

Edge detection



References

- <http://szeliski.org/Book/>
- <http://www.cs.cornell.edu/courses/cs5670/2019sp/lectures/lectures.html>
- <http://www.cs.cmu.edu/~16385/>

Contents

- **Intro to edges**
- Basic edge image
- Edge thinning
 - LoG
 - NMS
- Edge mask
- Canny edge detector
- Other edge related topics
 - Frequency representation
 - Unsharp filter

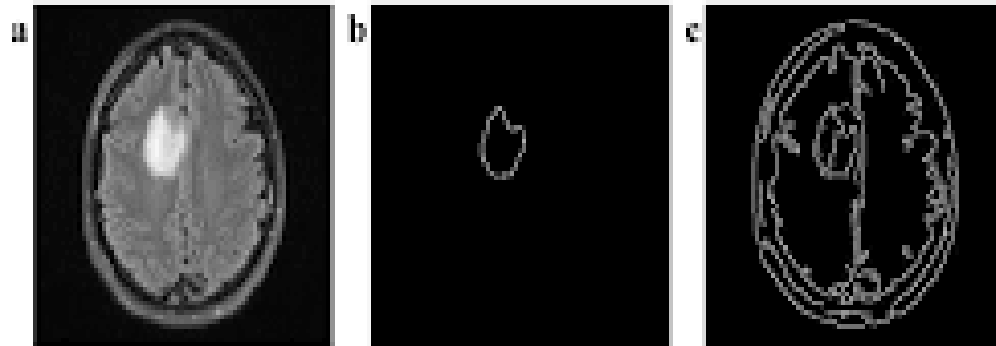
Some motivation



Art
(Instagram filters)



Robotics
(scene understanding)



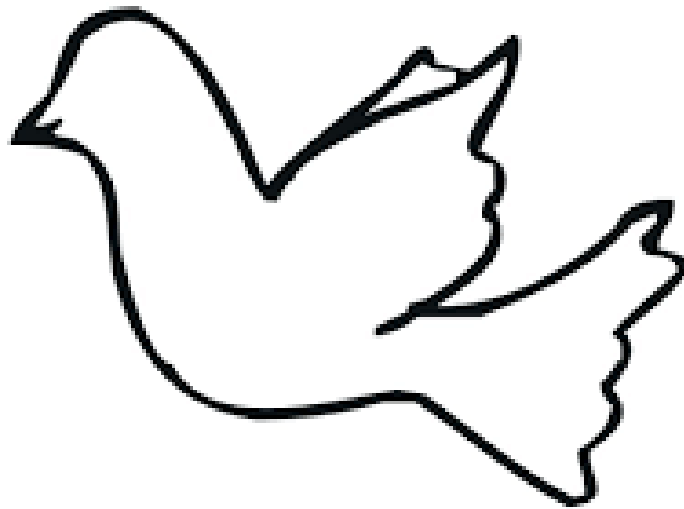
Medicine
(tumor detection)



Autonomous vehicles
(license plate detection)

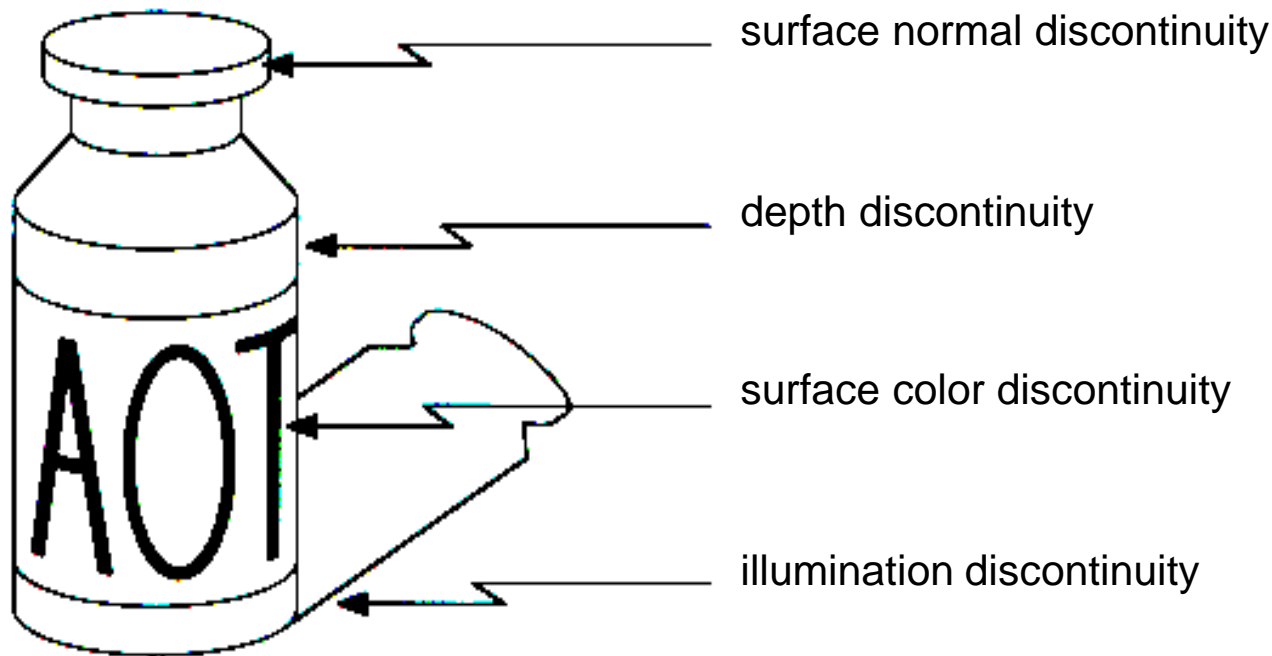
Why edges?

- Representation of objects can be done without full image representation- more compact.
- Edges are salient features (salient- “most noticeable or important”).



What are edges?

- “The outside limit of an object, area, or surface; a place or part farthest away from the center of something.”
- Edges can be caused from many reasons in images:



Representation in images

- Rapid changes in colors.
- Looks like steep edges if represented as a surface:

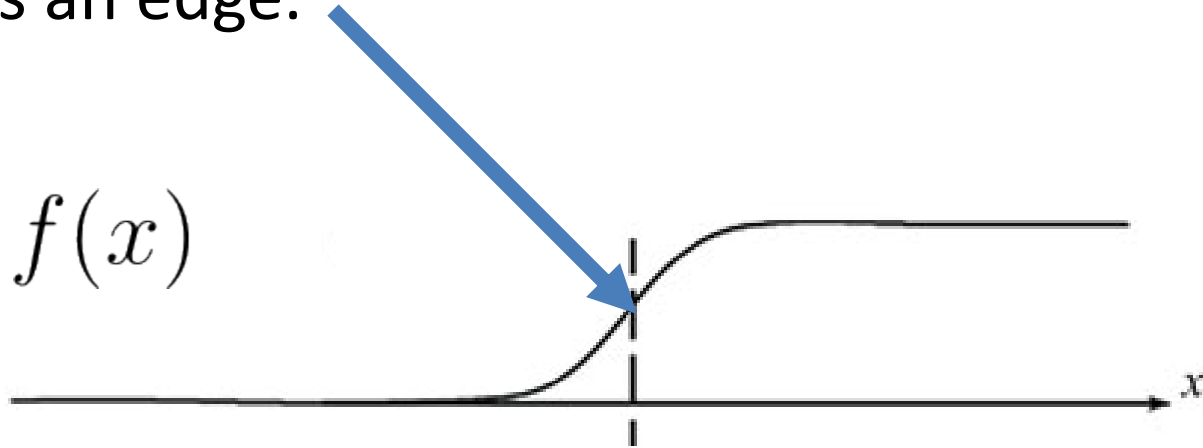


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How to find edge image?

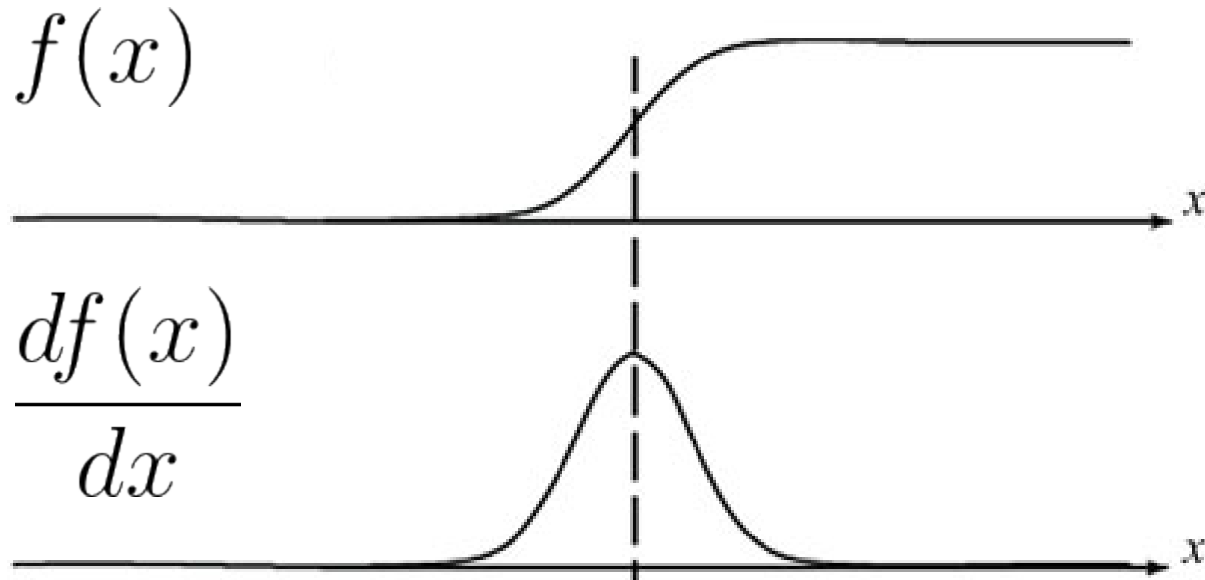
- We ultimately want image of a binary mask of where there is an edge.



