# Basic image processing

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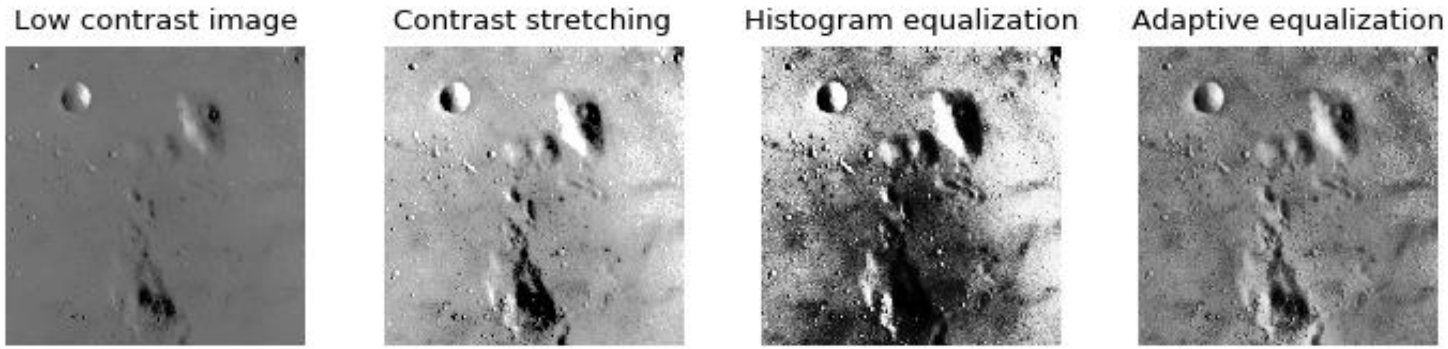
# References

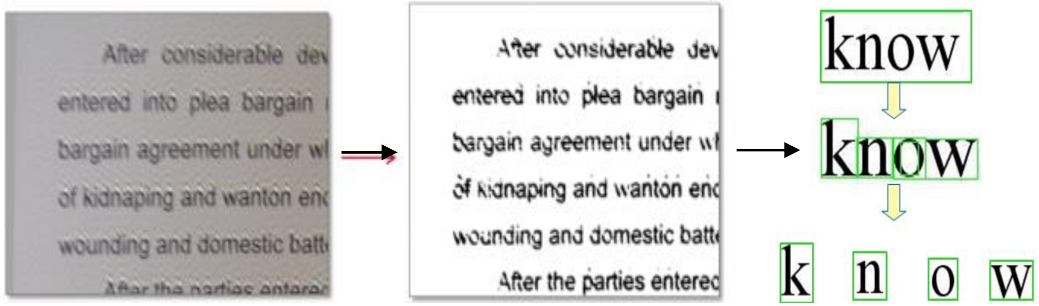

- http://szeliski.org/Book/
- http://www.cs.cornell.edu/courses/cs5670/2019sp/lectures/lectures.html
- http://www.cs.cmu.edu/~16385/

# Some motivation


Art
(Photoshop color grading)




Science and space
(image enhancement)




Robotics
(OCR – optical character recognition)

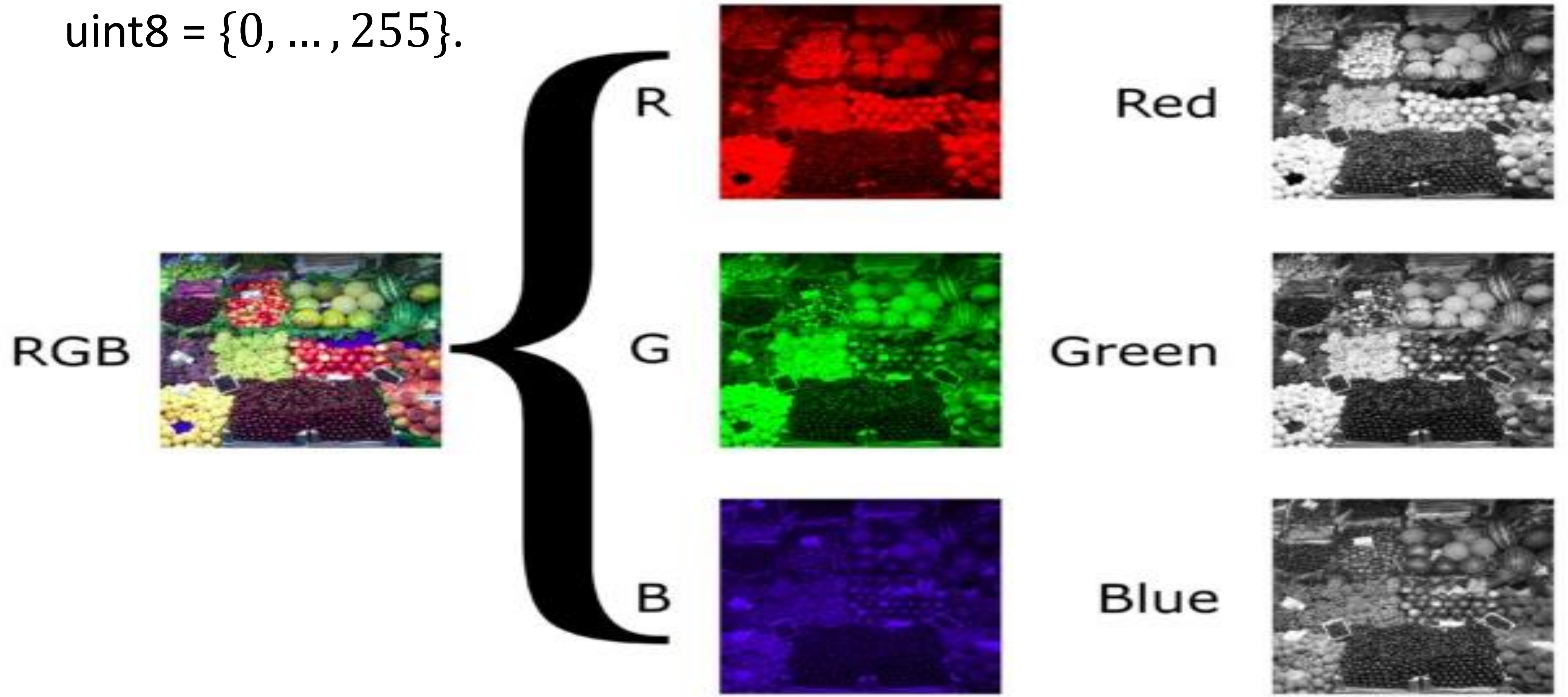
Agriculture
(color ripeness detection)

# contents

- **Image representation**
- Pixel-wise operations
- Histogram equalization
- Template matching
- Morphology operators
- Connected components
- Color space

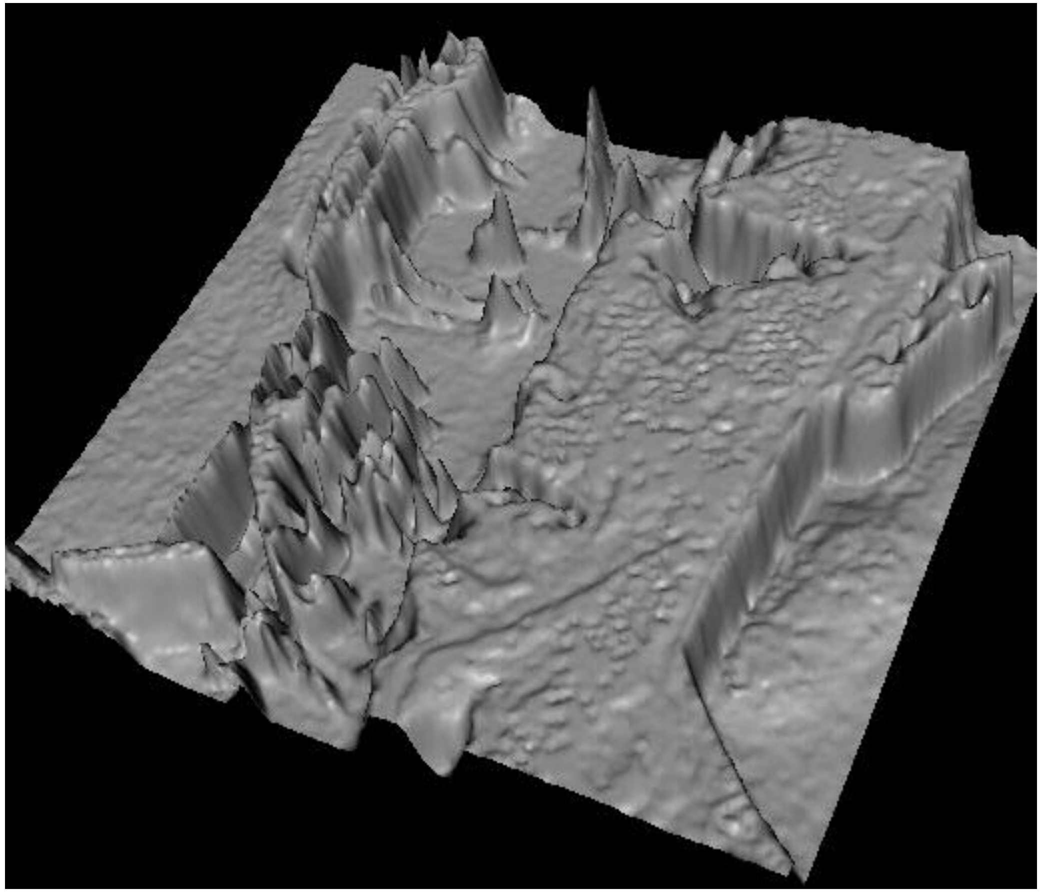
# Image representation

- We can think of an image as a 3d matrix of discrete RGB values.
- The values mark the intensity of each color channel and are usually of type uint8 = $\{0, \ldots, 255\}$.



# Image representation

- We can also think of an image as a function $f(x, y)$ .



# contents

- Image representation
- **Pixel-wise operations**
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# Pixel-wise operators

- Pixel-wise operators, or point operators, are defined as such that each output pixel’s value depends on only the corresponding input pixel value.

# Pixel-wise operators

original

$x$

darken

lower contrast

Gamma compression

invert

lighten

raise contrast

Gamma expansion

# Pixel-wise operators

original

$x$

darken

lower contrast

Gamma compression

invert

$255 - x$

lighten

raise contrast

Gamma expansion

# Pixel-wise operators

original

$x$

darken

$x - 128$

lower contrast

Gamma compression

invert

$255 - x$

lighten

raise contrast

Gamma expansion

# Pixel-wise operators

original

$x$

darken

$x - 128$

lower contrast

Gamma compression

invert

$255 - x$

lighten

$x + 128$

raise contrast

Gamma expansion

# Pixel-wise operators

original

$$x$$

darken

$$x - 128$$

lower contrast

$$\frac{x}{2}$$

Gamma compression

invert

$$255 - x$$

lighten

$$x + 128$$

raise contrast

Gamma expansion

# Pixel-wise operators

original

$x$

darken

$x - 128$

lower contrast

$\frac{x}{2}$

Gamma compression

invert

$255 - x$

lighten

$x + 128$

raise contrast

$x \times 2$

Gamma expansion

# Contrast

- **Contrast** in visual perception is the difference in appearance of two or more parts of a seen field.
- The human visual system is more sensitive to contrast than absolute luminance;
- **Contrast ratio**, or **dynamic range**, is the ratio between the largest and smallest values of the image or:

