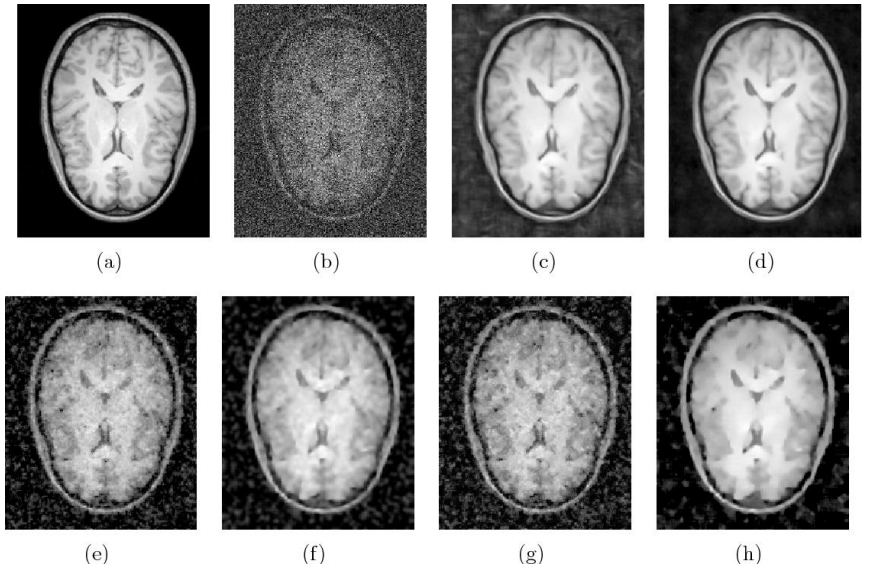
Image processing

References

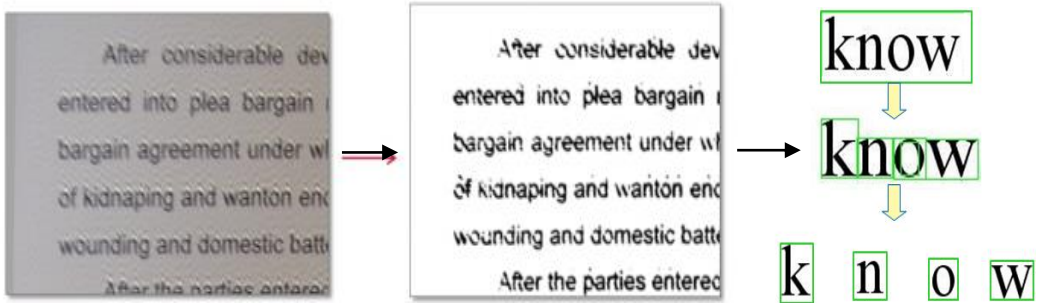
- http://szeliski.org/Book/
- http://www.cs.cornell.edu/courses/cs5670/2019sp/lectures/lectures.html
- http://www.cs.cmu.edu/~16385/

Some motivation

Art
(Photoshop image correction)



Medicine
(MRI denoising)



Robotics
(OCR – optical character recognition)

Agriculture
(color ripeness detection)

contents

- **Image representation**
- Pixel-wise operations
- Noise and filtering
- Frequency representation
- Decimation
- Interpolation
- Morphology operators
- Connected components

Image representation

- We cane think of image as a 3d matrix of discrete RGB values.
- The values mark the intensity of each color channel and are usually of type uint8 = $\{0, \dots, 255\}$.

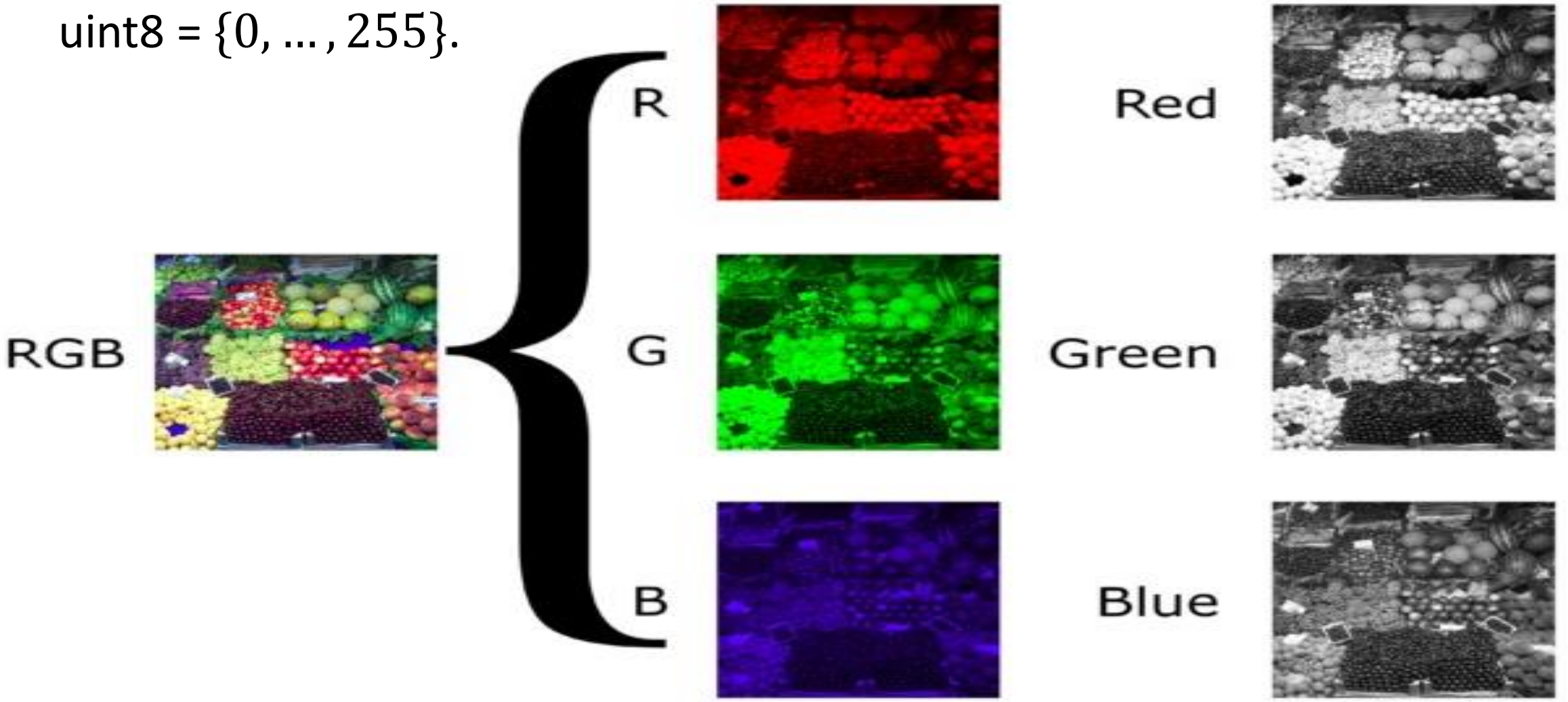
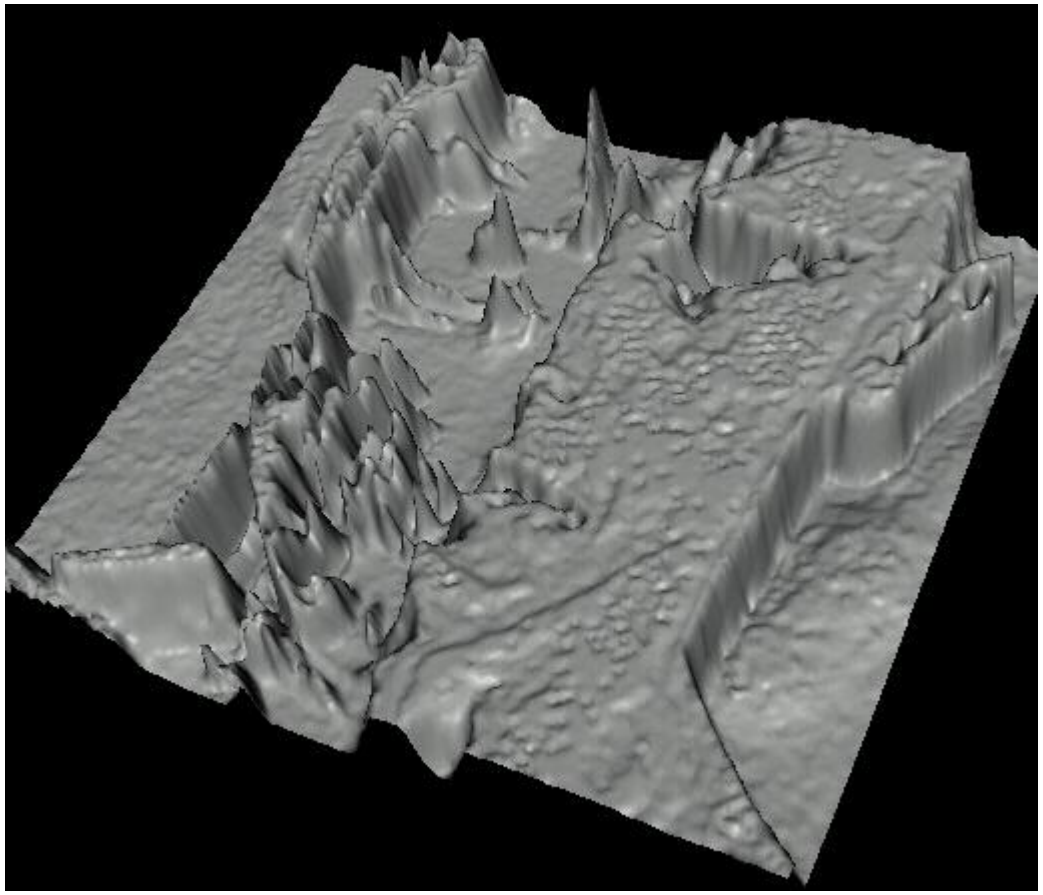


Image representation

- We can also think of an image as a function $f(x, y)$.



contents

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- **Pixel-wise operations**
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Pixel-wise operators

- Pixel-wise operators, or point operators, are defined as such that each output pixel’s value depends on only the corresponding input pixel value.

Pixel-wise operators

original

x

darken

lower contrast

Gamma compression

invert

lighten

raise contrast (?)

Gamma expansion

Pixel-wise operators

original

x

darken

lower contrast

Gamma compression

invert

$255 - x$

lighten

raise contrast (?)

Gamma expansion

Pixel-wise operators

original

x

darken

$x - 128$

lower contrast

Gamma compression

invert

$255 - x$

lighten

raise contrast (?)

Gamma expansion

Pixel-wise operators

original

x

darken

$x - 128$

lower contrast

Gamma compression

invert

$255 - x$

lighten

$x + 128$

raise contrast (?)

Gamma expansion

Pixel-wise operators

original

x

darken

$x - 128$

lower contrast

$\frac{x}{2}$

Gamma compression

invert

$255 - x$

lighten

$x + 128$

raise contrast (?)

Gamma expansion

Pixel-wise operators

original

x

darken

$x - 128$

lower contrast

$\frac{x}{2}$

Gamma compression

invert

$255 - x$

lighten

$x + 128$

raise contrast (?)

$x \times 2$

Gamma expansion

Contrast

- **Contrast** in visual perception is the difference in appearance of two or more parts of a field seen.
- The human visual system is more sensitive to contrast than absolute luminance;
- **Contrast ratio**, or **dynamic range**, is the ratio between the largest and smallest values of the image or :

$$CR = \frac{V_{max}}{V_{min}}$$

Pixel-wise operators

original

x

darken

$x - 128$

lower contrast

$\frac{x}{2}$

Gamma compression

invert

$255 - x$

lighten

$x + 128$

raise contrast (?)

$x \times 2$

Gamma expansion

Pixel-wise operators

original

x

darken

$x - 128$

lower contrast

$\frac{x}{2}$

Gamma compression

$\left(\frac{x}{255}\right)^{1/3} \times 255$

invert

$255 - x$

lighten

$x + 128$

raise contrast (?)

$x \times 2$

Gamma expansion

Pixel-wise operators

original

x

darken

$x - 128$

lower contrast

$\frac{x}{2}$

Gamma compression

$\left(\frac{x}{255}\right)^{1/3} \times 255$

invert

$255 - x$

lighten

$x + 128$

raise contrast (?)

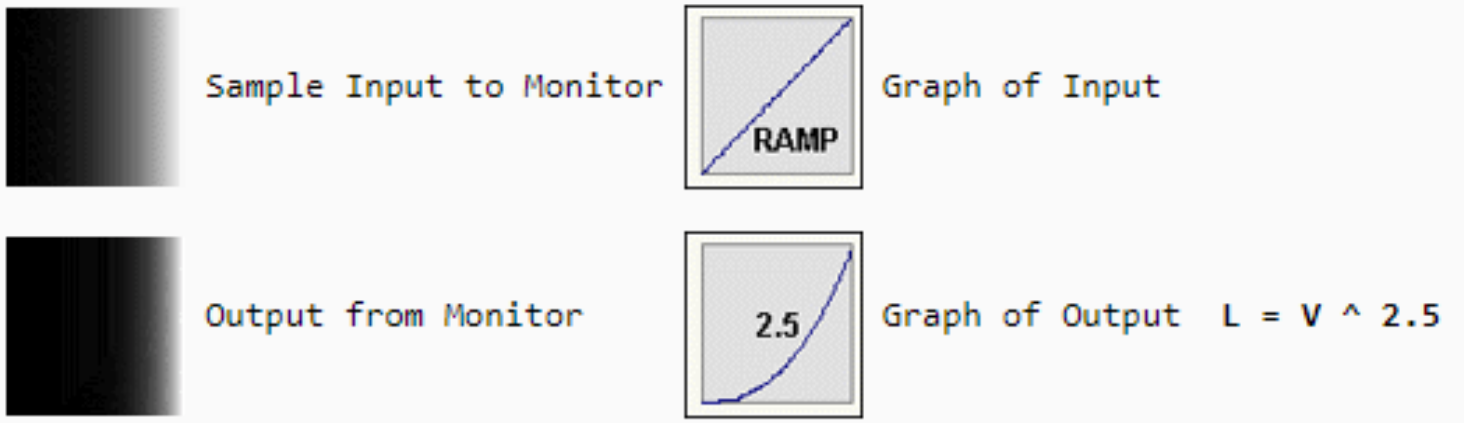
$x \times 2$

Gamma expansion

$\left(\frac{x}{255}\right)^{2} \times 255$

Gamma correction

- Originally, Due to non-linearities in the old CRT televisions, intensities was seen different then they are.

