

# Image processing



Yoni Chechik

[www.AliMath.com](http://www.AliMath.com)

# References

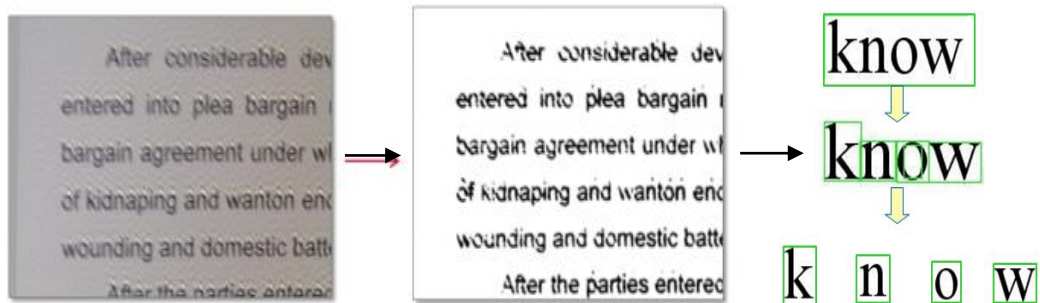
- <http://szeliski.org/Book/>
- <http://www.cs.cornell.edu/courses/cs5670/2019sp/lectures/lectures.html>
- <http://www.cs.cmu.edu/~16385/>

# Some motivation



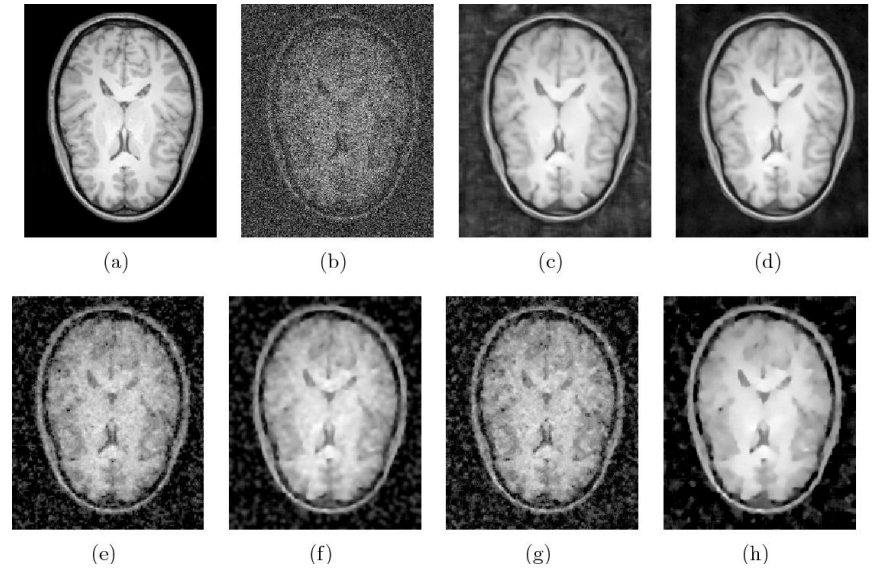
Art

(Photoshop image correction)



Robotics

(OCR – optical character recognition)



Medicine

(MRI denoising)



Agriculture

(color ripeness detection)

# contents

- **Image representation**
- Pixel-wise operations
- Noise and filtering
- Frequency representation
- Decimation
- Interpolation
- Morphology operators
- Connected components



# Image representation

- We can think of an image as a 3d matrix of discrete RGB values.
- The values mark the intensity of each color channel and are usually of type `uint8 = {0, ..., 255}`.



# Image representation

- We can also think of an image as a function  $f(x, y)$  .



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# Pixel-wise operators

- Pixel-wise operators, or point operators, are defined as such that each output pixel's value depends on only the corresponding input pixel value.



# Pixel-wise operators

original



$x$

darken



lower contrast



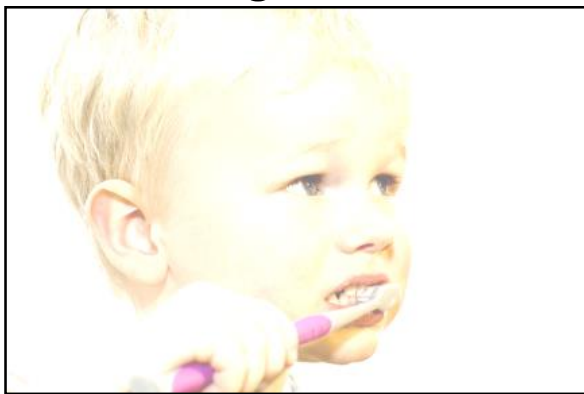
Gamma compression



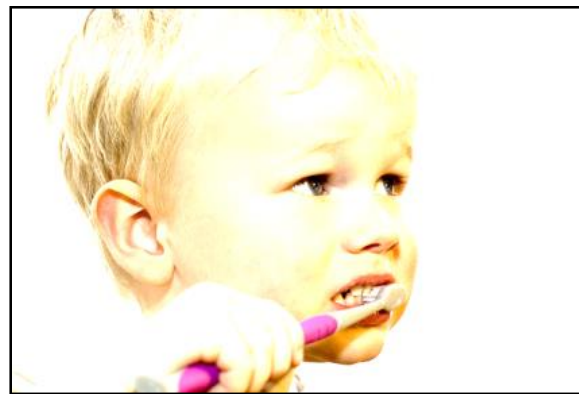
invert



lighten



raise contrast



Gamma expansion



# Pixel-wise operators

original



$x$

darken



lower contrast



Gamma compression

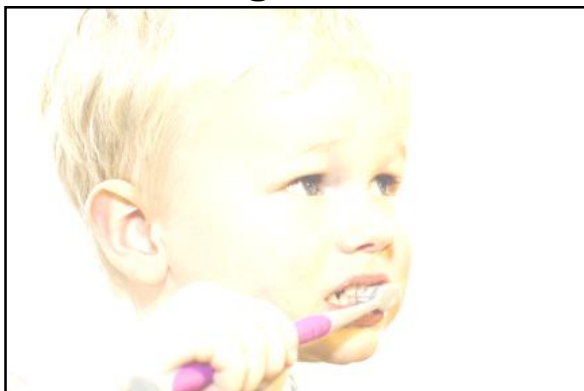


invert

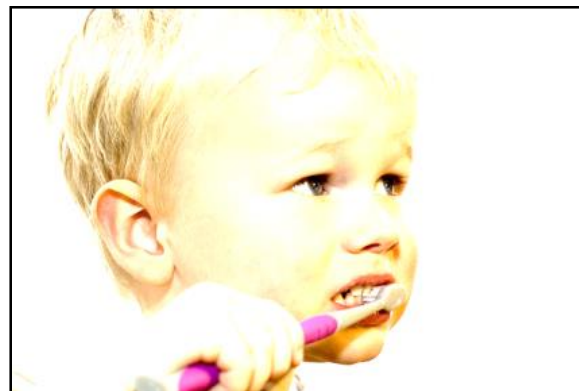


$255 - x$

lighten



raise contrast



Gamma expansion



# Pixel-wise operators

original



$$x$$

darken



$$x - 128$$

lower contrast



Gamma compression

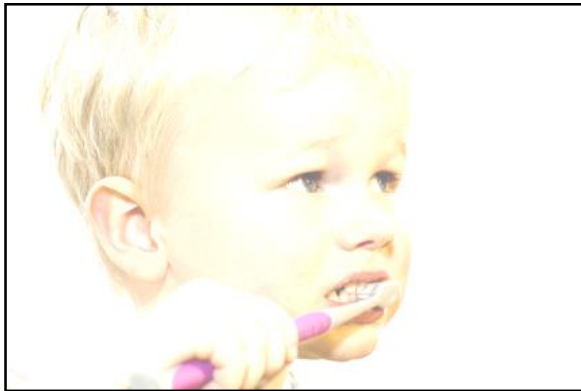


invert



$$255 - x$$

lighten



raise contrast



Gamma expansion





# Pixel-wise operators

original



$$x$$

darken



$$x - 128$$

lower contrast



Gamma compression

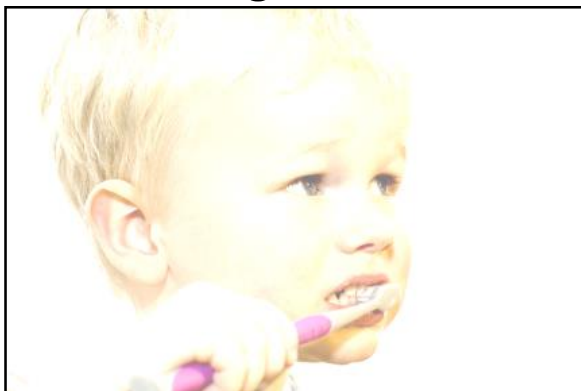


invert



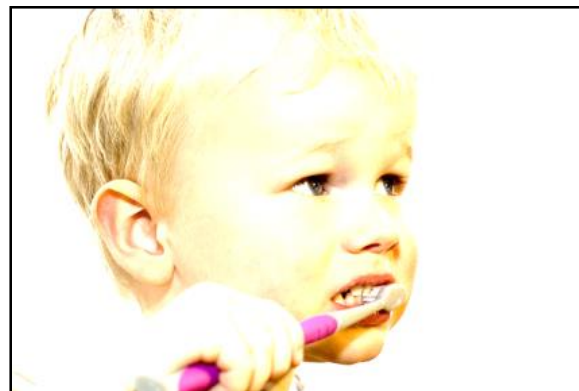
$$255 - x$$

lighten



$$x + 128$$

raise contrast



Gamma expansion



# Pixel-wise operators

original



$$x$$

darken



$$x - 128$$

lower contrast



$$\frac{x}{2}$$

Gamma compression



invert



$$255 - x$$

lighten



$$x + 128$$

raise contrast



Gamma expansion





# Pixel-wise operators

original



$$x$$

darken



$$x - 128$$

lower contrast



$$\frac{x}{2}$$

Gamma compression

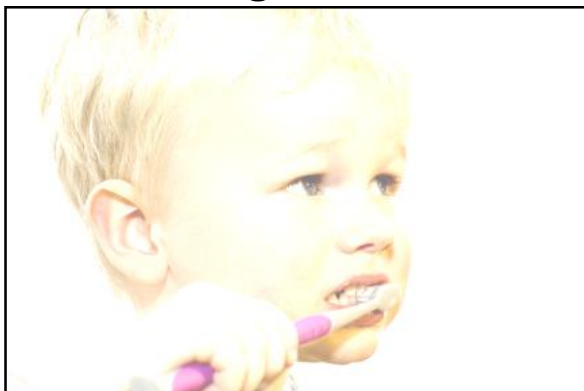


invert



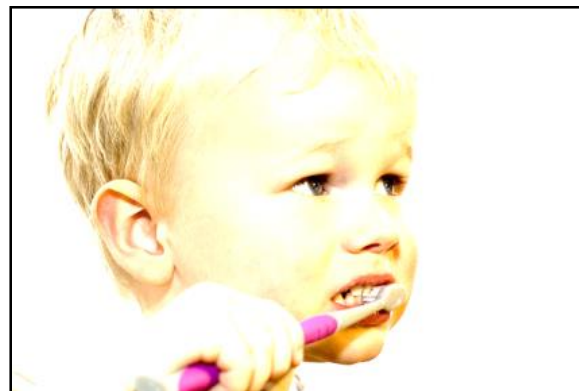
$$255 - x$$

lighten



$$x + 128$$

raise contrast



$$x \times 2$$

Gamma expansion



# Contrast

- **Contrast** in visual perception is the difference in appearance of two or more parts of a seen field.
- The human visual system is more sensitive to contrast than absolute luminance;
- **Contrast ratio**, or **dynamic range**, is the ratio between the largest and smallest values of the image or:

$$CR = \frac{V_{max}}{V_{min}}$$



# Pixel-wise operators

original



$$x$$

darken



$$x - 128$$

lower contrast



$$\frac{x}{2}$$

Gamma compression



invert



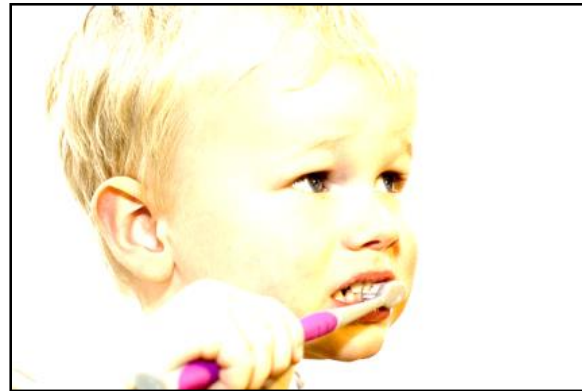
$$255 - x$$

lighten



$$x + 128$$

raise contrast



$$x \times 2$$

Gamma expansion





# Pixel-wise operators

original



$$x$$

darken



$$x - 128$$

lower contrast



$$\frac{x}{2}$$

Gamma compression



$$\left(\frac{x}{255}\right)^{1/3} \times 255$$

invert



$$255 - x$$

lighten



$$x + 128$$

raise contrast



$$x \times 2$$

Gamma expansion



# Pixel-wise operators

original



$$x$$

darken



$$x - 128$$

lower contrast



$$\frac{x}{2}$$

Gamma compression



$$\left(\frac{x}{255}\right)^{1/3} \times 255$$

invert



$$255 - x$$

lighten



$$x + 128$$

raise contrast



$$x \times 2$$

Gamma expansion

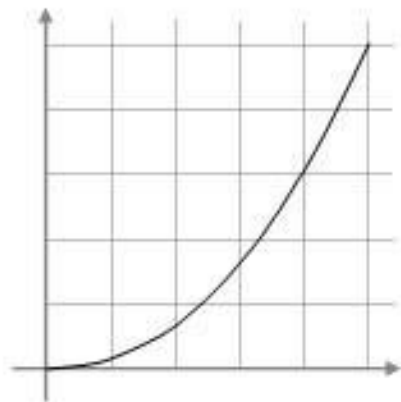


$$\left(\frac{x}{255}\right)^2 \times 255$$



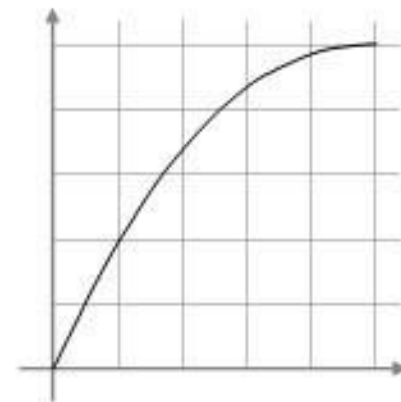
# Gamma correction

- Originally, Due to non-linearities in the old CRT televisions, intensities was seen different then they are.