$$CR = \frac{V_{max}}{V_{min}}$$

# Pixel-wise operators

original

$x$

darken

$x - 128$

lower contrast

$\frac{x}{2}$

Gamma compression

invert

$255 - x$

lighten

$x + 128$

raise contrast

$x \times 2$

Gamma expansion

# Pixel-wise operators

original

$$x$$

darken

$$x - 128$$

lower contrast

$$\frac{x}{2}$$

Gamma compression

$$\left(\frac{x}{255}\right)^{1/3} \times 255$$

invert

$$255 - x$$

lighten

$$x + 128$$

raise contrast

$$x \times 2$$

Gamma expansion

# Pixel-wise operators

original

$$x$$

darken

$$x - 128$$

lower contrast

$$\frac{x}{2}$$

Gamma compression

$$\left(\frac{x}{255}\right)^{1/3} \times 255$$

invert

$$255 - x$$

lighten

$$x + 128$$

raise contrast

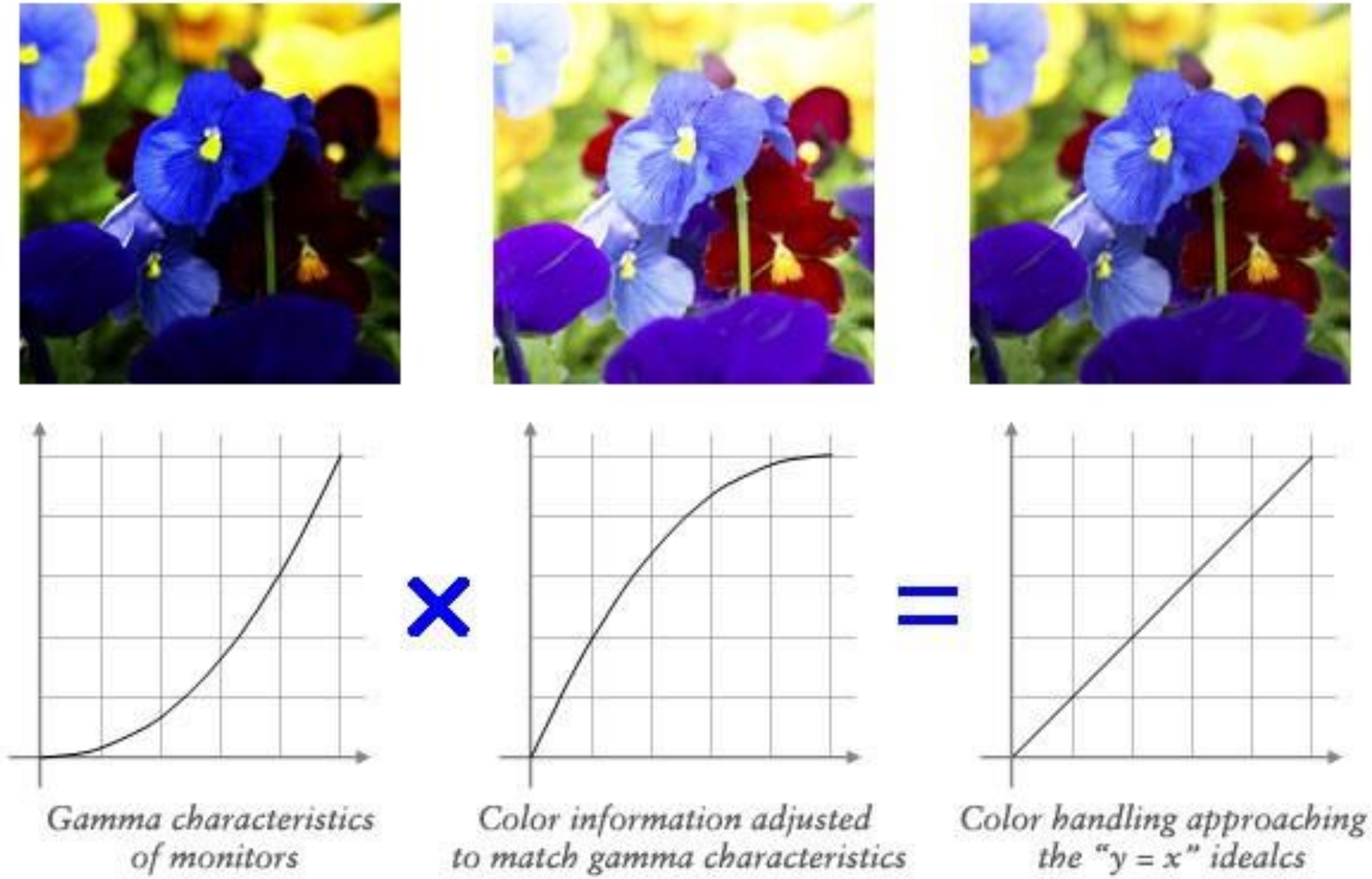
$$x \times 2$$

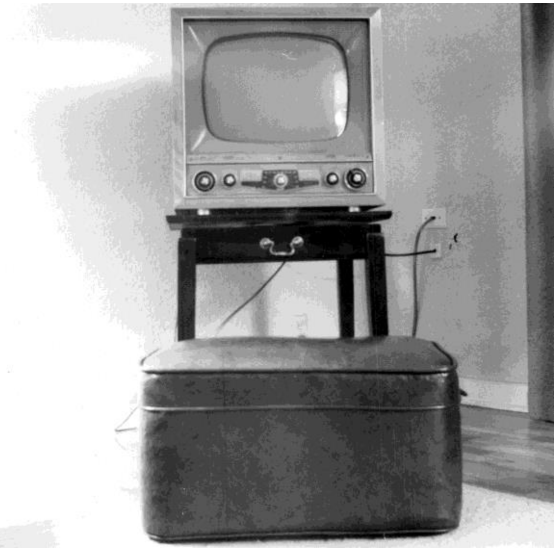
Gamma expansion

$$\left(\frac{x}{255}\right)^{2} \times 255$$

# Gamma correction

- Originally, Due to non-linearities in the old CRT televisions, intensities was seen different then they are.



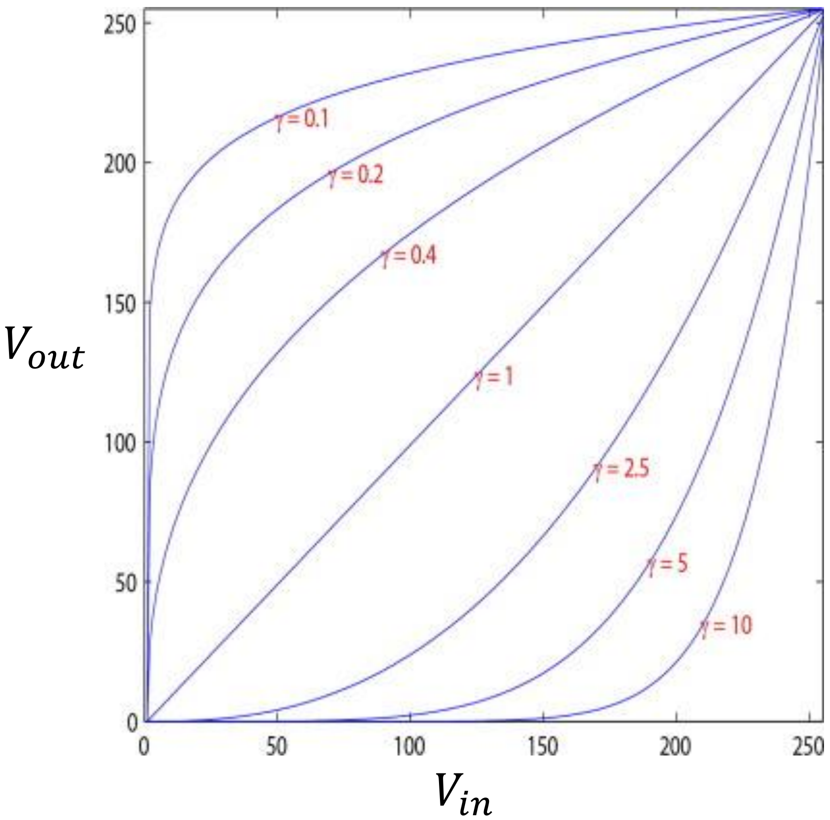



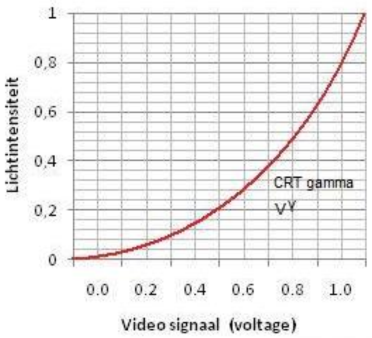
# Gamma correction

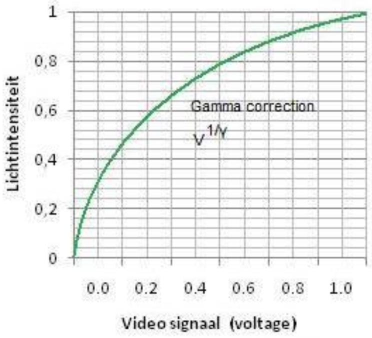
- To correct this non-linear transformation, gamma correction was done:

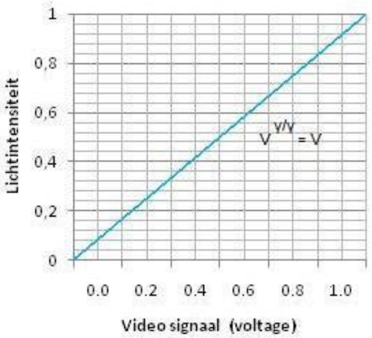
$$\boxed{V_{out} = \left(\frac{V_{in}}{255}\right)^{\gamma} \cdot 255} \qquad (V_{in}, V_{out} \in \{0,1,\dots,255\})$$

- This is, of course, also applicable for image enhancements.




# Some more point- wise operators

# contents

- Image representation
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- **Histogram equalization**
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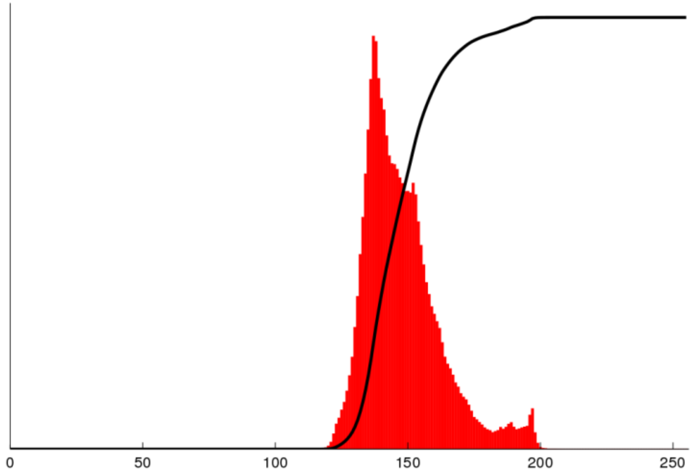
# Histogram equalization

- **Histogram equalization** is a method in image processing of contrast adjustment using the image's histogram.
- This method is used to increase the global contrast of an image and is useful in images with backgrounds and foregrounds that are both bright or both dark.
- Histogram equalization accomplishes this by effectively spreading out the most frequent intensity values.

# Histogram equalization

- A histogram is a discrete form representation of the distribution of numerical data.
- We will assume at first that our image is continues in the range [0,255] for better understanding.
- Instead of a histogram we will talk about the **probability density function** (**PDF**) $f_X(x)$ of the data.



# PDF and CDF

- **cumulative distribution function (CDF)** of a real-valued random variable $X$ is the probability that $X$ will take a value less than or equal to $x$:

$$F_X(x) = P(X \leq x)$$

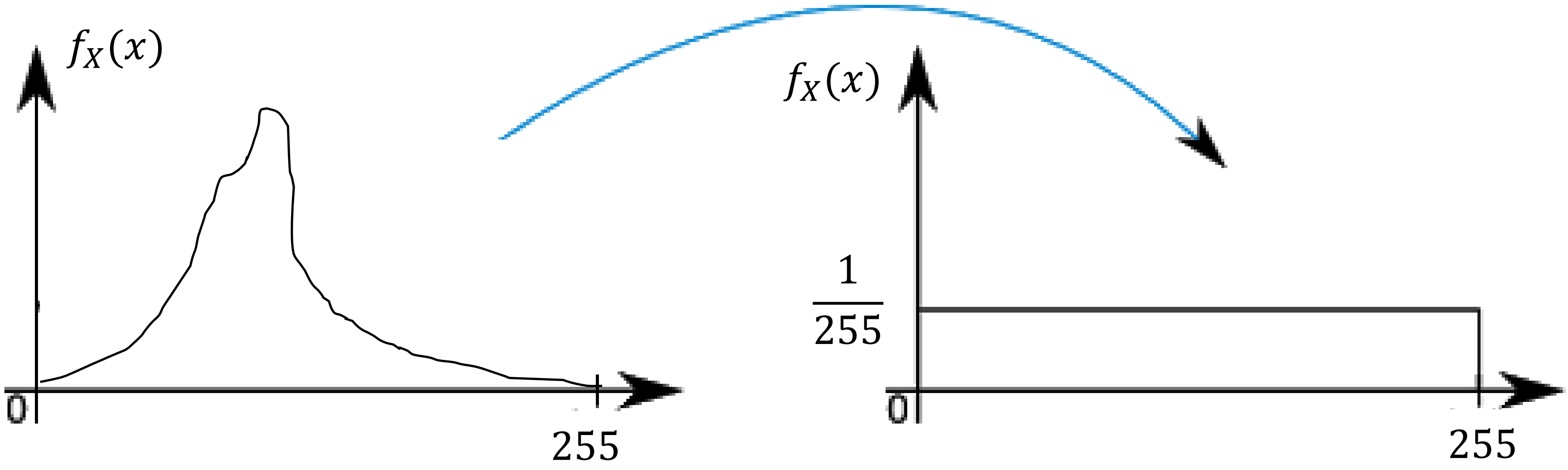
- Properties of CDF:
  - $\lim_{x \to -\infty} F_X(x) = 0, \quad \lim_{x \to +\infty} F_X(x) = 1$
  - Monotonically non decreasing.
- The **probability density function (PDF)** of a continuous random variable can be determined from the cumulative distribution function by differentiating.

$$f_X(x) = \frac{dF_X(x)}{dx} \quad \text{OR} \quad F_X(x) = \int_{-\infty}^{x} f_X(t)\, dt$$

# PDF equalization

- We want that our resulting PDF will be constant for any value in the range [0,255].
- If the PDF is constant, that means that the CDF is linear in [0,255], and so we get the final CDF as:

$$F_Y(y) = P(Y \leq y) = \begin{cases} 0 & : \ y < 0 \\ y & : \ 0 \leq y \leq 255 \\ 1 & : \ y > 255 \end{cases}$$



# PDF equalization

- So we are looking for a transformation function such that:

$$y = T(x)$$

- In the interesting area [0,1]:

$$P(Y \leq y) = y$$
$$P(T(X) \leq y) = y$$
$$assuming\ T\ is\ invertible\text{:} \quad P\left(X \leq T^{-1}(y)\right) = y$$
$$change\ of\ variables\ z = T^{-1}(y)\text{:} \quad P(X \leq z) = \mathrm{T(z)}$$