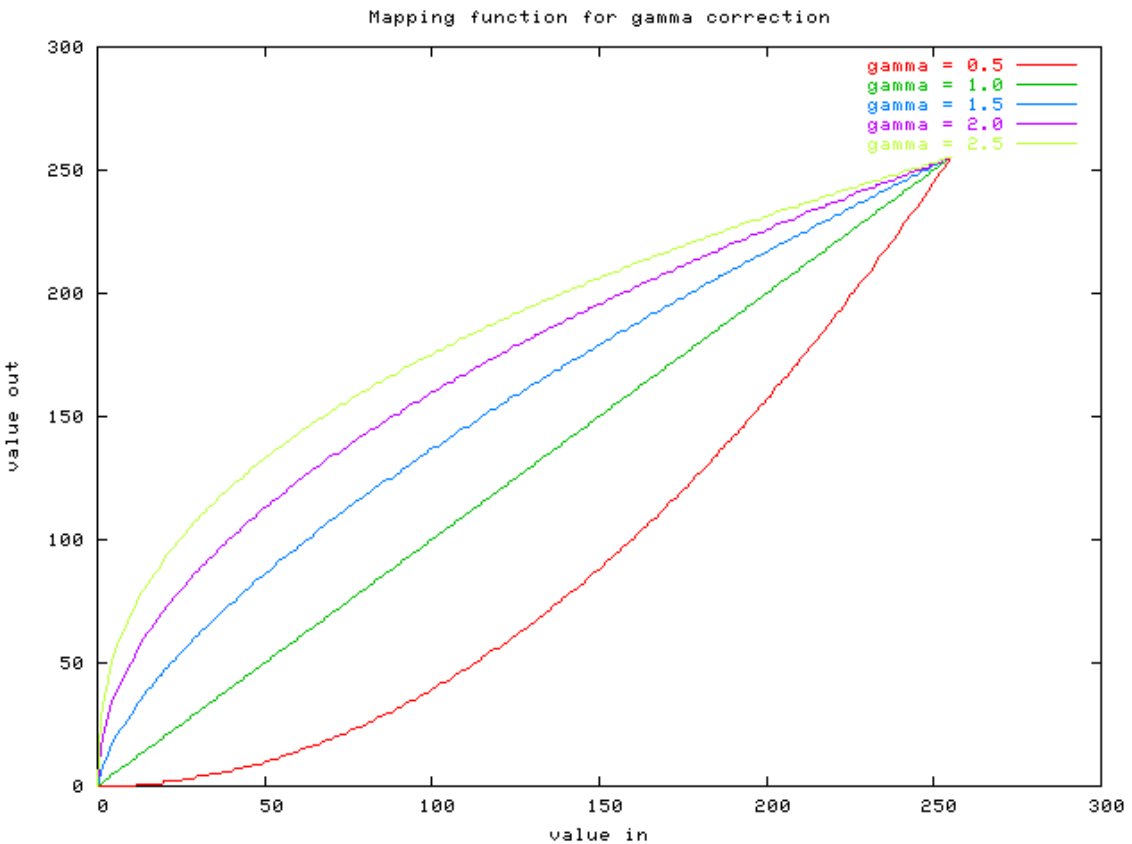


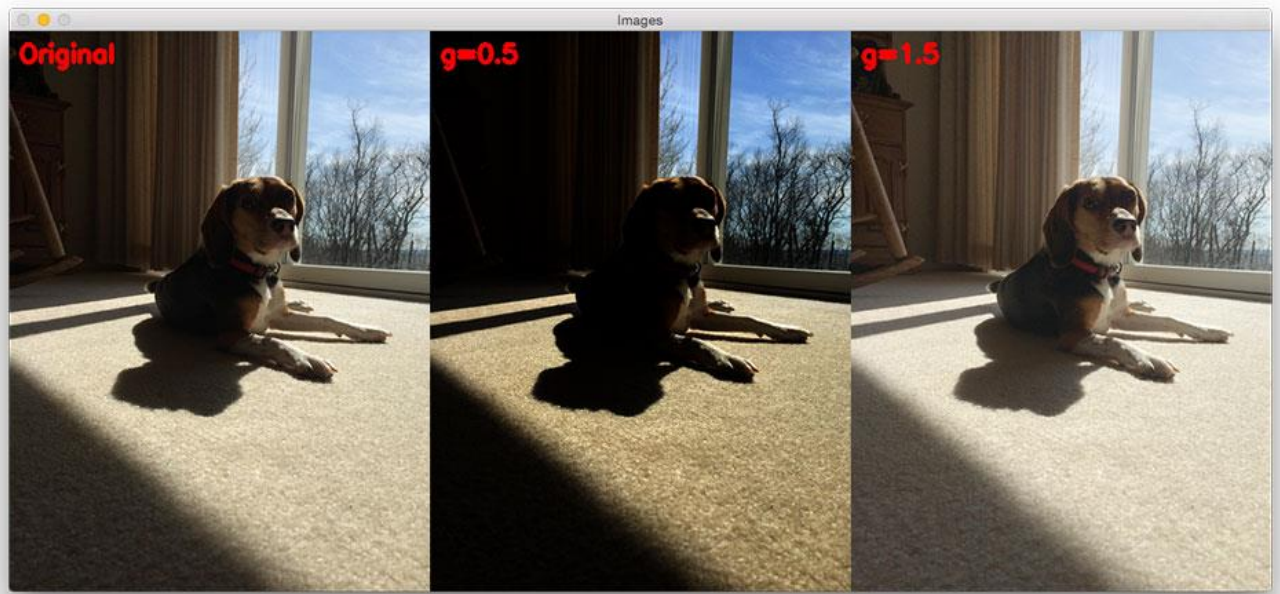
Gamma correction

- To correct this non linear transformation, gamma correction was done:

$$V_{out} = V_{in}^{\gamma}$$

- This is, of course, also applicable for image enhancements.





contents

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Gaussian Noise

- Gaussian noise is an additive noise that can appear in images due to the system electrical circuitry.
- This noise is independent of signal strength and independent at each pixel (IID).

$$p(z) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}$$

Gaussian Noise σ=0.01

SNR=34.0206

Gaussian Noise σ=0.05

SNR=19.1825

Gaussian Noise σ=0.1

SNR=13.7121

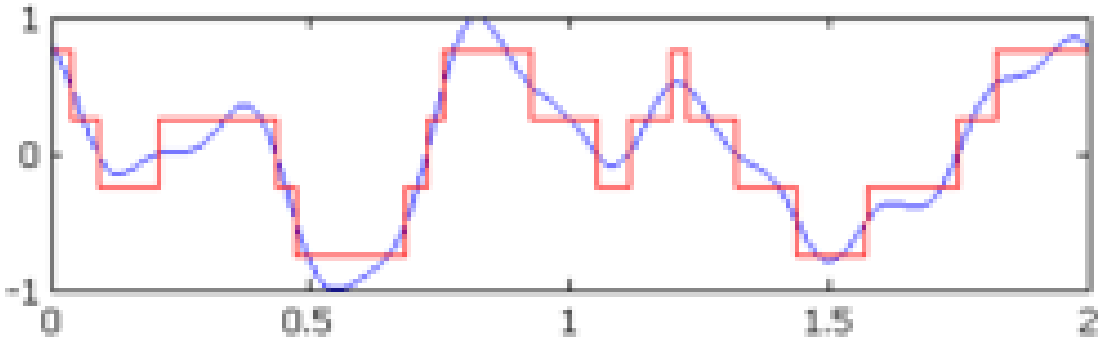
Salt & Pepper noise

- Noise that can be caused by analog-to-digital converter errors, bit errors in transmission, etc.
- This noise is **not** additive to the signal strength (a replacement of original value with noise value).
- This noise is independent of signal strength and independent at each pixel.

And some more noise

- **Shot noise** - caused by statistical quantum fluctuations, that is, variation in the number of photons sensed at a given exposure level in the darker parts of an image (where there are just few photons that enter each pixel “bin”). Modeled as Poisson noise.

- **Quantization noise** – caused by quantizing the pixels of a sensed image to several discrete levels (analog to digital conversion).



Noise reduction with LTI filters

- **linear filtering operators**, which involve weighted combinations of pixels in small neighborhood.
- The combination is determined by the filter's *kernel*.
- The same kernel is *shifted* to all pixel locations so that all pixels use the same linear combination of their neighbors.
- That's why it's called **linear shift-invariance** (LTI) filter.

Convolution

- Works on LTI filters.
- Let h be the image, f be the kernel (of size 2*k*+1 x 2*k*+1), and g be the output image:

$$g(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} h[u,v] f[i-u, j-v]$$

  - **Note:** by definition, this operator flips the kernel both horizontally and vertically.
- This operation is called **convolution operator** and is more compactly noted as:

$$g = h * f$$

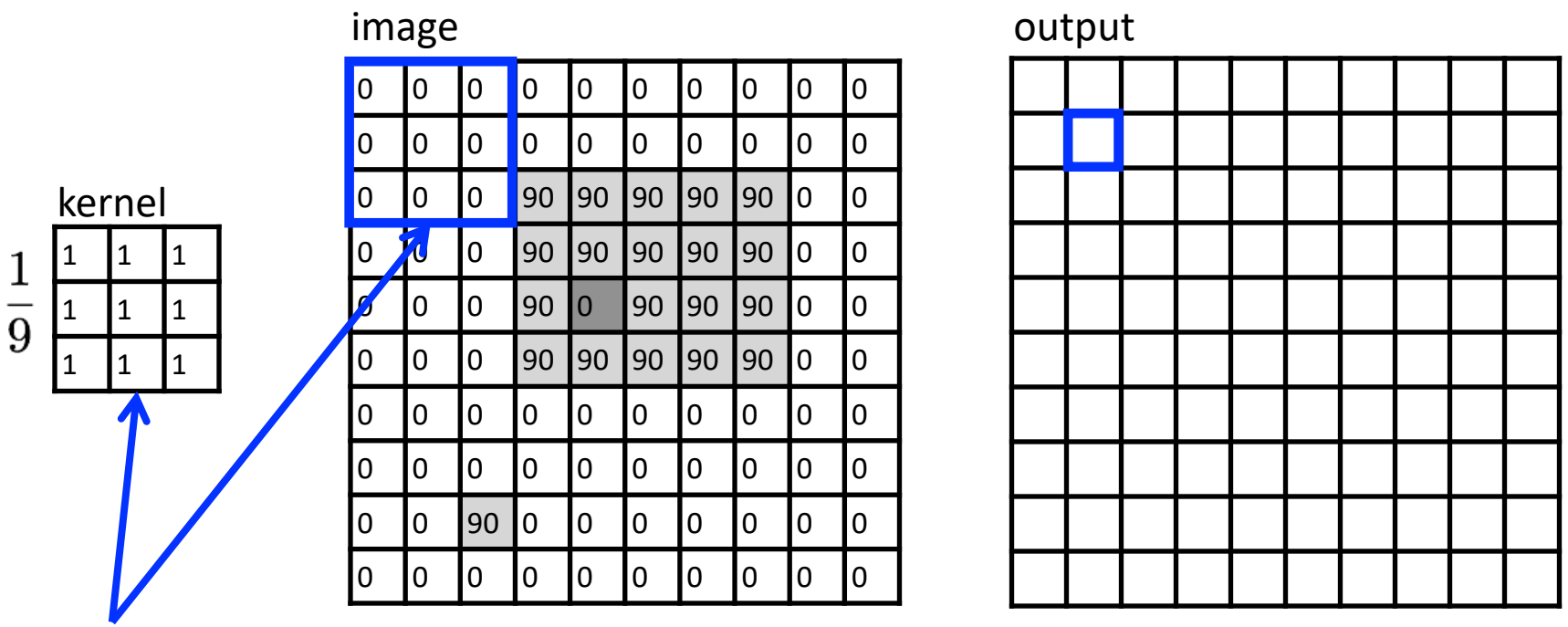
Example: mean filter

- The kernel is:

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

- Replaces pixel with local average.
- Has smoothing (blurring) effect
- The kernel can be in any other size as well (see .ipynb).

Run the filter



Note that we assume that the kernel coordinates are centered.
Here the kernel is symmetric horizontally and vertically, so the flipping is not noticeable.

Run the filter

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

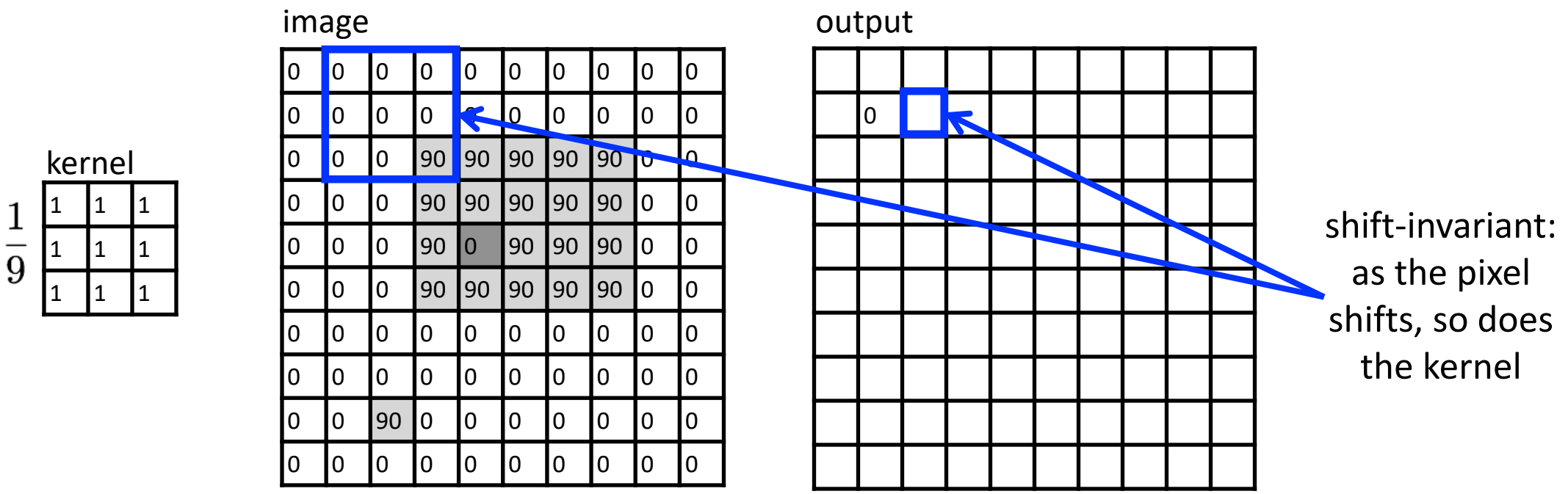
image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0								