- How to do so?

First order derivative

- Derivative of an edge:



- Finding maximum points in the derivative of an image is a possible way to find edges!**

Deriving the derivative

- Definition of derivative in continuous functions:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

- In discrete space we can set $h = 1$:

$$f'[x] = f[x + 1] - f[x]$$

- And in 2D space (derivative along x axis):

$$f'_x[x, y] = f[x + 1, y] - f[x, y]$$

Deriving the derivative

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1st derivative filter

$$f'_x[x, y] = f[x + 1, y] - f[x, y]$$

- We can mimic this derivative as a convolution operator:

$$f'_x = f * \begin{array}{|c|c|} \hline +1 & -1 \\ \hline \end{array}$$

- Note 1: when a kernel size is even in some dimension, the center of the kernel needs to be specified (above the center is -1).
- Note 2: remember that in the convolution operation **the kernel is flipped in both directions**.

Symmetric 1st derivative

- A more common approach is using the symmetric 1st derivative:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x-h)}{2h}$$

- Which translates to this kernel:

$$f'_x = f * \frac{1}{2} \begin{bmatrix} +1 & 0 & -1 \end{bmatrix}$$

- We'll use the kernel above without the $\frac{1}{2}$ constant, since we only care about the ratio between gradients.

Y direction

$$f'_y = f * \begin{array}{|c|} \hline +1 \\ \hline 0 \\ \hline -1 \\ \hline \end{array}$$

- The convolution kernel above is true for python/matlab/opencv image axis convention, where the positive y direction is down.

Image gradient

- The **gradient** of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$
- The gradient points in the direction of most rapid increase in intensity:

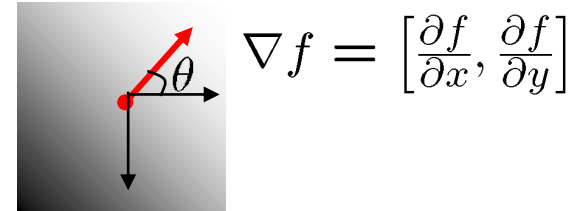
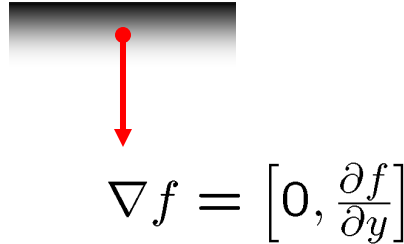
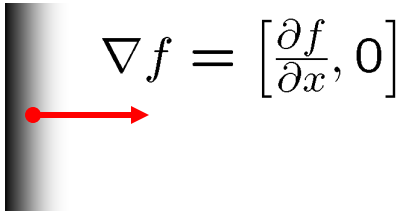
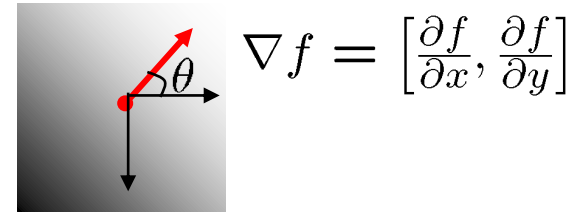
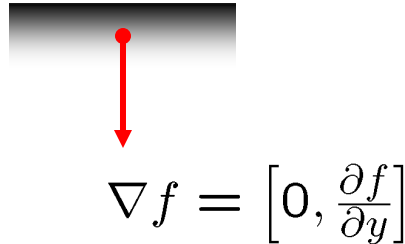
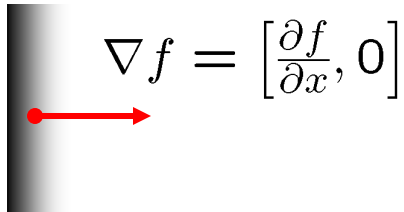


Image gradient

- The **gradient** of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$
- The gradient points in the direction of most rapid increase in intensity:



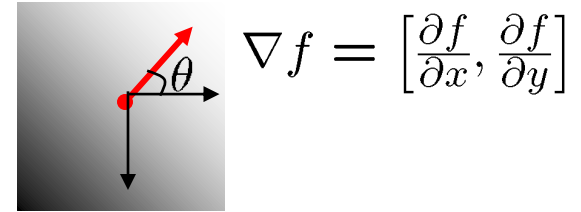
- The edge **strength** is given by the gradient magnitude:

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Gradient direction

- The gradient direction is given by:

$$\theta = \text{atan2}(-f'_y, f'_x)$$



- $\theta \in (-\pi, \pi]$ is determined by the right-hand rule from $+x$ axis.

$$\text{atan2}(y, x) = \begin{cases} \arctan(\frac{y}{x}) & \text{if } x > 0, \\ \arctan(\frac{y}{x}) + \pi & \text{if } x < 0 \text{ and } y \geq 0, \\ \arctan(\frac{y}{x}) - \pi & \text{if } x < 0 \text{ and } y < 0, \\ +\frac{\pi}{2} & \text{if } x = 0 \text{ and } y > 0, \\ -\frac{\pi}{2} & \text{if } x = 0 \text{ and } y < 0, \\ \text{undefined} & \text{if } x = 0 \text{ and } y = 0. \end{cases}$$

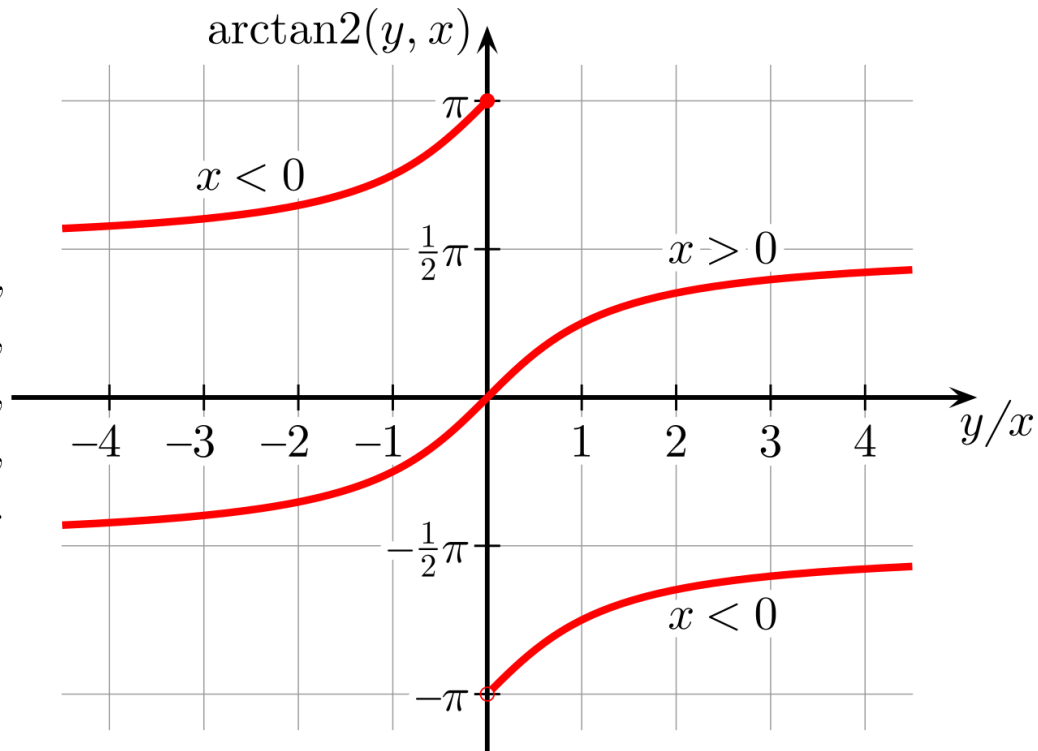
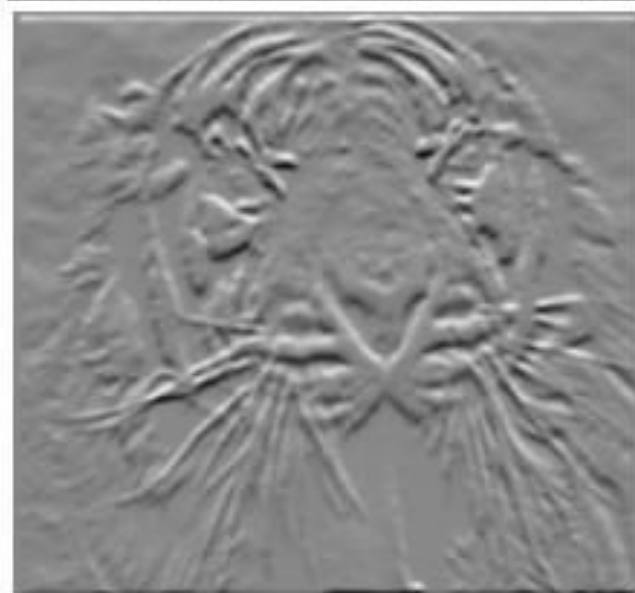
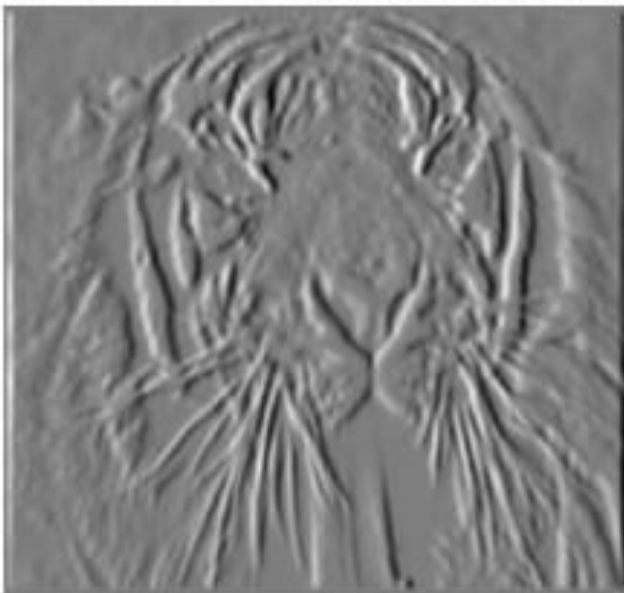