- The answer is $F_X(x) = T(x)$, and in fact T is invertible since $F_X$ is Monotonically non decreasing

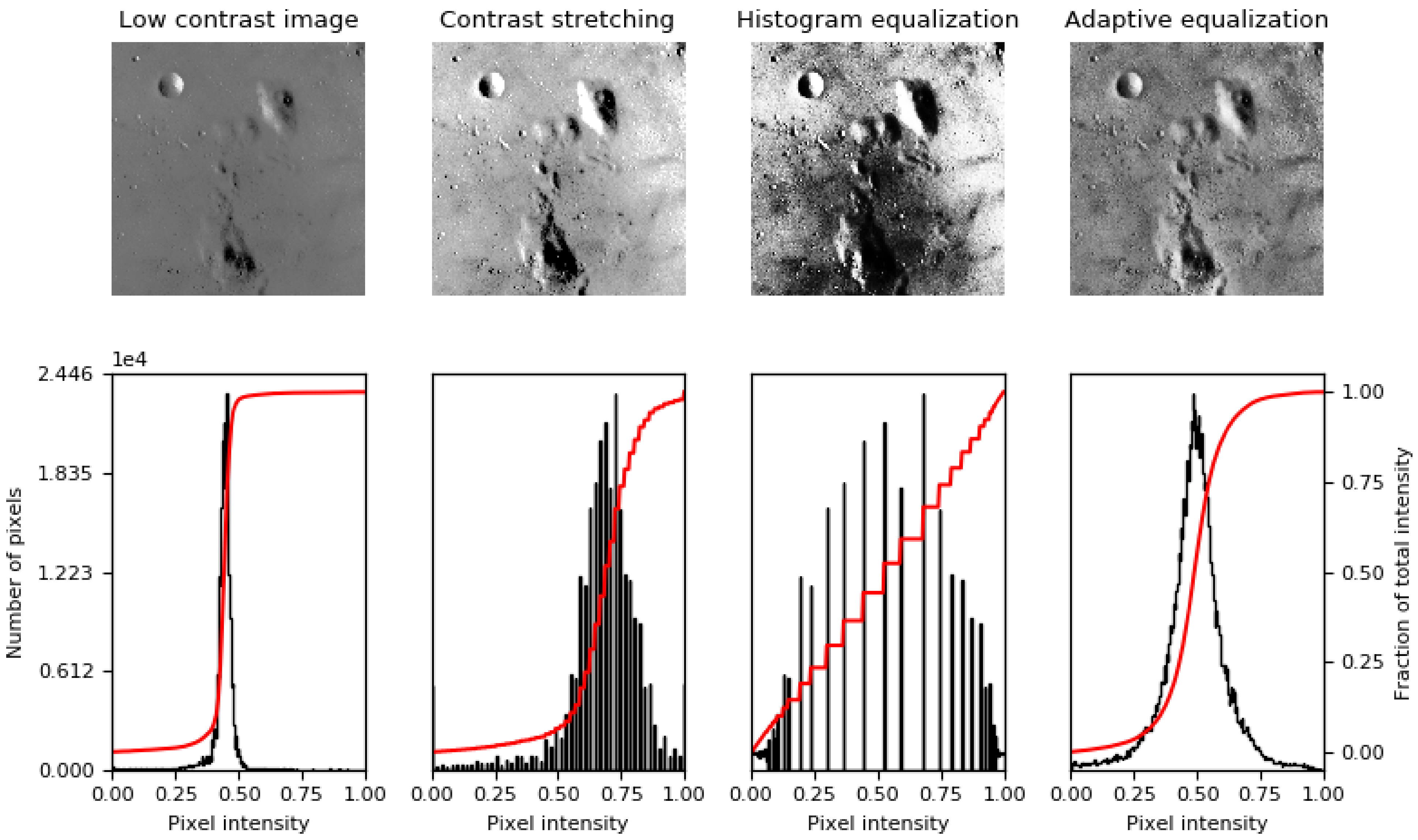
# Back to histogram equalization

- The same result is also applicable for discrete space like actual images and their histograms.

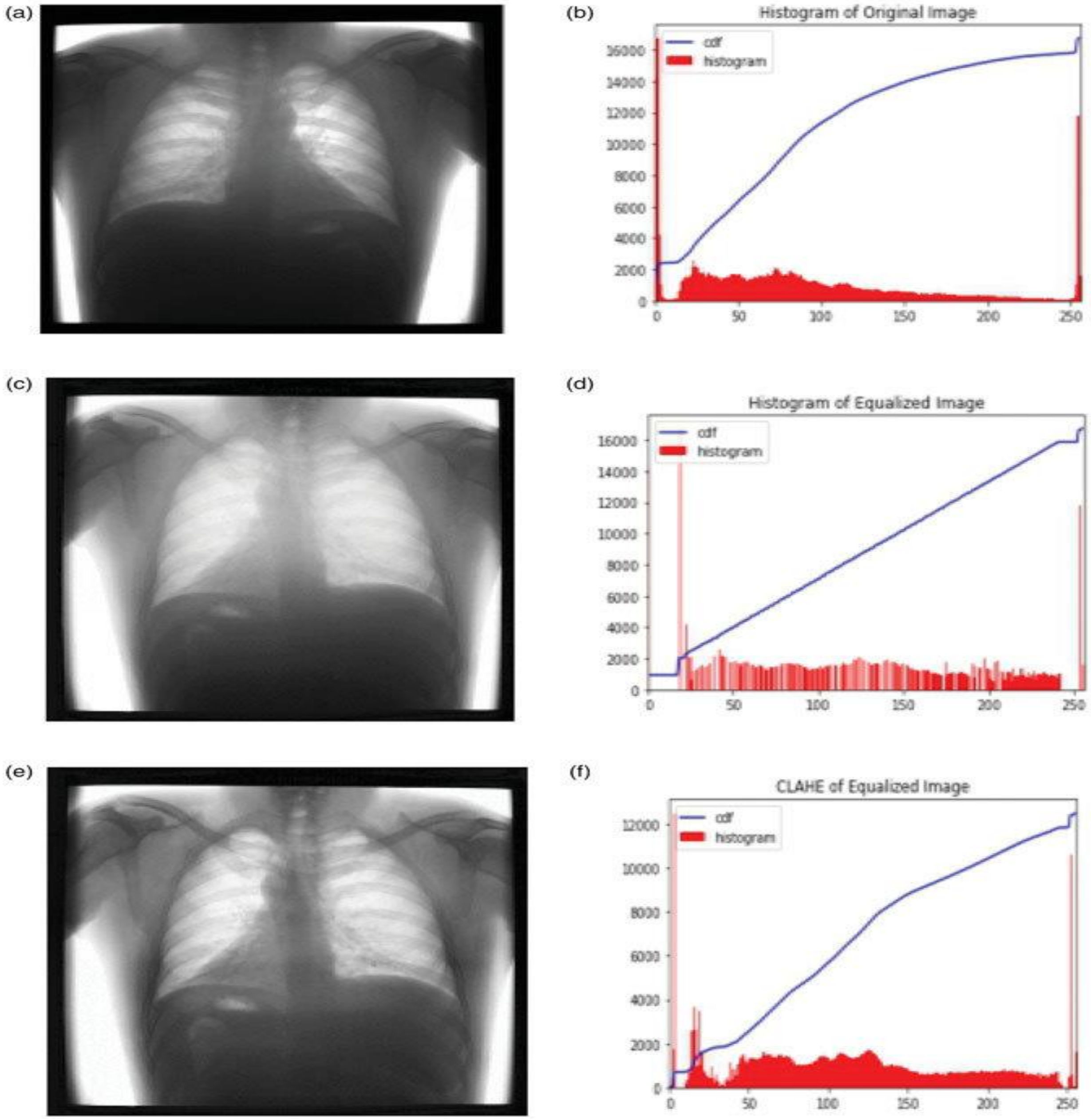
- Build histogram of a given image.
- To make the histogram act like a discrete pdf, we will normalize the PDF: divide each bin by the sum of all bins.
- Build the discrete CDF.
- Un-normalize the CDF and round the results back to uint8:

$$f\_eq(x) = round(CDF(x) * 255)$$

# Other variants of histogram equalization



# Other variants of histogram equalization



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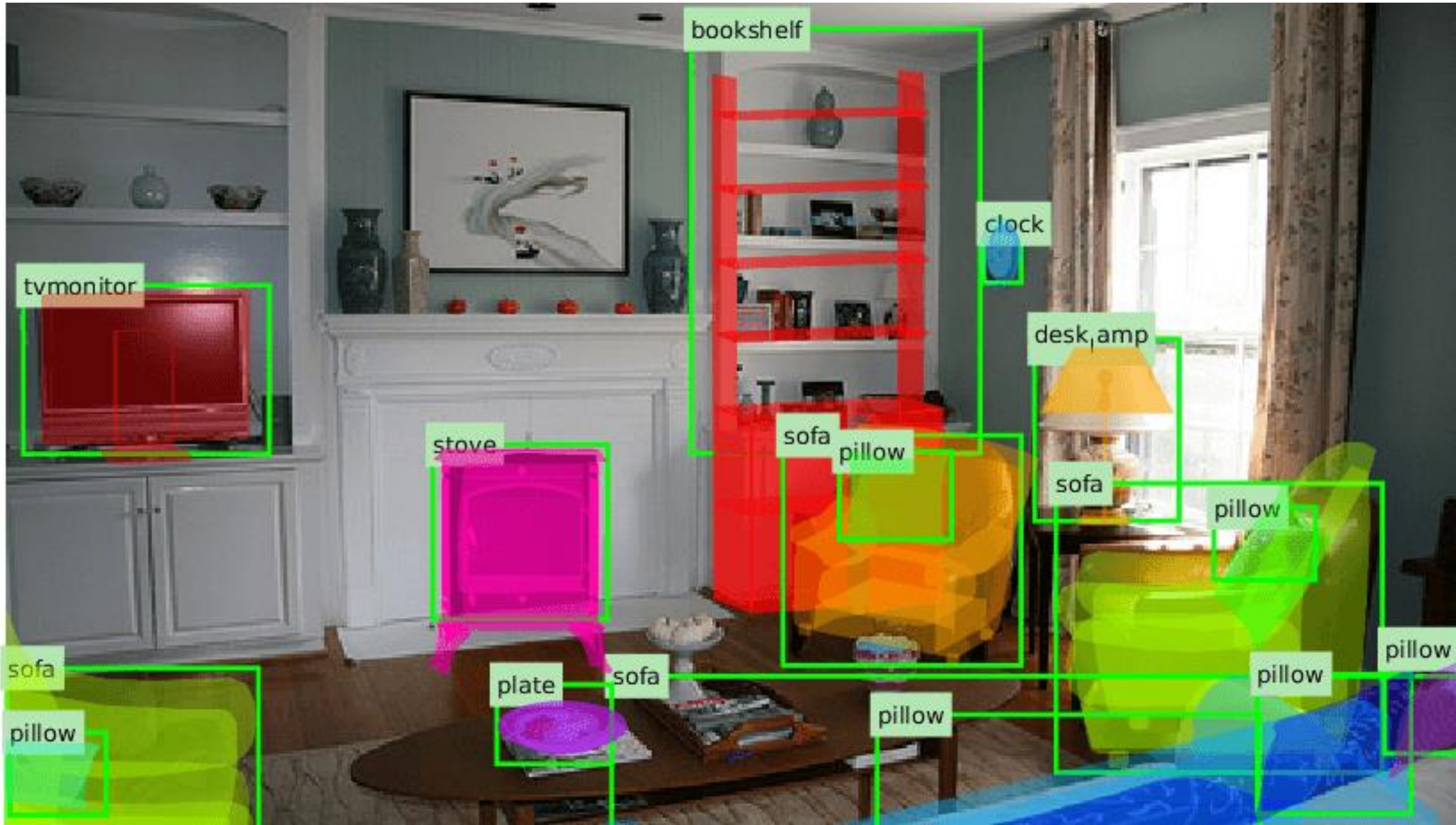
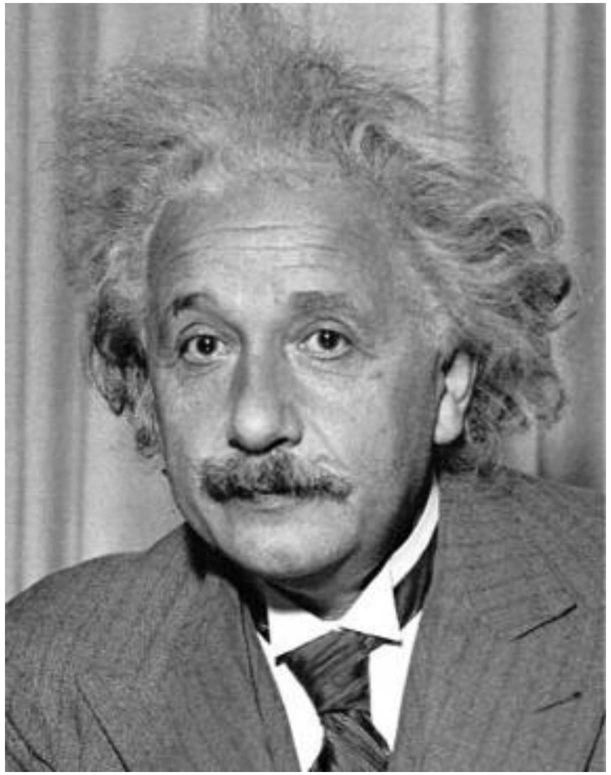
# Template matching