Run the filter



Run the filter

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10							

Run the filter

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10	20						

Run the filter

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10	20	30					

Run the filter

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10	20	30	30				

Run the filter

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10	20	30	30	30			

… and the result is

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

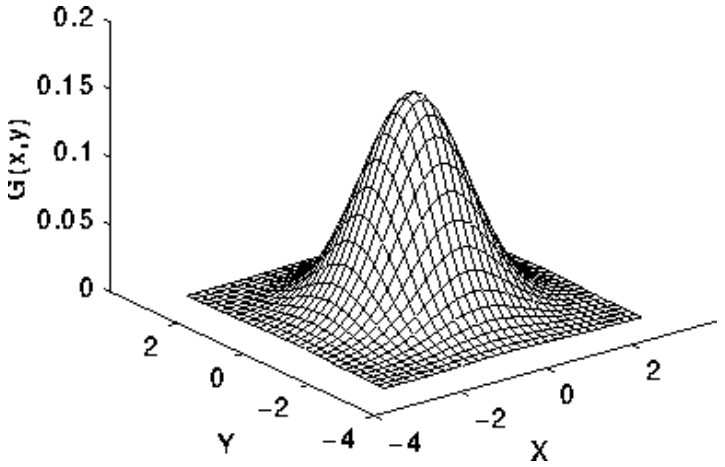
image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	0	10	20	30	30	30	20	10	
	10	10	10	10	0	0	0	0	
	10	10	10	10	0	0	0	0	

Gaussian filter



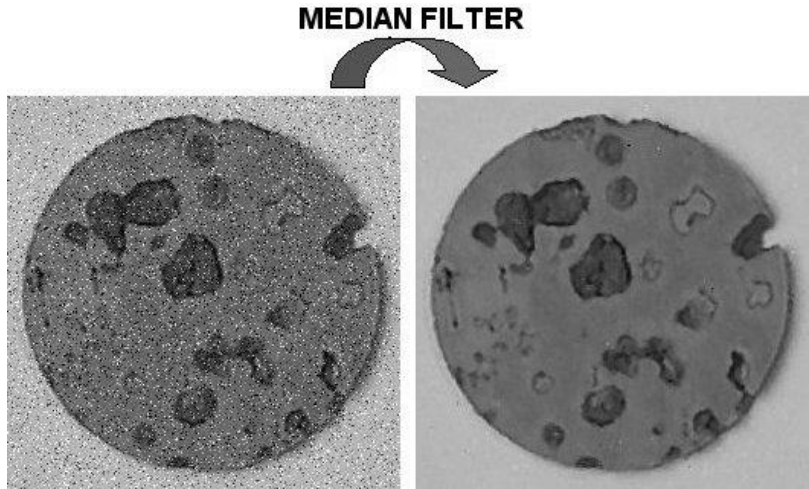
- Another kind of blur filter.
- this filter can be controlled by its size and STD.

$$h(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

- The kernel is discretized to bins according to the wanted kernel size.
- Isn't Gaussian function infinite?
  - Most often, the kernel cuts out the remaining lower bins (usually at 2-3 σ).
- **Both Gaussian and mean filters are good against Gaussian noise, but not effective against S&P noise.**

Median filter

- Takes the median value from the given neighbors (and hence can also be considered as a “blurring effect”).
- For example:
  - The median of [1, 0, 100] is 1.
- **Median filter is good against salt and pepper noise, and against Gaussian noise (but not as effective).**
- **Median filter is also more computationally expansive.**
- This filter is not LTI because it’s not linear on its weights.



Example in .ipynb

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- Image representation
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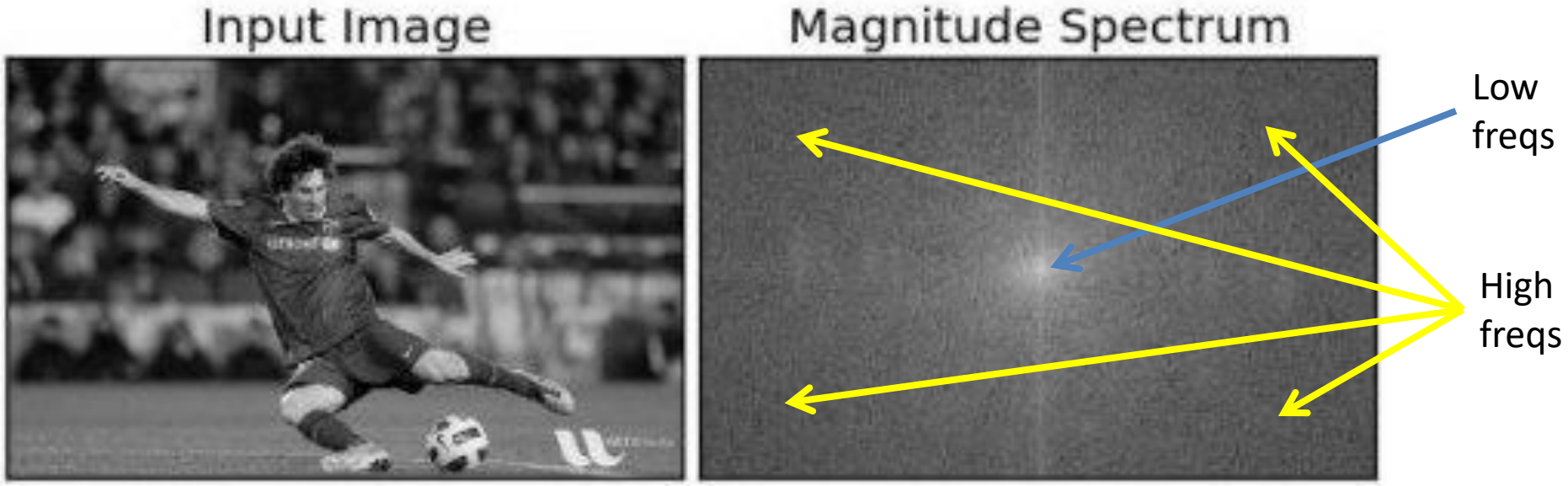
FFT of an image

- Frequency analysis is not in the scope of the course and is not needed for the rest of the course, but here it can give some intuition.
- In audio signals (or any other 1D signals) Lower frequencies change less over time than higher frequencies.
  - In images, the change is represented in change in distance, so images that changes slowly from pixel to pixel has more lower frequencies then others.
- Natural images are mainly built from low frequencies.



Convolution in frequency domain

- Recall: in time (space) domain:

$$g(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} h[u,v] f[i-u, j-v]$$

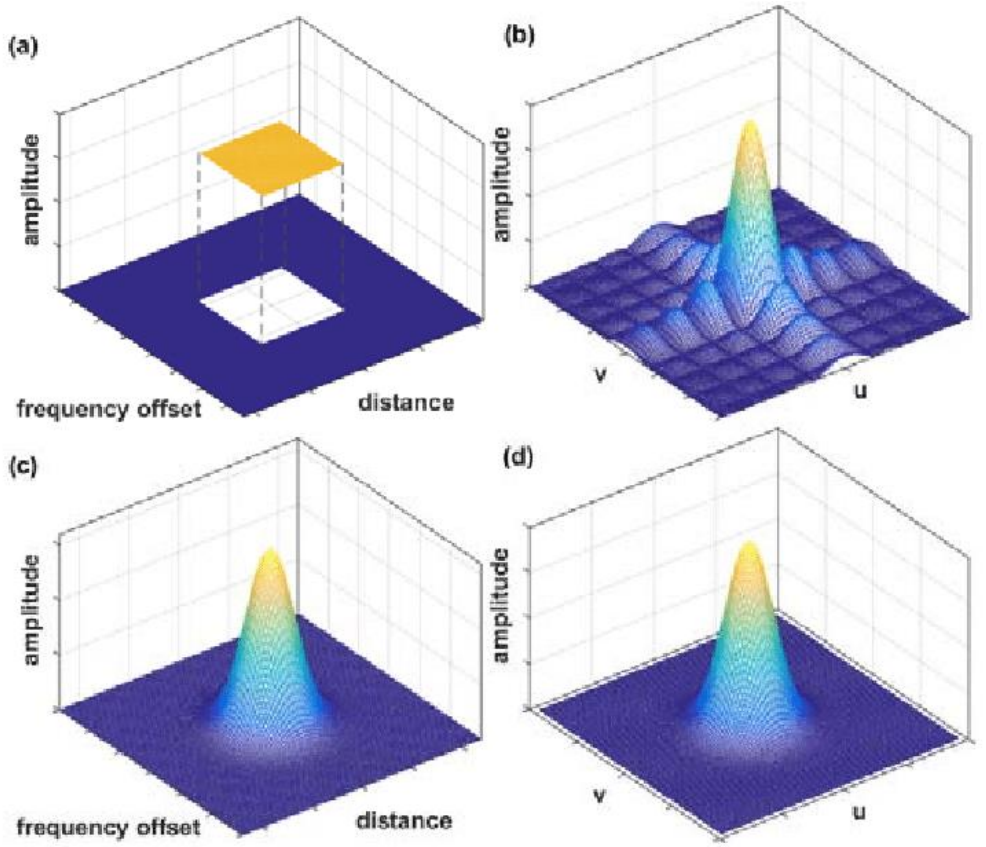
$$g = h * f$$

- In frequency domain- simple multiplication:

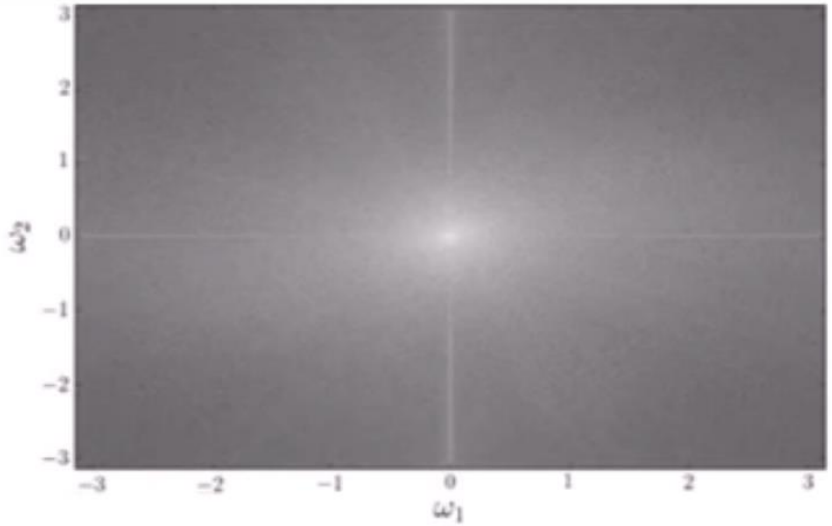
$$G = H \cdot F$$

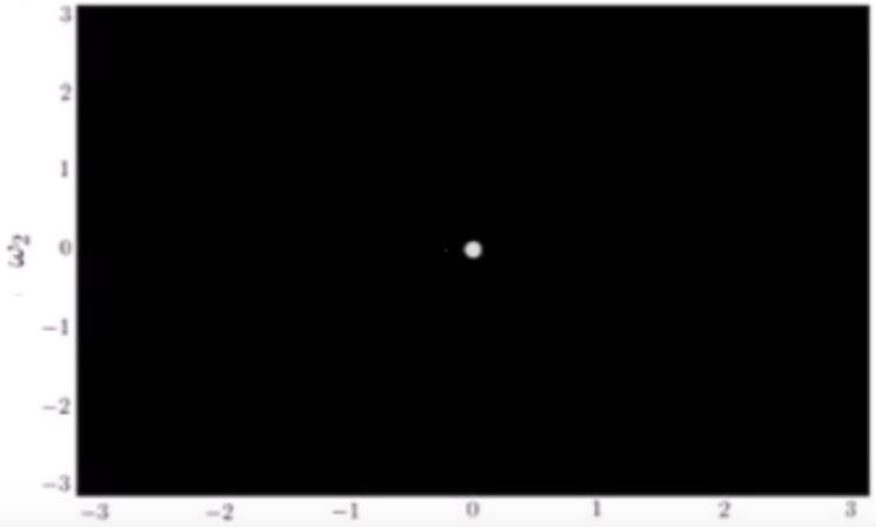
Low-pass filters

- Both mean and Gaussian filters are considered low-pass filters because in the frequency domain, they have higher values in the lower frequencies- and when multiplied with frequency spectrums, the high frequencies get smaller.
- When image is left only with the lower frequencies, the rapidly changes parts of the image (e.g.: edges, noise) are smoothen.