Image gradient example



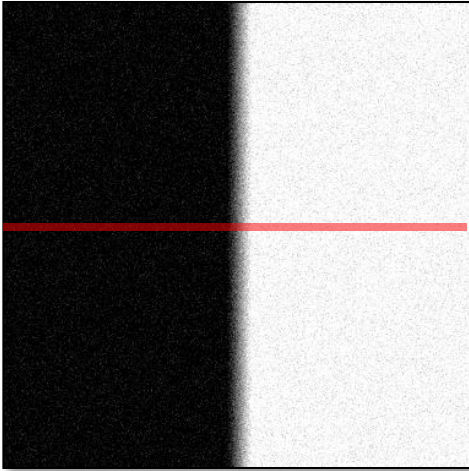
Prewitt filter

$$f'_x = f * \begin{array}{|c|c|c|} \hline +1 & 0 & -1 \\ \hline +1 & 0 & -1 \\ \hline +1 & 0 & -1 \\ \hline \end{array}$$

- The same as before but more robust to noise since it uses the diagonal neighbors as well.

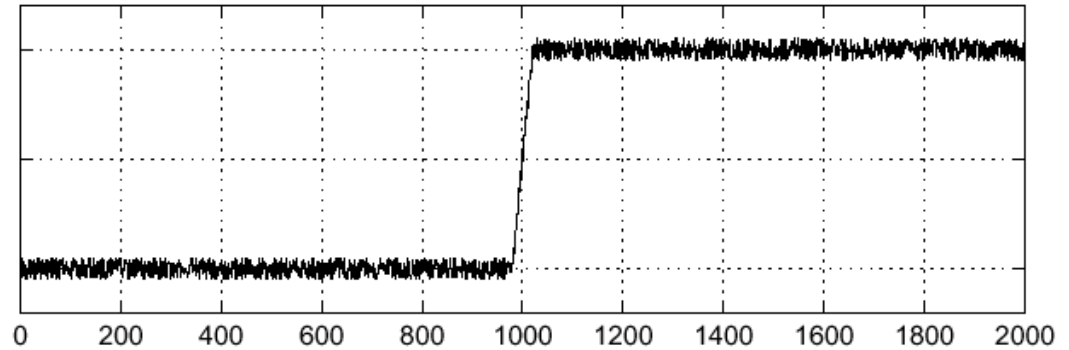
Noise effects

- How to find maximum of derivative in noisy environment?

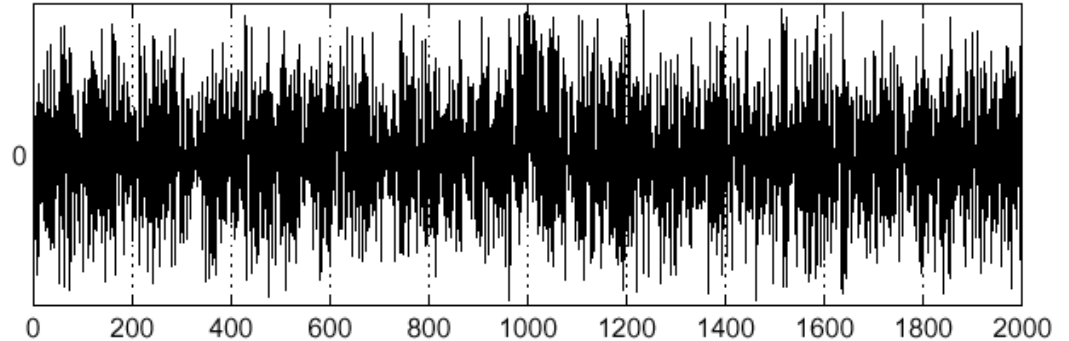


Noisy input image

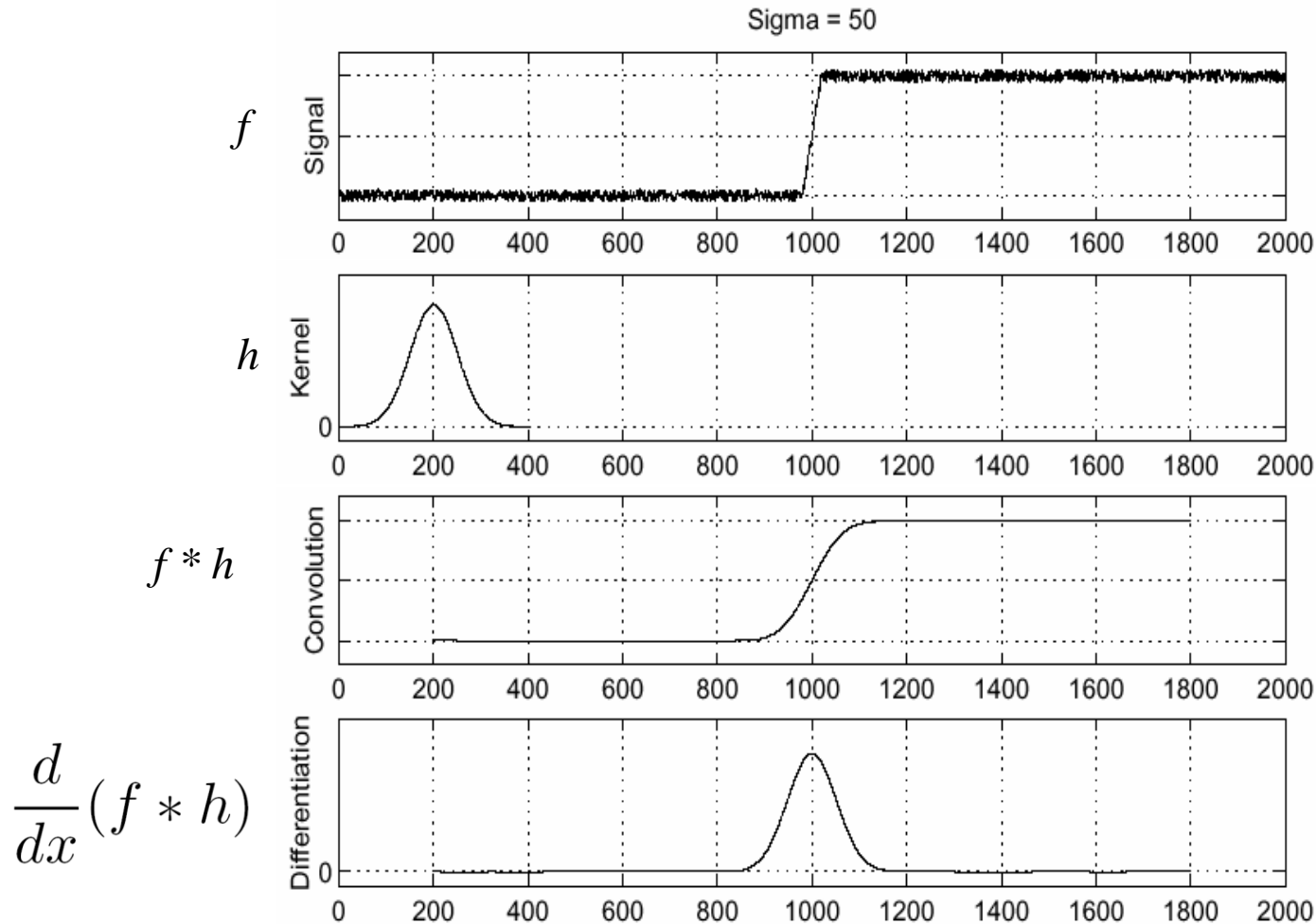
$$f(x)$$



$$\frac{d}{dx}f(x)$$



Solution: smoothing the noise



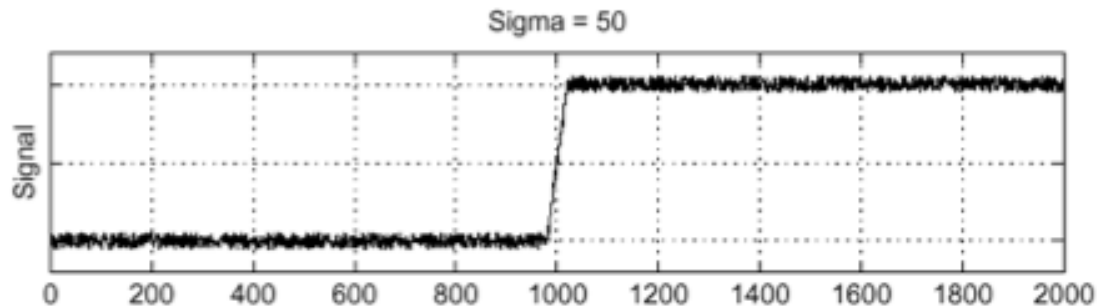
- Search for the maximum in the smooth image!

