Stitching of arbitrary parametric surface patches with G1 continuity using B-splines

Article *in* International Journal of Applied Mathematics and Statistics · January 2013

2 authors, including:

Ali Akgunduz
Concordia University Montreal

International Journal of Applied Mathematics and Statistics,
Int. J. Appl. Math. Stat.; Vol. 51; Issue No. 21; Year 2013,
ISSN 0973-1377 (Print), ISSN 0973-7545 (Online)

Stitching of Arbitrary Parametric Surface Patches with G^1 Continuity Using B-splines

Lan Wu[1] and **Ali Akgunduz**[2]

Department of Mechanical and Industrial Engineering, Concordia University,
1455 de Maisonneuve Blvd. West, H3G 1M8 Montreal, Quebec (Canada)

[1]wulan1971@yahoo.com

[2]ali.akgunduz@concordia.ca

ABSTRACT

In computer graphics, the construction of complete 3-dimensional objects requires the blending of several incompatible (mathematical properties are different) surface patches such as Bezier, B-spline, and NURBS surfaces. The blending process often results in undesirable geometric properties along the connected boundaries such as sharp edges, gaps, overlaps. In this paper, we propose an efficient surface blending method for stitching of arbitrary parametric surfaces that guarantees G^1 continuity along stitching boundaries. The surface patching problem between two-surface patches is investigated for following special cases: sharp edges, gaps and overlaps. The proposed surface stitching method is to construct a B-spline blending surface between base surfaces through incorporating the following conditions: i) Determine stitching boundaries on base surfaces; ii) Control points of B-spline blending surface are constructed from the base surfaces' data points and the interpolation scheme; iii) The knot vectors of the B-spline blending surfaces are organized in such way that full multiplicity is achieved at their end-knots; iv) The surface order along the stitching boundary directions is set to 2; and v) Parametric conditions along stitching boundary directions for both B-spline blending surface and base surfaces are forced to be uniform. The main advantages of the proposed methodology are: i) Capability of blending surfaces at the local level; and ii) Blending different kinds of surface patches such as Bezier and B-spline.

Keywords: B-spline, Bezier surface, G^1blending, sharp edges.

Mathematics Subject Classification: 68U05, 65D18

1. INTRODUCTION

In multidisciplinary areas, parametric curves and surfaces have a wide range of applications (Wan Din et al. 2013, and Zakaria et al. 2013). In computer graphics, the construction of complete 3-dimensional objects requires the blending of several incompatible (mathematical properties are different) surface patches such as Bezier, B-spline, and NURBS surfaces. The blending of several incompatible surfaces often results in undesirable geometric properties such as sharp edges, gaps, overlaps, etc. To tackle such problems, the early works focus on methodologies based on parametric representations of the geometries (Barsky et al., 1989 and 1990). In order to blend surface patches, Cheng et al. (2003) used Bezier curve properties. Shi et al. (2004) proposed a local boundary strip scheme. The proposed method achieves G^1smooth connection between two adjacent 6^{th} order-B-spline patches. The work by Lai et al. (2005) focuses on establishing G^1 smooth connection between two adjacent C^1 Bi-Cubic B-spline surfaces. Pungotra et al. (2010) proposed a three-step method to achieve smooth blending between two B-spline surfaces. A surface blending method based on cubic Hermit interpolation was presented by Yang et al. (2011). Their interpolation technique enables a better tangential continuity across the boundary curves that lead to a smoother blending along the stitching lines. Another line of research in surface blending is based on rolling-ball blending methods (Lukacs, 1998, and Huang et al. 2000). Blending methods based on the Finite Element Method (FEM) (Xu, 2008 and 2009) and subdivision methods (Deng et al., 2011, and Wallner et al., 2005) are other techniques. Li et al. (2002) introduced a subdivision surface blending method where the blending is achieved by simultaneously removing boundary and applying Catmull-Clark subdivision method on a pre-constructed mesh that satisfies the smooth continuity conditions. By removing the redundant control points and adding the fairness term in the parametric representation, another smooth technique was proposed by Zhang et al. (2013).

In this paper, we develop a two-surface blending methodology. The method focuses on blending surface patches using B-spline. The proposed method stitches a variety of parametric surfaces and maintains G^1 continuity along the stitching boundaries. The sharp edge, gap and overlapping problems are also handled by this proposed method. Details of the proposed method are discussed in the methodology section.