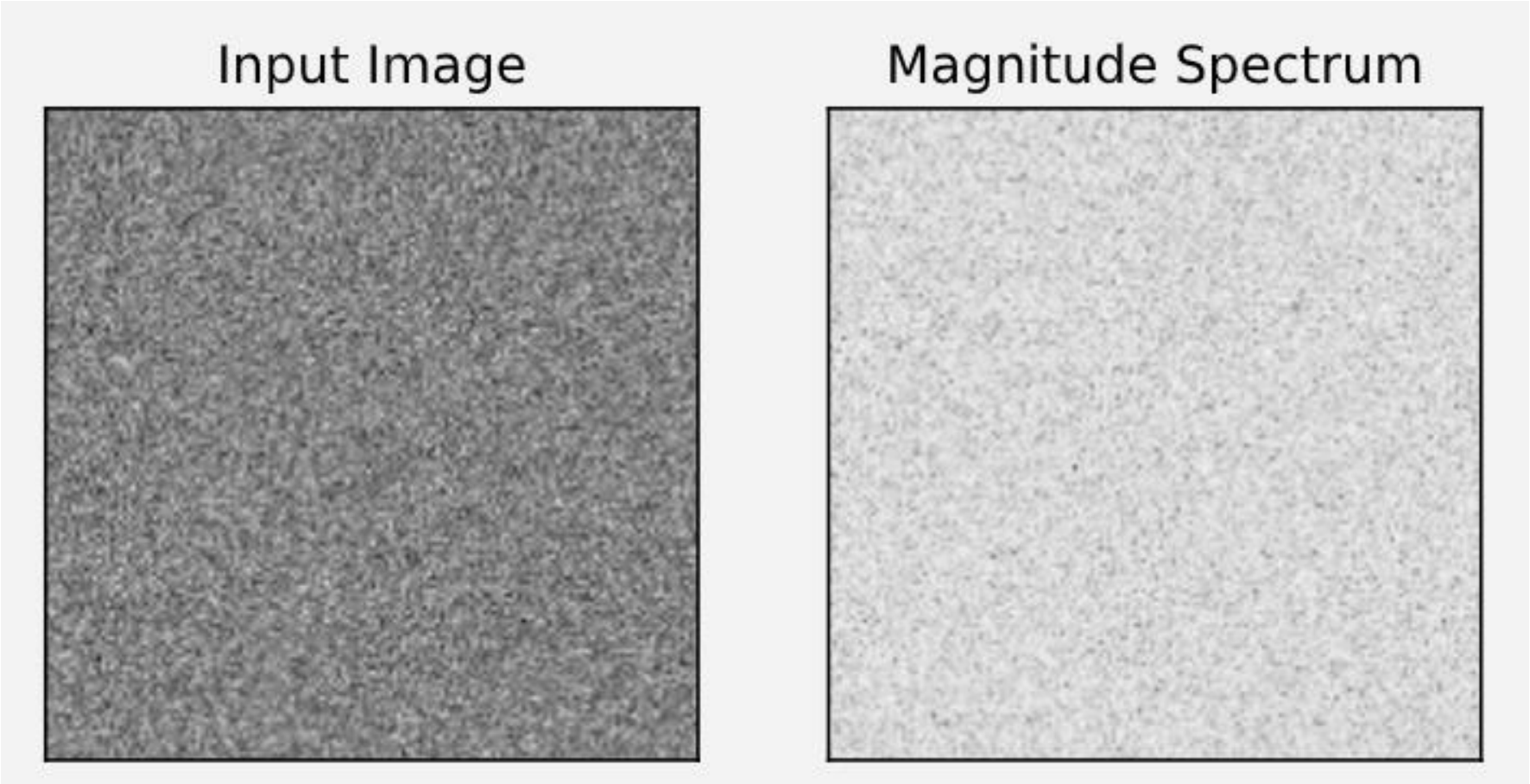
LP example





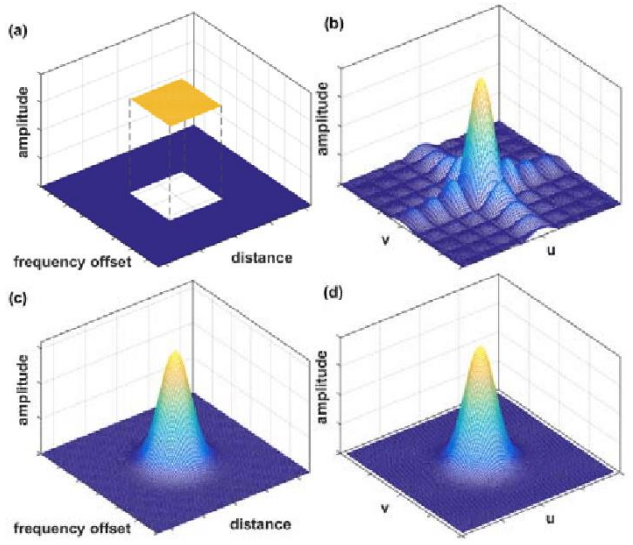
FFT of gaussian noise

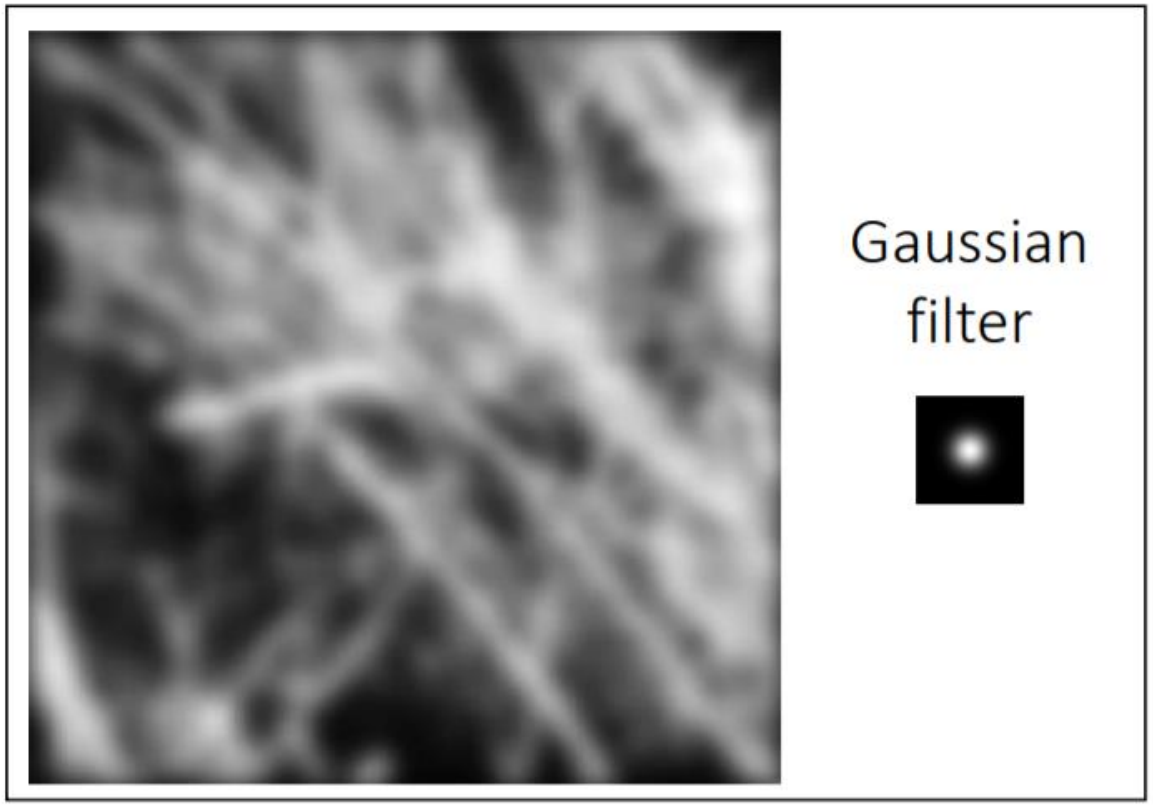
- Since gaussian noise (AWGN) is distributed along all frequencies, LP filter reduce this kind of noise significantly.



Mean vs. Gaussian filter

- Since Mean filter has higher values in the higher frequencies, edge artifacts sometimes remains.





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Imag

This image is too big to fit on the screen. How can we generate a half-sized version?

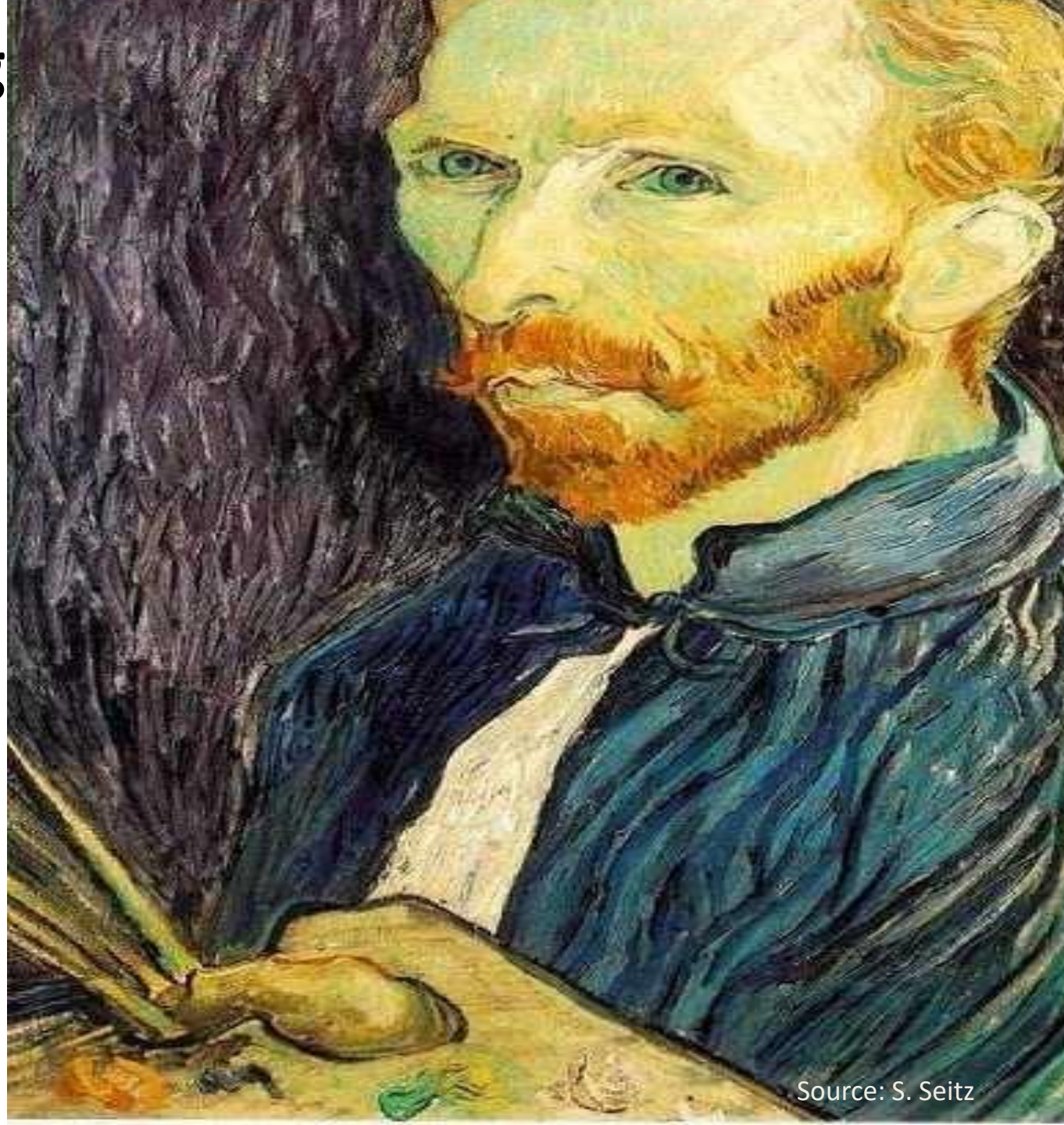
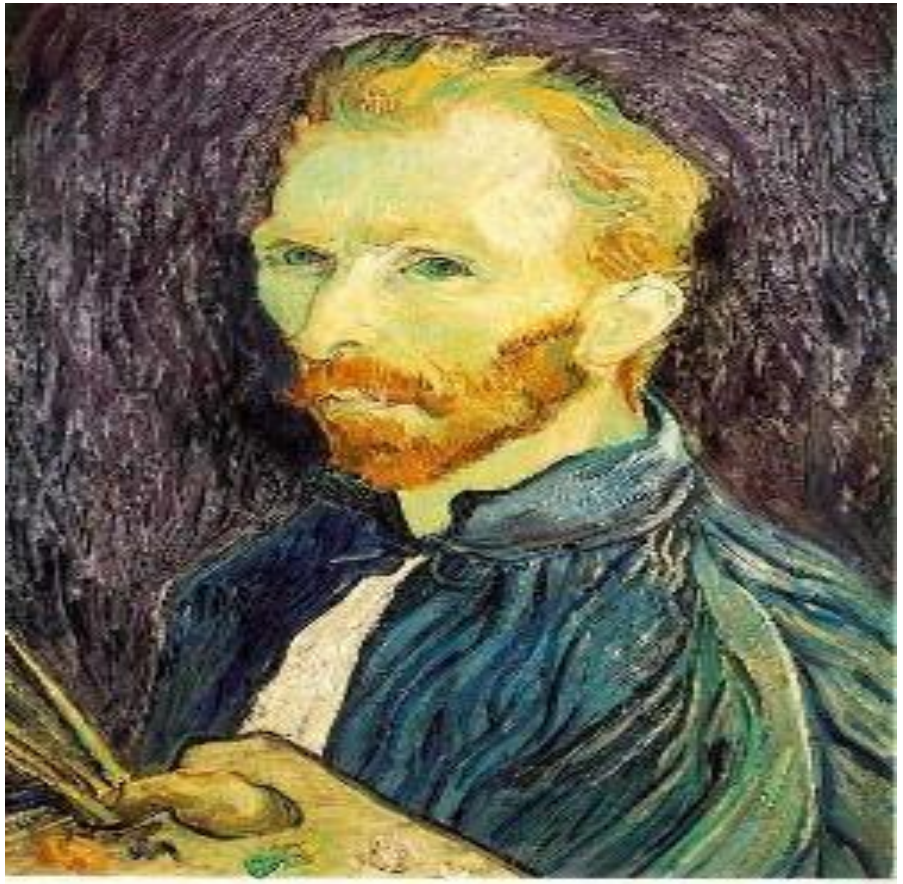


Image sub-sampling



1/4

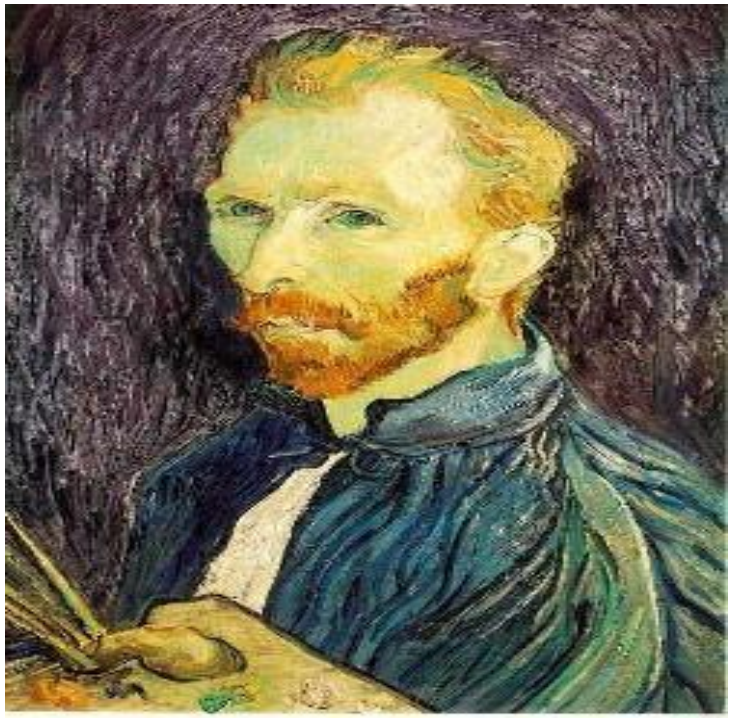
1/8

Throw away every other row and column to create a 1/2 size image

- called **image sub-sampling** or **decimation**

Source: S. Seitz

Image sub-sampling



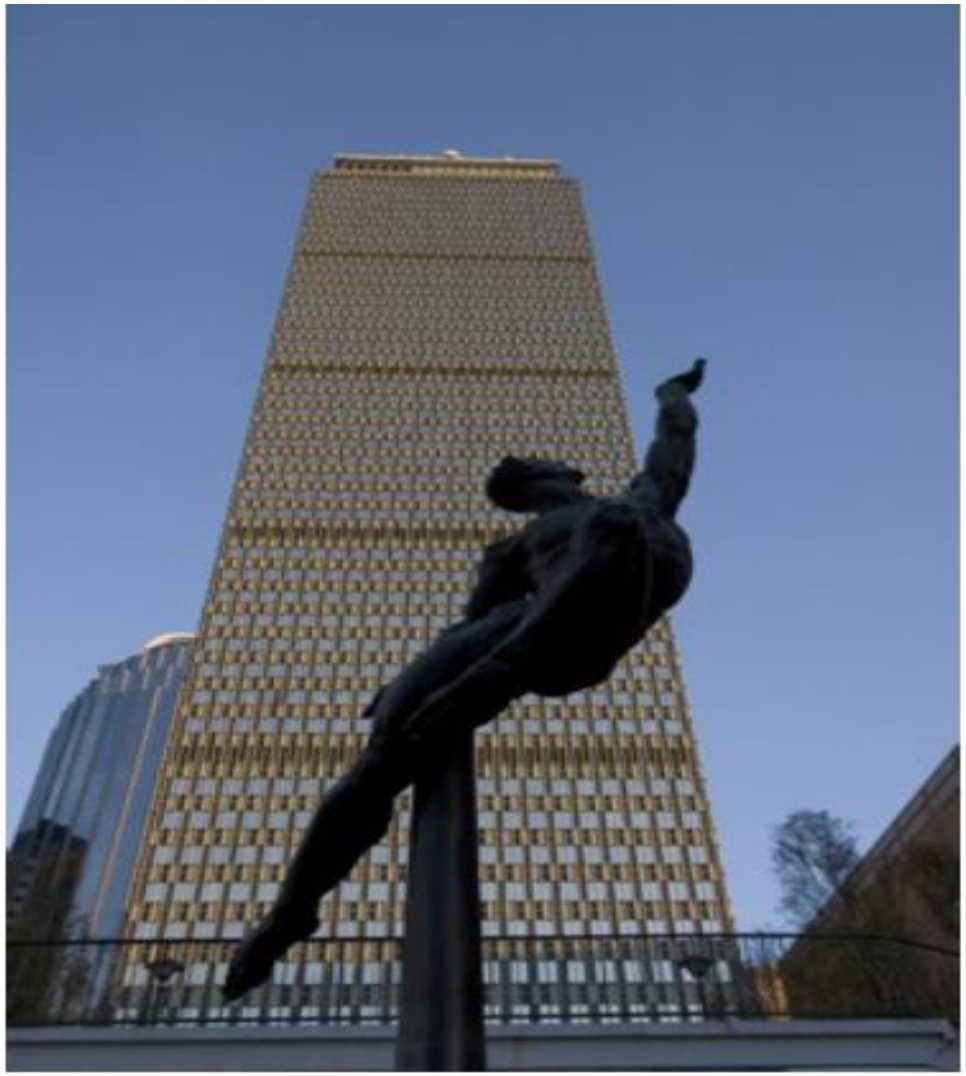
1/2

1/4 (2x zoom)