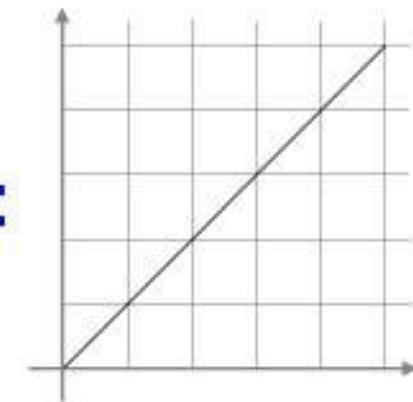
Gamma characteristics of monitors

$\times$

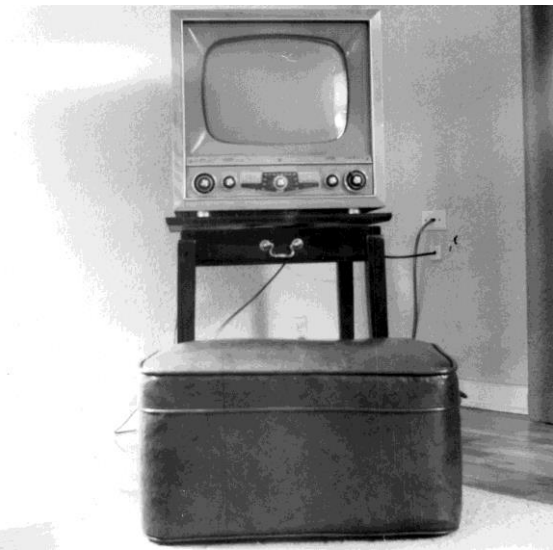


Color information adjusted to match gamma characteristics

$=$



Color handling approaching the "y = x" idealcs

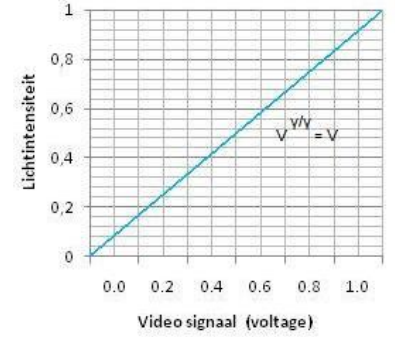
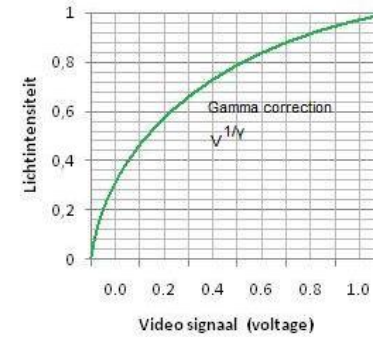
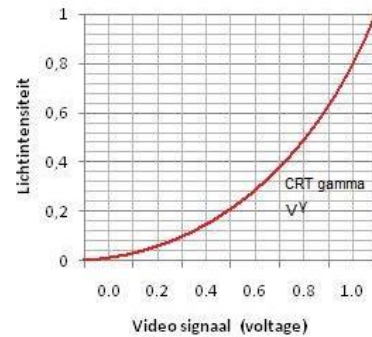
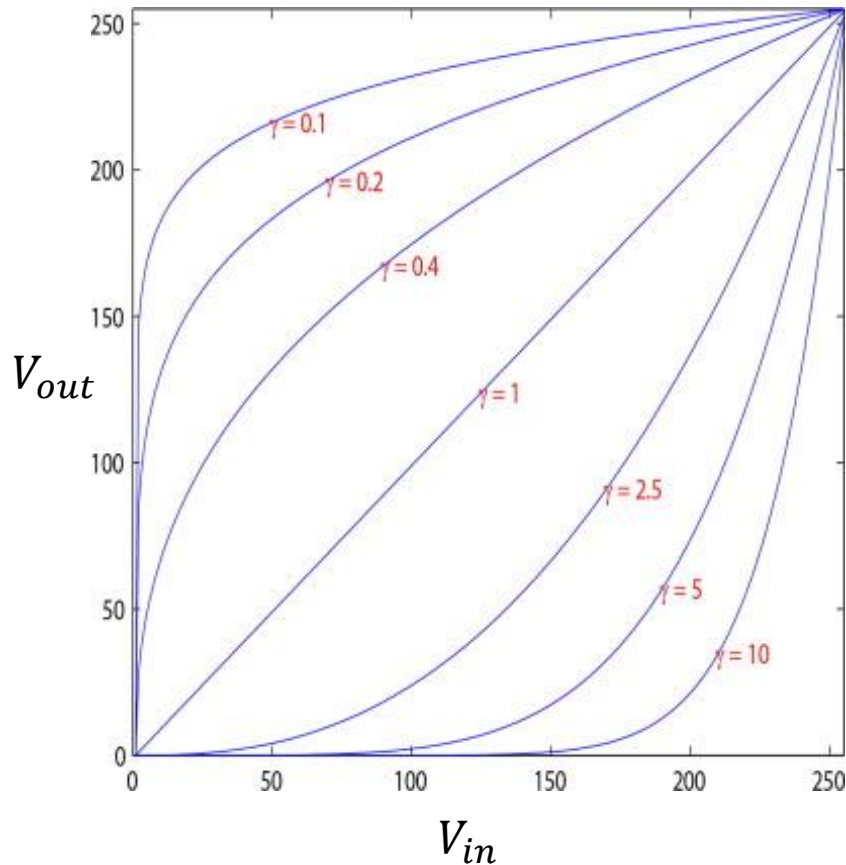


# Gamma correction

- To correct this non-linear transformation, gamma correction was done:

$$V_{out} = V_{in}^{\gamma} \quad (V_{in} \in [0,1])$$

- This is, of course, also applicable for image enhancements.



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# Gaussian Noise

- Gaussian noise is an additive noise that can appear in images due to the system electrical circuitry.
- This noise is independent of signal strength and independent at each pixel (IID- independent and identically distributed).

$$p(z) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}$$

Gaussian Noise  $\sigma=0.01$



SNR=34.0206

Gaussian Noise  $\sigma=0.05$



SNR=19.1825

Gaussian Noise  $\sigma=0.1$



SNR=13.7121

# Salt & Pepper noise

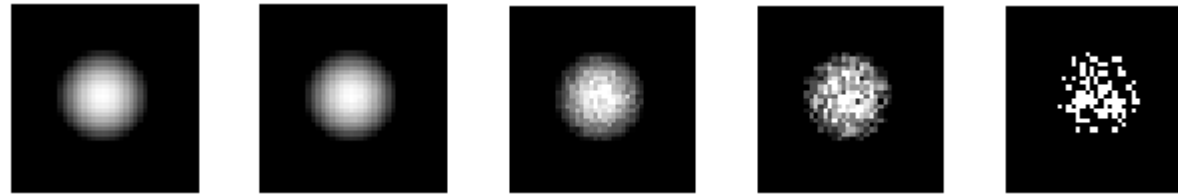
- Noise that can be caused by analog-to-digital converter errors, bit errors in transmission, etc.
- This noise is **not** additive to the signal strength (a replacement of original value with noise value).
- This noise is independent of signal strength and independent at each pixel.



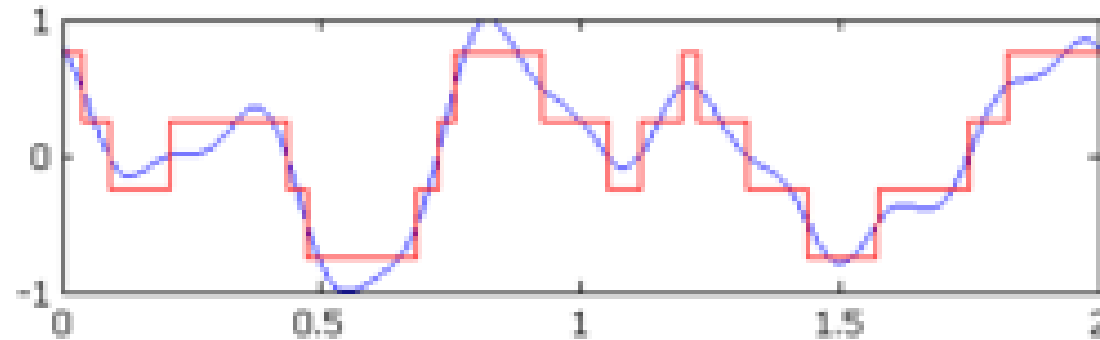


# And some more noise

- **Shot noise** - caused by statistical quantum fluctuations, that is, variation in the number of photons sensed at a given exposure level in the darker parts of an image (where there are just few photons that enter each pixel “bin”). Modeled as Poisson noise.



- **Quantization noise** – caused by quantizing the pixels of a sensed image to several discrete levels (analog to digital conversion).



# Noise reduction with LTI filters

- **LTI (LSI) filters** are also known as **convolution filters** or **kernel filters** and are a known solution for the noise problem.
- They are **linear** operators, which involve weighted combinations of pixels in small neighborhood, The combination is determined by the filter's *kernel*.
- The same kernel is **shifted** to all pixel locations so that all pixels use the same linear combination of their neighbors.
- Shifting the output after the weighted combination vs. shifting the input and then doing weighted combination is the same- so **shift-invariance**.
- That's why it's called **linear shift-invariance** filter.
  - LSI for short, but more commonly known by the name of the 1D signal filter- LTI, linear time-invariant.

# Convolution

- Works on LTI filters.
- Let  $f$  be the image,  $h$  be the kernel of size  $(2k + 1) \times (2k + 1)$  ( $k$  is a chosen integer), and  $g$  be the output image:

$$g(i, j) = \sum_{u=-k}^k \sum_{v=-k}^k h[u, v] f[i - u, j - v]$$

- **Note:** by definition, this operator flips the kernel both horizontally and vertically.
- This operation is called **convolution operator** and is more compactly notated as:

$$g = h * f$$

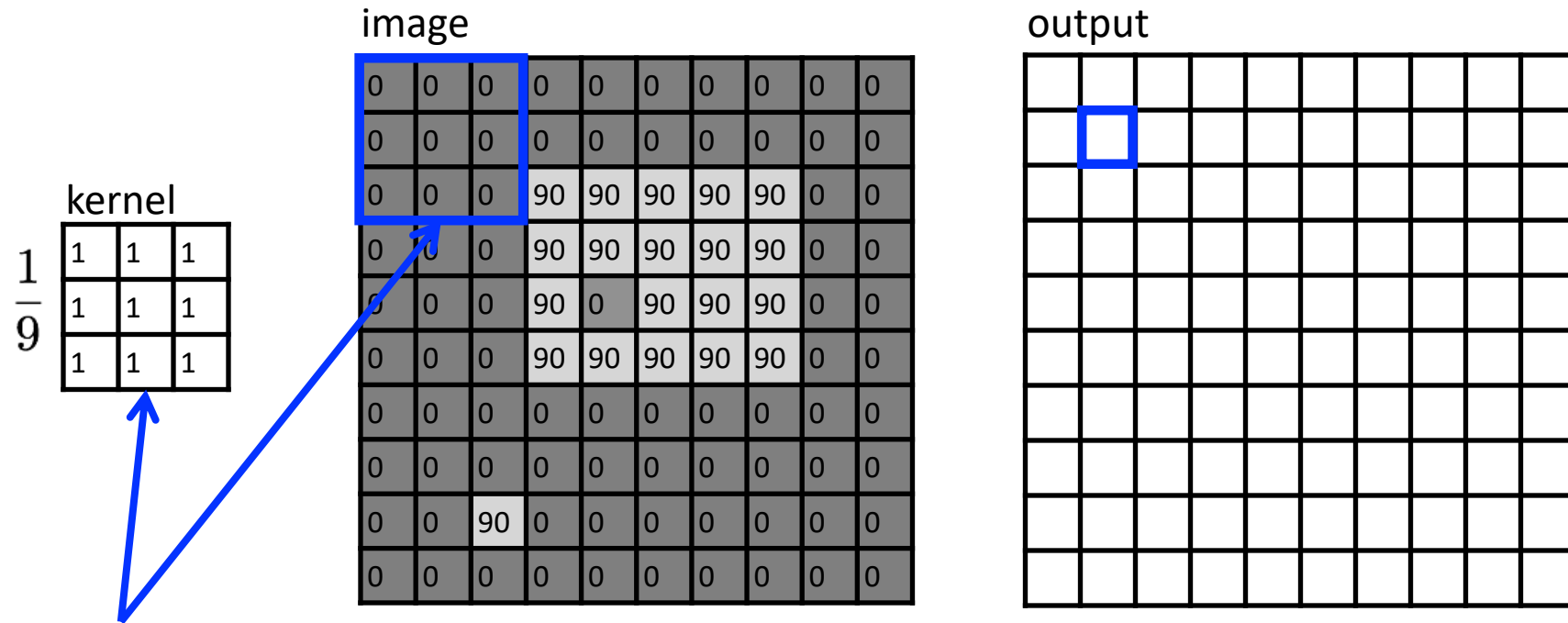
# Example: mean filter

- The kernel is:

$$\frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

- Replaces pixel with local average.
- Has smoothing (blurring) effect.
- The kernel can be in any other size as well (see .ipynb).

# Run the filter



Note that we assume that the kernel coordinates are centered.

Here the kernel is symmetric horizontally and vertically, so the flipping is not noticeable.



## Run the filter

$$\frac{1}{9} \begin{array}{|c|c|c|} \hline \text{kernel} & & \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

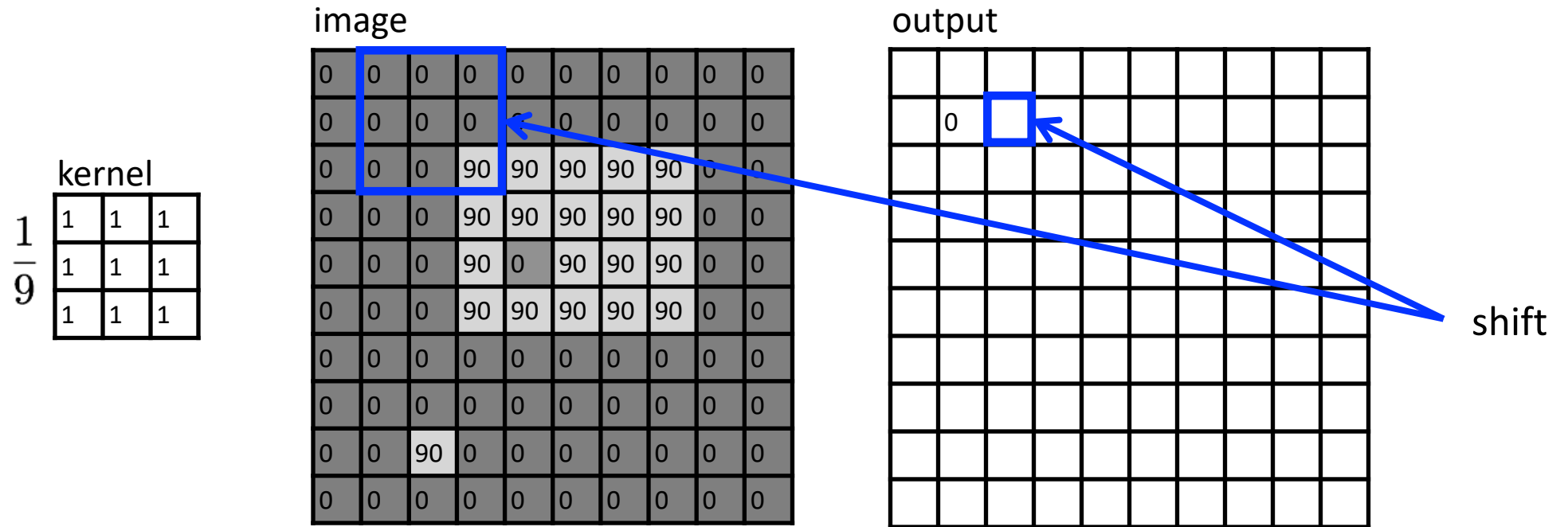
image

[illegible]

output

A 10x10 grid where the top-left cell contains the number 0.

