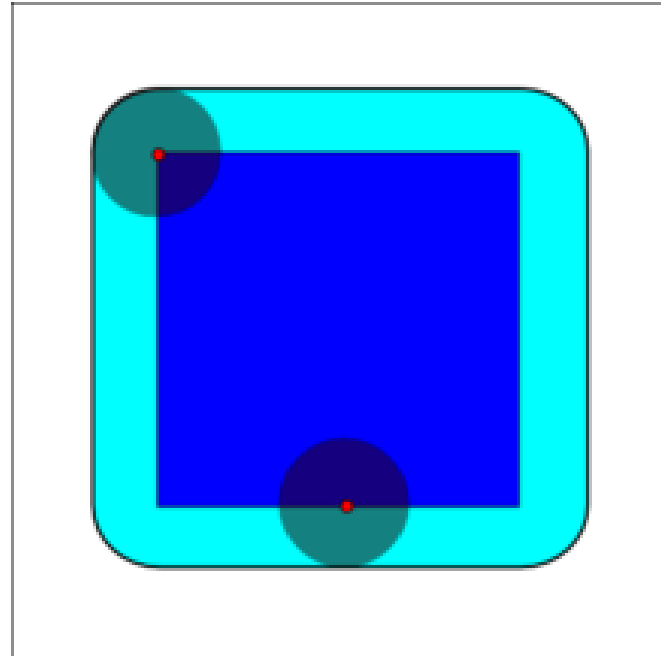
1. Select some kind of structure element (binary kernel)
2. Convolve with input binary image $g = f * s$
3. Threshold the output

$$\theta_{TH}(x, t) = \begin{cases} 1 & if \quad x \geq t, \\ 0 & else \end{cases}$$

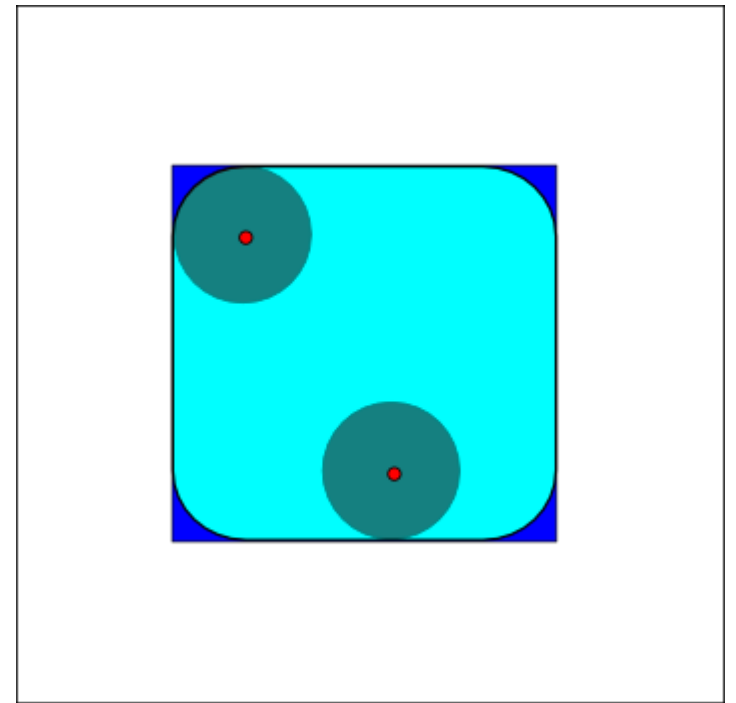- Dilation: $t = 1$
  - Skinny lines will get thicker

Erosion

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

1. Select some kind of structure element (binary kernel)
2. Convolve with input binary image $g = f * s$
3. Threshold the output

$$\theta_{TH}(x, t) = \begin{cases} 1 & if \quad x \geq t, \\ 0 & else \end{cases}$$

- Erosion: $t = sum(s)$
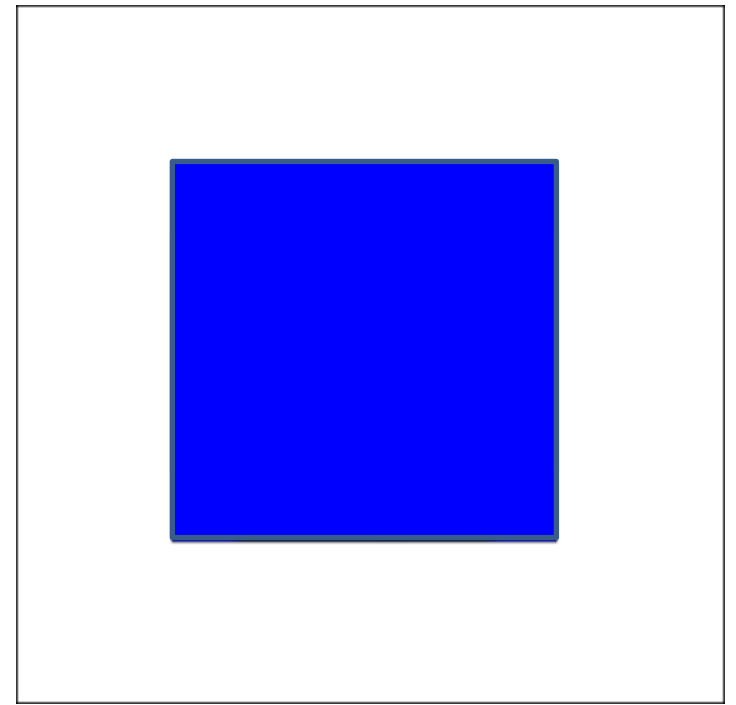 - Thicker lines will get skinny
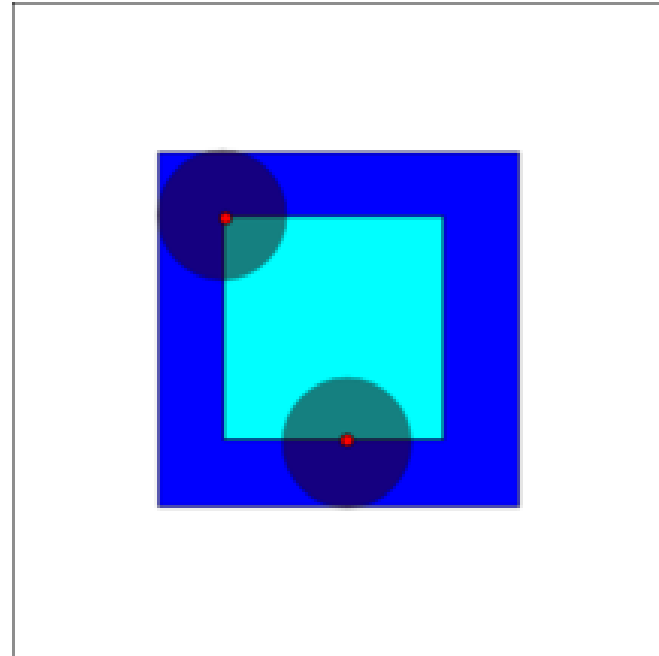
Opening

- Erosion followed by dilation.
  - The effect is of rounding off sharp edges.

Closing

- Dilation followed by erosion.
  - The effect is of closing of narrow gaps and holes.

contents

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- **Connected components**

Connected components

- Defined as regions of adjacent pixels that have the same value.
- Commonly used with binary images to find stand alone objects.
  - e.g.: letters in a document.