- Given an image template- find it in another image.
- Template matching is a sub-field in **object recognition**.
  - We will see it **a lot** of this topic in this course:
    - SSD
    - Cross correlation
    - Feature based – SIFT
    - Neural networks



# SSD – sum squared distances

How do we detect the template  in he following image?



filter 

output

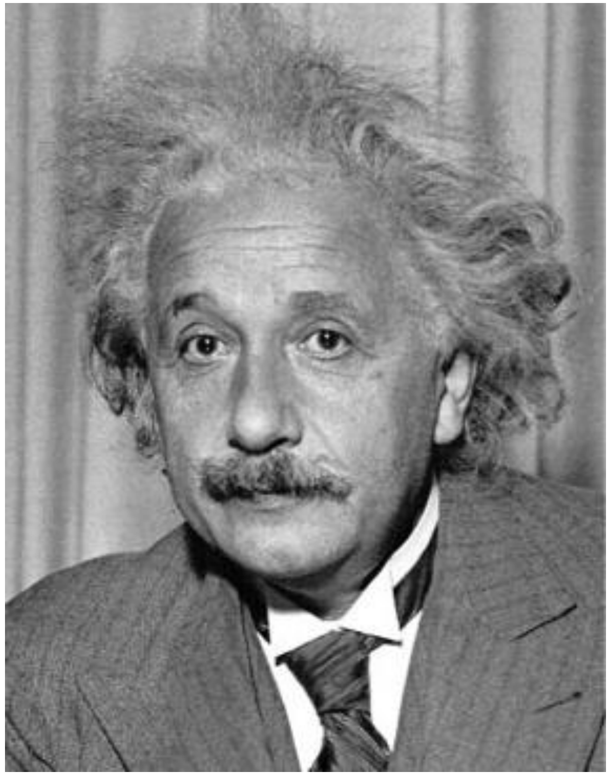
$$h[m,n] = \sum_{k,l} (g[k,l] - f[m+k, n+l])^2$$

image

What will the output look like?

# SSD – sum squared distances

How do we detect the template  in he following image?



filter 

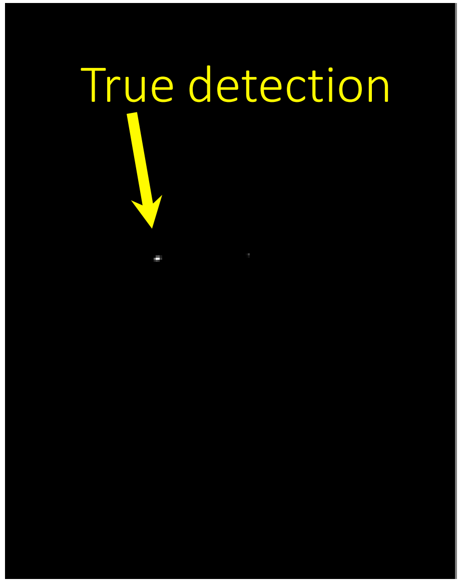
output

$$h[m,n] = \sum_{k,l} (g[k,l] - f[m+k, n+l])^2$$

image

1-output

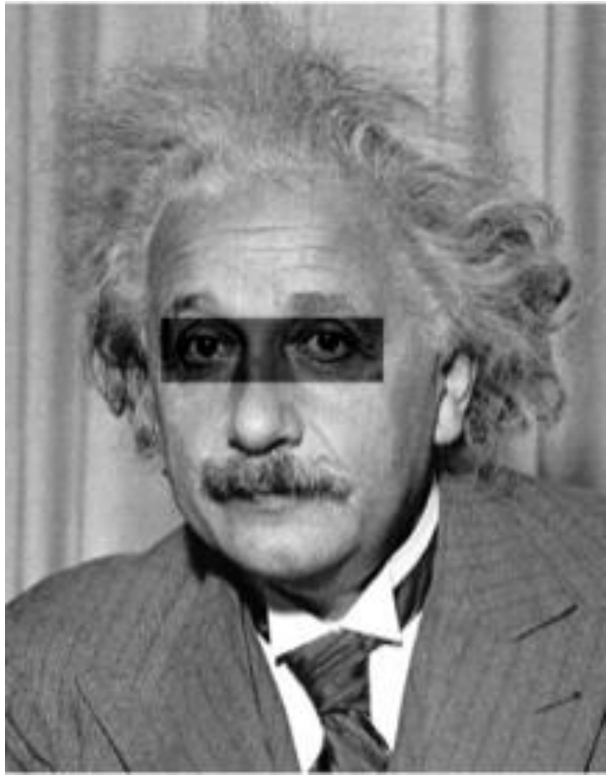
thresholding




What could go wrong?

# SSD – sum squared distances

How do we detect the template  in he following image?



1-output

filter 

output

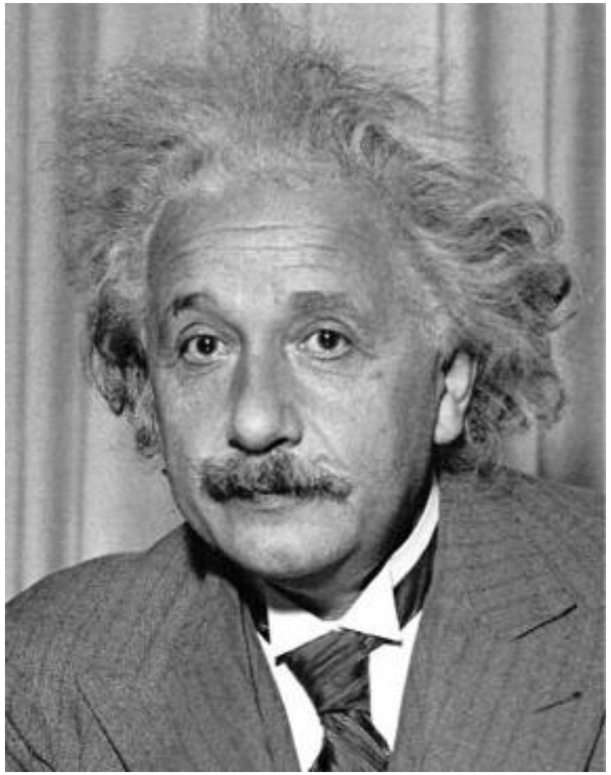
$$h[m,n] = \sum_{k,l} (g[k,l] - f[m+k, n+l])^2$$

image

Not robust to local intensity changes

# CC – cross correlation

How do we detect the template  in he following image?



filter 

output

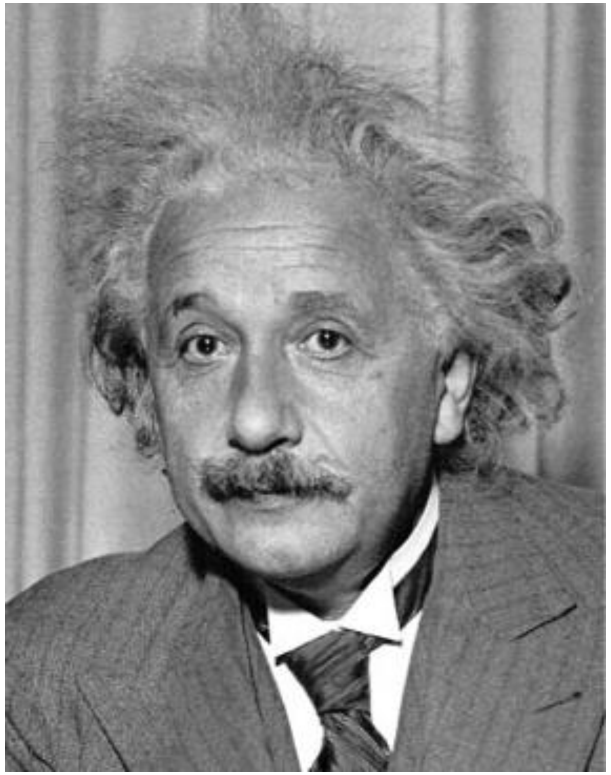
$$h[m,n] = \sum_{k,j} g[k,l] f[m+k, n+l]$$

image

What will the output look like?

# CC – cross correlation

How do we detect the template  in he following image?

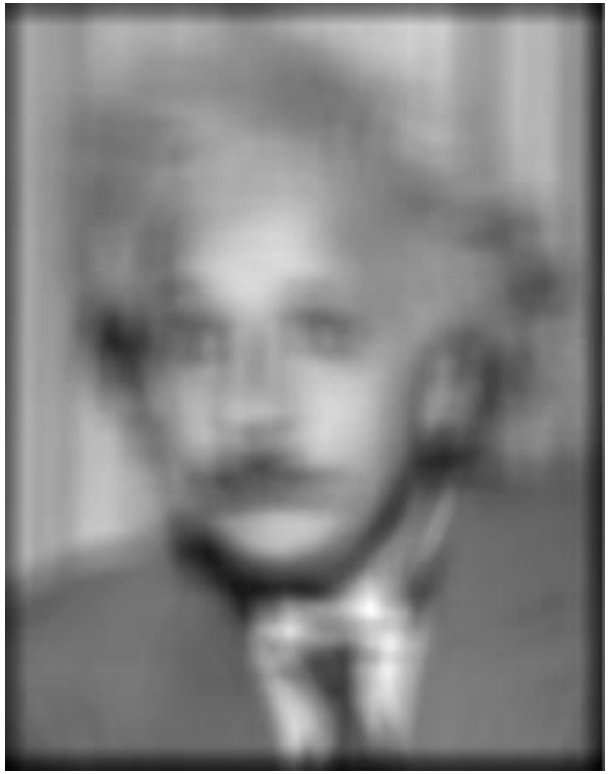


filter 

output

$$h[m,n] = \sum_{k,j} g[k,l] f[m+k, n+l]$$

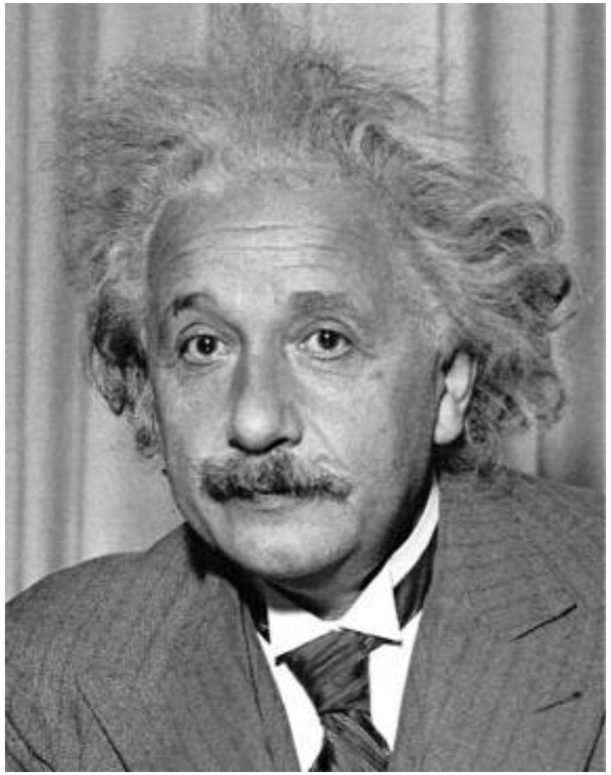
image



What went wrong?

# CC – cross correlation

How do we detect the template  in he following image?

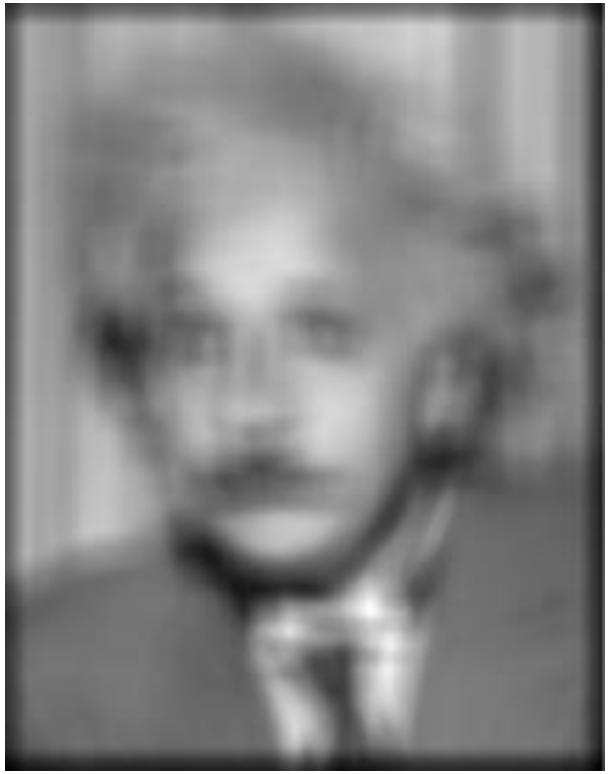


filter 

output

$$h[m,n] = \sum_{k,j} g[k,l]\, f[m+k, n+l]$$

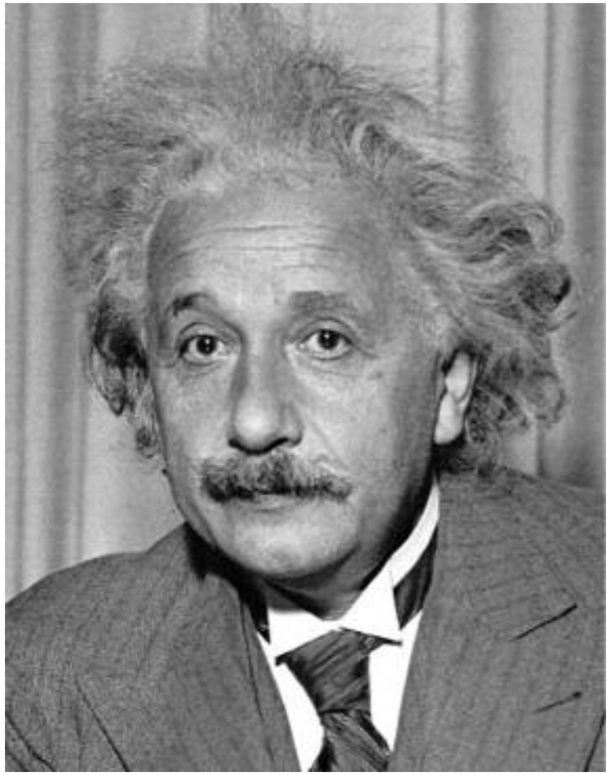
image



Increases for higher local intensities.