# Run the filter



## Run the filter

$$\frac{1}{9} \begin{array}{|c|c|c|} \hline \text{kernel} & & \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

image

[illegible]

output

[illegible]

## Run the filter

$$\frac{1}{9} \begin{array}{|c|c|c|} \hline \text{kernel} & & \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

image

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0
0	0	0	90	90	90	90	90	0
0	0	0	90	0	90	90	90	0
0	0	0	90	90	90	90	90	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

output

[illegible]

## Run the filter

	kernel		
1	1	1	1
9	1	1	1
9	1	1	1

image

[illegible]

output

[illegible]

## Run the filter

$$\frac{1}{9} \begin{array}{|c|c|c|} \hline \text{kernel} & & \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

image

[illegible]

output

[illegible]



## Run the filter

$$\frac{1}{9} \begin{array}{|c|c|c|} \hline \text{kernel} & & \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

image

[illegible]

output

[illegible]

## ... and the result is

	kernel		
$\frac{1}{9}$	1	1	1
	1	1	1
	1	1	1

image

[illegible]

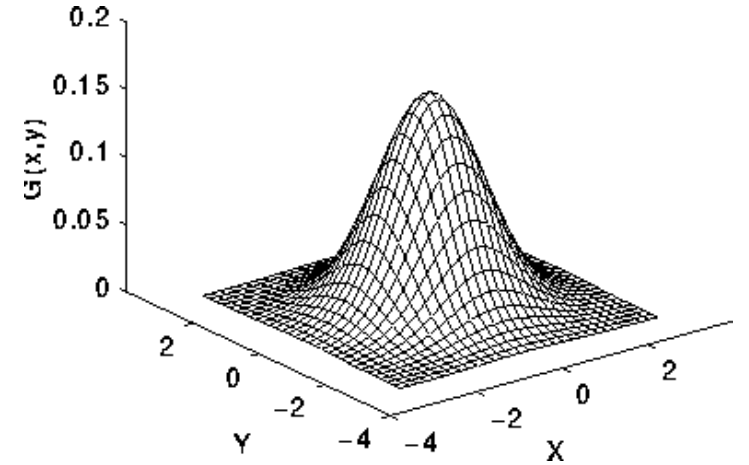
output

[illegible]

# Gaussian filter

- Another kind of blur filter.
- this filter can be controlled by its size and STD.

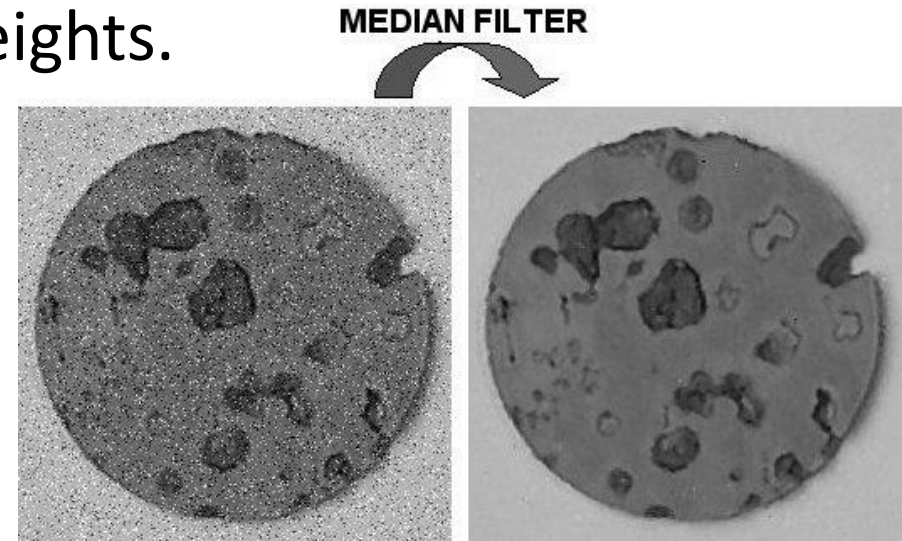
$$h(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



- The kernel is discretized to bins according to the wanted kernel size.
- Isn't Gaussian function infinite?
  - Most often, the kernel cuts out the remaining lower bins (usually at 2-3  $\sigma$ ).
- **Both Gaussian and mean filters are good against Gaussian noise, but not effective against S&P noise.**

# Median filter

- Takes the median value from the given neighbors.
- For example:
  - The median of  $[1, 0, 100]$  is 1.
- **Median filter is good against salt and pepper noise & against Gaussian noise (but not as effective).**
  - Can be considered as a blur filter, but also has edge preserving properties.
- Median filter is more computationally expansive than mean/gaussian.
- This filter is not LTI because it's not linear on its weights.



# Example in .ipynb

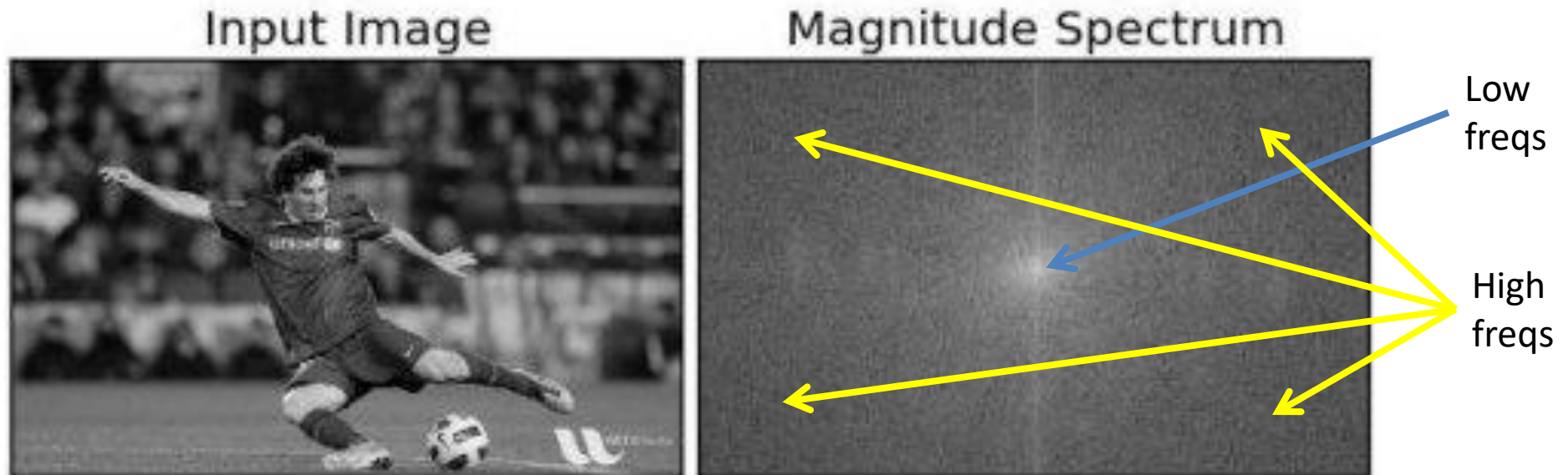
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# FFT of an image

- Frequency analysis is not in the scope of the course and is not needed for the rest of the course, but here it can give some intuition.
- In audio signals (or any other 1D signals) Lower frequencies change less over time than higher frequencies.
  - In images, the change is represented in change in distance, so images that changes slowly from pixel to pixel has more lower frequencies than others.
- Natural images are mainly built from low frequencies.



# Convolution in frequency domain

- Recall: in time (space) domain:

$$g(i, j) = \sum_{u=-k}^k \sum_{v=-k}^k h[u, v] f[i - u, j - v]$$

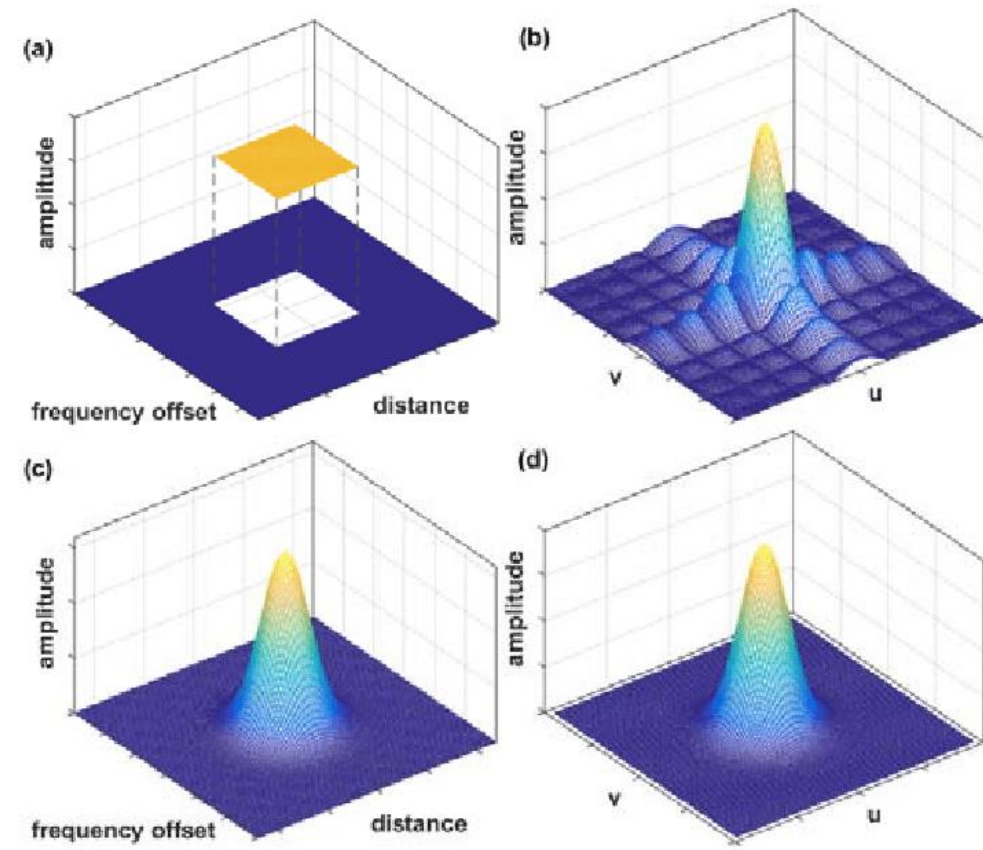
$$g = h * f$$

- In frequency domain- simple multiplication:

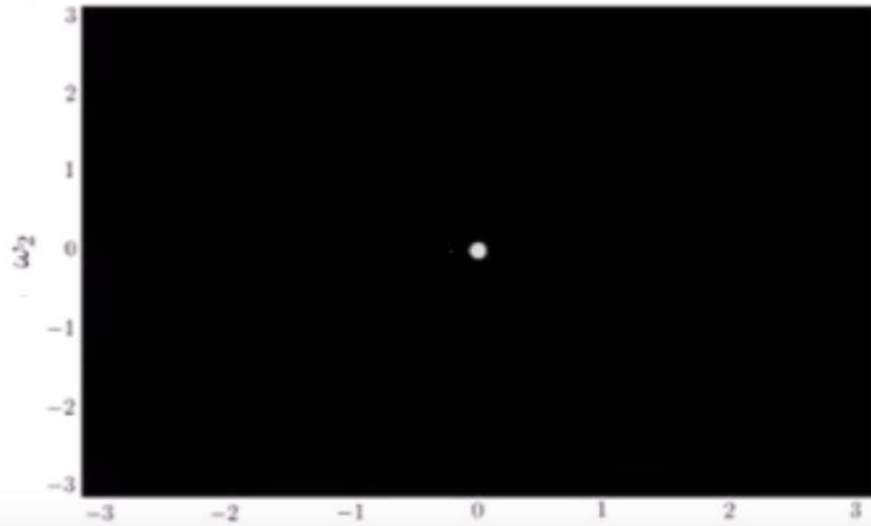
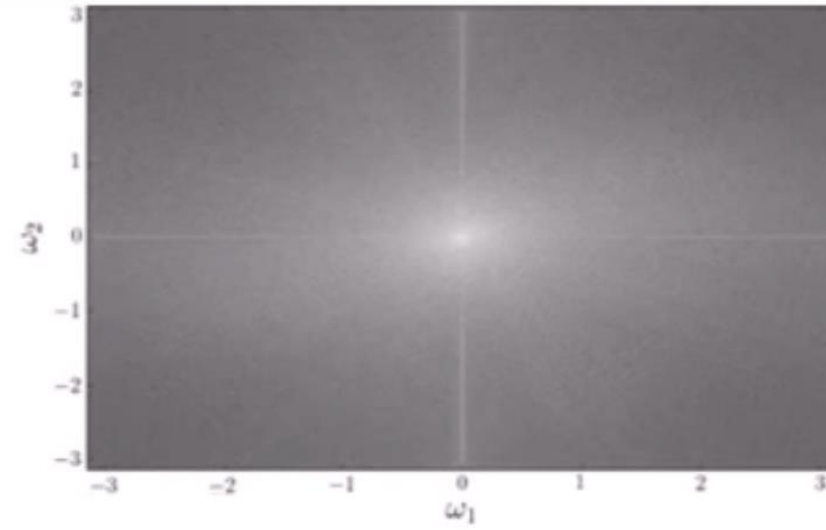
$$G = H \cdot F$$

# Low-pass filters

- Both mean and Gaussian filters are considered low-pass filters because in the frequency domain, they have higher values in the lower frequencies- and when multiplied with frequency spectrums, the high frequencies get smaller.
- When image is left only with the lower frequencies, the rapidly changes parts of the image (e.g.: edges, noise) are smoothen.