Gaussian derivative kernel

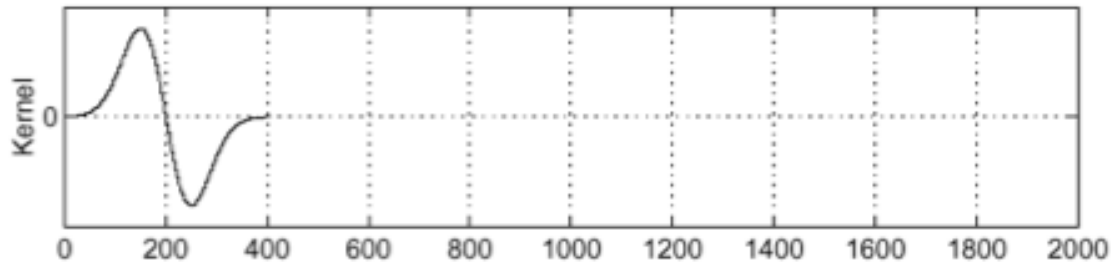
- Using this convolution trick:

$$\frac{d}{dx}(h * f) = \left(\frac{d}{dx}h\right) * f$$

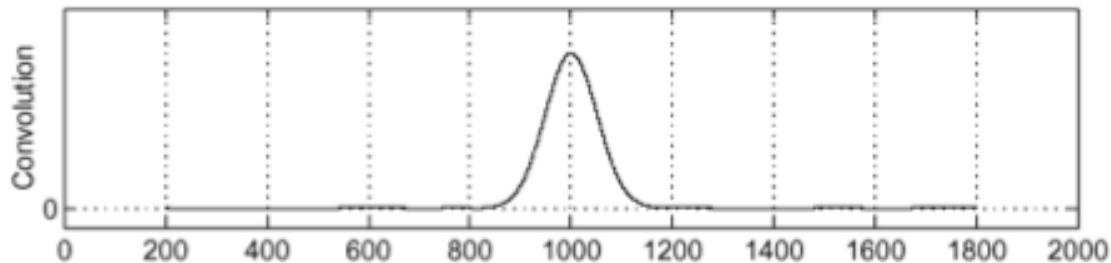
input



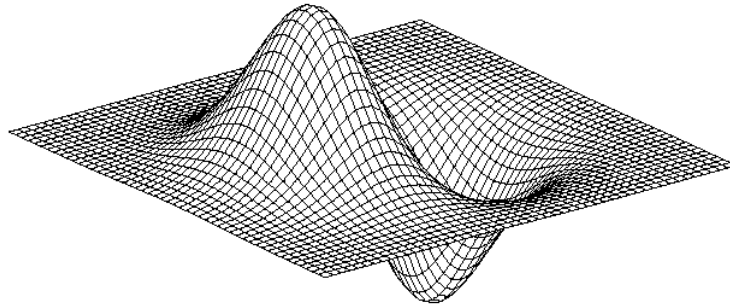
derivative of
Gaussian



output (same
as before)

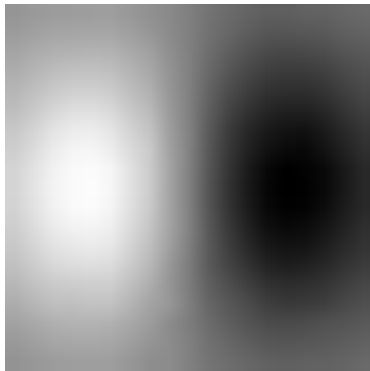


Gaussian derivative kernel 2D

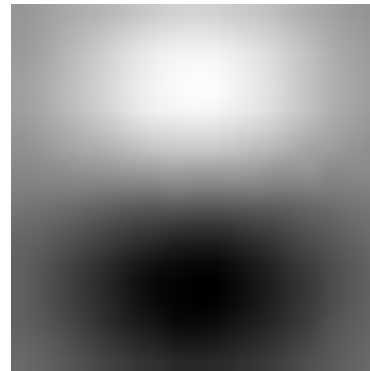


Derivative of Gaussian

x-direction

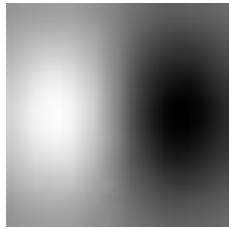


y-direction



Sobel filter

- Common approximation of derivative of Gaussian

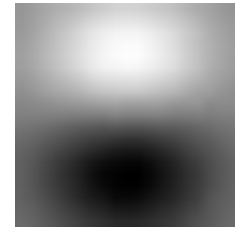


| | | |
|---|---|----|
| 1 | 0 | -1 |
| 2 | 0 | -2 |
| 1 | 0 | -1 |

s_x

| | | |
|----|----|----|
| 1 | 2 | 1 |
| 0 | 0 | 0 |
| -1 | -2 | -1 |

s_y



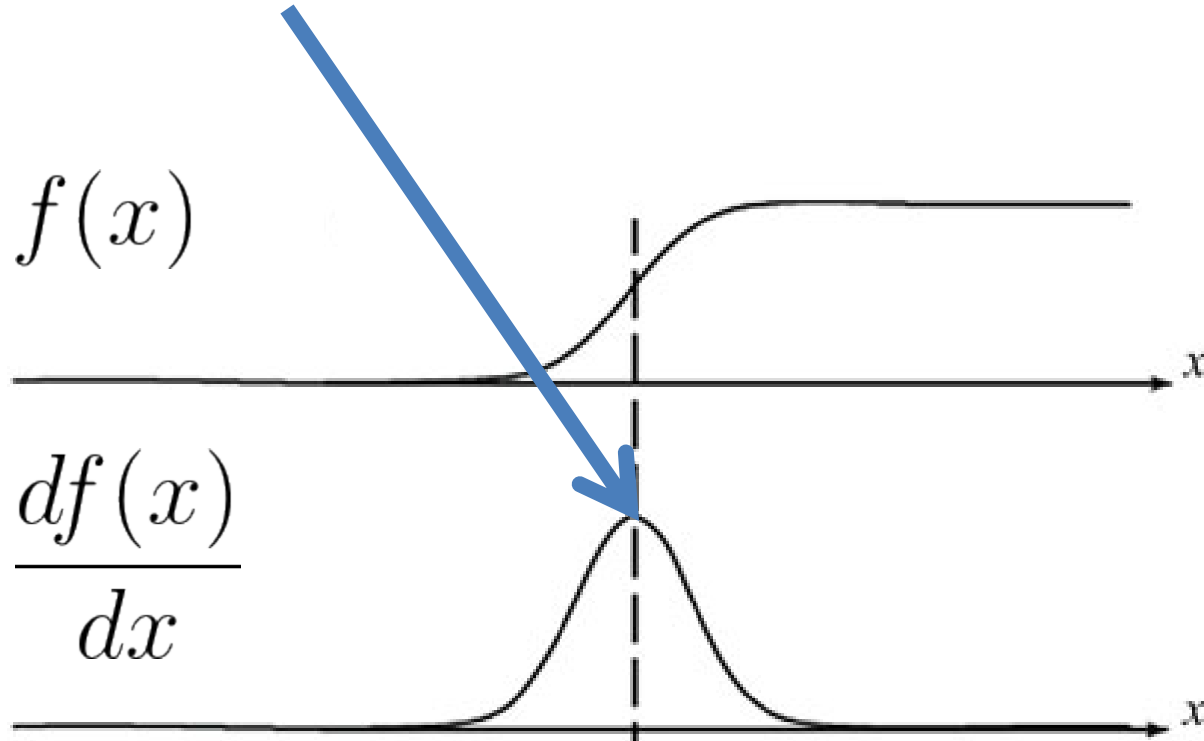
- Can also be thought of as Prewitt with higher weighting for closer neighbors.
- In theory-** should give better performance relative to Prewitt since it includes smoothing effect.
- In practice-** non definitive superiority to Prewitt (3X3 kernel is a rough approximation...).