# Zero mean cross correlation

How do we detect the template  in he following image?



filter 

template mean

output

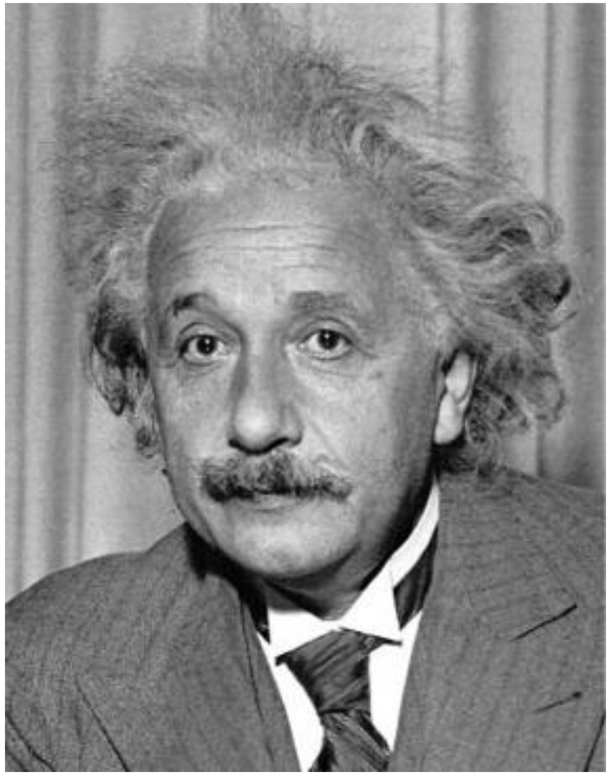
$$h[m,n] = \sum_{k,l} (g[k,l] - \bar{g}) f[m+k, n+l]$$

image

What will the output look like?

# Zero mean cross correlation

How do we detect the template  in he following image?



filter 

template mean

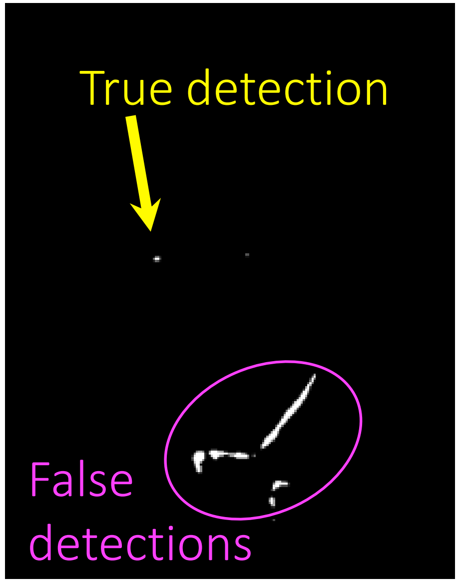
output

$$h[m,n] = \sum_{k,l} (g[k,l] - \bar{g}) f[m+k, n+l]$$

image

output

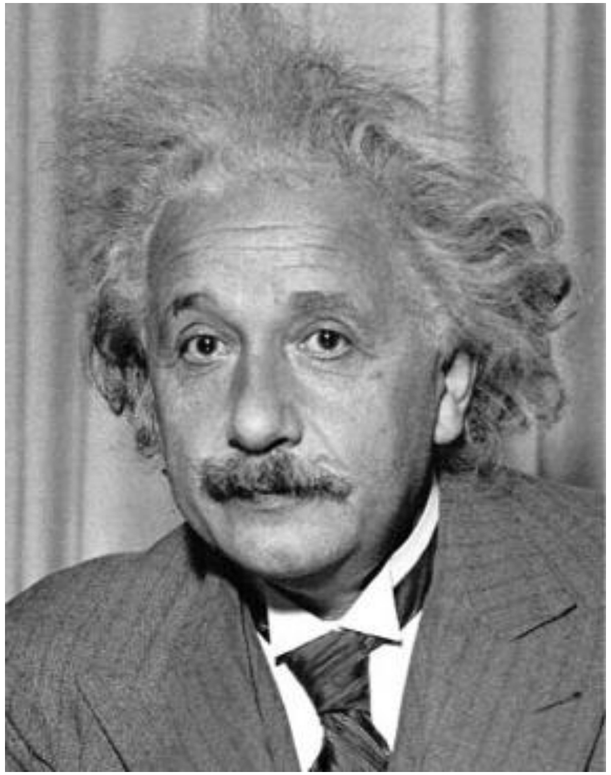
thresholding




What went wrong?

# Zero mean cross correlation

How do we detect the template  in he following image?



filter 

template mean

output

$$h[m,n] = \sum_{k,l} (g[k,l] - \bar{g}) f[m+k, n+l]$$

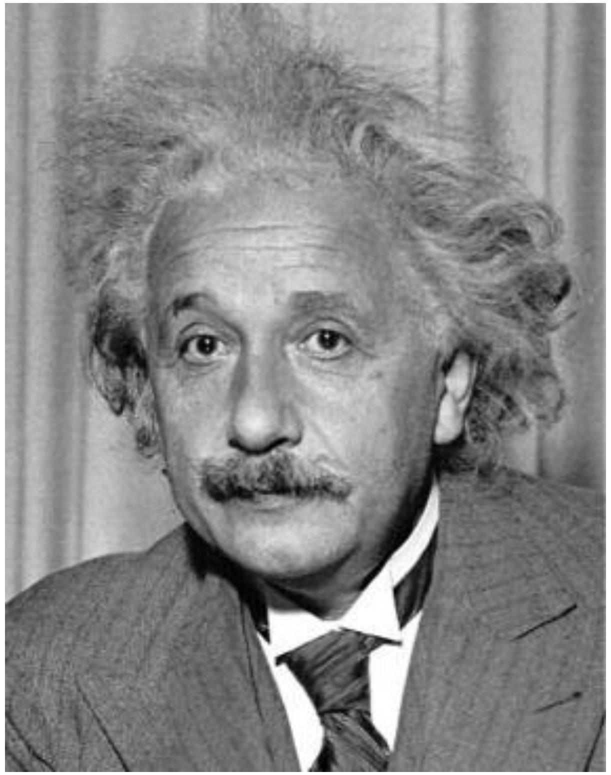
image

output

Not robust to high-contrast areas

# ZNCC – zero mean normalized cross correlation

How do we detect the template  in he following image?



What will the output look like?

filter 

template mean

output

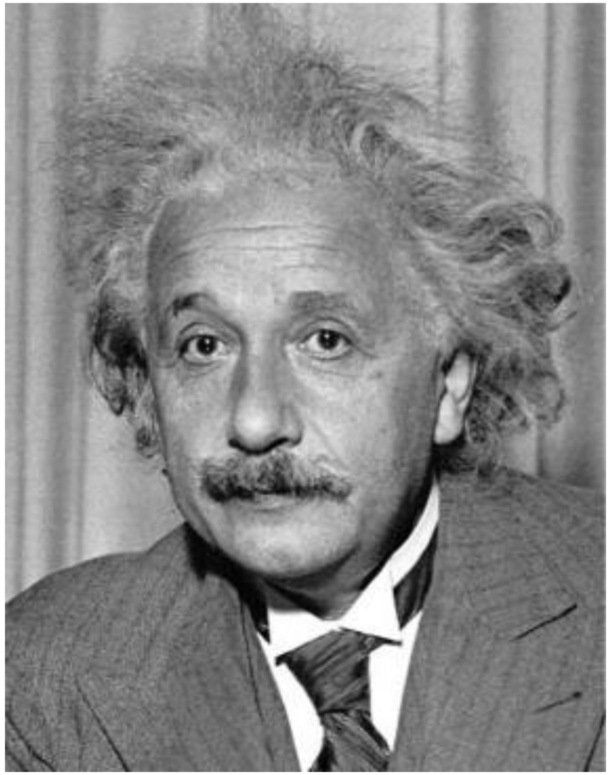
$$h[m,n] = \frac{\sum_{k,l}(g[k,l]-\bar{g})(f[m+k,n+l]-\bar{f}_{m,n})}{\sqrt{(\sum_{k,l}(g[k,l]-\bar{g})^2 \sum_{k,l}(f[m+k,n+l]-\bar{f}_{m,n})^2)}}$$

image

local patch mean

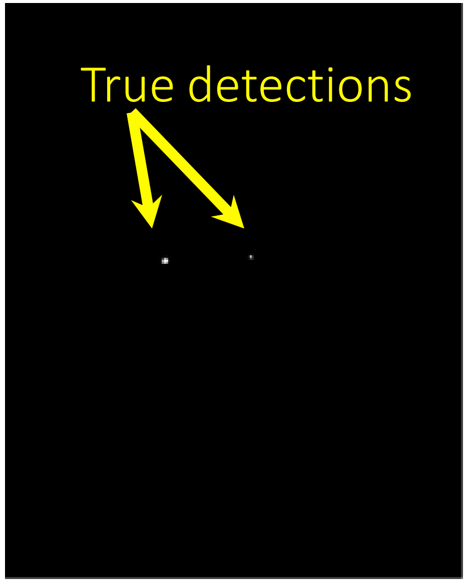
# ZNCC – zero mean normalized cross correlation

How do we detect the template  in he following image?



1-output

thresholding



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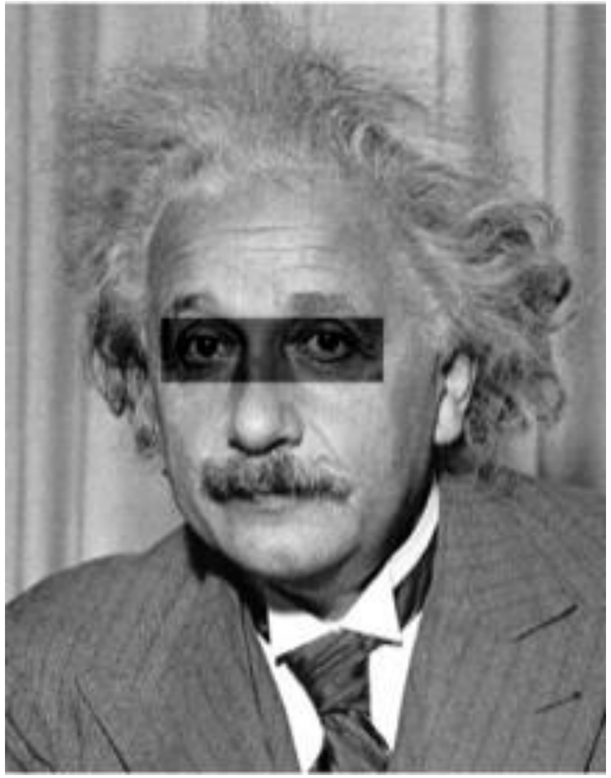
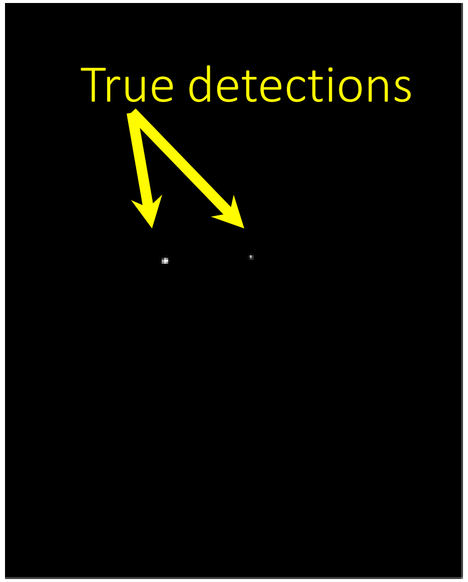