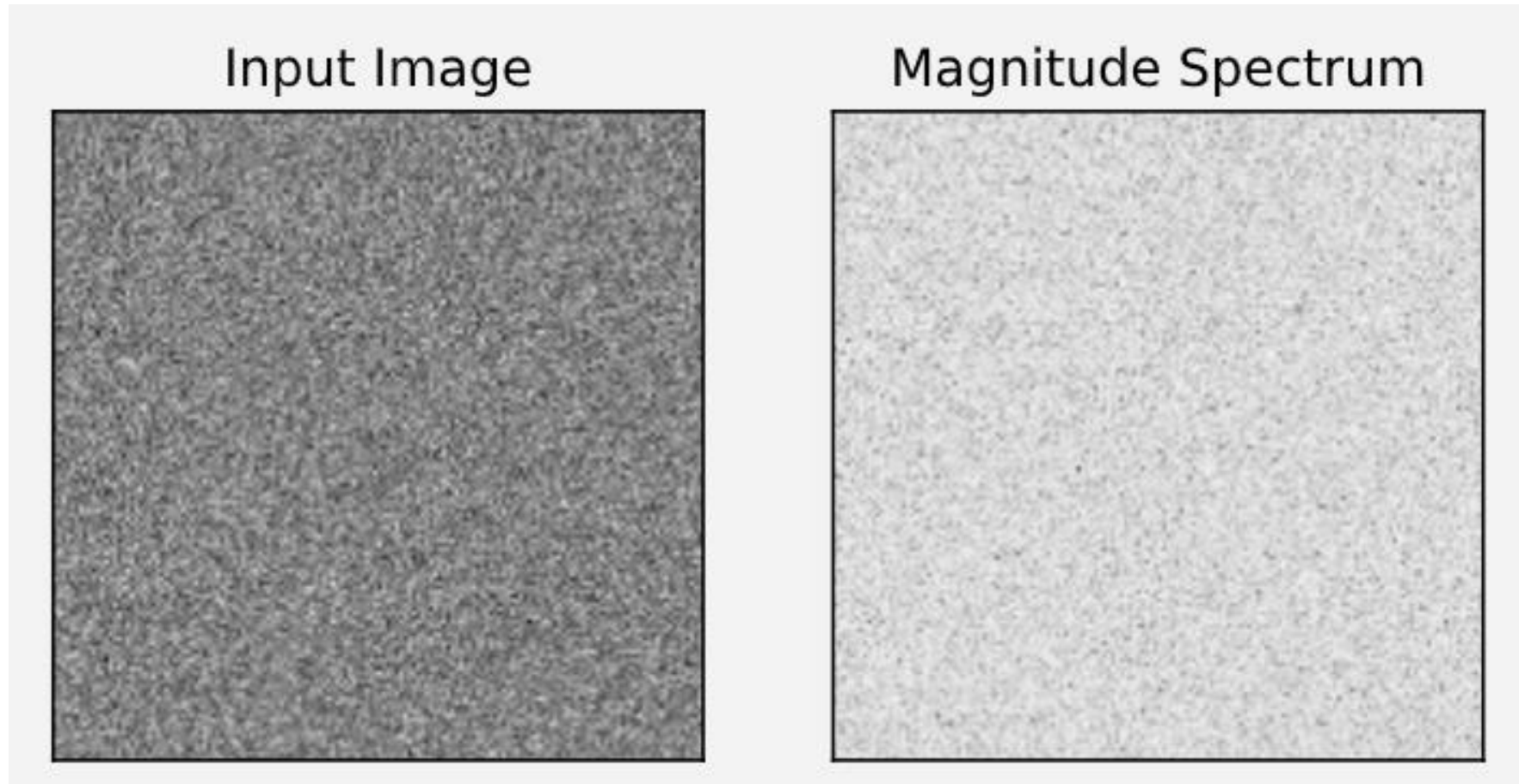
# LP example





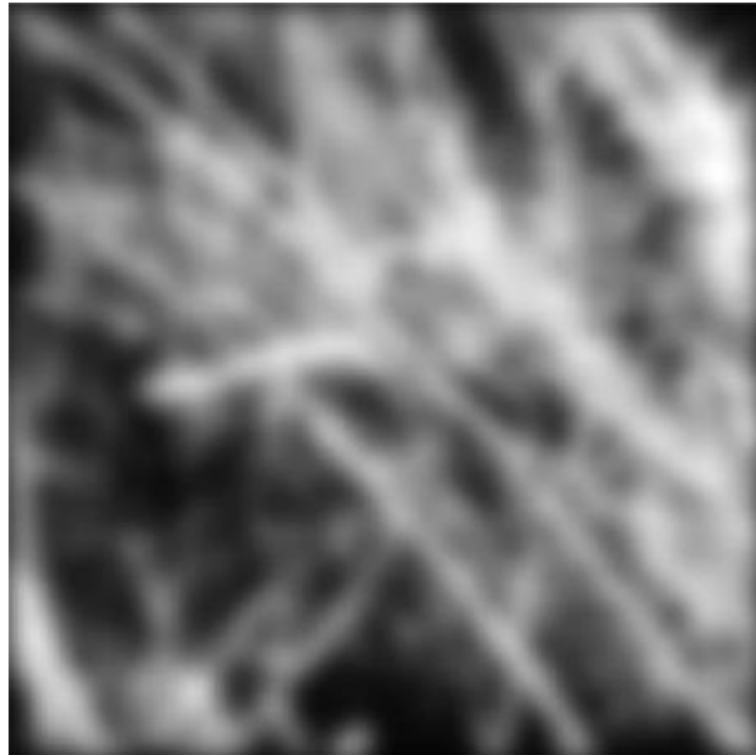
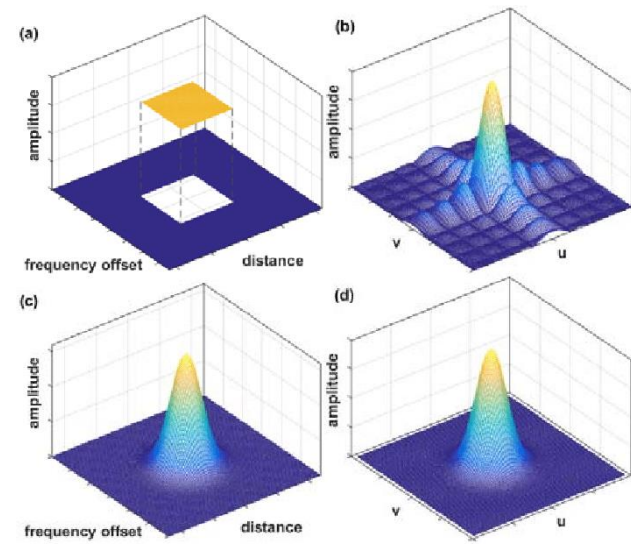
# FFT of gaussian noise

- Since gaussian noise (AWGN) is distributed along all frequencies, LP filter reduce this kind of noise significantly.

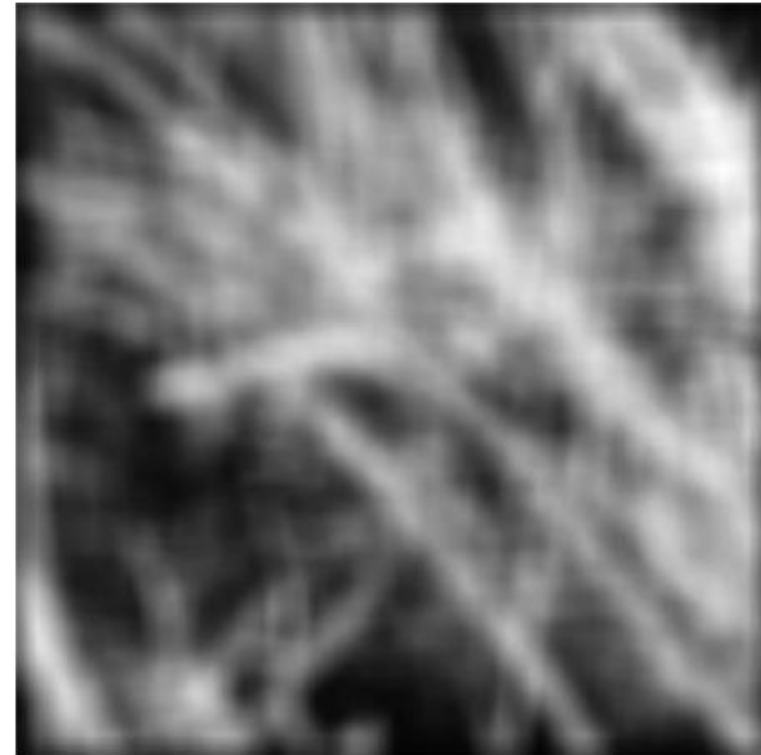
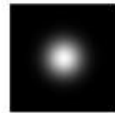


# Mean vs. Gaussian filter

- Since Mean filter has higher values in the higher frequencies, edge artifacts sometimes remains.



Gaussian  
filter



Box  
filter

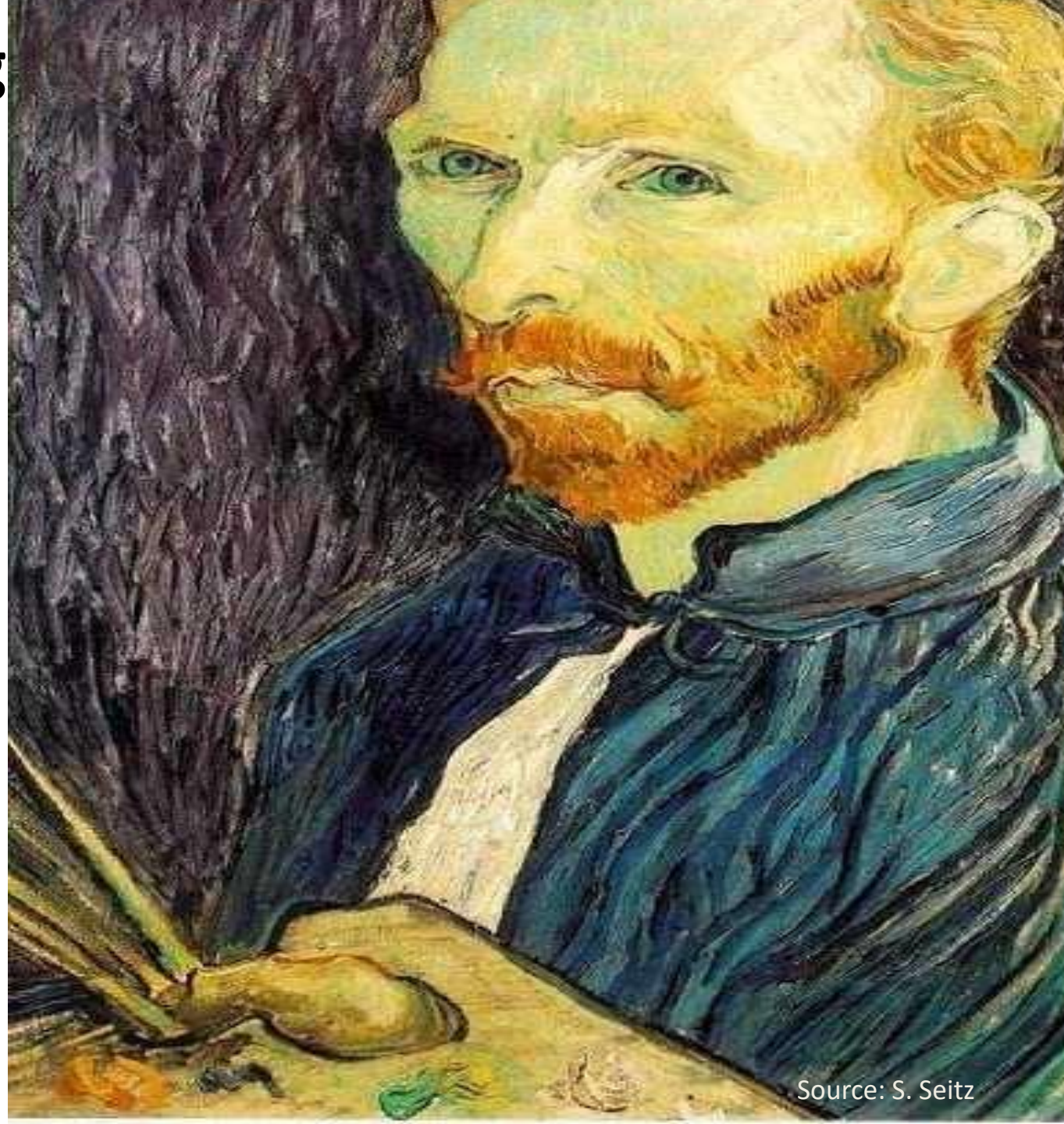


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# Image

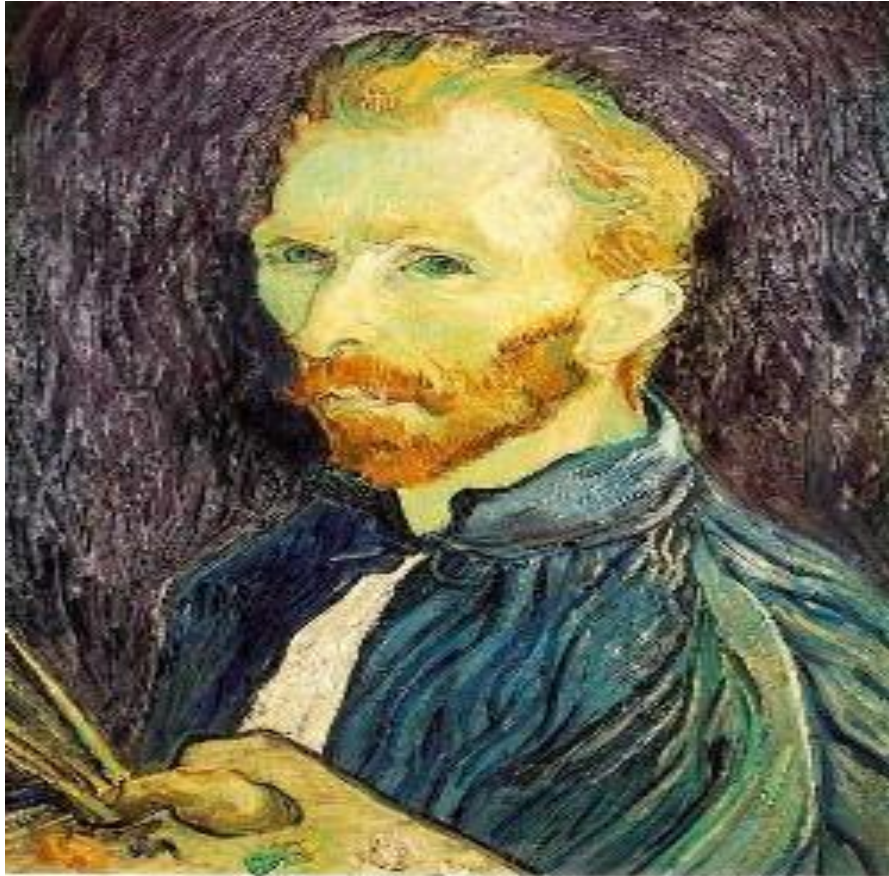
This image is too big to fit on the screen. How can we generate a half-sized version?