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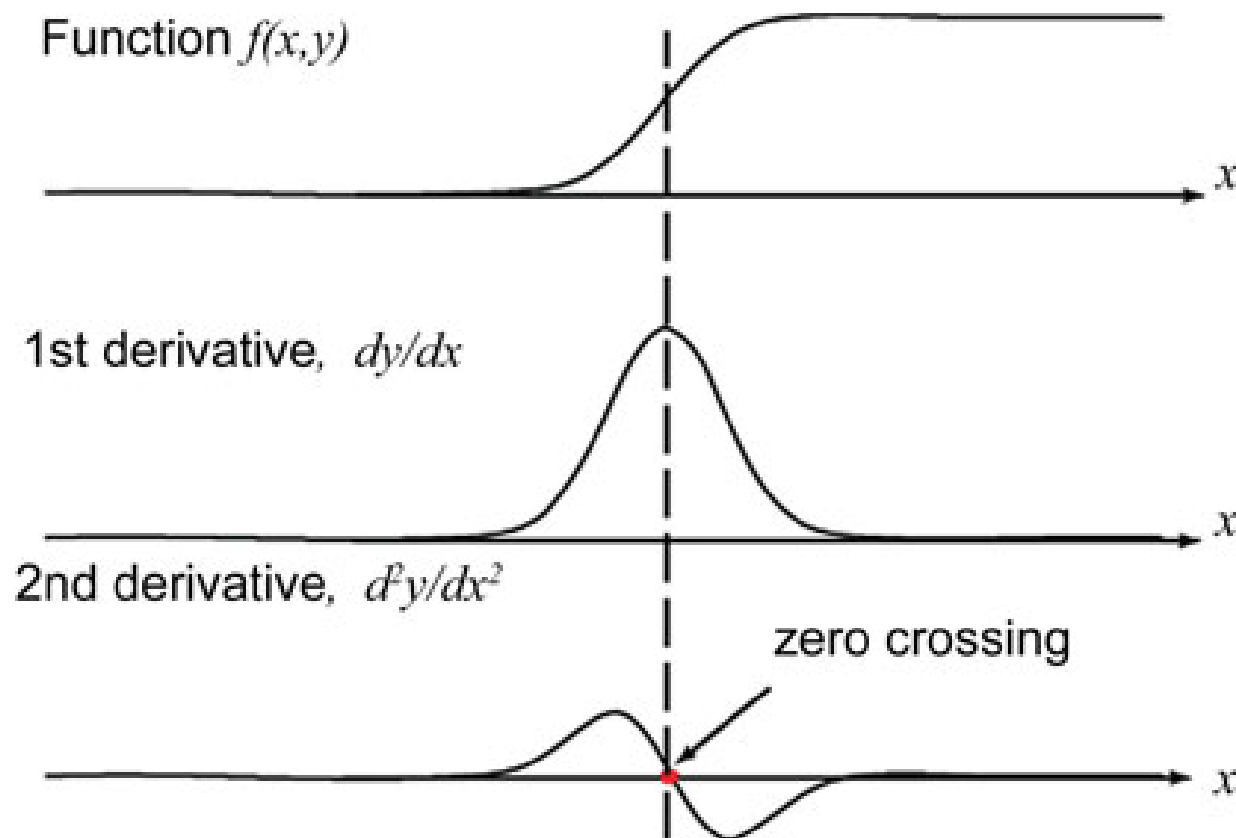
Edge thinning

- I have the edge filter result, but I want only one pixel to represent the edge in a binary mask.
- How do I find this?



Naïve approach: 2nd derivative

- Let's try to find the zero crossing of the 2nd derivative.
- Only single zero crossing- should produce thinner edge



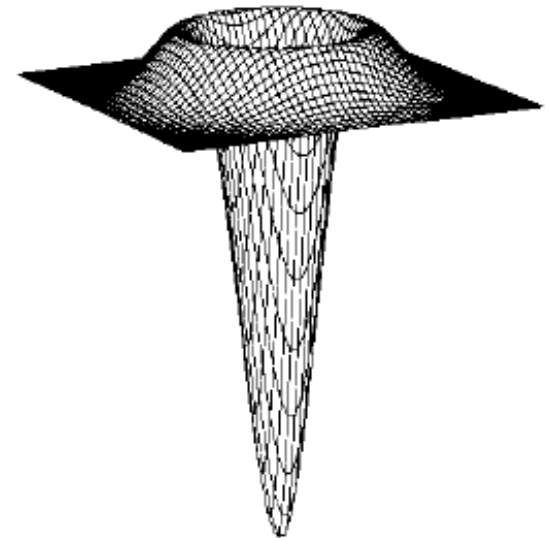
LoG

- Let's take the 2nd derivative of the Gaussian (Laplacian of Gaussian: LoG) kernel so smoothing will help with noise reduction:

$$\Delta f = \nabla^2 f = \nabla \cdot \nabla f = \nabla \cdot \left[\frac{df}{dx}, \frac{df}{dy} \right] = \frac{d^2 f}{dx^2} + \frac{d^2 f}{dy^2}$$

$$\nabla^2 h_\sigma(u, v)$$

$$\text{LoG}(x, y) = -\frac{1}{\pi\sigma^4} \left[1 - \frac{x^2 + y^2}{2\sigma^2} \right] e^{-\frac{x^2 + y^2}{2\sigma^2}}$$

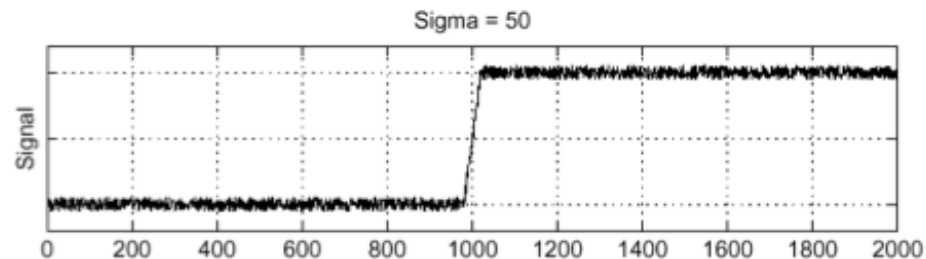


Laplacian of Gaussian

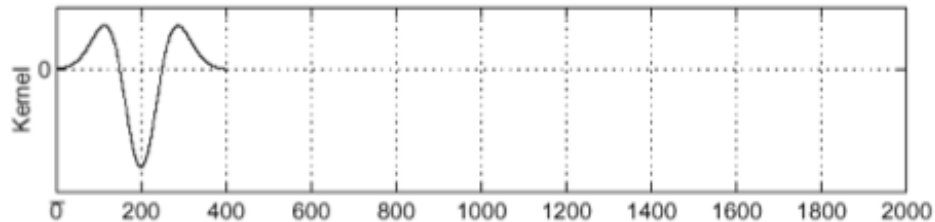
Find edge in noise signal: LoG

- Input is noisy step signal, output is zero crossing at the step.

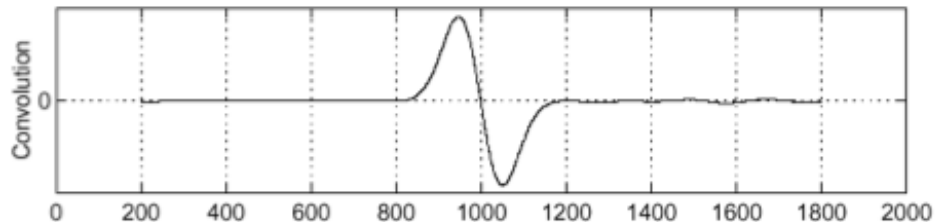
input



Laplacian of
Gaussian



output



LoG quantization

- Can be filter of different sizes:

– 3X3:

| | | |
|---|----|---|
| 0 | 1 | 0 |
| 1 | -4 | 1 |
| 0 | 1 | 0 |

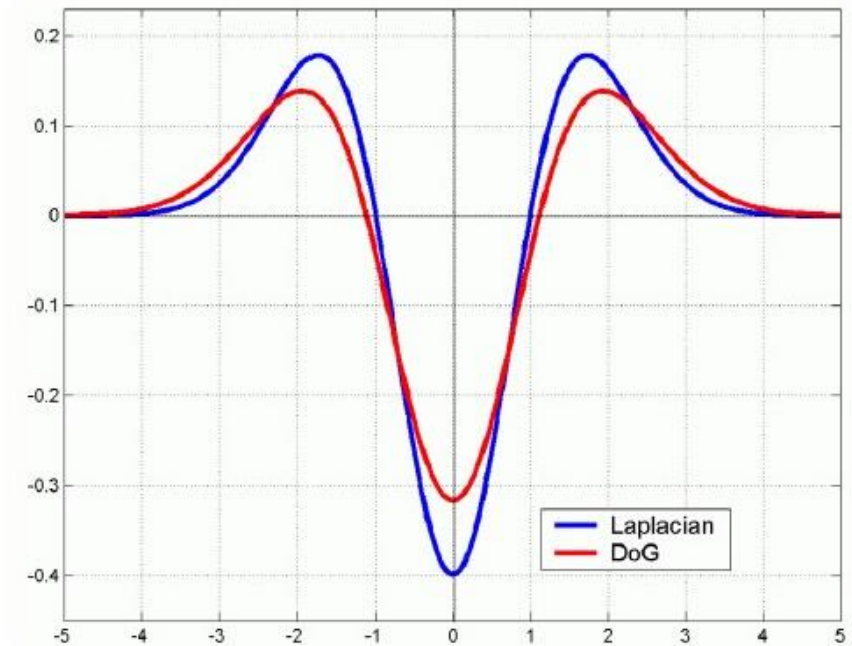
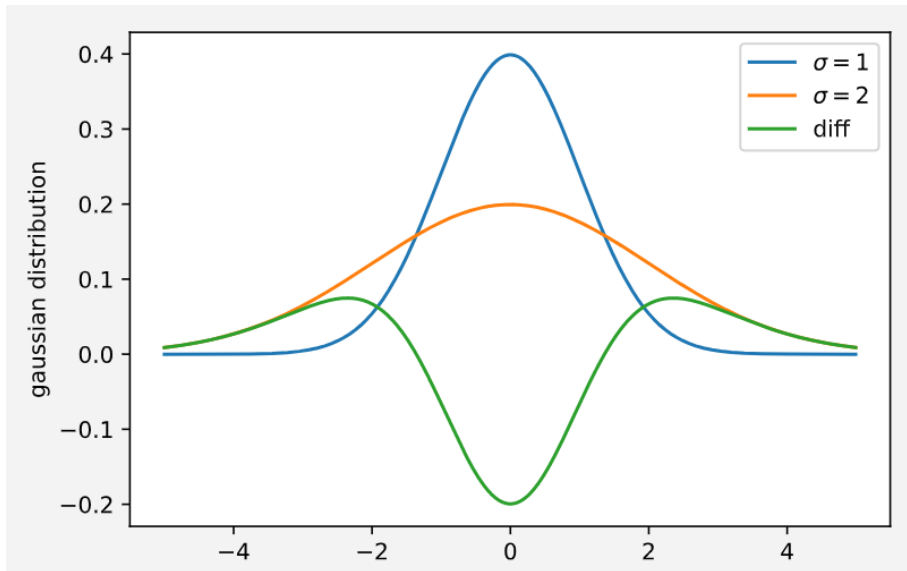
| | | |
|---|----|---|
| 1 | 1 | 1 |
| 1 | -8 | 1 |
| 1 | 1 | 1 |