1-output

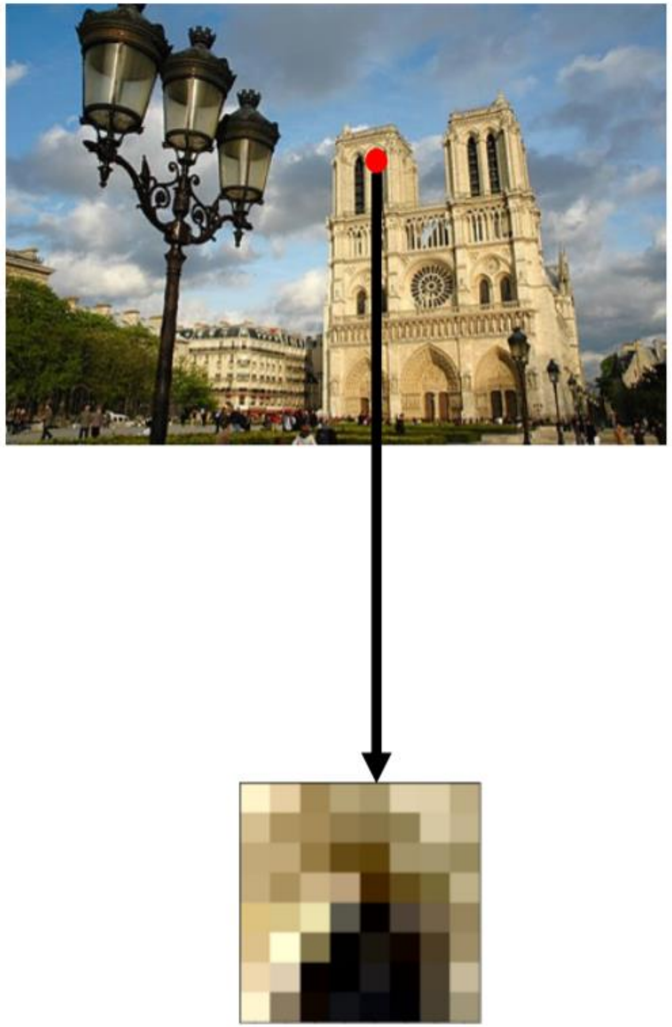
thresholding

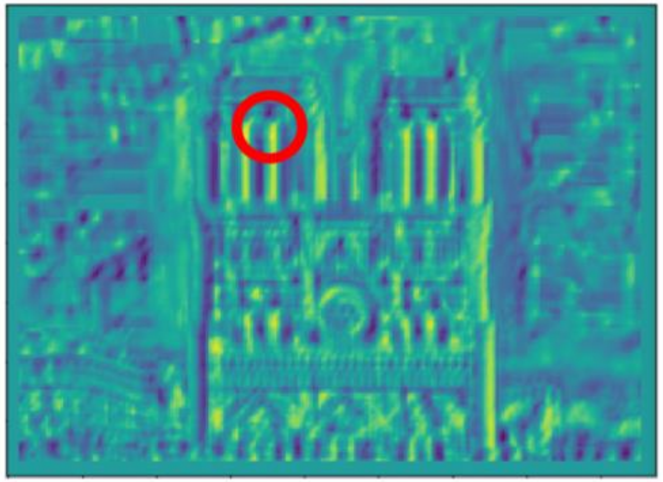



robust to change in intensities

# Template matching- SSD, ZNCC

- Good for very carefully constructed scenarios.
- Can’t handle change in rotation and scale.





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- **Morphology operators**
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# Morphology

- Handy tool whenever needed to clean up binary images.
- Each morphology operator is constructed as such:
  1. Select a structure element (binary kernel)

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

# Morphology

- Handy tool whenever needed to clean up binary images.
- Each morphology operator is constructed as such:
  1. Select a structure element (binary kernel)
  2. Cross-correlate with input binary image $g = f \star s$

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

# Morphology

- Handy tool whenever needed to clean up binary images.
- Each morphology operator is constructed as such:
  1. Select a structure element (binary kernel)
  2. Cross-correlate with input binary image $g = f \star s$
  3. Threshold the output

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\theta_{TH}(x,t) = \begin{cases} 1 & if \quad x \geq t, \\ 0 & else \end{cases}$$

# Morphology

- Handy tool whenever needed to clean up binary images.
- Each morphology operator is constructed as such:
  1. Select a structure element (binary kernel)
  2. Cross-correlate with input binary image $g = f \star s$
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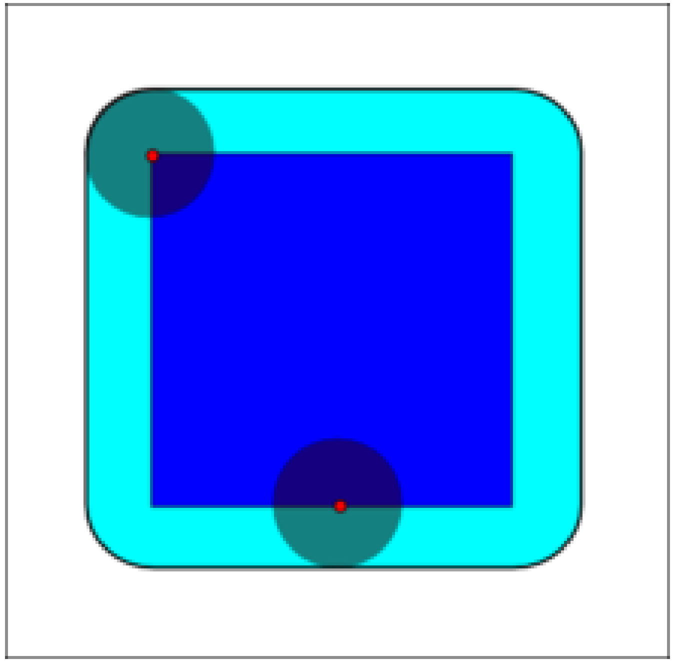
$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\theta_{TH}(x,t) = \begin{cases} 1 & if \quad x \geq t, \\ 0 & else \end{cases}$$

- Overall morphologic operation should look like so:

$$k = \theta_{TH}(f \star s, t)$$

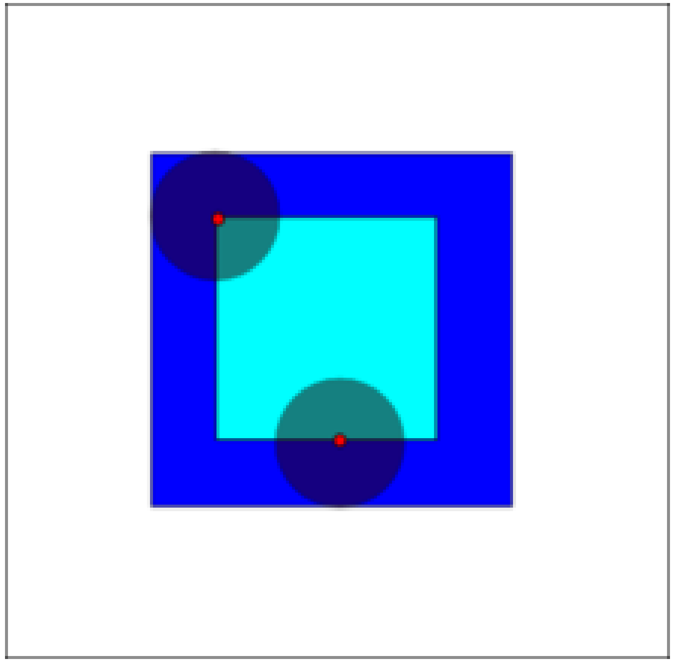
# Dilation



- $k = \theta_{TH}(f \star s, t)$

- Dilation: $t = 1$
  – Skinny lines will get thicker

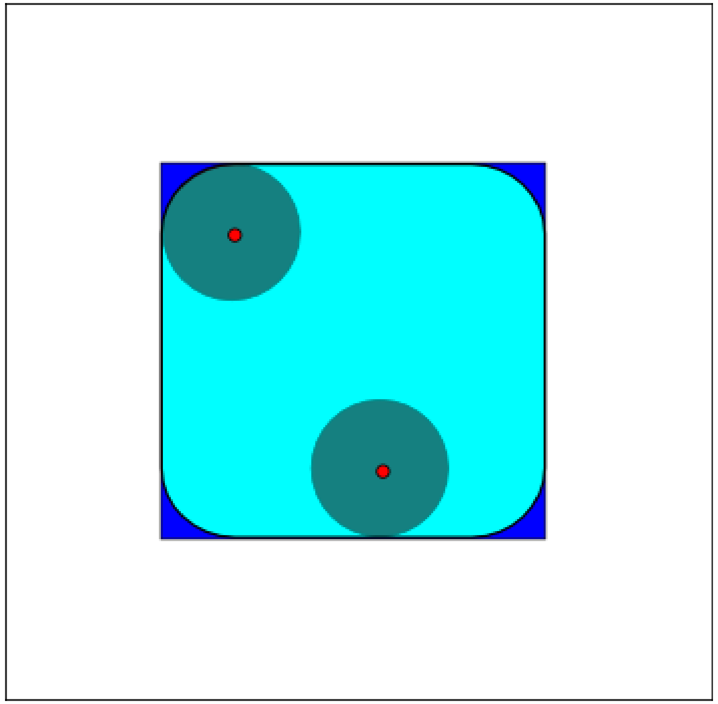
# Erosion



- $k = \theta_{TH}(f \star s, t)$

- Erosion: $t = sum(s)$
  - Thicker lines will get skinny

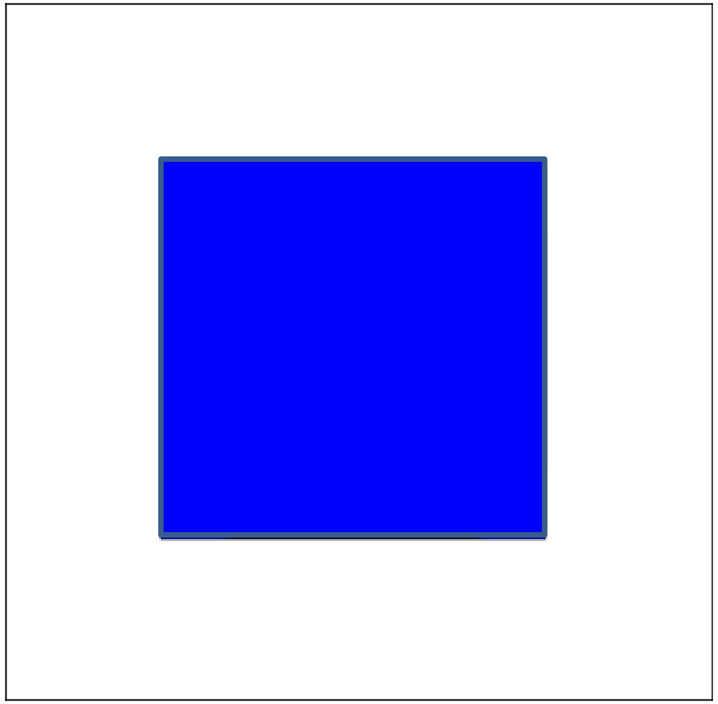
# Opening



- Erosion followed by dilation.
  - The effect is of rounding off sharp edges.

# Closing

- Dilation followed by erosion.
  - The effect is of closing of narrow gaps and holes.



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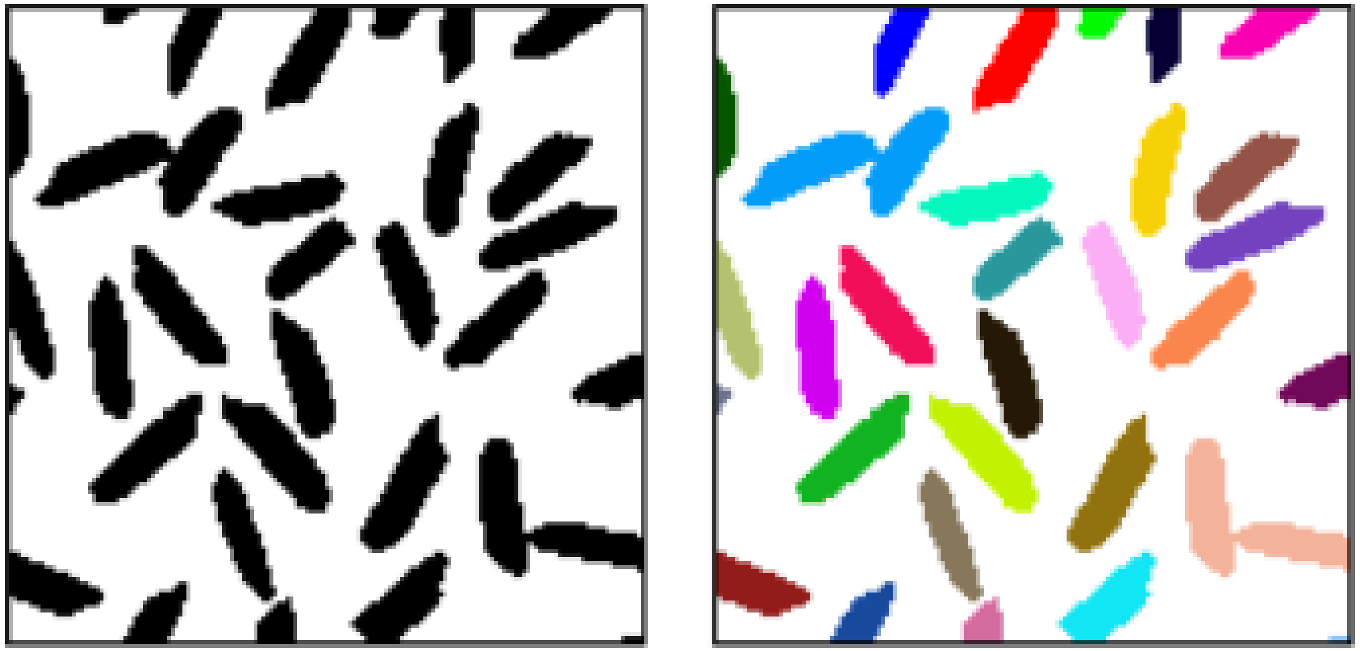
# Connected components