Source: S. Seitz



# Image sub-sampling



1/4

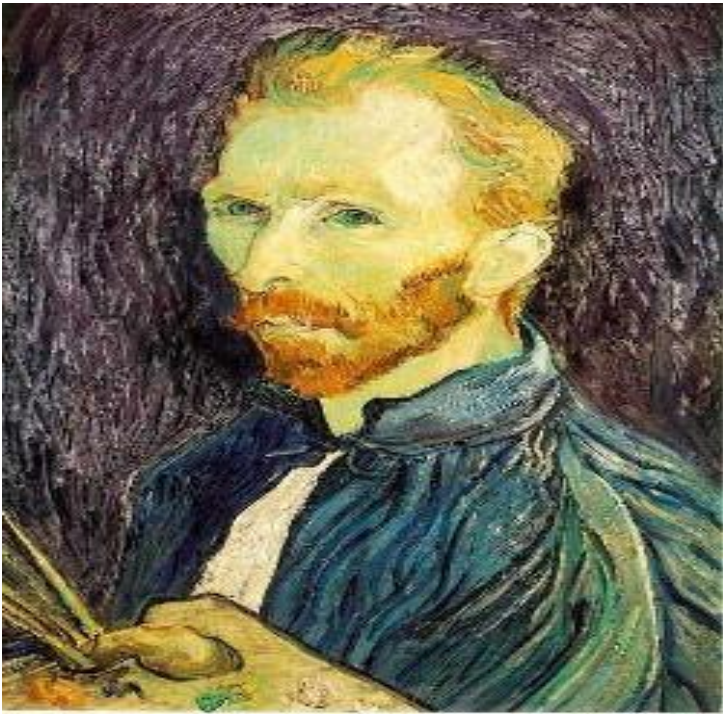


1/8

Throw away every other row and column to create a 1/2 size image

- called **image sub-sampling** or **decimation**

# Image sub-sampling



$1/2$



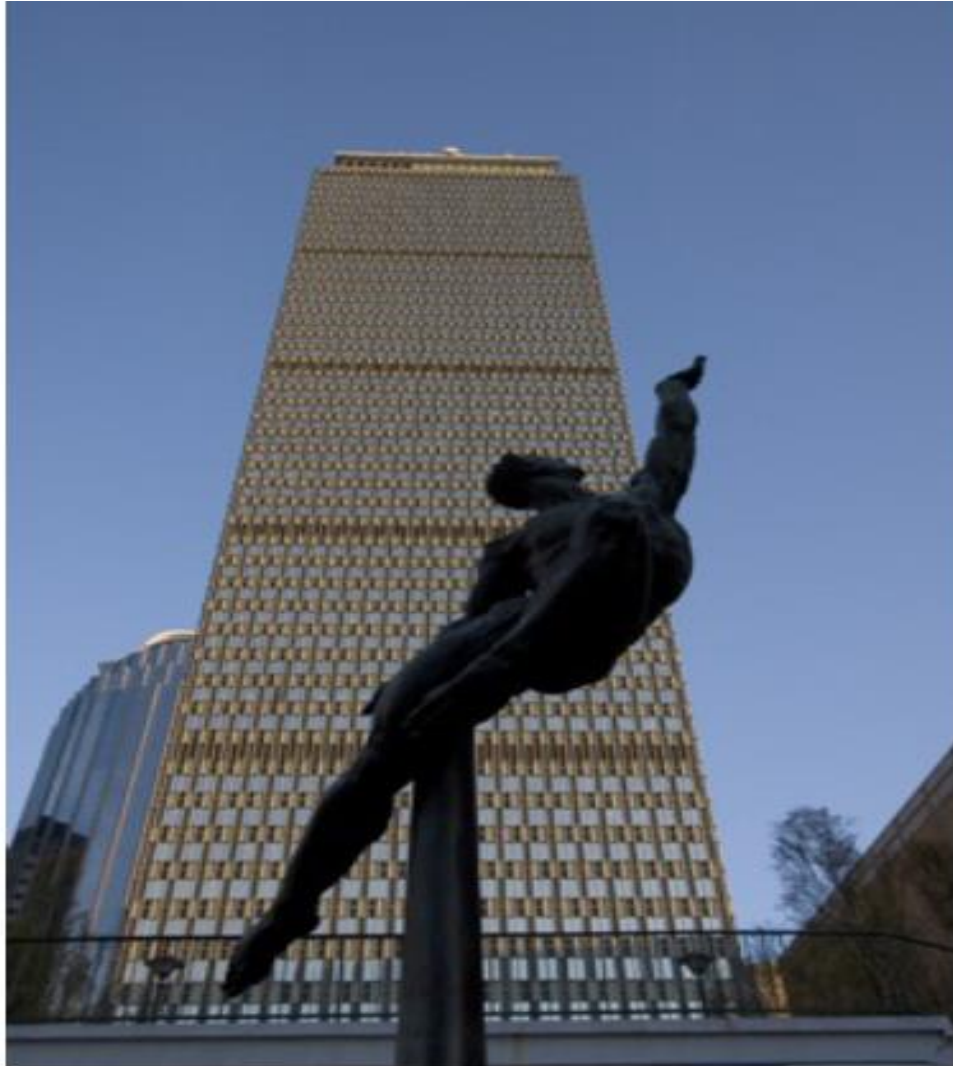
$1/4$  (2x zoom)



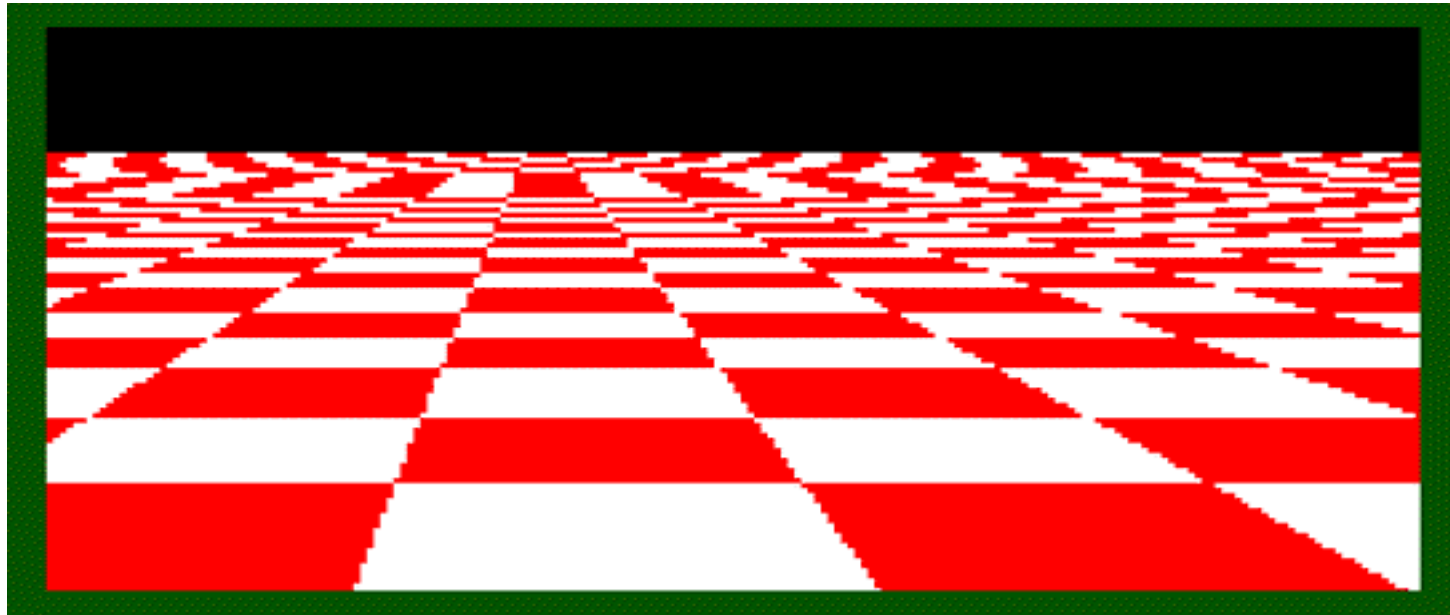
$1/8$  (4x zoom)



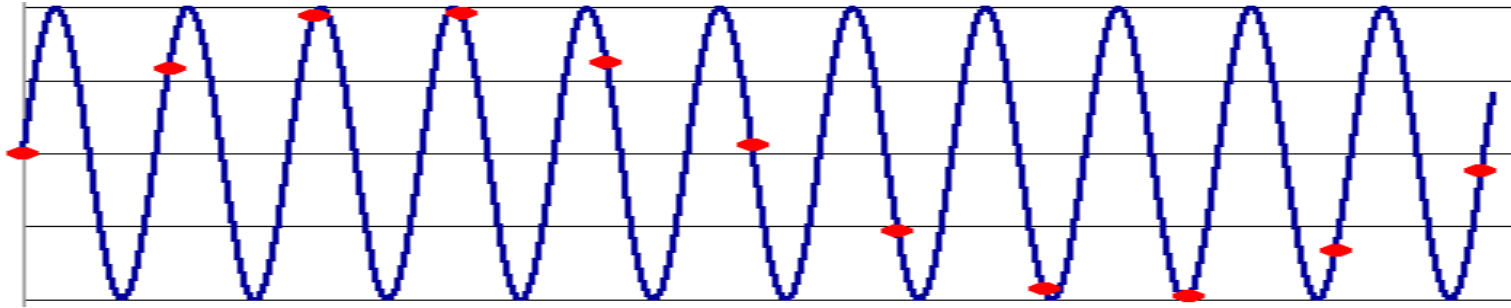
# Image sub-sampling



# Even worse for synthetic images

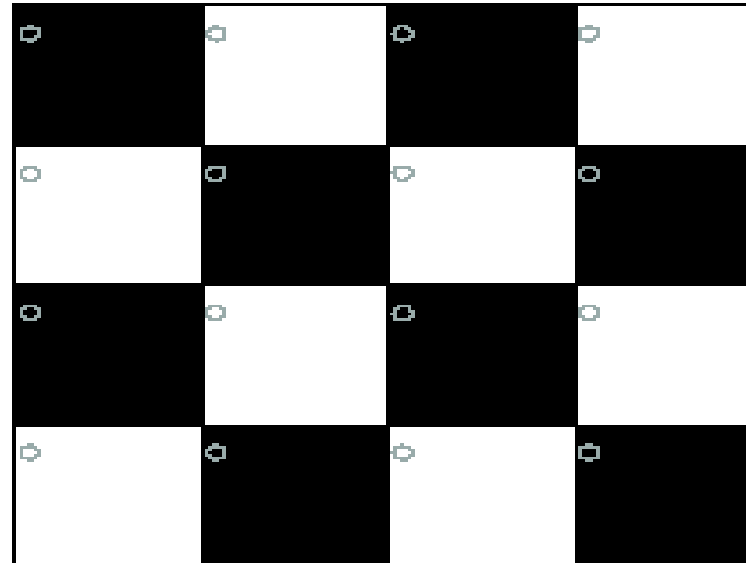
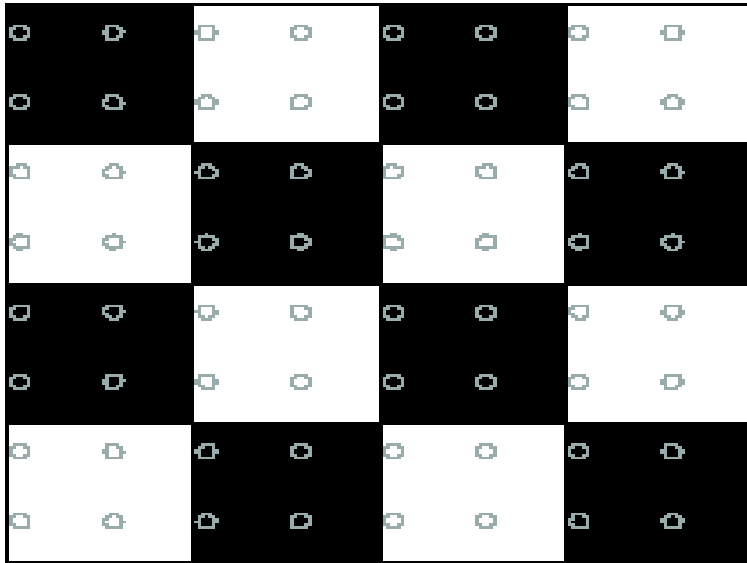


# Aliasing

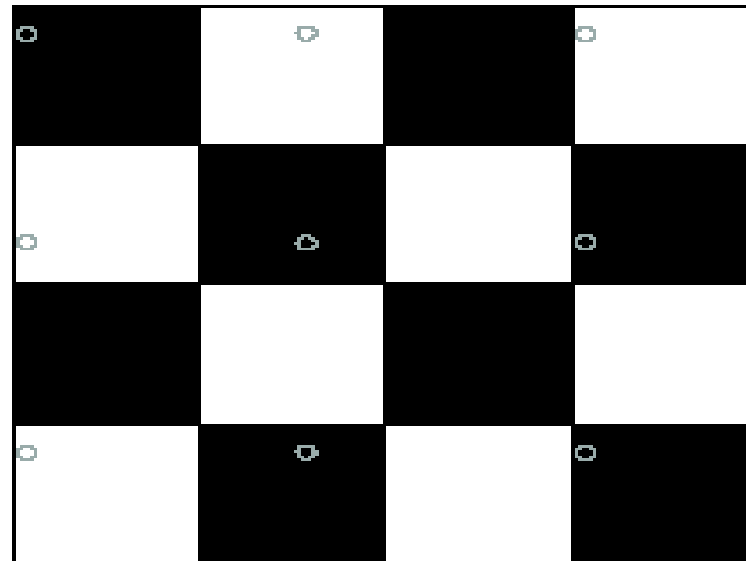
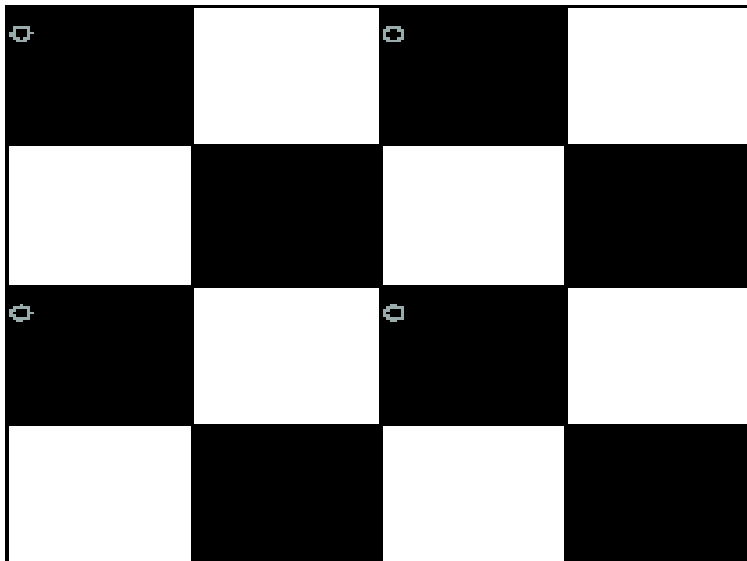


- Occurs when your sampling rate is not high enough to capture the amount of detail in your image
- Can give you the wrong signal/image—an *alias*
- To do sampling right, need to understand the structure of your signal/image
- To avoid aliasing:
  - **sampling rate  $\geq 2 * \text{max frequency}$**  in the image
  - This minimum sampling rate is called the **Nyquist rate**