– 9X9:

| | | | | | | | | |
|---|---|---|-----|-----|-----|---|---|---|
| 0 | 0 | 3 | 2 | 2 | 2 | 3 | 0 | 0 |
| 0 | 2 | 3 | 5 | 5 | 5 | 3 | 2 | 0 |
| 3 | 3 | 5 | 3 | 0 | 3 | 5 | 3 | 3 |
| 2 | 5 | 3 | -12 | -23 | -12 | 3 | 5 | 2 |
| 2 | 5 | 0 | -23 | -40 | -23 | 0 | 5 | 2 |
| 2 | 5 | 3 | -12 | -23 | -12 | 3 | 5 | 2 |
| 3 | 3 | 5 | 3 | 0 | 3 | 5 | 3 | 3 |
| 0 | 2 | 3 | 5 | 5 | 5 | 3 | 2 | 0 |
| 0 | 0 | 3 | 2 | 2 | 2 | 3 | 0 | 0 |

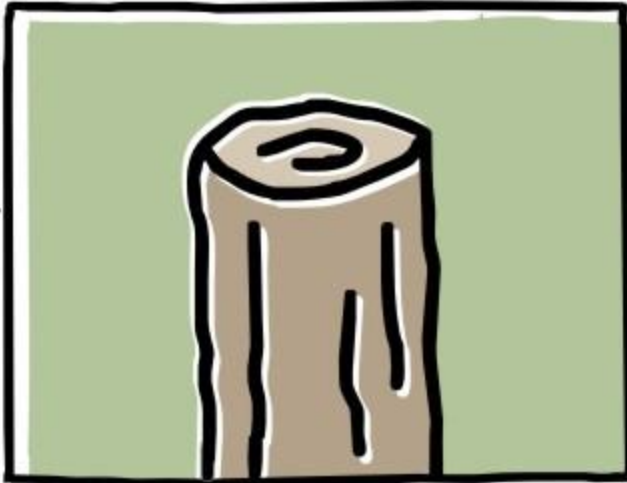
DoG

- Can also use difference of Gaussians (DoG) to mimic LoG.
- Why do we want to do this? Because we can...



log vs. dog

Dist. by Universal Uclick



likes to be outside
found near the fireplace
gives warmth and comfort
plays dead
bark
doesn't have a tail

© John Atkinson, Wrong Hands



likes to be outside
found near the fireplace
gives warmth and comfort
plays dead
bark
can't be made into shelves

Example: LoG



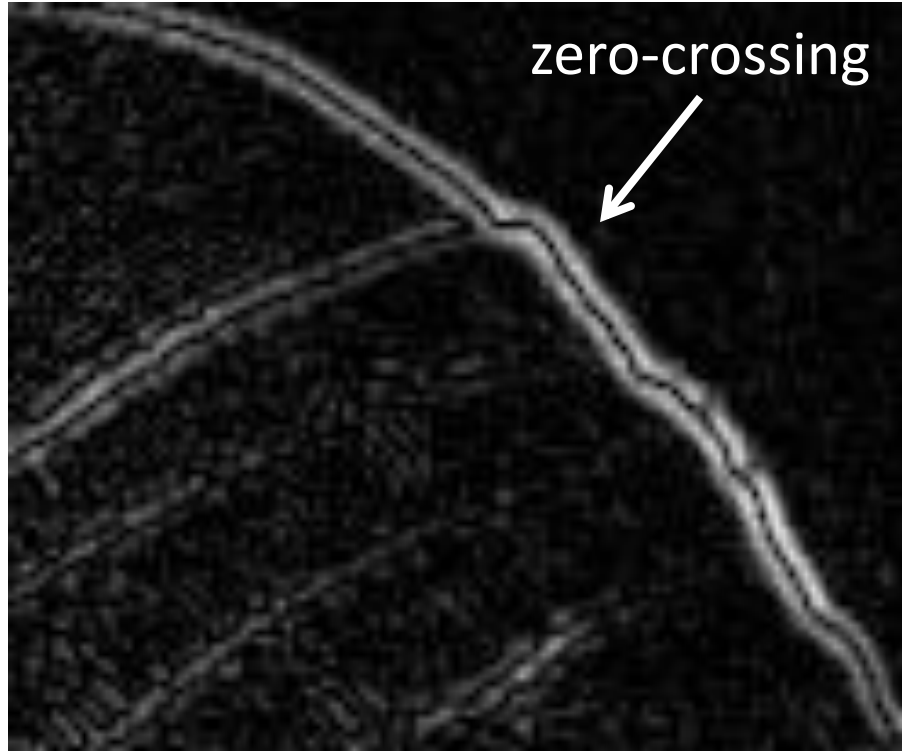
Laplacian of Gaussian filtering



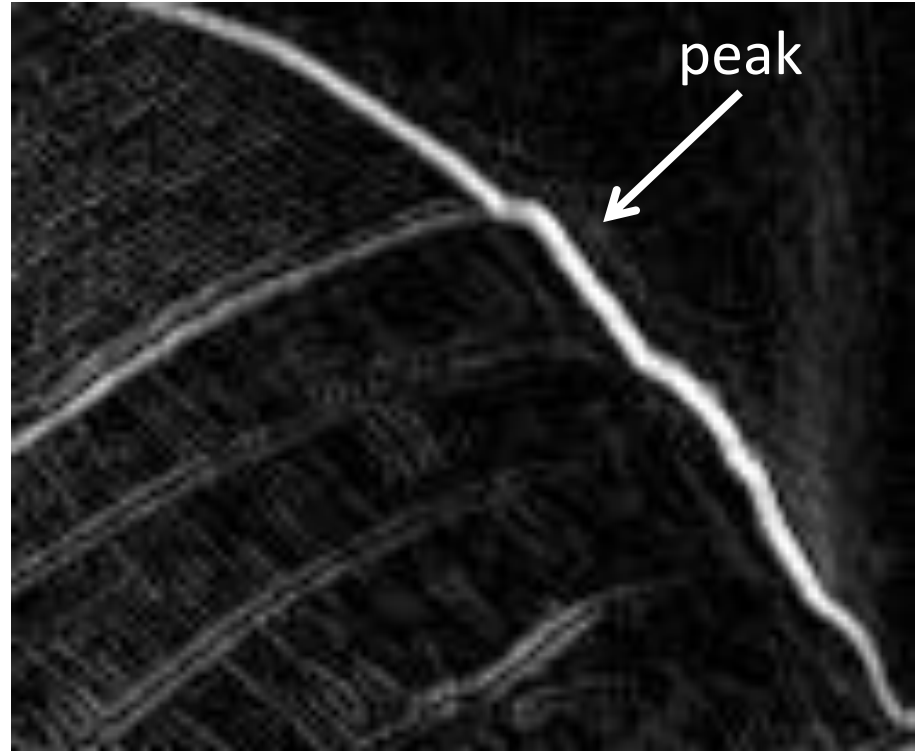
Derivative of Gaussian filtering

Example: LoG

- Note: both images are after absolute value.