1/8 (4x zoom)

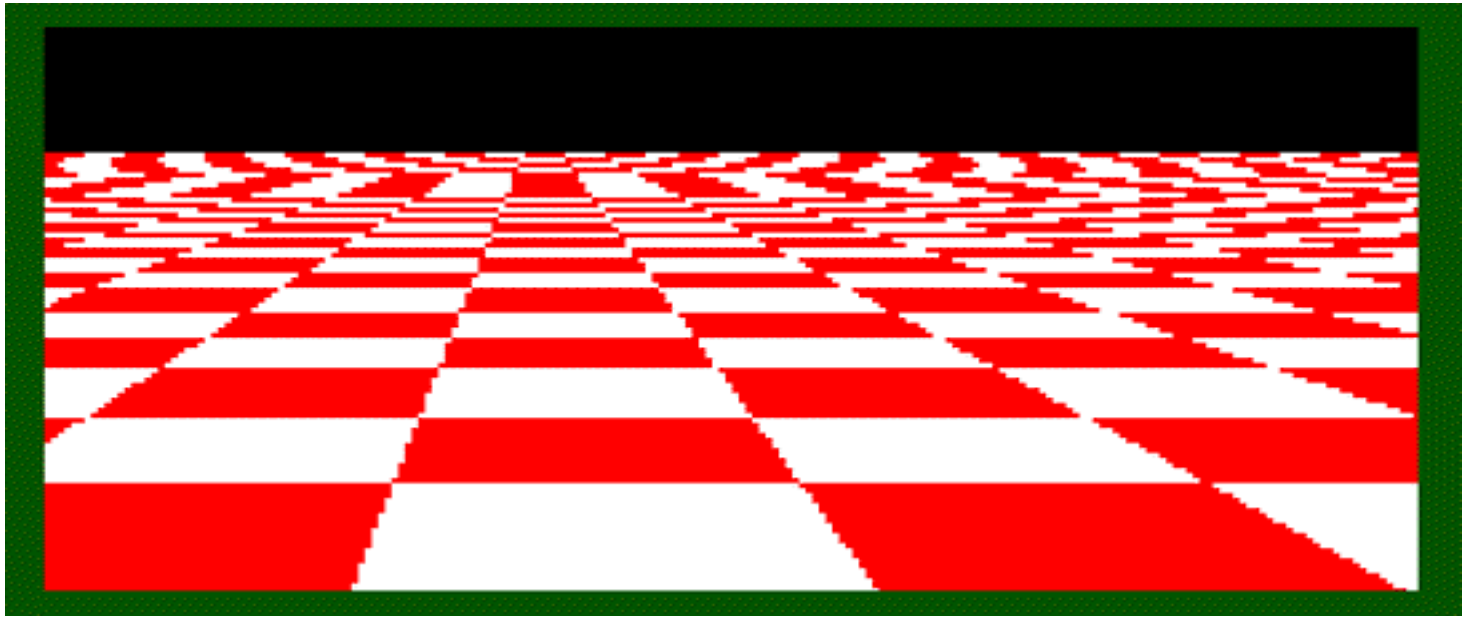
Source: S. Seitz

Image sub-sampling



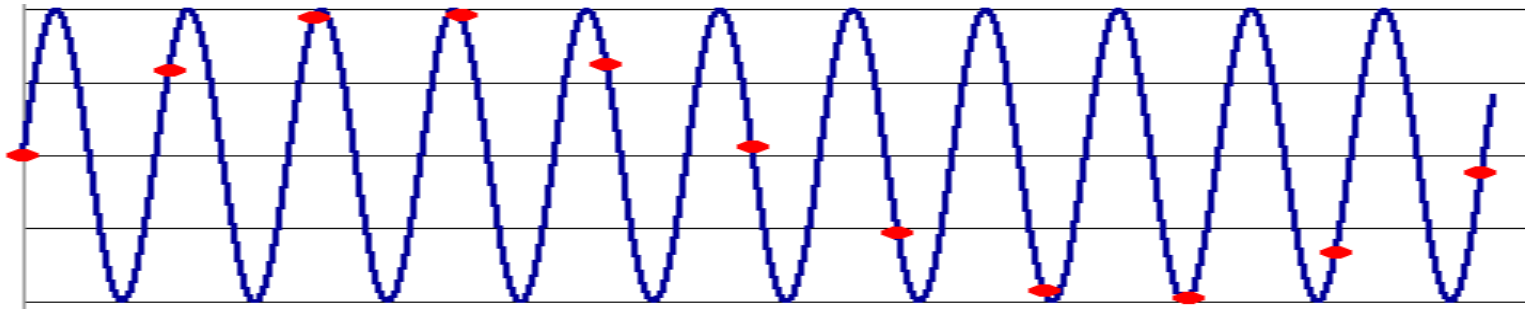
Source: F. Durand

Even worse for synthetic images



Source: L. Zhang

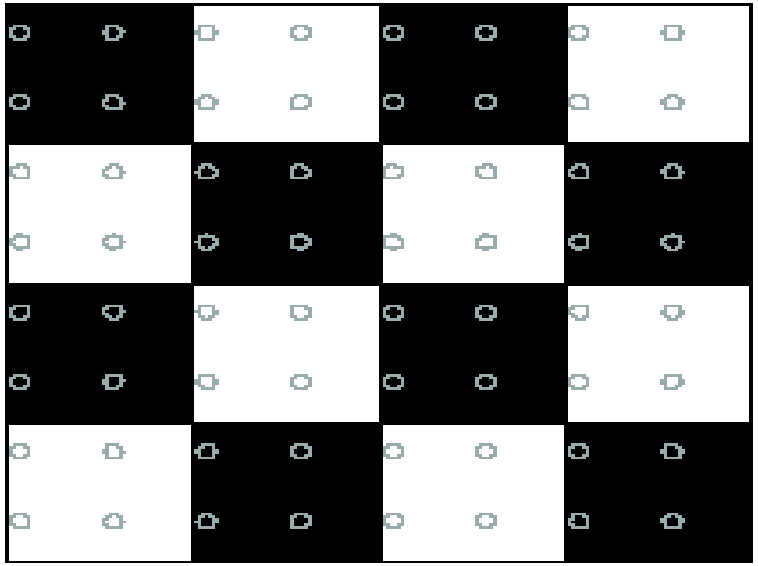
Aliasing

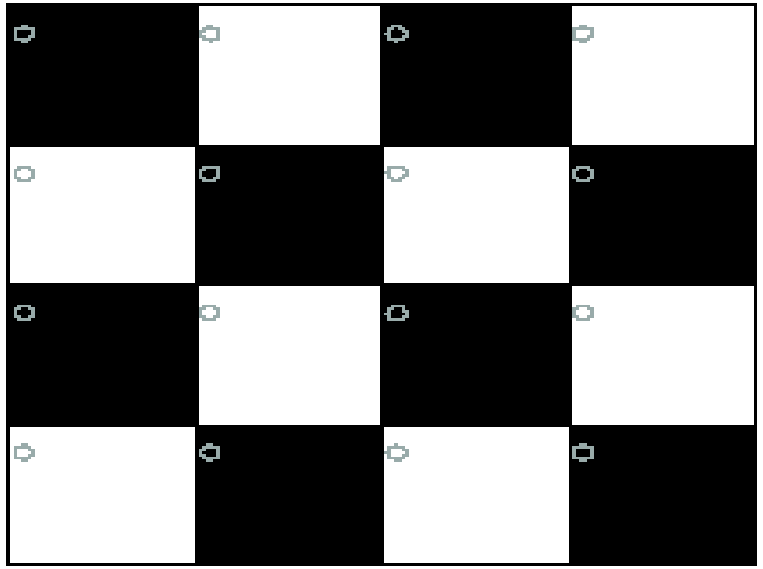


- Occurs when your sampling rate is not high enough to capture the amount of detail in your image
- Can give you the wrong signal/image—an *alias*
- To do sampling right, need to understand the structure of your signal/image
- To avoid aliasing:
  - **sampling rate ≥ 2 * max frequency** in the image
  - This minimum sampling rate is called the **Nyquist rate**