# Nyquist limit – 2D example



Good sampling

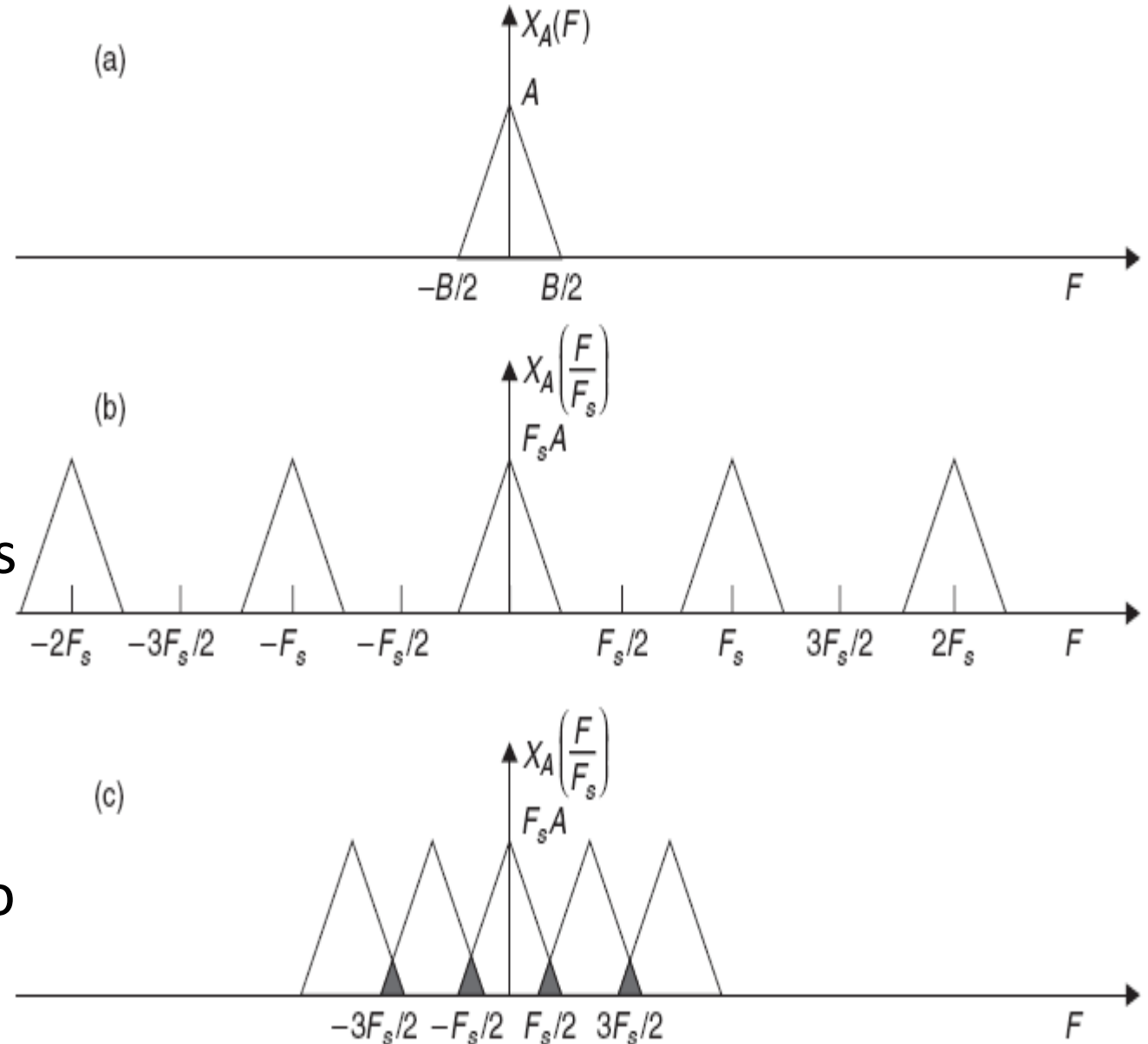


Bad sampling



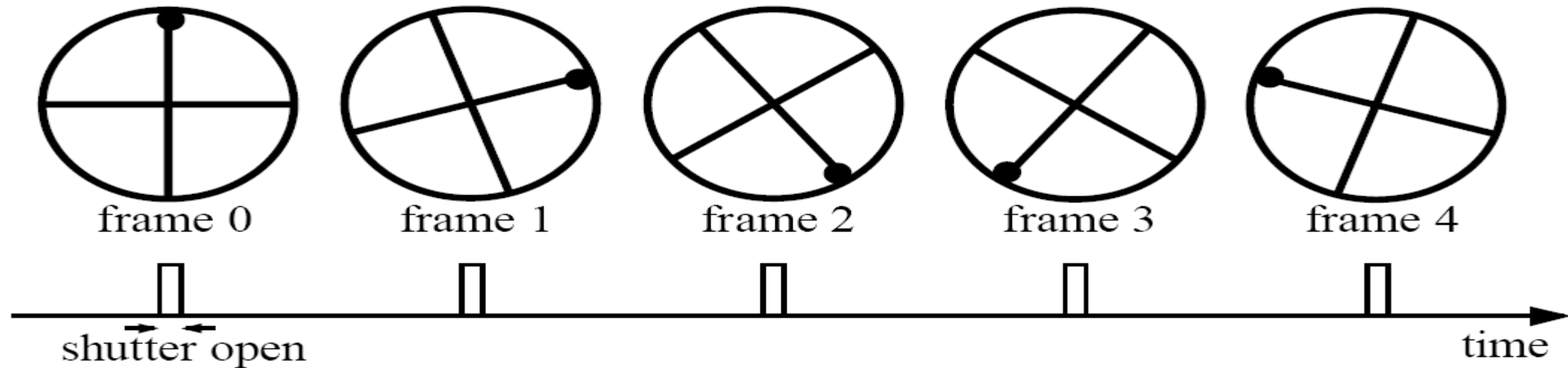
# Nyquist limit- frequency response

- Original freq representation of signal.
- Regular sampling above nyquist rate- can recreate the original frequencies of the image. (copies are from sampling).
- Sampling below nyquist- original frequencies are destroyed due to the copies overlap- **this is the aliasing.**



# Example: wagon-wheel effect

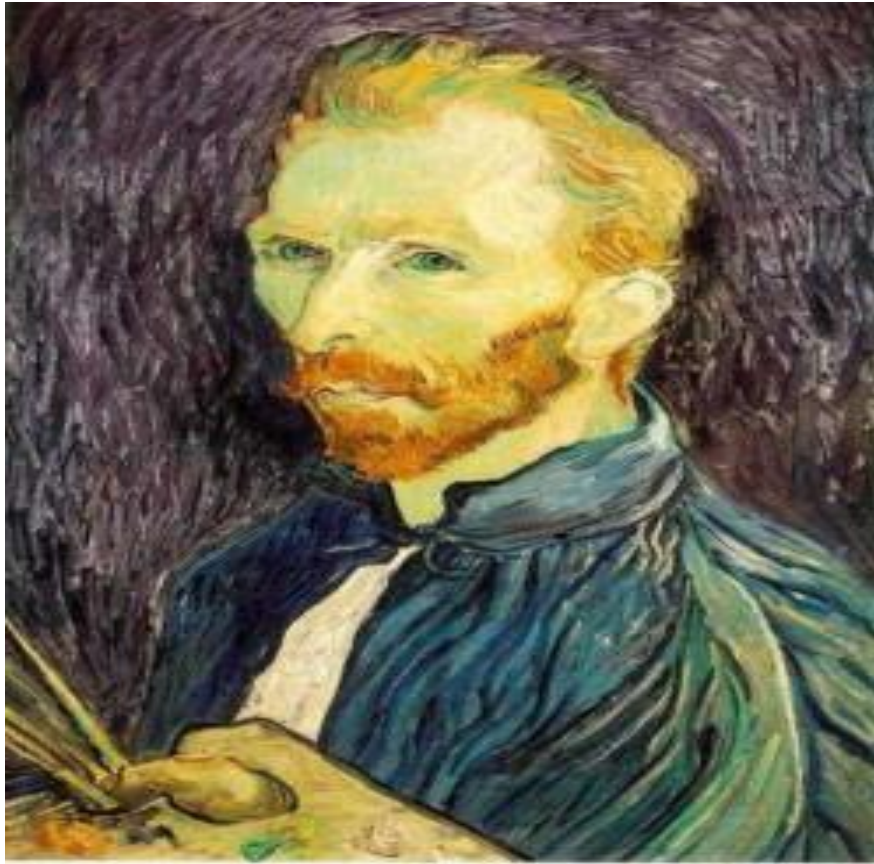
- An example of sub sampling in time domain (instead of spatially like before).



Without dot, wheel appears to be rotating slowly backwards!  
(counterclockwise)

- [https://en.wikipedia.org/wiki/Wagon-wheel\\_effect](https://en.wikipedia.org/wiki/Wagon-wheel_effect)

# Gaussian pre-filtering



Gaussian 1/2



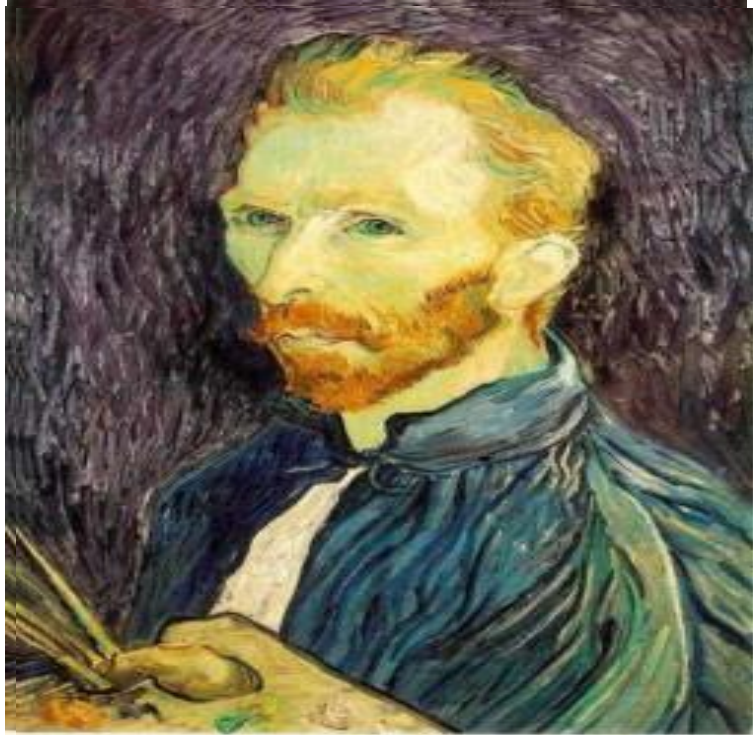
G 1/4



G 1/8

- Solution: filter the image, *then* subsample

# Subsampling with Gaussian pre-filtering



Gaussian 1/2



G 1/4

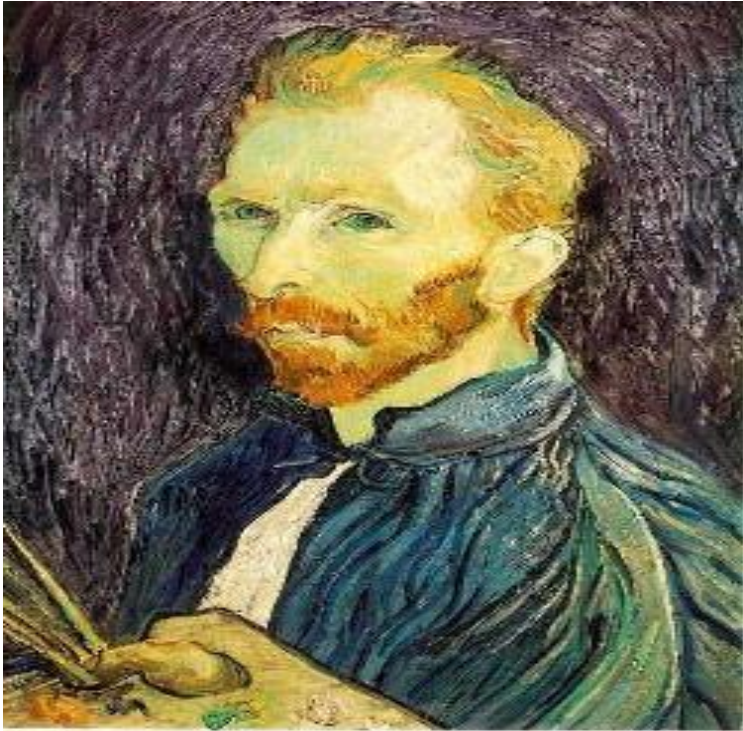


G 1/8

- Solution: filter the image, *then* subsample



# Compare with...



1/2

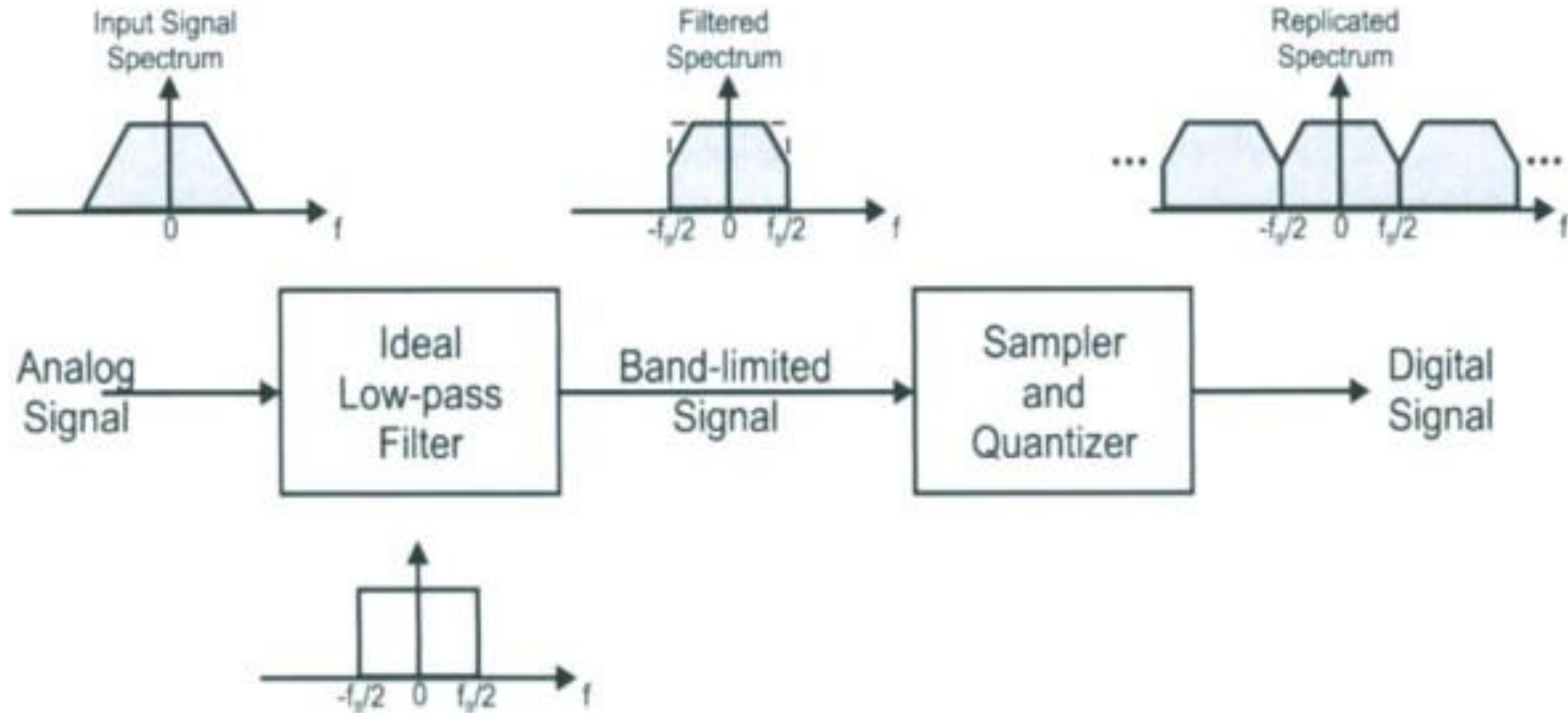


1/4 (2x zoom)