Laplacian of Gaussian filtering



Derivative of Gaussian filtering

Zero crossing

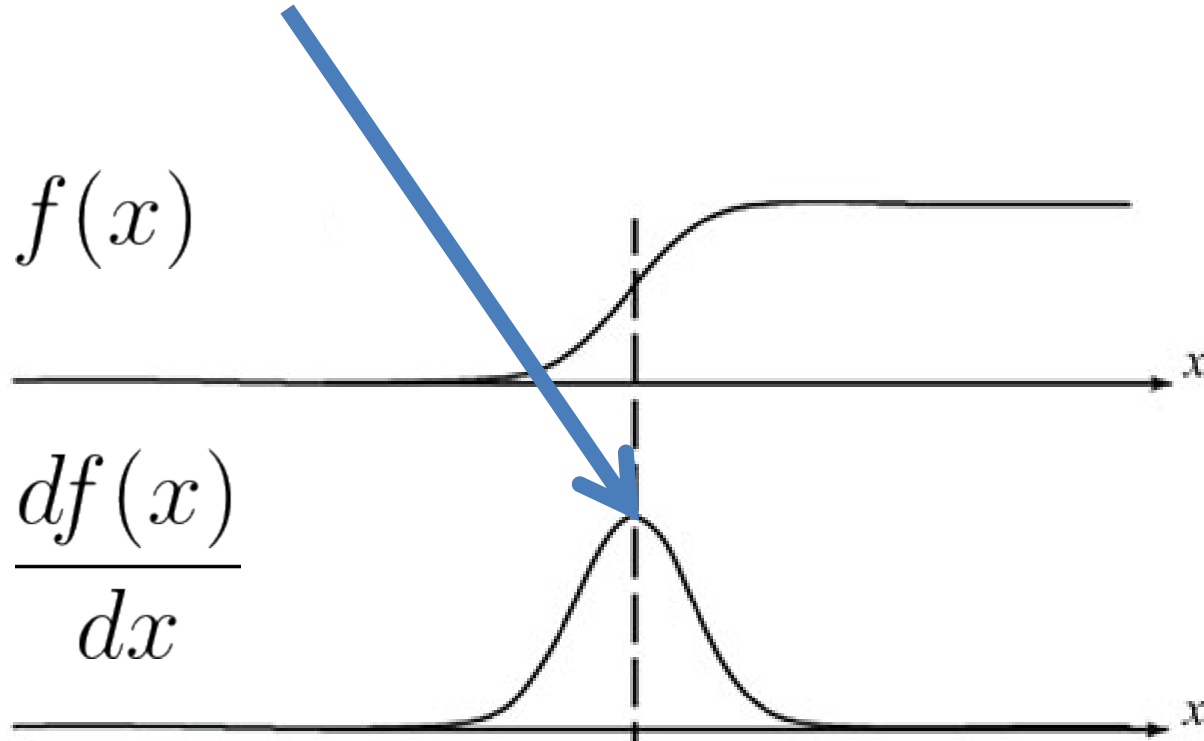
- The new problem arising from the LoG filter is: how to mark the zero crossings?
- Answer: no easy algorithm to detect zero crossings.
 - E.g.: planes with minor noise will also produce zero crossing artifacts.

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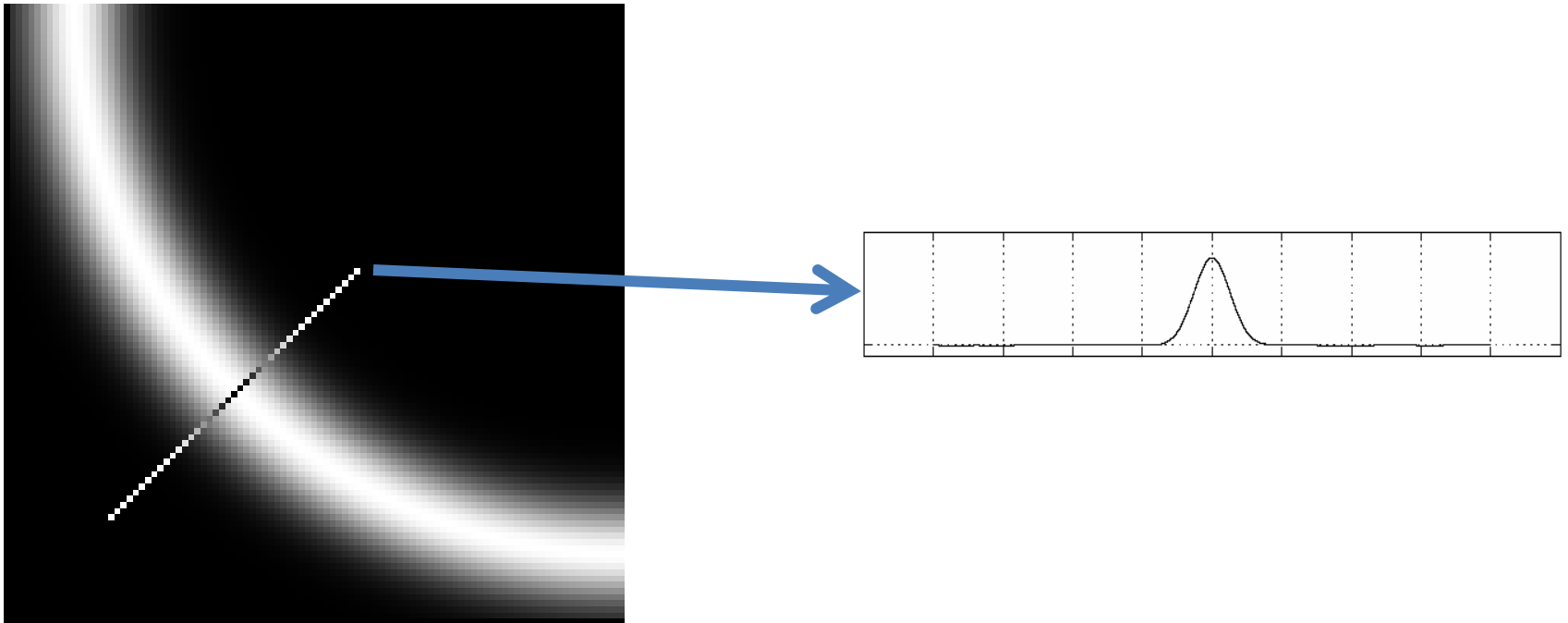
Edge thinning

- I have the edge filter result, but I want only one pixel to represent the edge in a binary mask.
- How do I find this?



Non maximum suppression

- NMS
- Find the gradient magnitude + orientation of each pixel and search on this 1D line for maximum point.



NMS algorithm

get image gradient magnitude + orientation using 1D 3X3 gradient filter (e.g.: Sobel).

for each pixel p_0 :

Quantize $\angle p_0$ to one of four possibilities:
 $[0^\circ, 45^\circ, 90^\circ, 135^\circ]$.

In 3X3 neighborhood of p_0 , find two neighbors in quantized gradient orientation $\{p_1, p_2\}$.

If $||p_0|| < ||p_1||$ or $||p_0|| < ||p_2||$:
 $||p_0|| \leftarrow 0$

NMS results

Before NMS



After NMS



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Edge mask

- How do we transform this integer image to a binary mask of where there is/ isn't an edge?

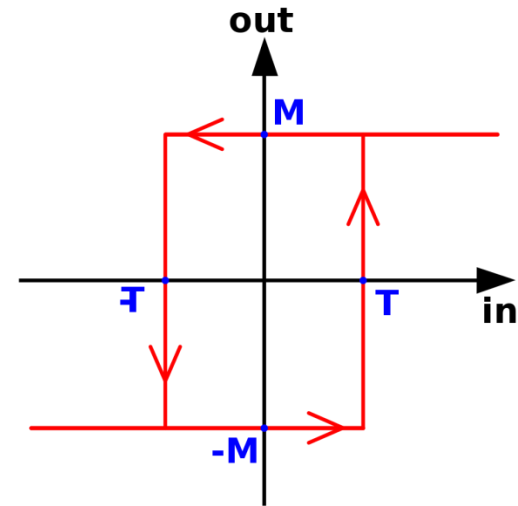


First try: single threshold edge mask

- Mask == binary image.
- Possible 1st solution- thresholding:
 - Choose an TH edge value, above which the pixel mask is 1, 0 otherwise.
 - The value can be a constant or percentile of the maximum edge value exists in the image.
 - Low TH: will get extra edges, but also input noise.
 - High TH: can miss lower valued edge pixels, less noise.
- How can we difference between low value edge pixels and noise?

Hysteresis motivation

- Weak edges are usually neighbors of strong edges, while noise can be at any pixel.
 - Usually “neighbors” means 3X3 square of adjacent pixels.
- If we know that a neighbor of a weak edge is a strong edge, then **the weak edge is a strong edge!**



Schmitt trigger
hysteresis example plot

hysteresis

Choose two thresholds: $\{TH_h, TH_l | TH_h > TH_l\}$

For each pixel p_i :

If $p_i \geq TH_h$:

$p_i \leftarrow 1$

elif $TH_l \leq p_i < TH_h$:

$p_i \leftarrow \text{weak_edge_pixel}$

Else: $// p_i < TH_l$

$p_i \leftarrow 0$

While *weak_edge_pixels* that are neighbors of 1 exists:

for each *weak_edge_pixel* p_i :

If *weak_edge_pixel* p_i neighbor of 1:

$\text{weak_edge_pixel } p_i \leftarrow 1$

All remaining *weak_edge_pixels* $\leftarrow 0$

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Canny edge detector

- Canny edge detector is one of the most known and used CV algorithms, still highly used even today (developed in 1986):

Gaussian filter

Find image gradient magnitude and orientation

NMS

Hysteresis

Example output



Different Gaussians



original



Canny with $\sigma = 1$



Canny with $\sigma = 2$

- Stronger smoothing will cause only “large-scale” gradients to remain (more on scaling- later in class).

Important note: tradeoffs

- It's a common **misconception** in CV to think that one algorithm is **always** better than another.
- In CV, algorithms are highly dependent in the given environment in which they are executed. Each environment can vary in:
 - Noise.
 - Needed computation efficiency.
 - Overall problem variance.
 - Etc...