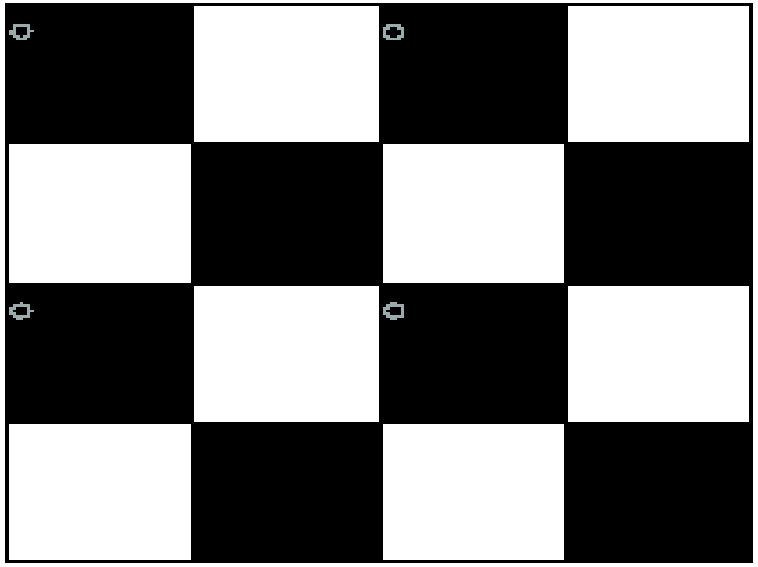
Source: L. Zhang

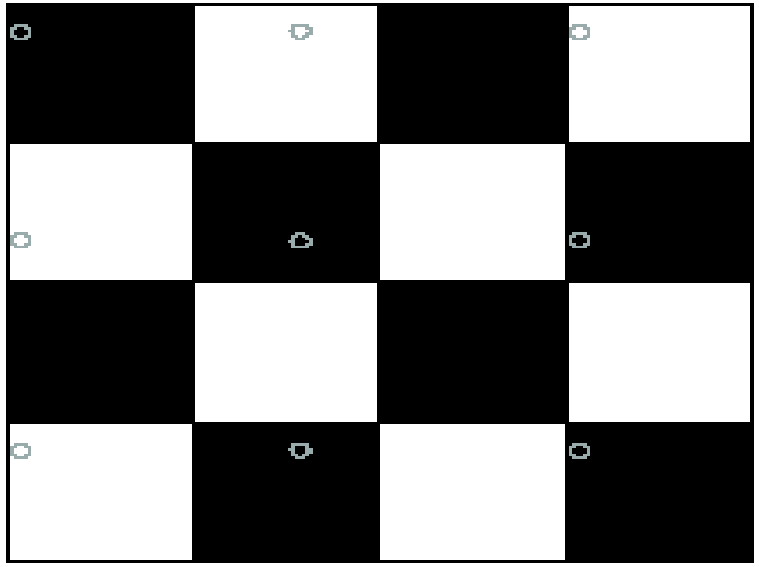
Nyquist limit – 2D example





Good sampling

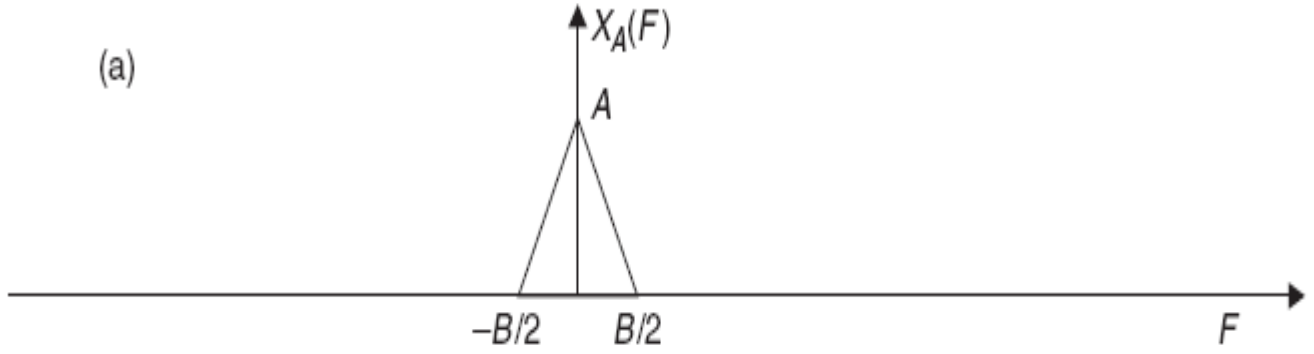




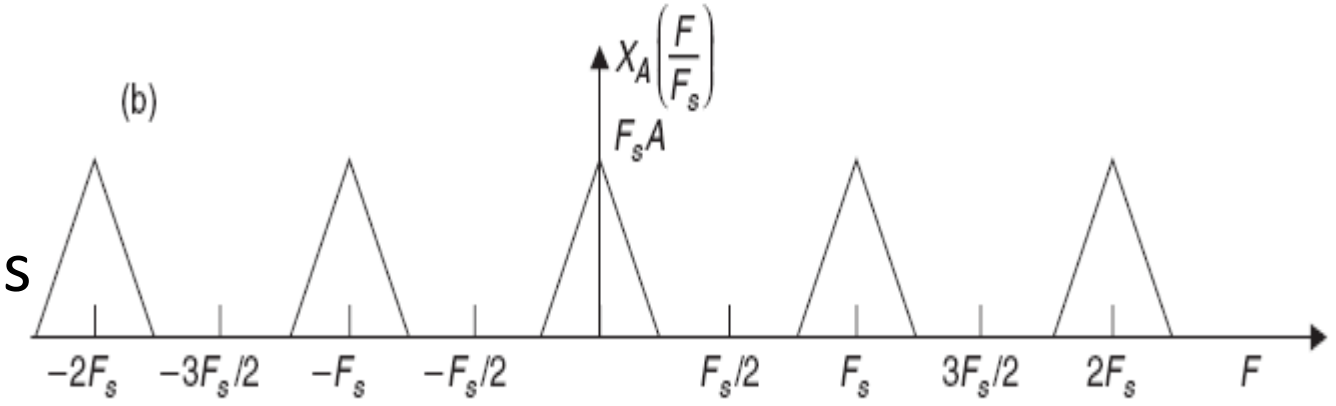
Bad sampling

Nyquist limit- frequency response

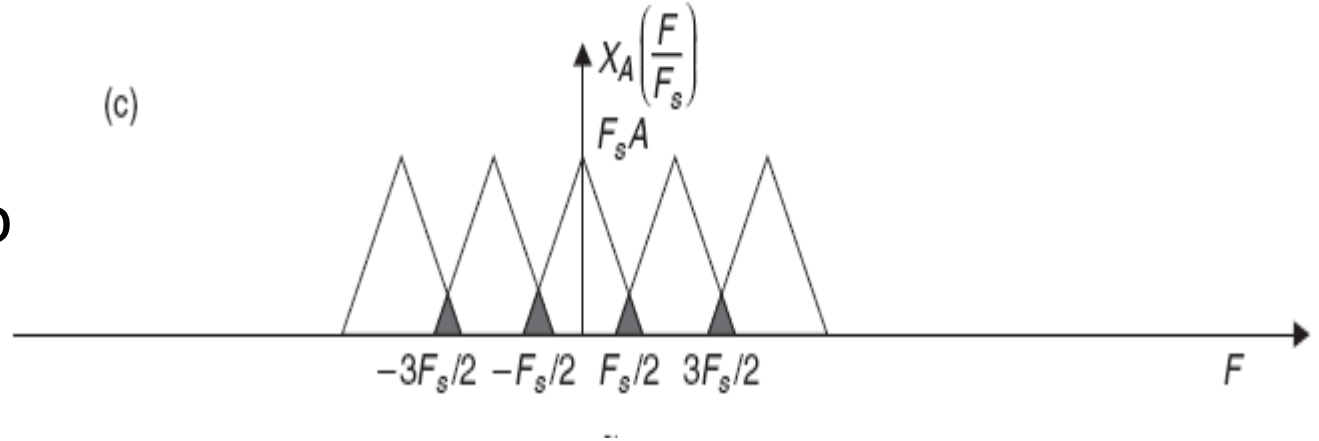
- Original freq representation of signal.



- Regular sampling above nyquist rate- can recreate the original frequencies of the image. (copies are from sampling).

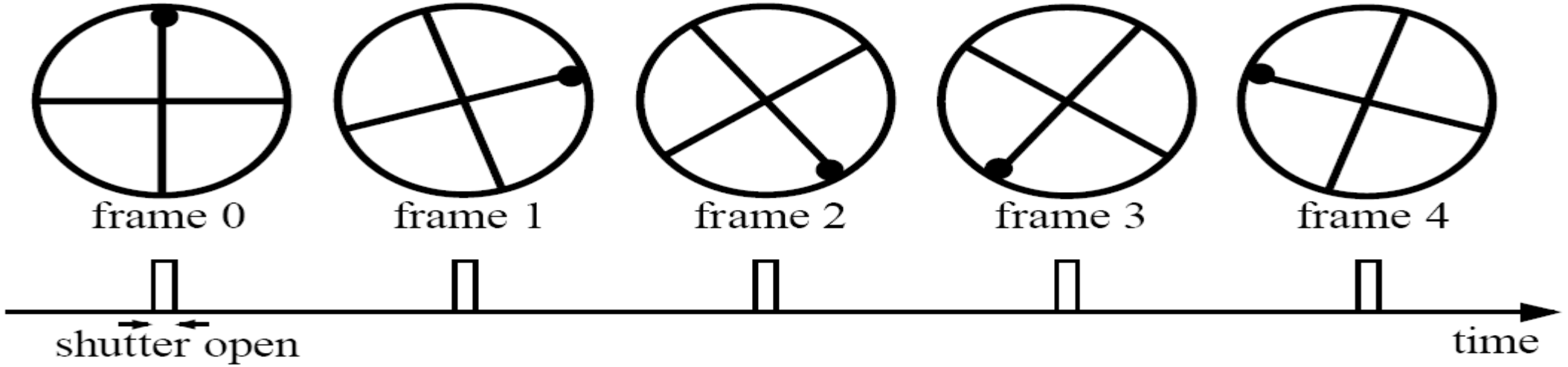


- Sampling below nyquist- original frequencies are destroyed due to the copies overlap- **this is the aliasing**.



Example: wagon-wheel effect

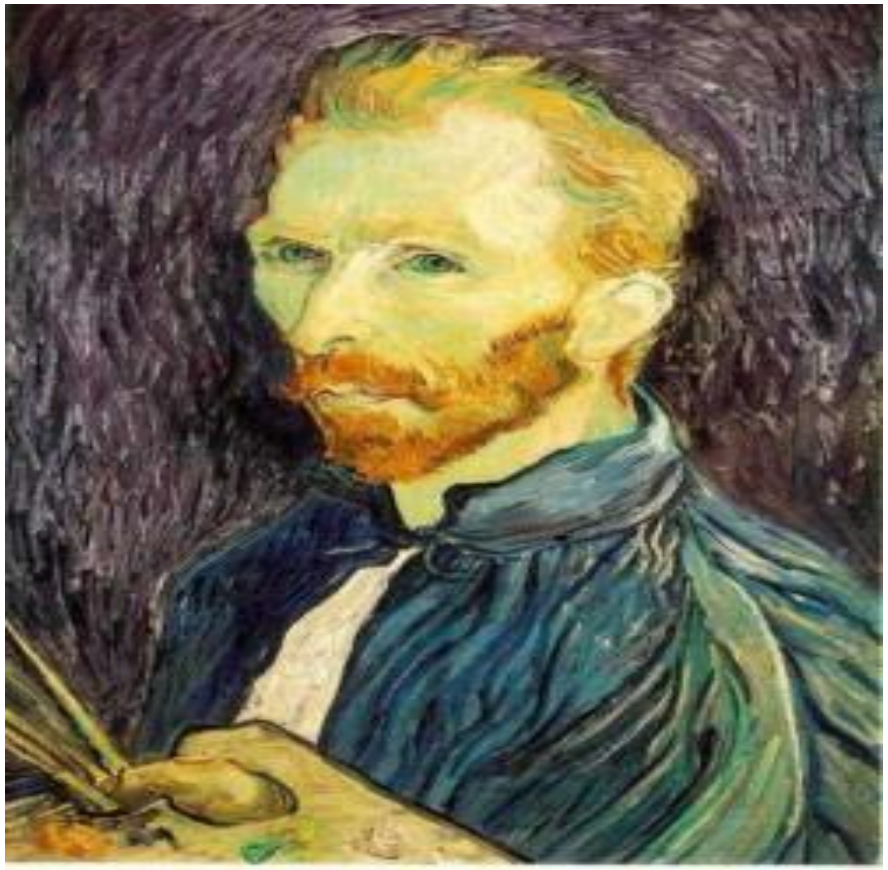
- An example of sub sampling in time domain (instead of spatially like before).



Without dot, wheel appears to be rotating slowly backwards!
(counterclockwise)

- https://en.wikipedia.org/wiki/Wagon-wheel_effect

Gaussian pre-filtering



Gaussian 1/2

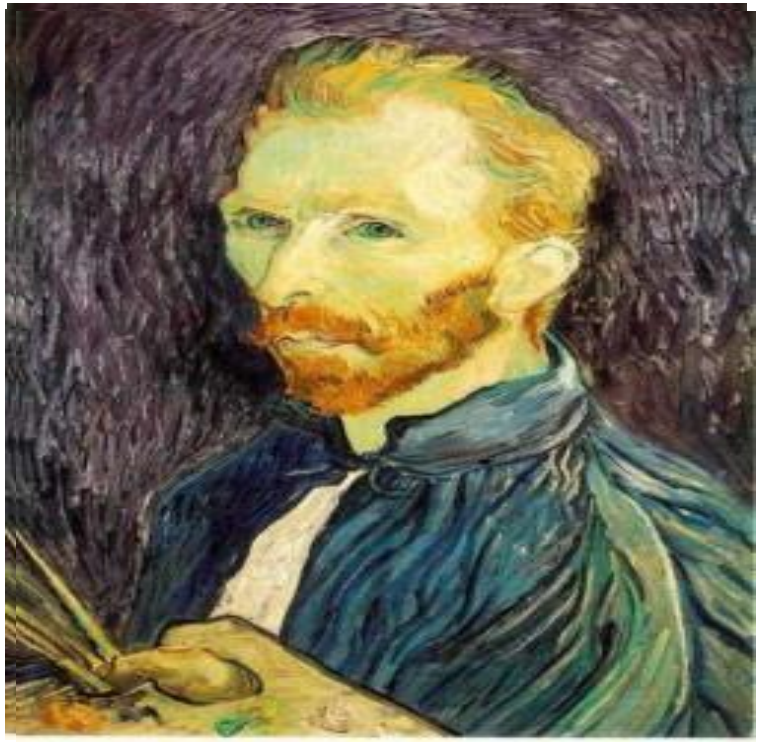
G 1/4

G 1/8

- Solution: filter the image, *then* subsample

Source: S. Seitz

Subsampling with Gaussian pre-filtering



Gaussian 1/2

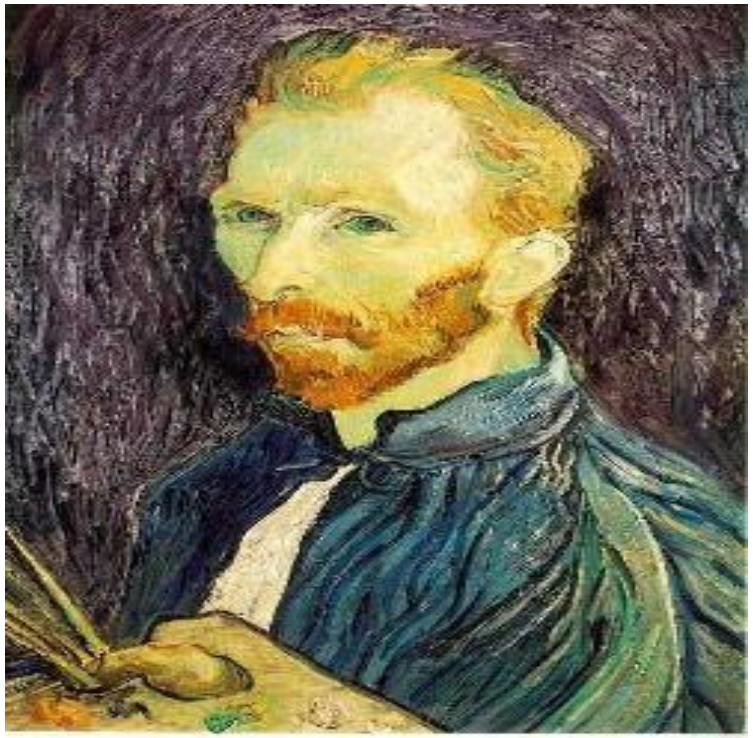
G 1/4

G 1/8

- Solution: filter the image, *then* subsample

Source: S. Seitz

Compare with...



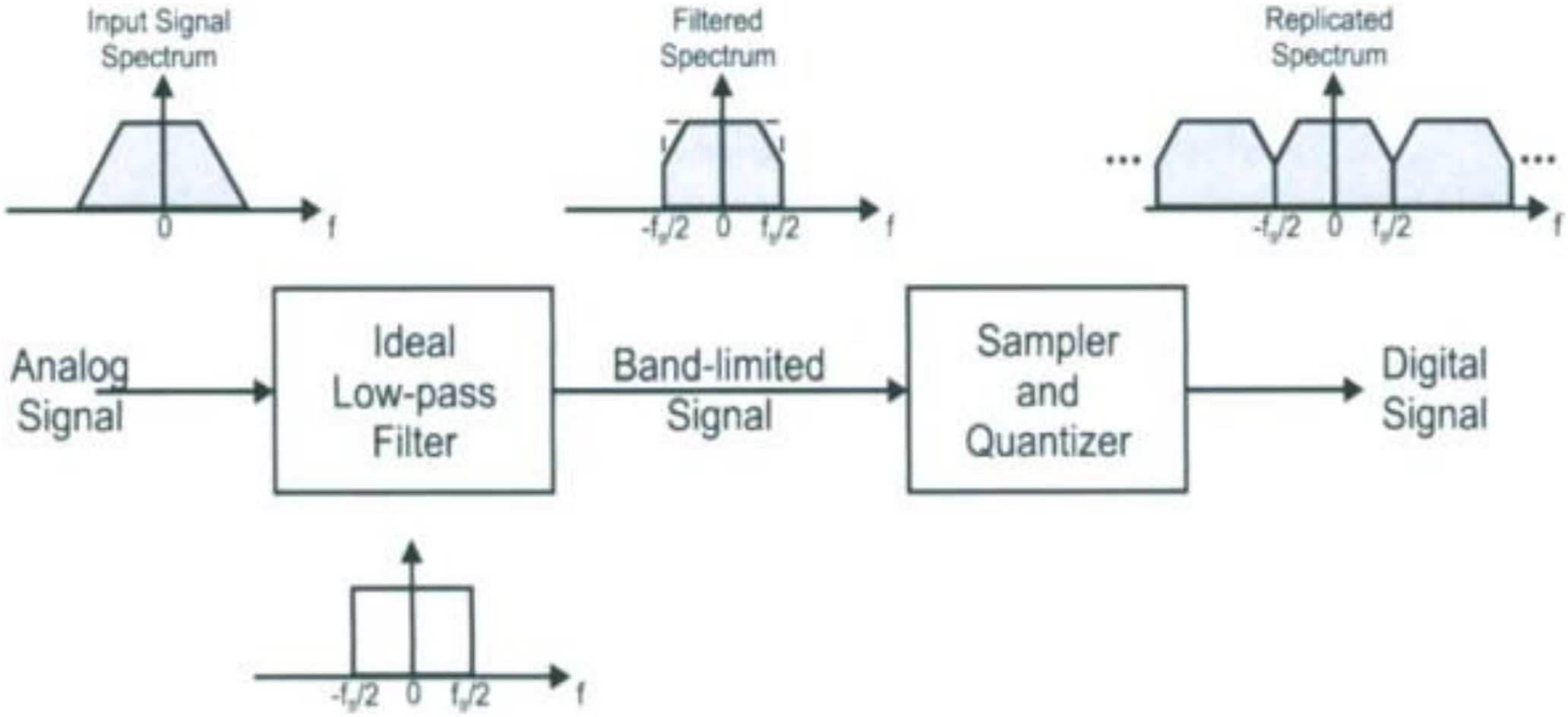
1/2

1/4 (2x zoom)