1/8 (4x zoom)

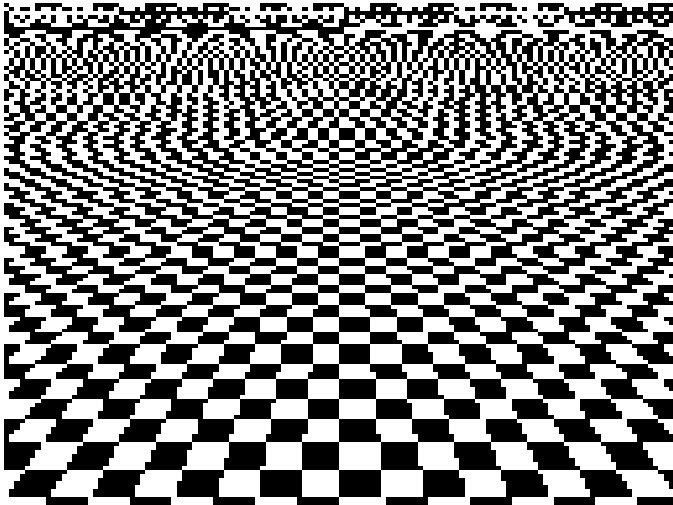
# Low pass filtering- frequency response



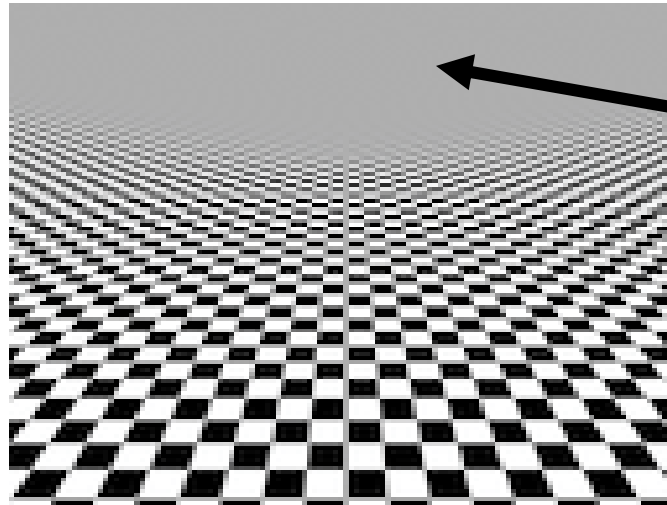


# Back to the checkerboard

- What should happen when you make the checkerboard smaller and smaller?



Naïve subsampling

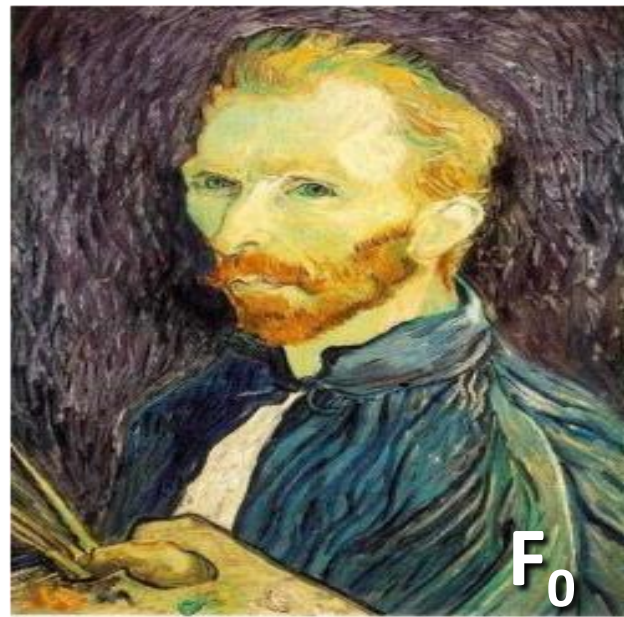


Proper prefiltering  
("antialiasing")

Image turns grey! (Average of black and white squares, because each pixel contains both.)

# Gaussian pre-filtering

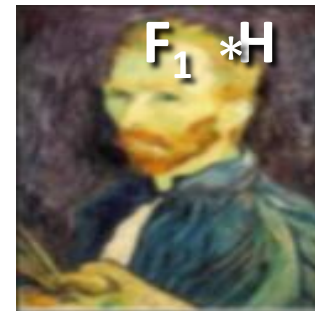
- Solution: filter the image, *then* subsample



blur      subsample



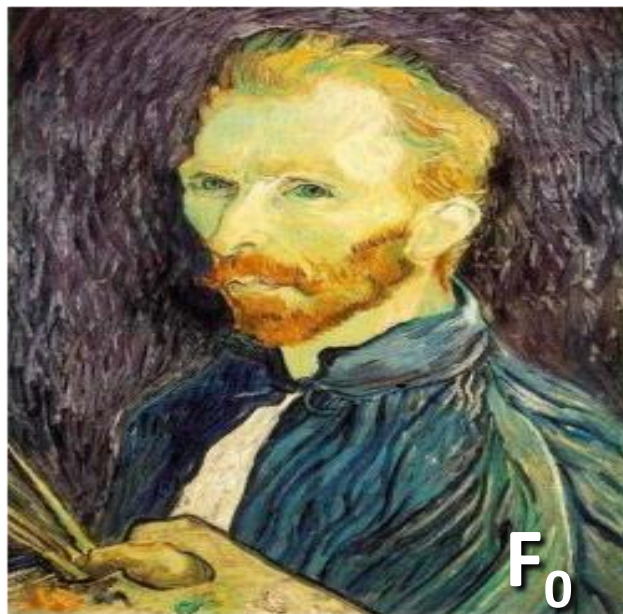
blur      subsample



...



*Gaussian pyramid*



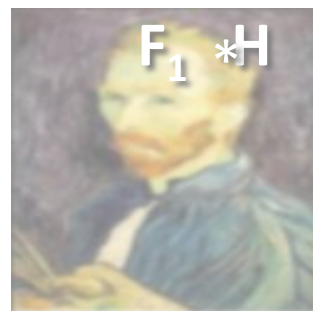
$F_0$



$F_0 * H$



$F_1$



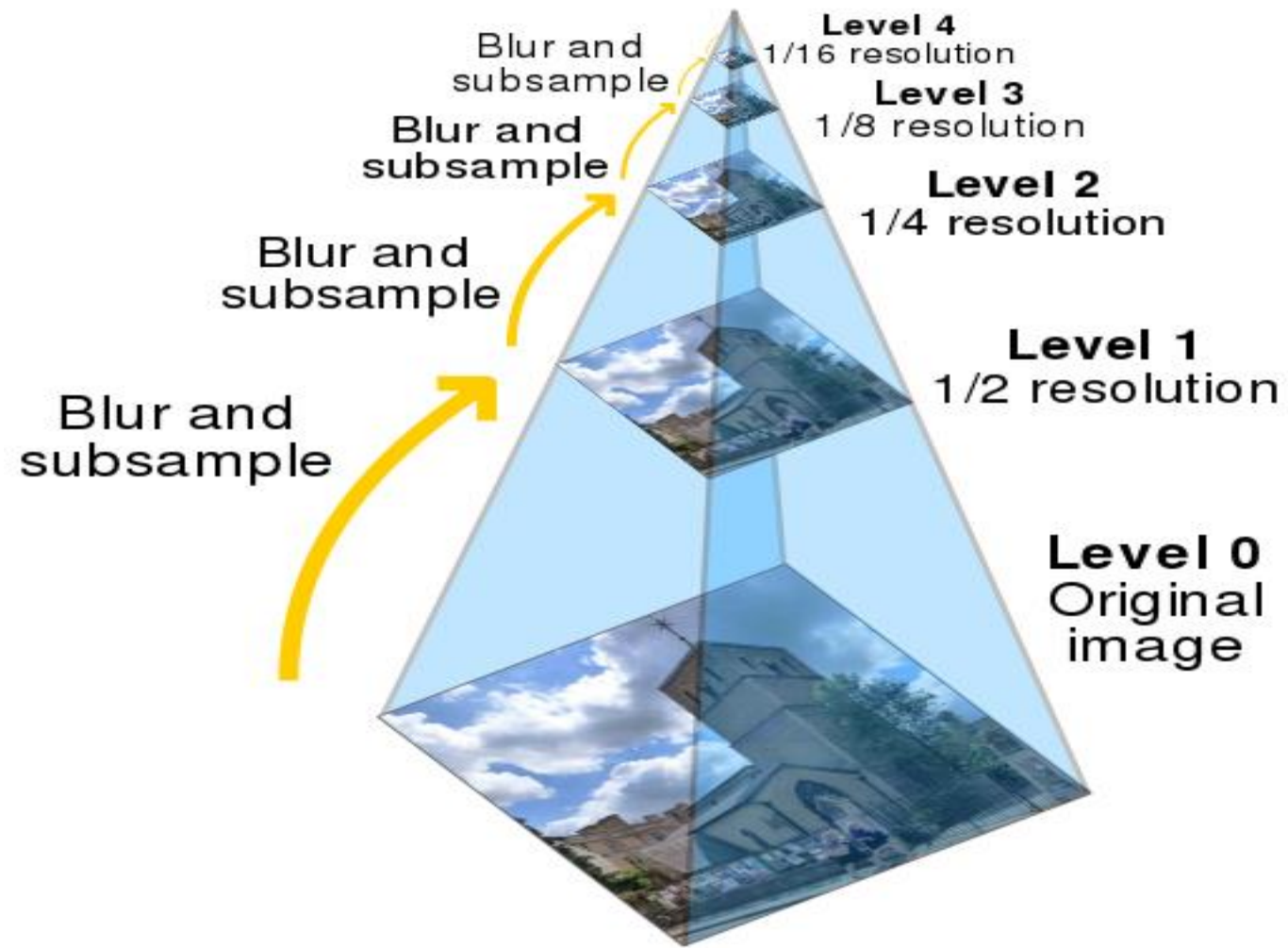
$F_1 * H$



$F_2$

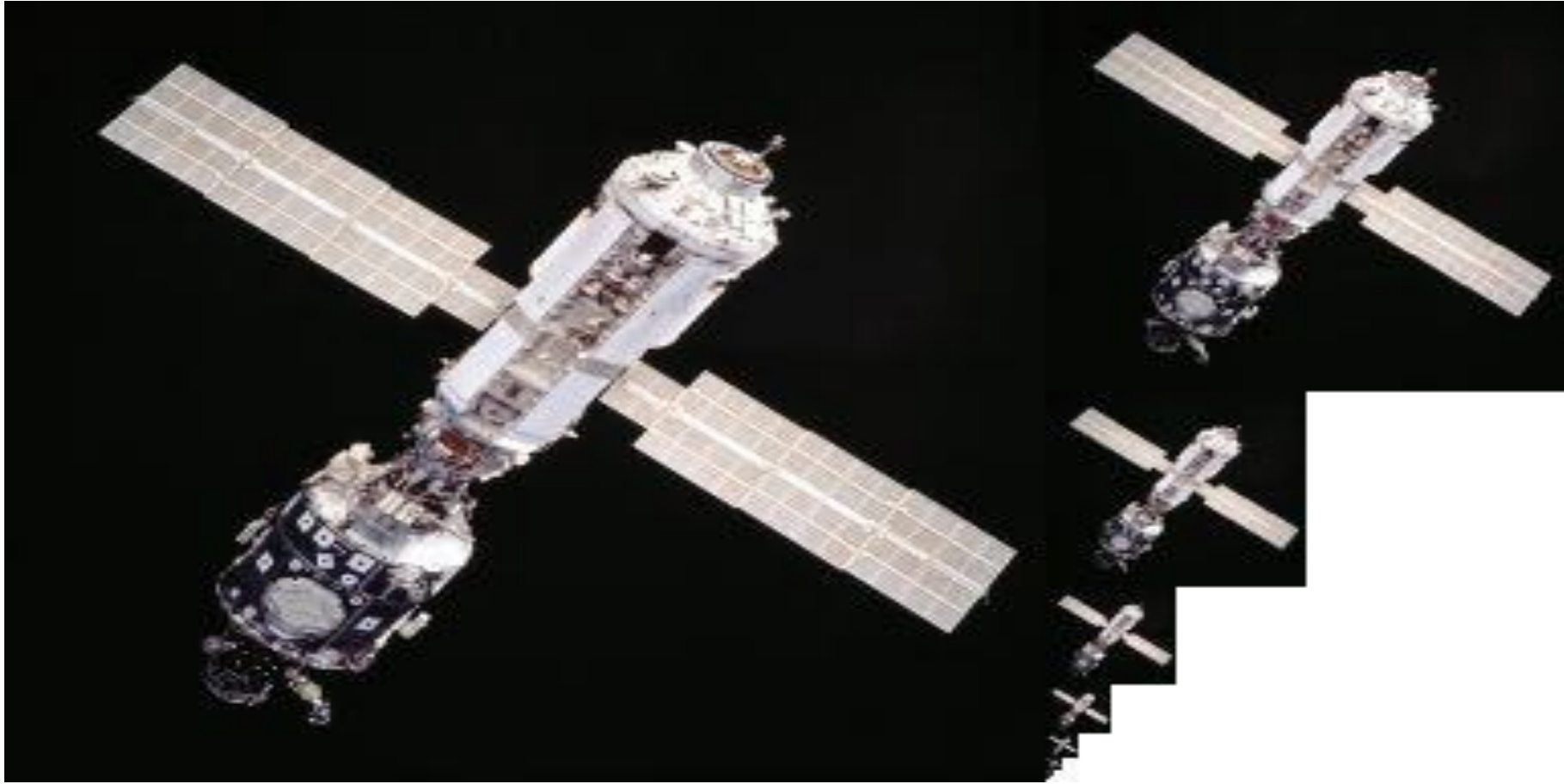
...







# Gaussian Pyramid



# contents

- Image representation
- Pixel-wise operations
- Noise and filtering
- Frequency representation
- Decimation
- **Interpolation**
- Morphology operators
- Connected components

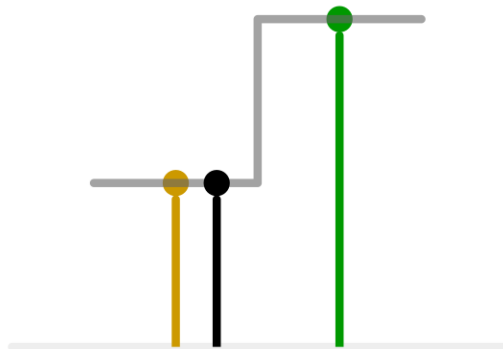


# Upsampling

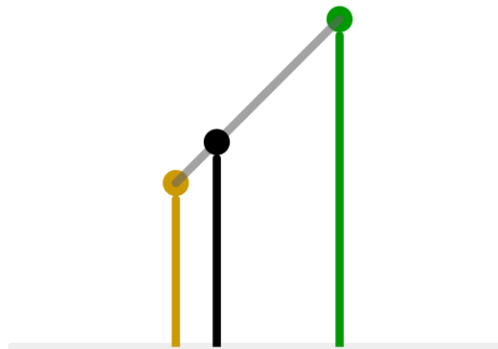
- This image is too small for this screen:
- How can we make it 10 times as big?
- Simplest approach: repeat each row and column 10 times (“**Nearest neighbor interpolation**”)
- This operation is known as **upsampling** or **interpolation**.