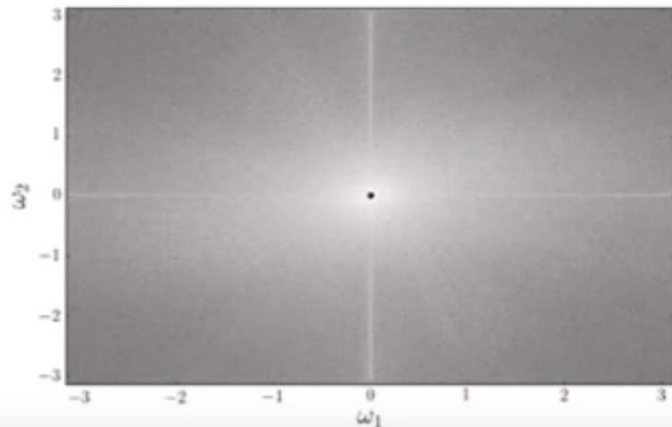
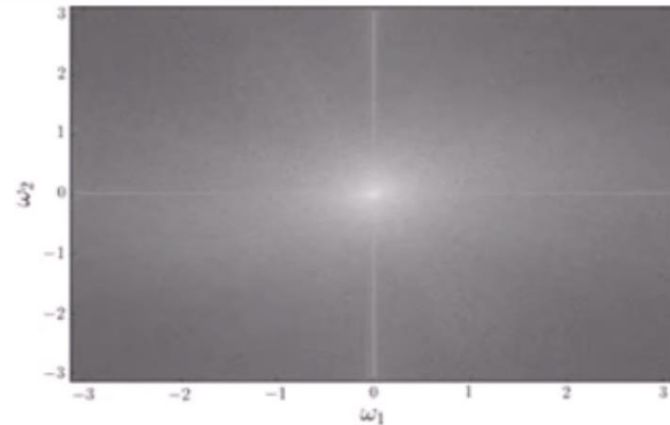
CV is the
land Of
tradeoffs

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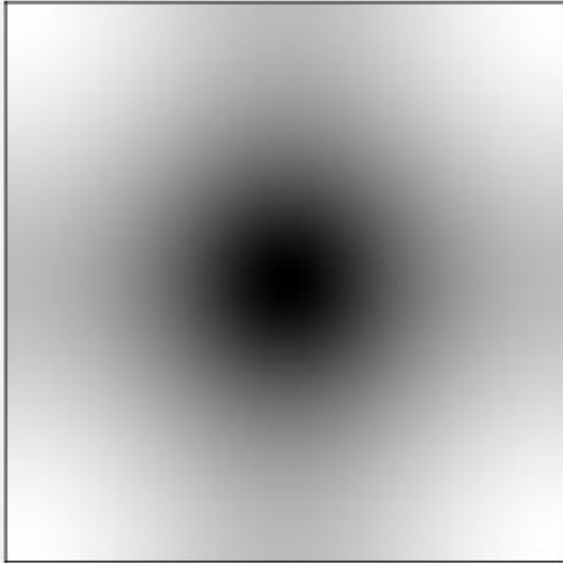
HP filter

- Higher frequencies represents the edges of images.
- Removing the lower frequencies of an image will result in edge image!

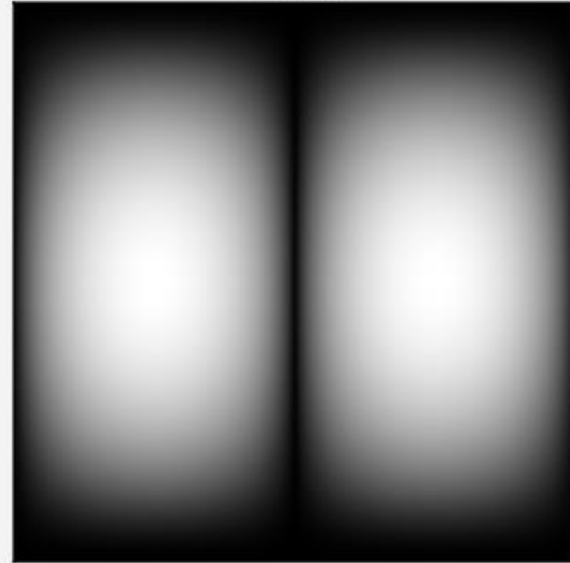


Different edge filters

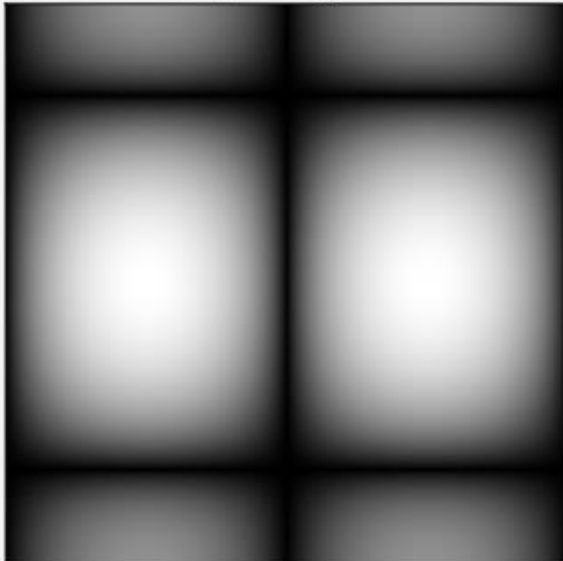
LoG



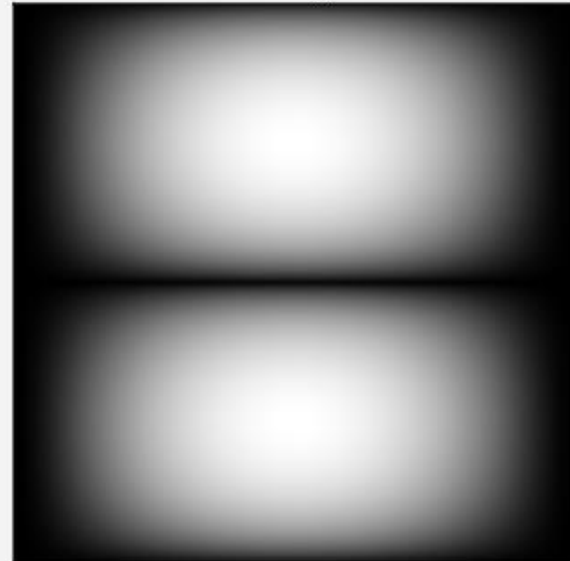
sobel_x



prewitt_x

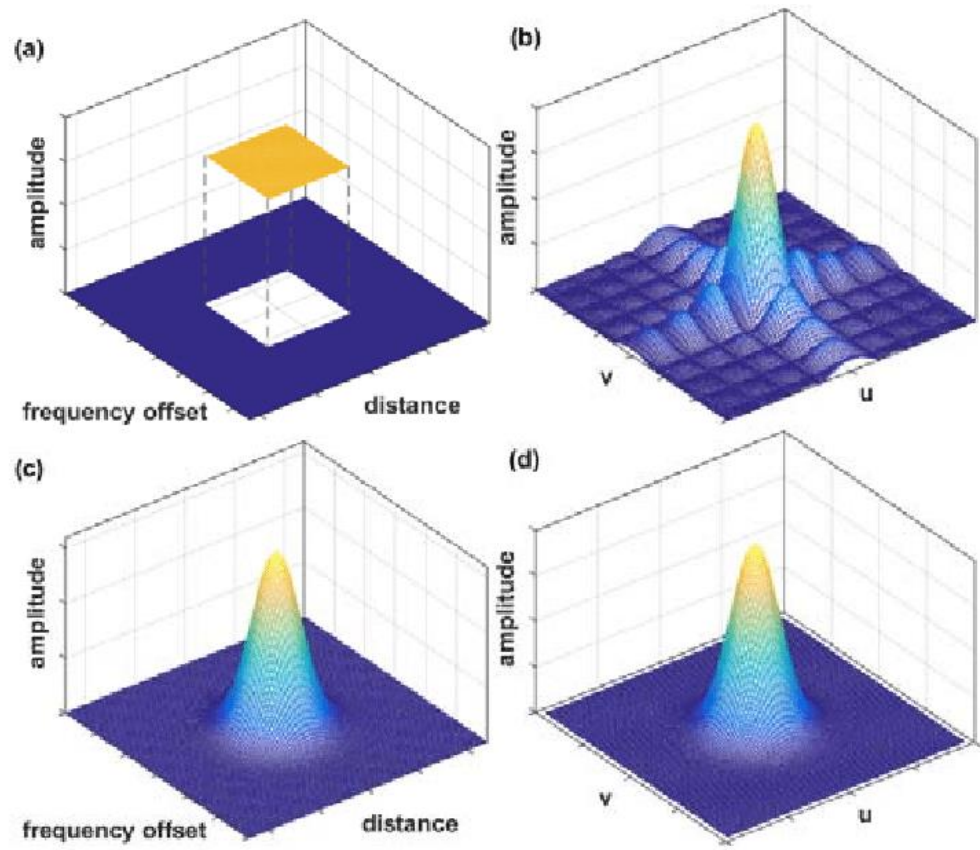


sobel_y



Why prewitt has waves?

- Recalling the mean filter – we can say that prewitt is like two side by side rectangles.
- Sobel is like two gaussians side by side!



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Image sharpening

Obtain the high frequencies magnitude image.

Enhance the edges (e.g. by multiplying with a constant >1).

Add the enhanced edges back to the original image.

- Or- one liner:

$$f_{sharpen} = f + \gamma \cdot \|\nabla f\|$$



Unsharp filter

- The former can also be done with only low pass filtering!

$$f_{unsharp} = f + \gamma(f - h_{blur} * f)$$

- This was also the way that photographers enhanced edges before CV (dates to the 1930s). More on this topic here:

[https://en.wikipedia.org/wiki/Unsharp_masking#Photographic darkroom unsharp masking](https://en.wikipedia.org/wiki/Unsharp_masking#Photographic_darkroom_unsharp_masking)