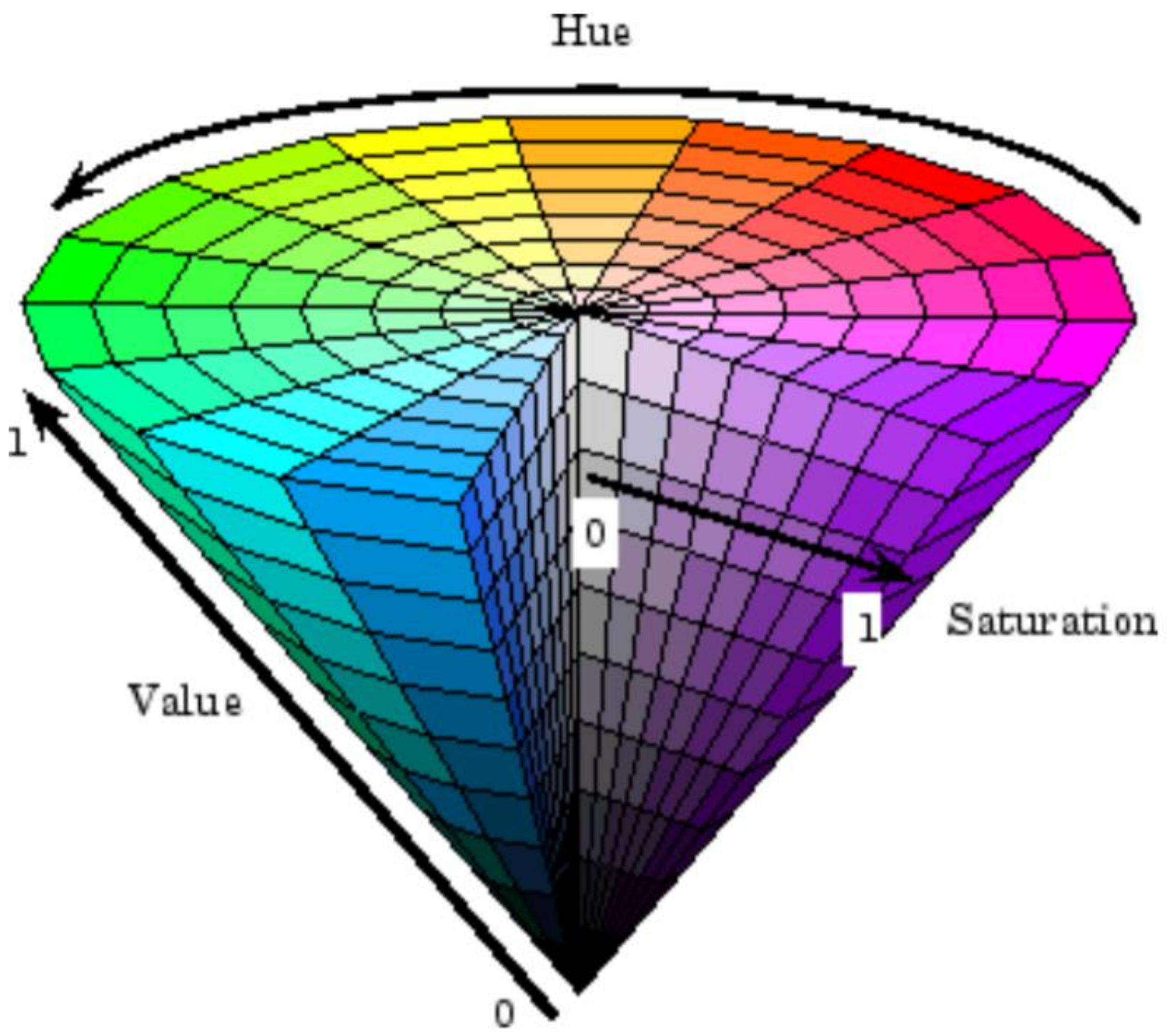
- Defined as regions of adjacent pixels that have the same value.
- Commonly used with binary images to find stand alone objects.
  - e.g.: letters in a document.



# contents

- Image representation
- Pixel-wise operations
- Histogram equalization
- Template matching
- Morphology operators
- Connected components
- **Color space**

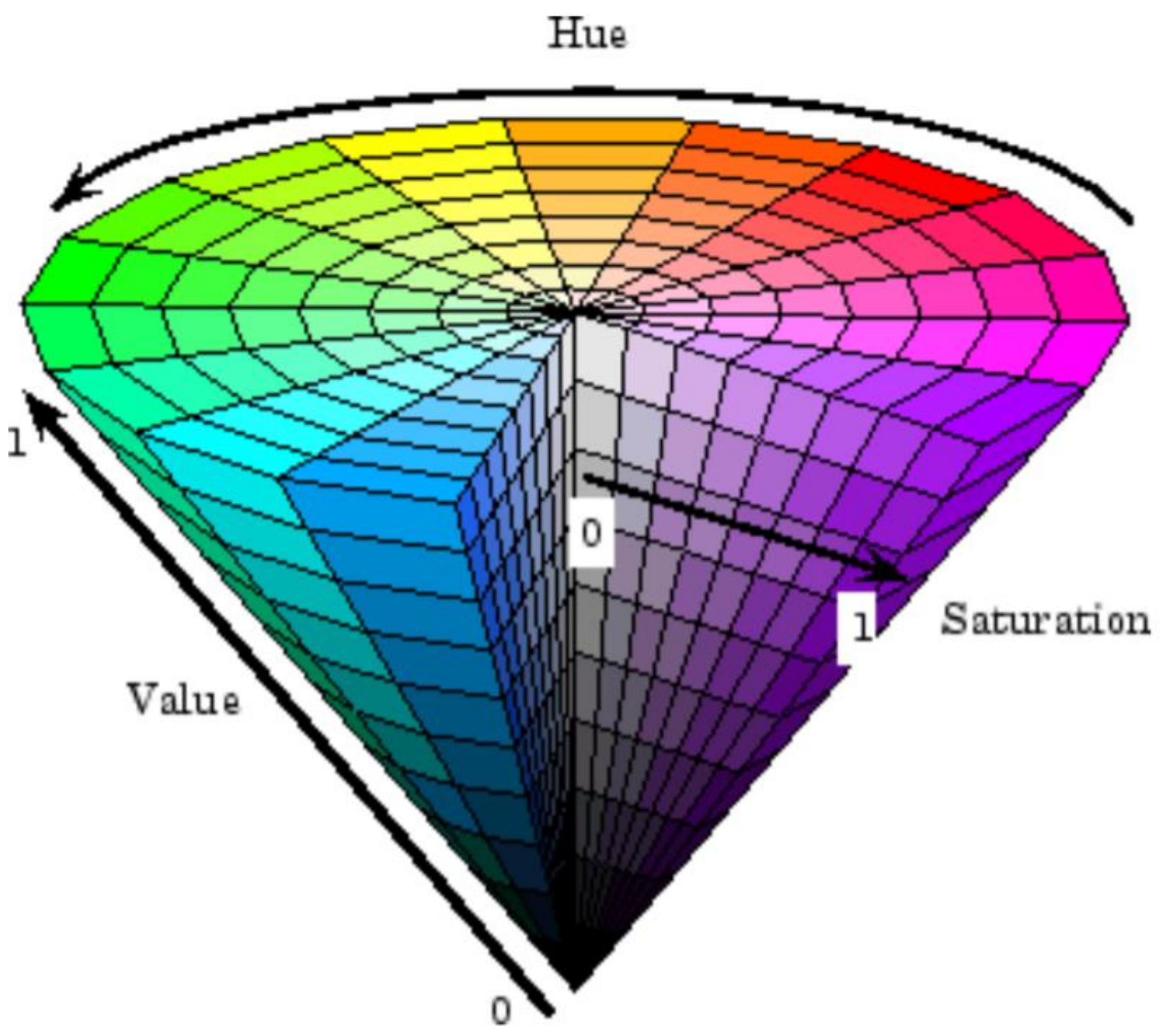
# HSV

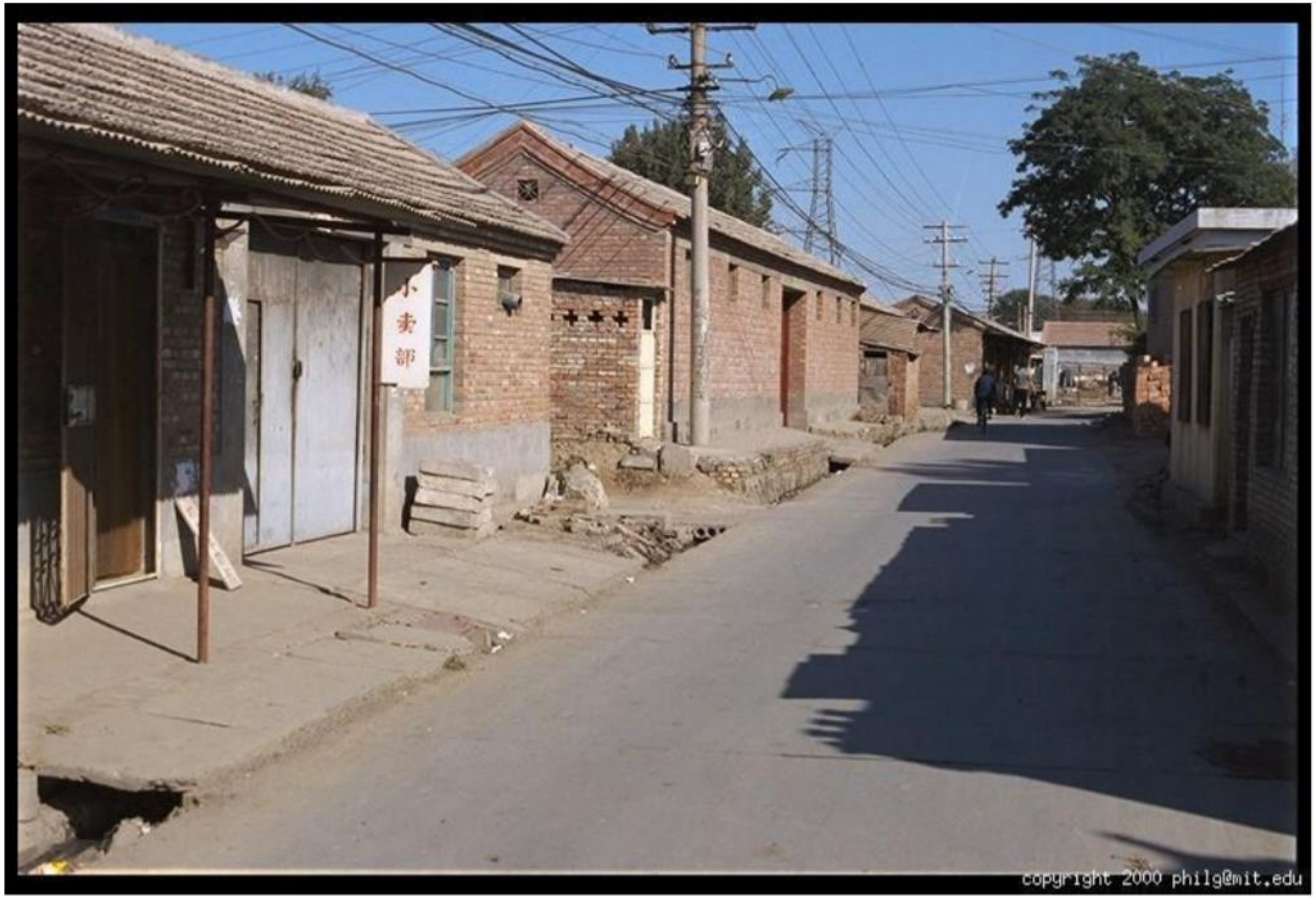



https://cs.brown.edu/courses/csci1430/lectures/2020Spring_08_CamerasLightAndColor.pdf