1/8 (4x zoom)

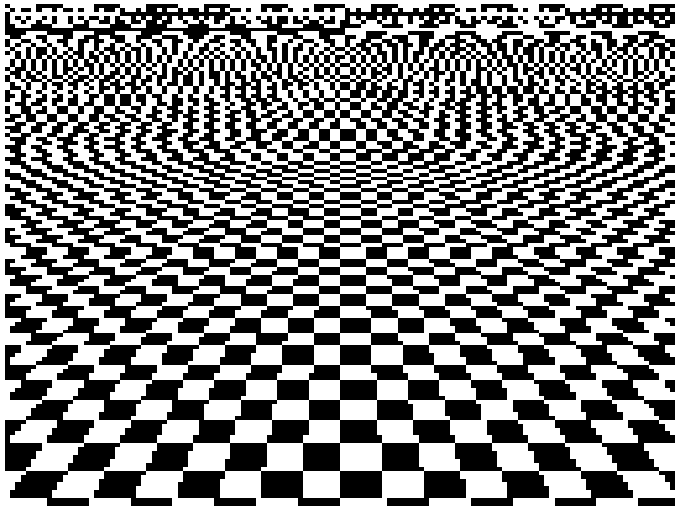
Source: S. Seitz

Low pass filtering- frequency response

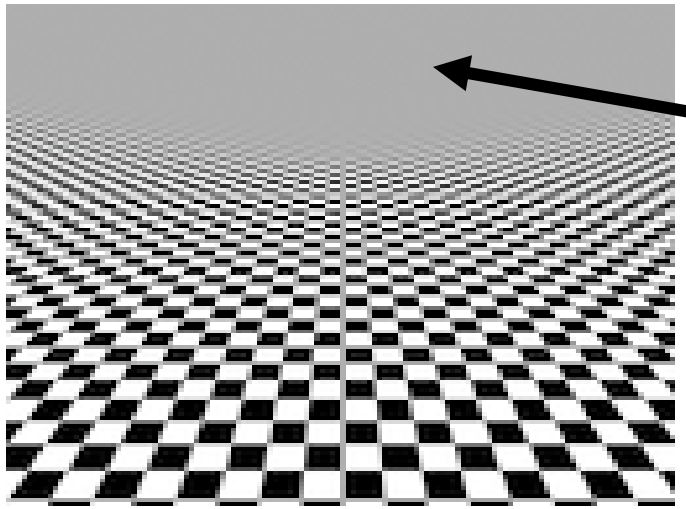


Back to the checkerboard

- What should happen when you make the checkerboard smaller and smaller?



Naïve subsampling



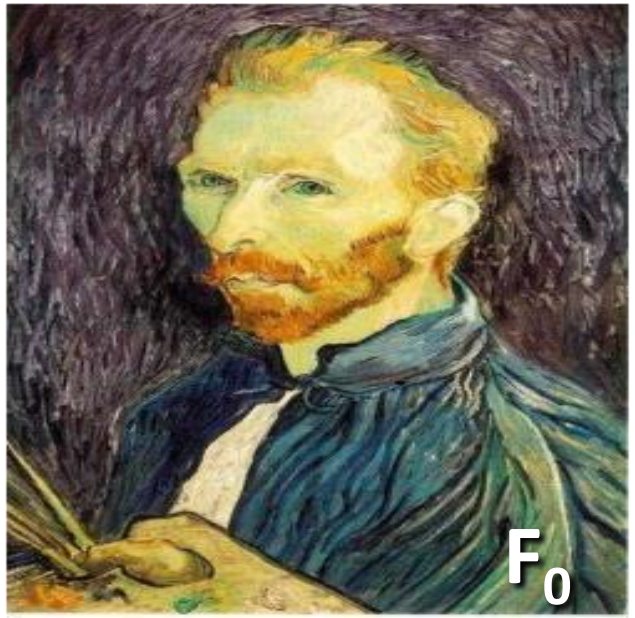
Proper prefiltering
(“antialiasing”)

Image turns grey! (Average of black and white squares, because each pixel contains both.)

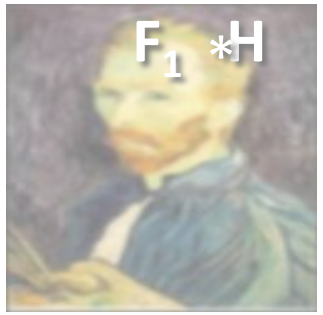
Gaussian pre-filtering

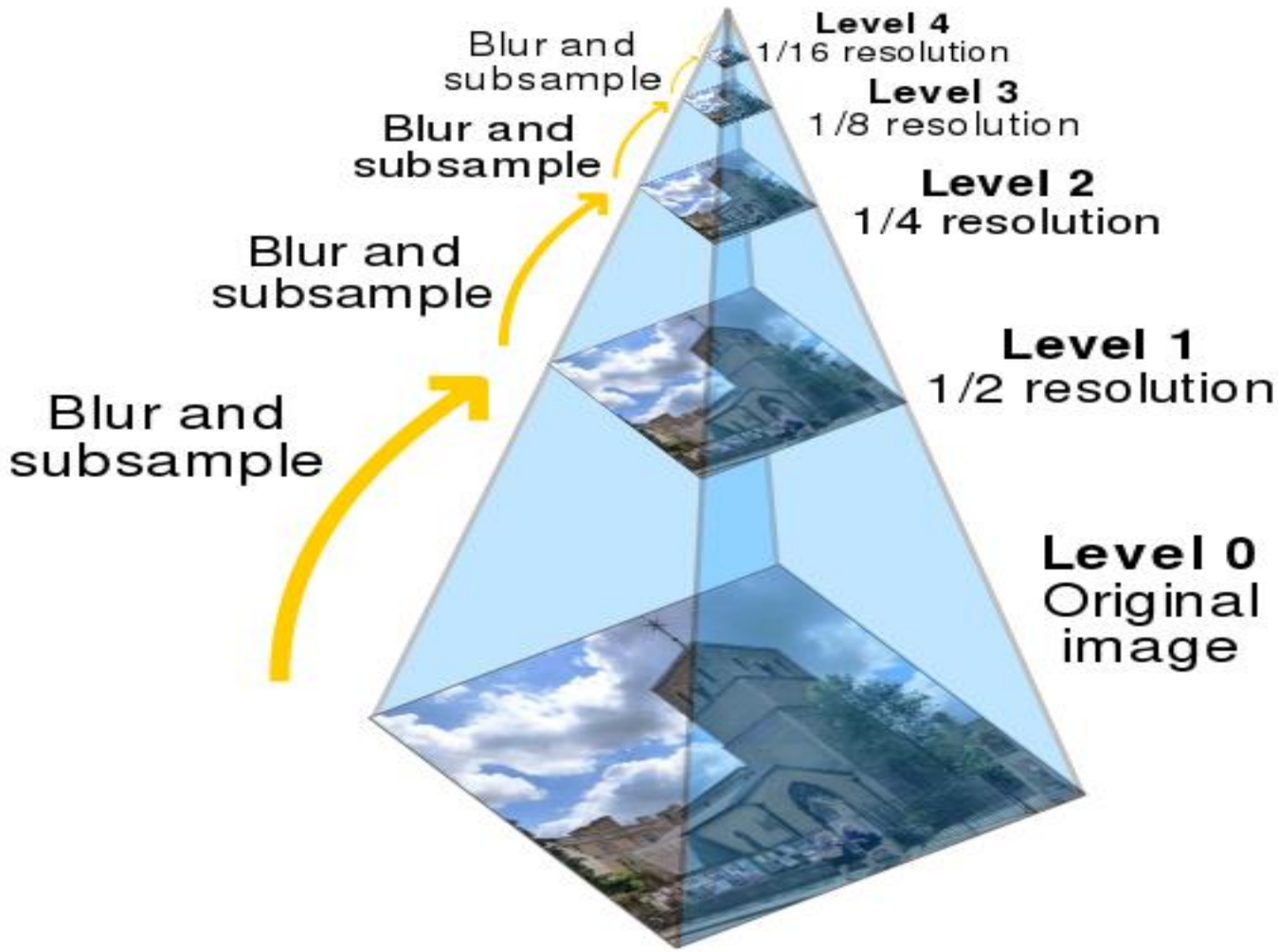
- Solution: filter the image, *then* subsample

Gaussian pyramid

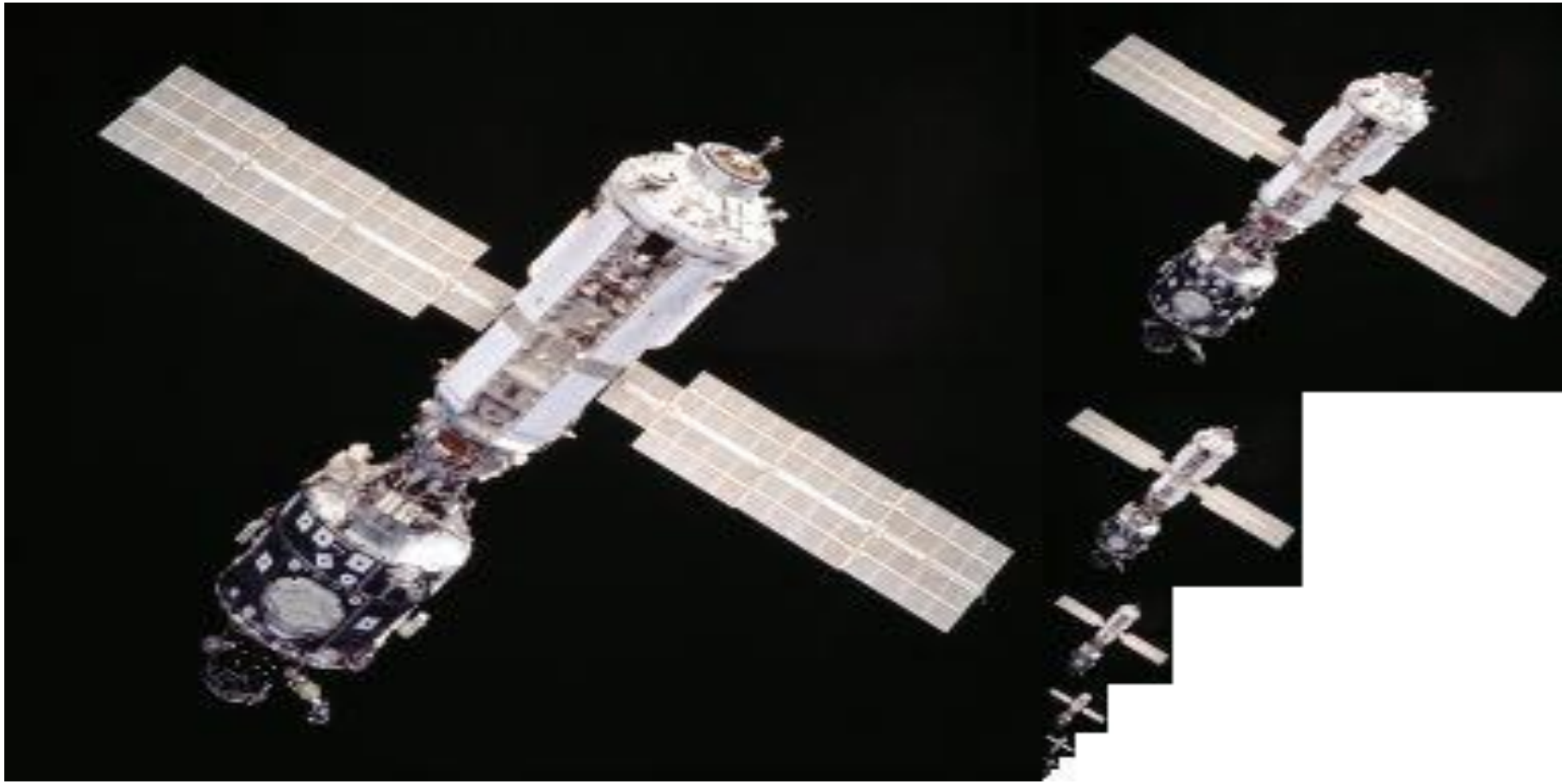


blur subsample blur subsample • • •




Blur and subsample
Blur and subsample
Blur and subsample
Blur and subsample
Level 4
1/16 resolution
Level 3
1/8 resolution
Level 2
1/4 resolution
Level 1
1/2 resolution
Level 0
Original image

Gaussian Pyramid



contents

- Image representation
- Pixel-wise operations
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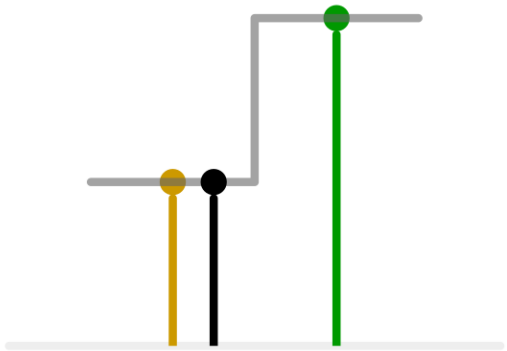
- **Interpolation**
- Morphology operators
- Connected components

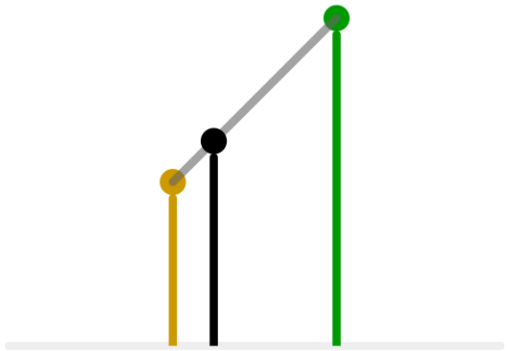
Upsampling

- This image is too small for this screen:
- How can we make it 10 times as big?
- Simplest approach: repeat each row and column 10 times (“**Nearest neighbor interpolation**”)
- This operation is known as **upsampling** or **interpolation.**