2. METHODOLOGY

In this section, the problem of stitching base surfaces with G^1continuity is tackled. Let $S^0 and S^1$ be two base-surfaces and $\emptyset$ be a B-spline surface where$S^t = t^t(u,v)$ and $\emptyset^t = l^t(u,v)$for $t = \{0,1\}, 0 \leq u \leq 1, and\ 0 \leq v \leq 1$. The objective is to use $\emptyset$ to stitch two base surfaces $S^0$and S^1 with G^1continuity along the stitching boundaries and no sharp edges, gaps or overlapping regions remaining on the final surface. In order to obtain $\emptyset$, a three-step approach has been adopted: i) The construction of control polygons; ii) The arrangement of knot vectors and orders as well as the determination of parameterizations (number of parametric intervals); iii) The generation of blending surfaces. The details of the proposed method are discussed on several classes of stitching problems in the following section.

In general, for a $\emptyset$ with $(m+3)\times(n+1)$ control points, it is defined as:

$$l(u,v)=\sum_{i=0}^{m+2}\sum_{j=0}^{n} p_{ij}N_{il}(u)N_{jk}(v) \qquad u=\{0,1\}, v=\left\{0,\frac{1}{n^{\emptyset}},\frac{2}{n^{\emptyset}}\cdots\frac{n^{\emptyset}-1}{n^{\emptyset}},1\right\}$$

Where

- $l(u,v)$ is a data point on $\emptyset$.Let $n^{\emptyset}$ be the parameterization of $\emptyset$ along **v** direction and $n^{\emptyset}$ is equal to the parameterization (n) of original base surfaces along the stitching boundaries. Let $m^{\emptyset}$ be the parameterization of $\emptyset$ along **u** direction. As $n^{\emptyset}=n\to\infty$, C^0 connection is guaranteed along the stitching boundaries.
- $p_{ij}$is a control point on the i^{th}row and j^{th}column of $\emptyset$ for $0\le i\le m+2, 0\le j\le n$ where $m+2$and n are the parameterizations of control polygon mesh of $\emptyset$ along **u** and **v** directions respectively; m is the number of rows of data points from both base surfaces, which are used to form the part of the control polygon of $\emptyset$.
- $N_{il}(u)$ and $N_{jk}(v)$ are basis functions of B-spline surfaces, where l and k are orders of B-spline surfaces along **u** and **v** directions respectively.

The following notations are used throughout the paper.

S^t	: A base surface
n^t	: Parameterization of a surface
$\emptyset$	:A B-spline blending surface
d_{ij}^t	: Data point on the i^{th} row and j^{th} column of S^t
C	: Control point matrix of B-spline blending surface
p_{ij}	:A control point in matrix C as $C=\begin{bmatrix} p_{00} & \cdots & p_{0n} \\ \vdots & \ddots & \vdots \\ p_{(m+2)0} & \cdots & p_{(m+2)n}\end{bmatrix}$
l_{ij}	: Data point on the i^{th} row and j^{th} column of $\emptyset$
$N_{il}(u)$ and $N_{jk}(v)$	:Basis functions of B-spline surfaces along **u** and **v** directions respectively
T_j^t	:A row of interpolation control points of $\emptyset$
α_j^t	:A set of proportional constant for the interpolation control points of $\emptyset$
m and n	: Surface parameterizations along **u** and **v** directions respectively
r	: An arbitrary row on a base surface
π	: An arbitrary row on another base surface

Edge blending

Edge blending is defined as the process of constructing a B-spline blending surface $\emptyset$ between two base surfaces $S^0$and S^1, that ensures G^1continuity along the common boundaries between $S^0$and $\emptyset$, S^1 and $\emptyset$. Three different edge blending cases are discussed below.

For all discussed cases, d^t_{ij} is a data point on the i^{th} row and j^{th} column of $S^t$for $i = \{0, 1, \cdots, m^t\}$, $j = \{0, 1, \cdots, n^t\}$, where m^t is the parameterization of S^talong **u** direction; and n^t is the parameterization of S^talong **v** direction ($n^t = n$ for all base surfaces). Let p_{ij}ln $(0 \leq i \leq m + 2, 0 \leq j \leq n)$be a control point and$l(u, v)$be a data point on $\emptyset$. For the two-surface blending problem, $t = \{0, 1\}$.