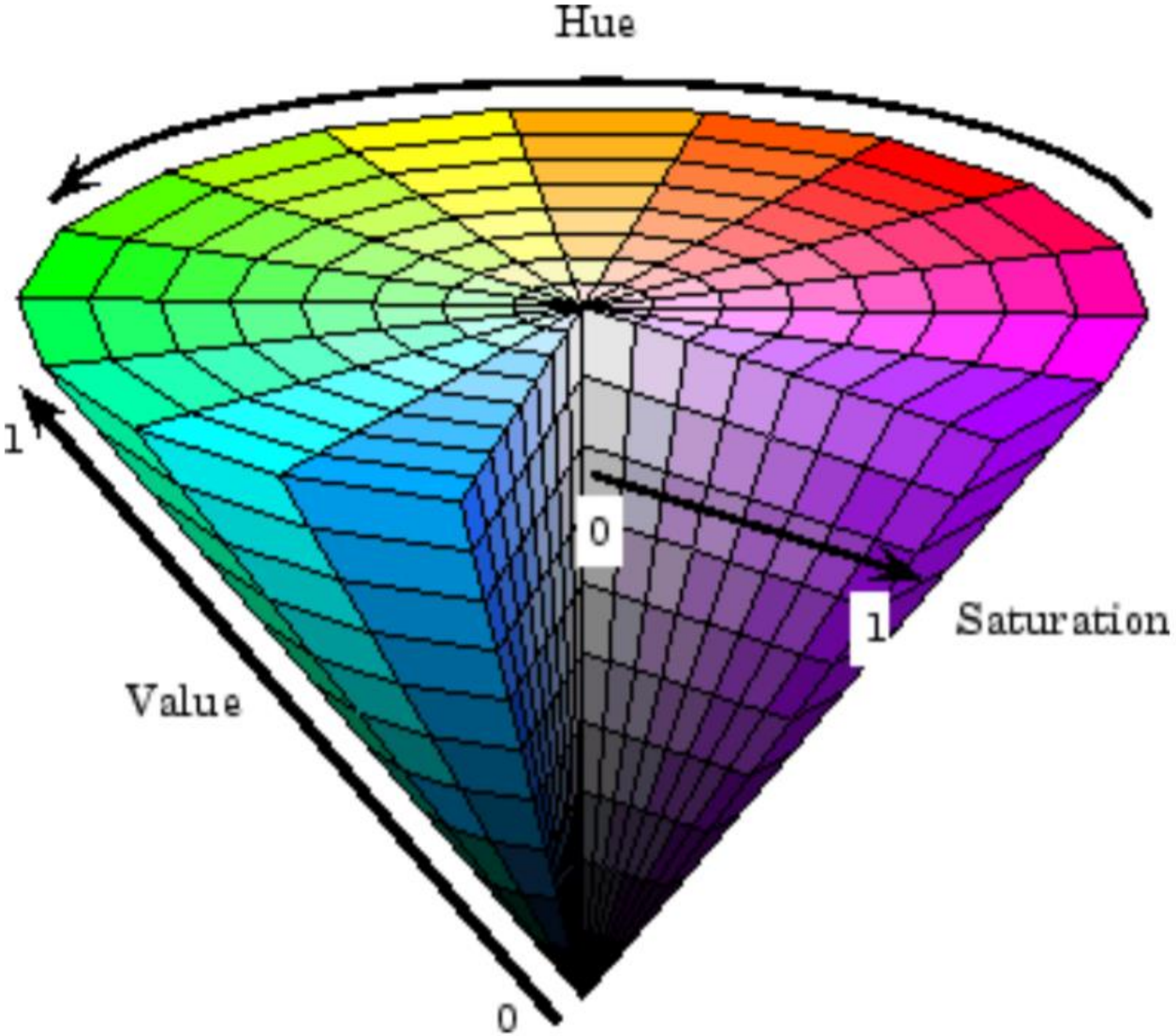
# HSV

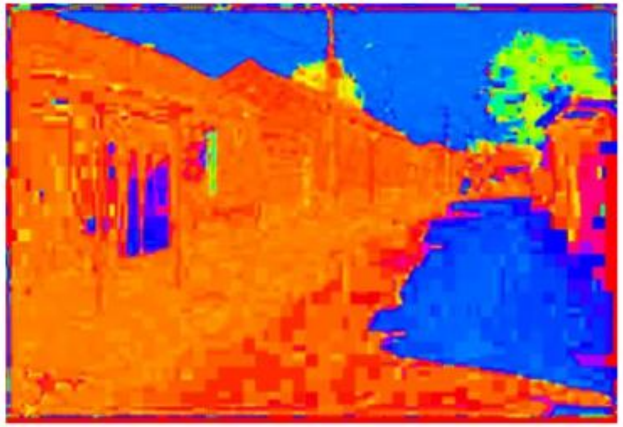
- **Hue**: The "attribute of a visual sensation according to which an area appears to be similar to one of the perceived colors: red, yellow, green, and blue, or to a combination of two of them"
- **Saturation**: The "colorfulness of a stimulus relative to its own brightness"
- **Value**: The "brightness relative to the brightness of a similarly illuminated white“. Can also be called **brightness or intensity**.
  – [Wikipedia]






Original image



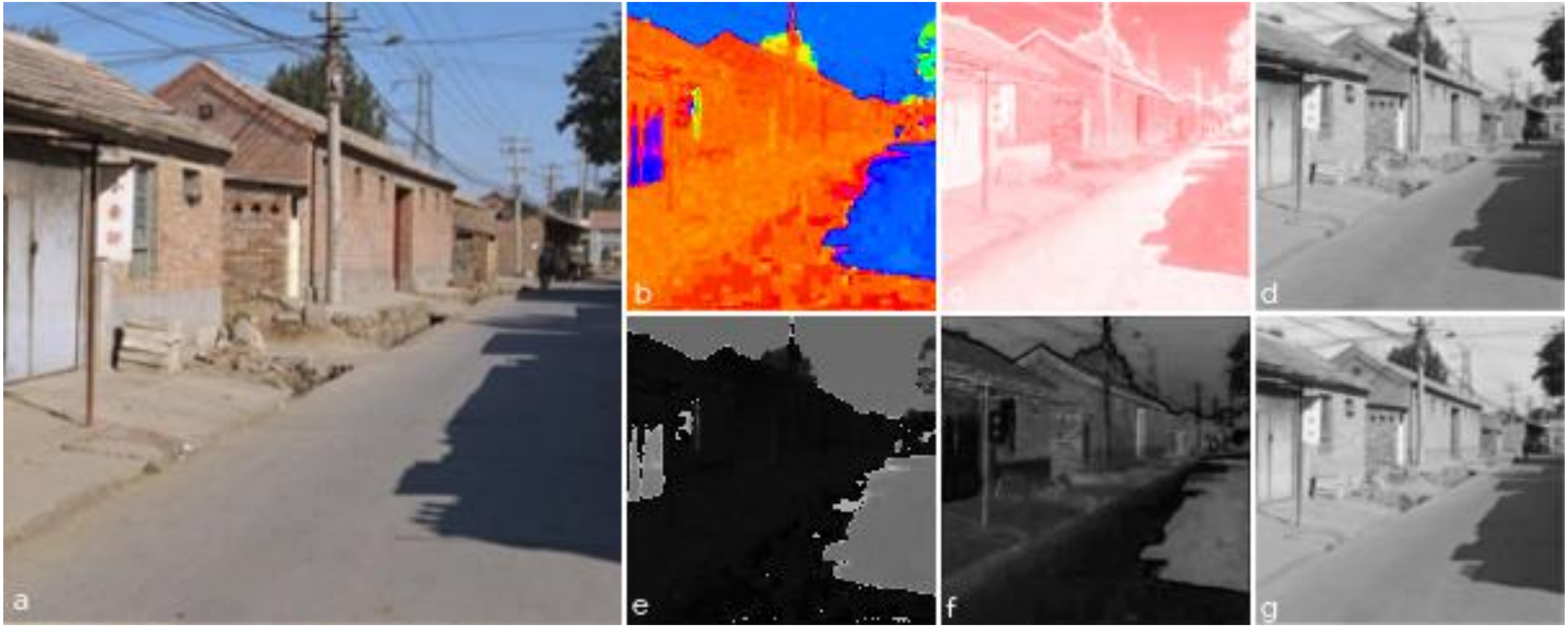
**H**
(S=1,V=1)

**S**
(H=1,V=1)

**V**
(H=1,S=0)

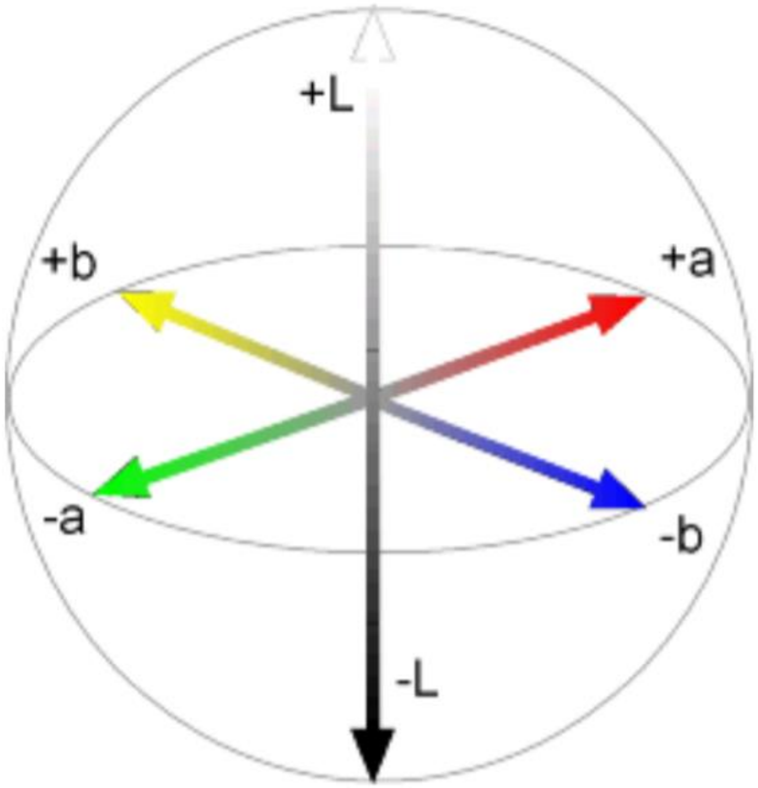
# HSV

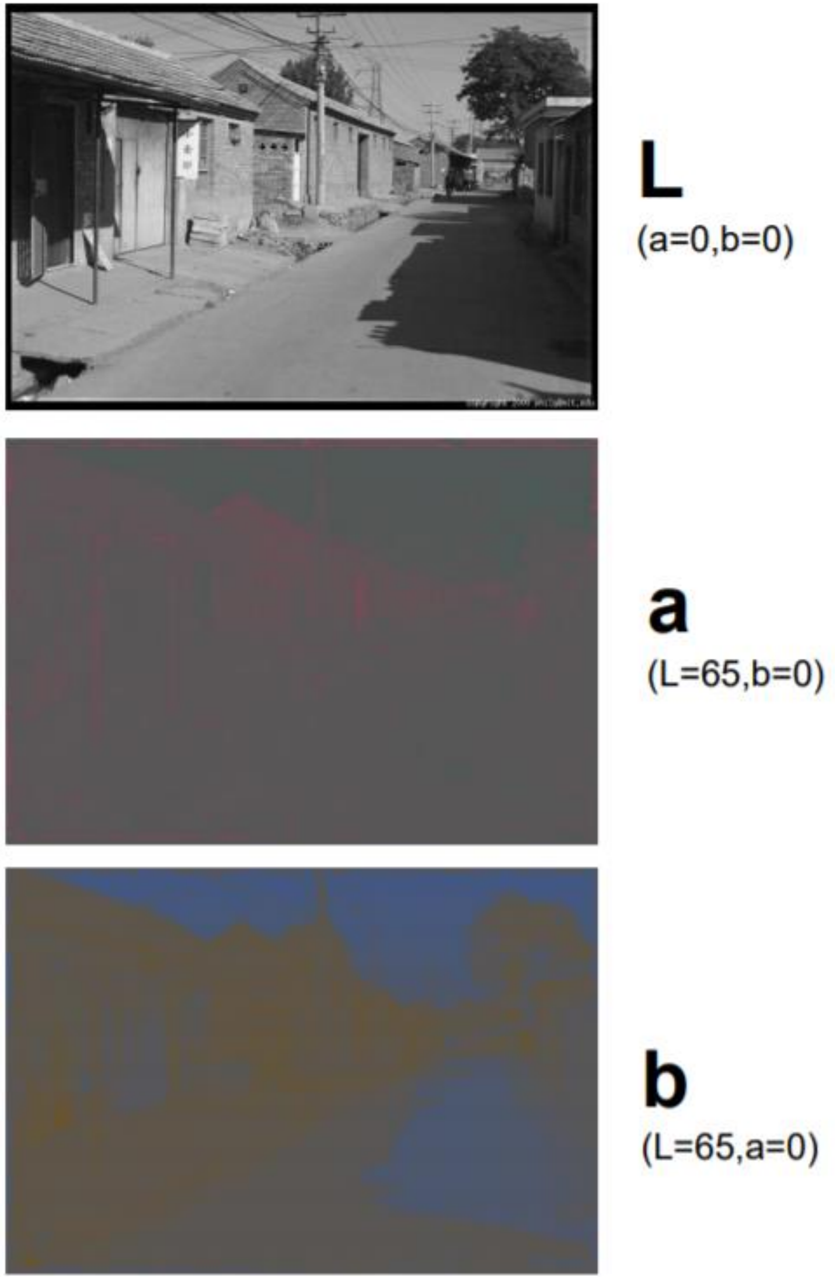
- In e, f, g: single channel image representation.
- Conclusion: people are much more responsive to intensity then chroma.



# More color spaces: LAB

- L: lightness from black (0) to white (100).
- A: from green (−) to red (+).
- B: from blue (−) to yellow (+).



# More color spaces: YUV

- Y: brightness/ intensity.
- U: blue projection.
- V: red projection.

- [Similar to YCbCr]