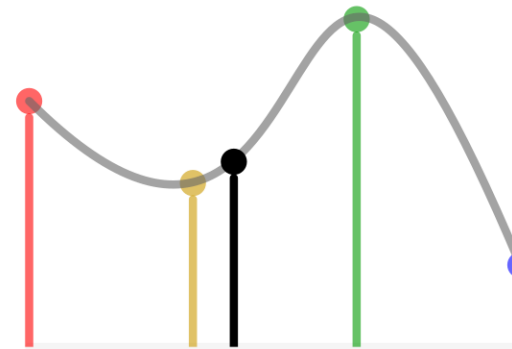
# Upsampling



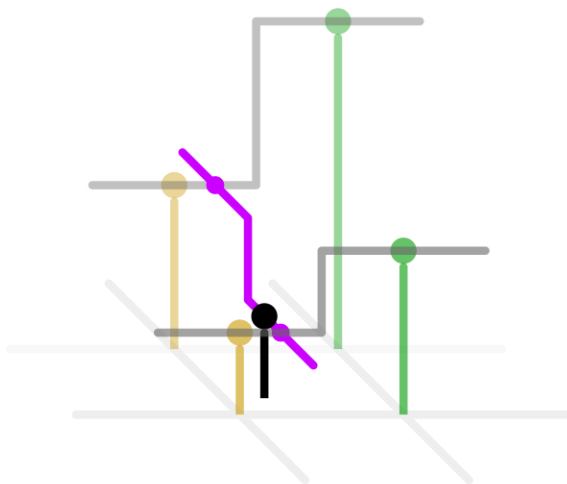
1D nearest-neighbour



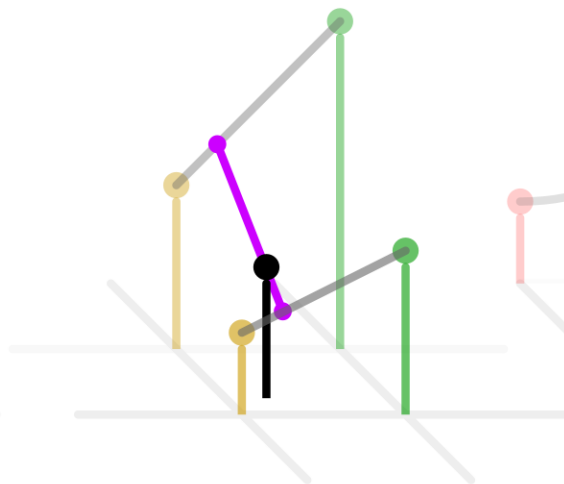
Linear



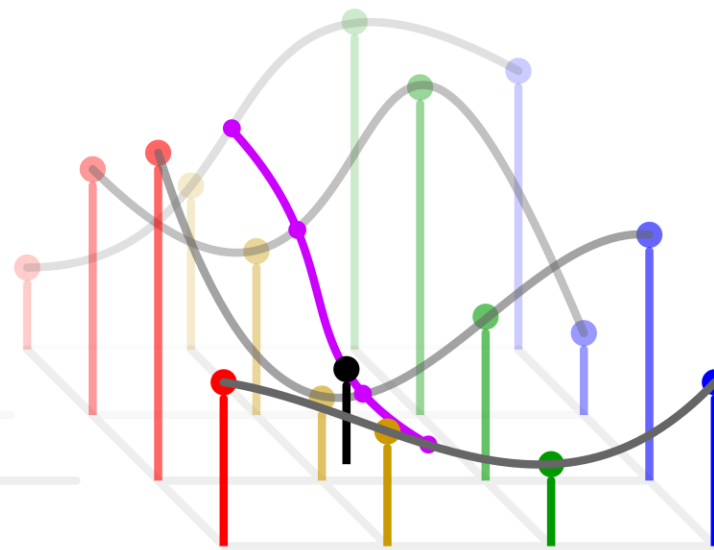
Cubic



2D nearest-neighbour



Bilinear



Bicubic

# Image interpolation

Original image:  x 10



Nearest-neighbor interpolation



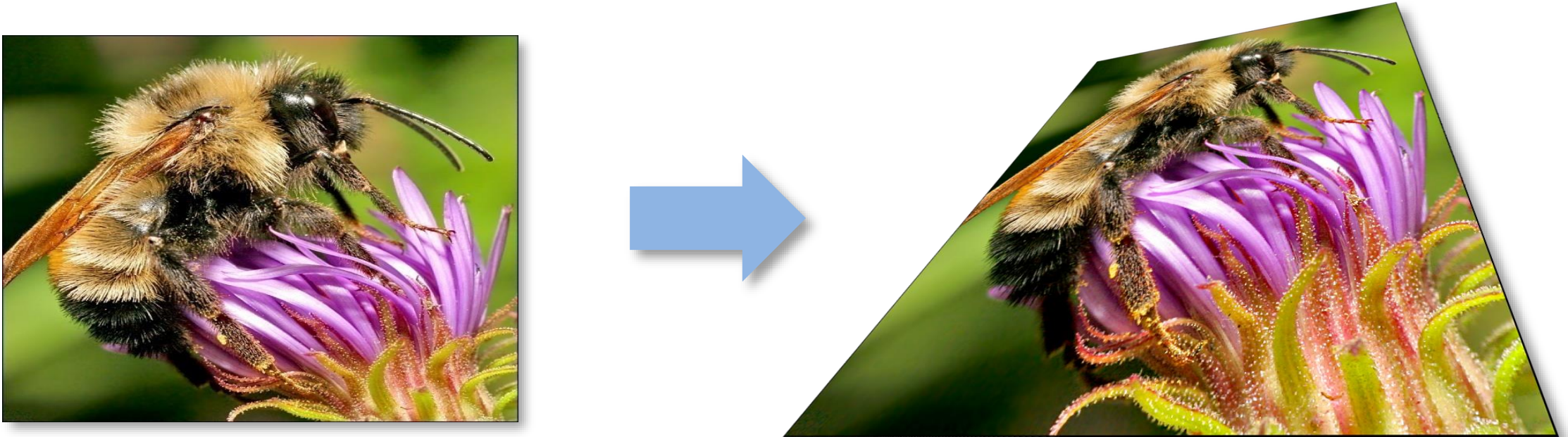
Bilinear interpolation



Bicubic interpolation

# Image interpolation

Also used for image warping



# contents

- Image representation
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- **Morphology operators**
- Connected components

# Morphology

- Handy tool whenever needed to clean up binary images.
- Each morphology operator is constructed as such:
  1. Select some kind of structure element (binary kernel)

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$



# Morphology

- Handy tool whenever needed to clean up binary images.

- Each morphology operator is constructed as such:

1. Select some kind of structure element (binary kernel)

2. Convolve with input binary image  $g = f * s$

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2. Convolve with input binary image  $g = f * s$

3. Threshold the output

$$\theta_{TH}(x, t) = \begin{cases} 1 & \text{if } x \geq t, \\ 0 & \text{else} \end{cases}$$

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$$\theta_{TH}(x, t) = \begin{cases} 1 & \text{if } x \geq t, \\ 0 & \text{else} \end{cases}$$

- Overall morphologic operation should look like so:

$$k = \theta_{TH}(f * s, t)$$

# Dilation

1. Select some kind of structure element (binary kernel)

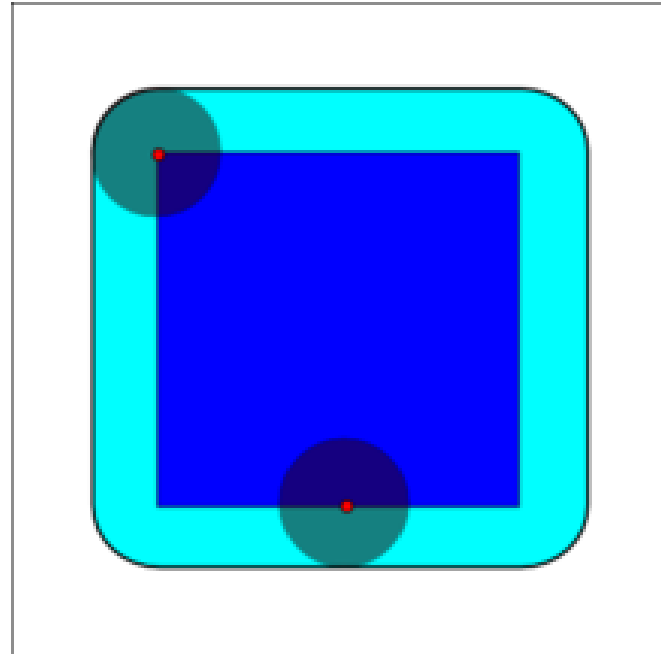
2. Convolve with input binary image  $g = f * s$

3. Threshold the output

$$\theta_{TH}(x, t) = \begin{cases} 1 & \text{if } x \geq t, \\ 0 & \text{else} \end{cases}$$

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- Dilation:  $t = 1$ 
  - Skinny lines will get thicker



# Erosion

1. Select some kind of structure element (binary kernel)

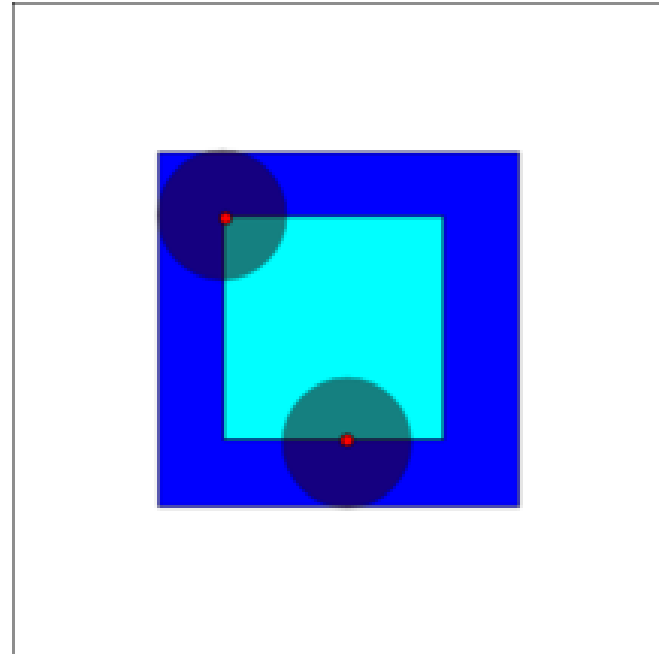
2. Convolve with input binary image  $g = f * s$

3. Threshold the output

$$\theta_{TH}(x, t) = \begin{cases} 1 & \text{if } x \geq t, \\ 0 & \text{else} \end{cases}$$

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

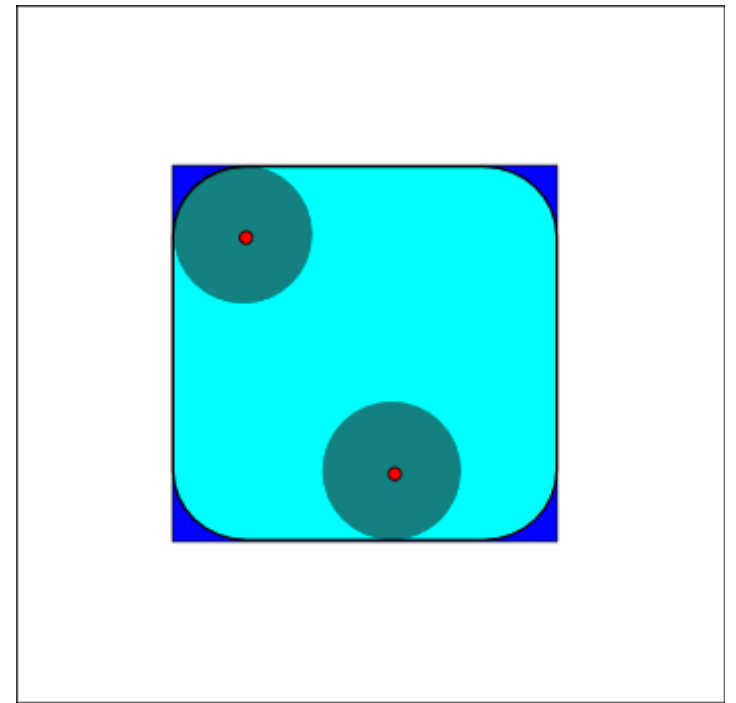
- Erosion:  $t = \text{sum}(s)$ 
  - Thicker lines will get skinny





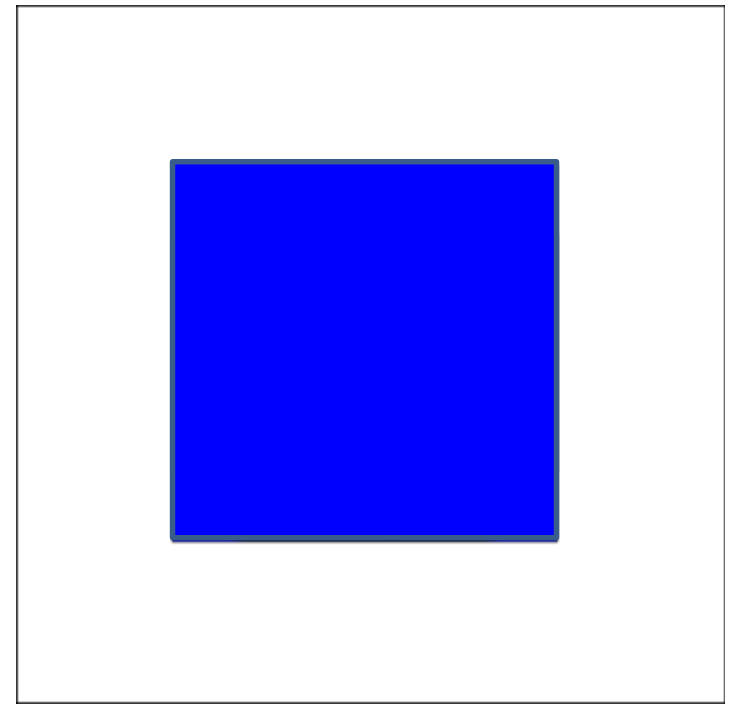
# Opening

- Erosion followed by dilation.
  - The effect is of rounding off sharp edges.



# Closing

- Dilation followed by erosion.
  - The effect is of closing of narrow gaps and holes.



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# Connected components

- Defined as regions of adjacent pixels that have the same value.
- Commonly used with binary images to find stand alone objects.
  - e.g.: letters in a document.