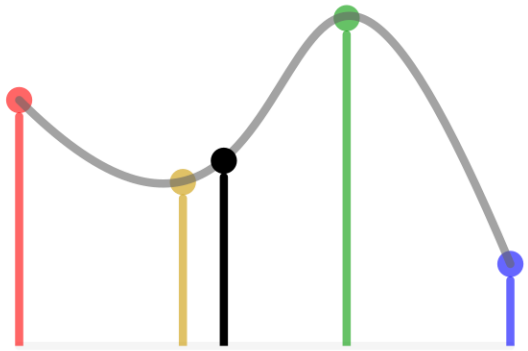
Upsampling



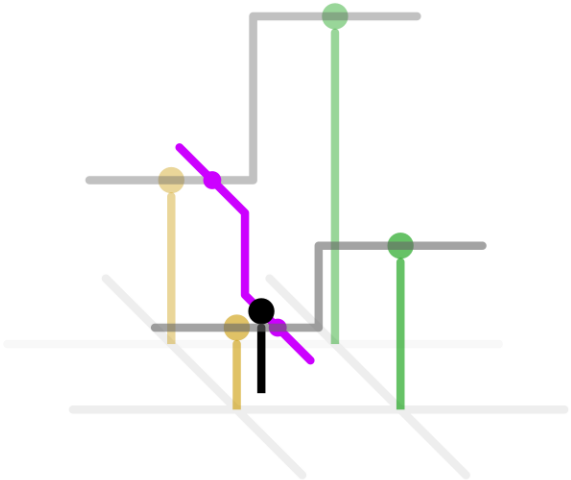
1D nearest-neighbour



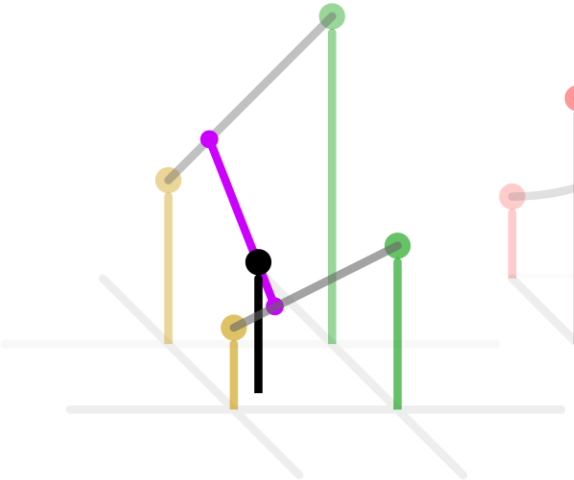
Linear



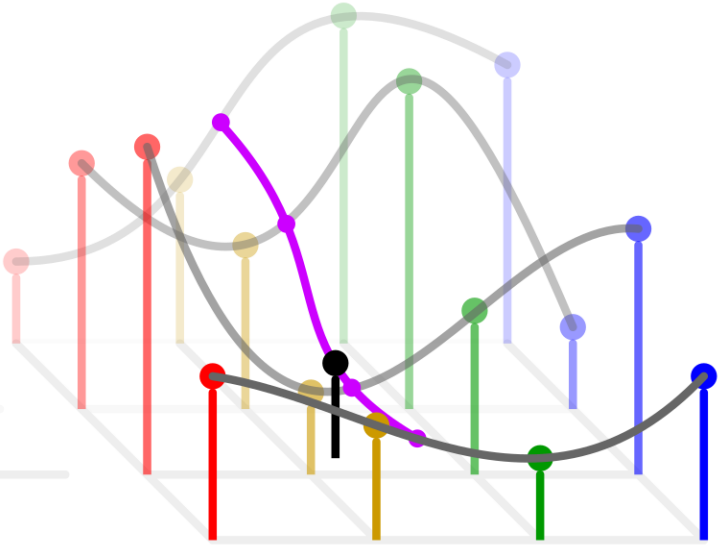
Cubic



2D nearest-neighbour



Bilinear



Bicubic

Image interpolation

Original image: x 10

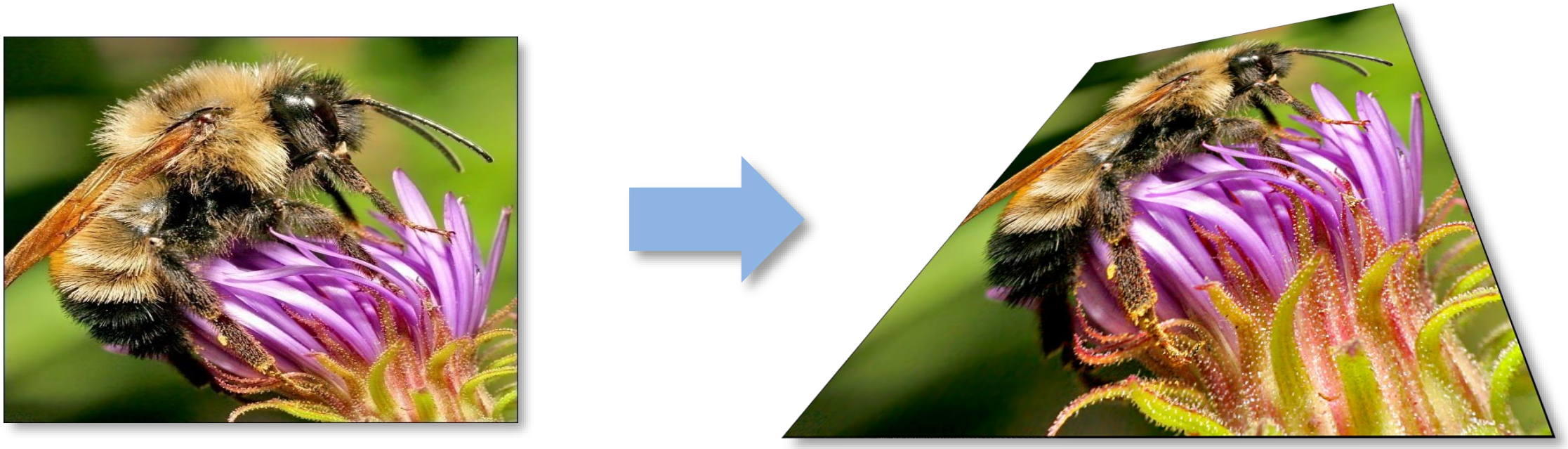
Nearest-neighbor interpolation

Bilinear interpolation

Bicubic interpolation

Image interpolation

Also used for image warping



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Morphology

- Handy tool whenever needed to clean up binary images.
- Each morphology operator is constructed as such:
  1. Select some kind of structure element (binary kernel)

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

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  3. Threshold the output $\theta_{TH}(x,t) = \begin{cases} 1 & if \quad x \geq t, \\ 0 & else \end{cases}$

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

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- Overall morphologic operation should look like so:

$$k = \theta_{TH}(f * s, t)$$

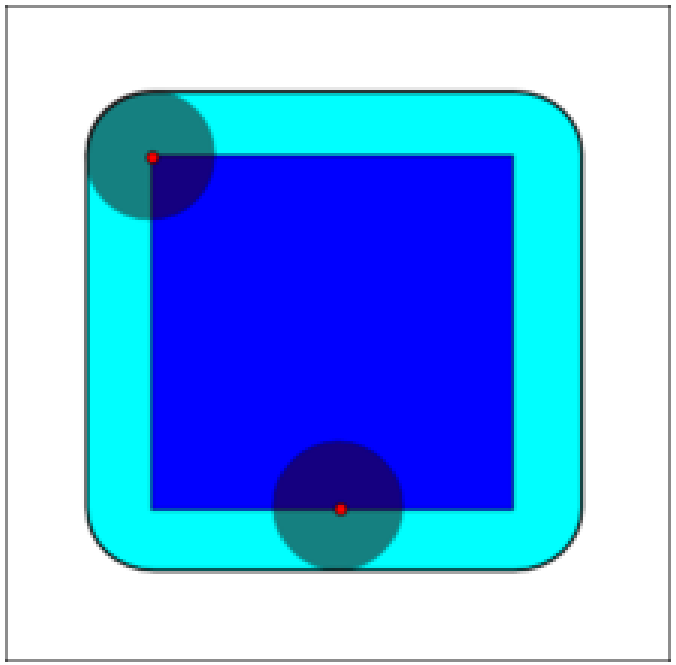
Dilation

1. Select some kind of structure element (binary kernel)
2. Convolve with input binary image $g = f * s$
3. Threshold the output

$$\theta_{TH}(x,t) = \begin{cases} 1 & if \quad x \geq t, \\ 0 & else \end{cases}$$

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

- Dilation: $t = 1$
  - Skinny lines will get thicker



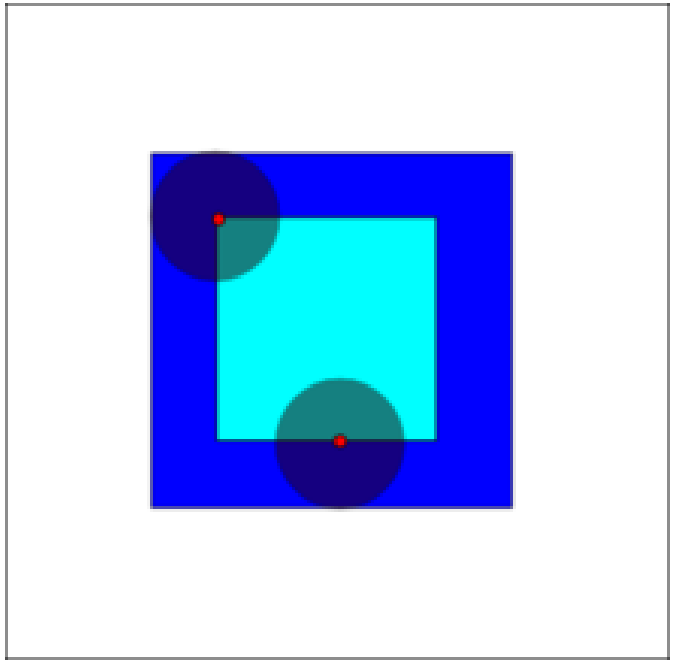
Erosion

1. Select some kind of structure element (binary kernel)
2. Convolve with input binary image $g = f * s$
3. Threshold the output

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

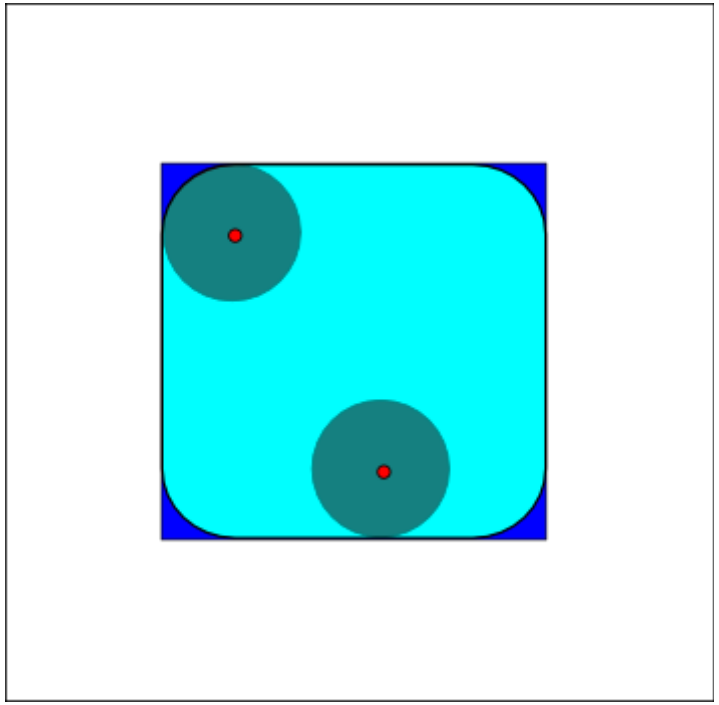
$$\theta_{TH}(x,t) = \begin{cases} 1 & if \quad x \geq t, \\ 0 & else \end{cases}$$

- Erosion: $t = sum(s)$
  - Thicker lines will get skinny



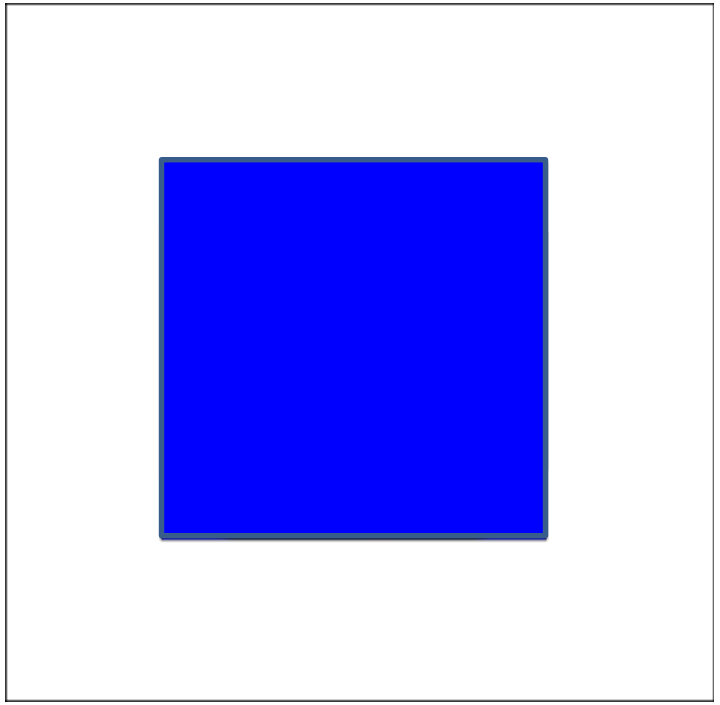
Opening

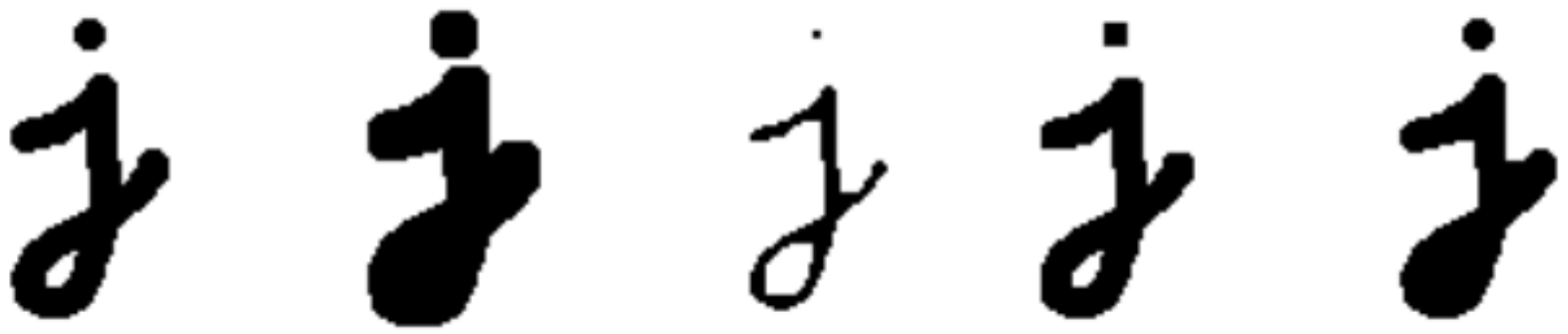
- Erosion followed by dilation.
  - The effect is of rounding off sharp edges.



Closing

- Dilation followed by erosion.
  - The effect is of closing of narrow gaps and holes.





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Connected components

- Defined as regions of adjacent pixels that have the same value.
- Commonly used with binary images to find stand alone objects.
  - e.g.: letters in a document.