Case 1: Edge blending between two adjacent base surfaces

<u>Objective:</u> Given two base-surfaces $S^0$and S^1which are connected with G^0 continuity (see Figure 1-(a), construct a $\emptyset$ with G^1continuity along the stitching boundaries.

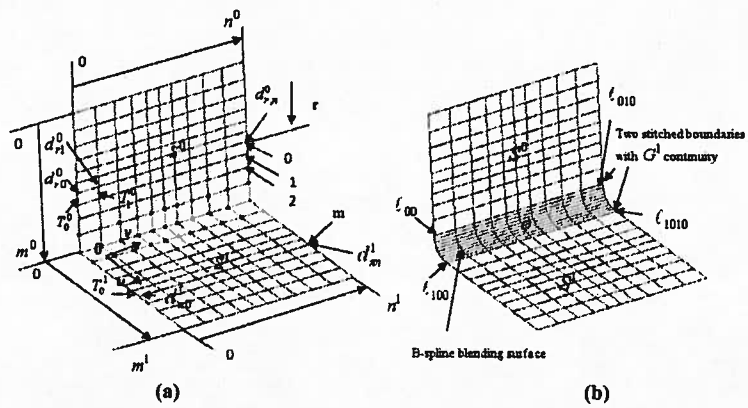


Figure 1.(a) Construction of the control polygon for B-spline blending surface;**(b)** Illustration of blended surface with G^1 continuity for $r = 9$; $m = 2$; $m^0 = m^1 = m^\emptyset = 10$; and $n^0 = n^1 = n^\emptyset = n = 10$

<u>Solution:</u>

Steps:

1.1 Constructing the control polygon for $\emptyset$.

1.1.1 Calculate derivatives of two new selected stitching boundaries along **u** directions.

Let r be a new selected stitching boundary which can be an arbitrary row on $S^0$for $(0 \leq r \leq m^0 - 1)$. Since, with respect to **u**, the first order derivatives of $S^0$is:

$$S^0_u(u, v) = \frac{\partial}{\partial u} S^0(u, v)$$

Then, at new stitching boundary r where $u = u_r$, the derivatives of S_u^0 is:

$$S_u^0(u,v)|_{u=u_r} = \frac{\partial}{\partial u} S^0(u,v)|_{u=u_r}$$

Similarly, let π be another selected stitching boundary $(\pi = m - m^0 + r)$ which can be an arbitrary row on S^1 for $(m^0 - r + 1 \le m \le m^0 + m^1 - r)$. Since, the first order derivatives of S^1 along **u** direction can be calculated as

$$S_u^1(u,v) = \frac{\partial}{\partial u} S^1(u,v)$$

Then, at new stitching boundary π where $u = u_\pi$, the derivatives of S_u^1 is:

$$S_u^1(u,v)|_{u=u_\pi} = \frac{\partial}{\partial u} S^1(u,v)|_{u=u_\pi}$$

1.1.2 Interpolating to obtain two rows of control points

Denote these two rows of control points as T_j^0 and T_j^1 which are adjacent to d_{rj}^0 and $d_{\pi j}^1$ respectively.

- ***Determine*** T_j^0 ***and*** T_j^1

  In order to satisfy the G^1 continuity along **u** directions, T_j^0 and T_j^1 are interpolated along the directions of $S_u^0(u,v)|_{u=u_r}$ and $\pm S_u^1(u,v)|_{u=u_\pi}$. Here $\pm$sign implies that the direction of the tangent vector points towards another base surface. Here,

$$T_j^0 = d_{rj}^0 + \alpha_j^0 S_u^0(u,v)|_{u=u_r}$$
$$T_j^1 = d_{\pi j}^0 - \alpha_j^1 S_u^1(u,v)|_{u=u_\pi}$$

  Where, α_j^0 and α_j^1 are proportional constants and

$$0 \le \alpha_j^0 \le \frac{\left(S_u^0(u,v)|_{u=u_r}\right) \bullet \left(d_{(r+1)j}^0 - d_{rj}^0\right)}{\left|S_u^0(u,v)|_{u=u_r}\right|}$$
$$0 \le \alpha_j^1 \le \frac{\left(-S_u^1(u,v)|_{u=u_\pi}\right) \bullet \left(d_{(\pi-1)j}^1 - d_{\pi j}^1\right)}{\left|-S_u^1(u,v)|_{u=u_\pi}\right|}$$

1.1.3 Then the control points of $\emptyset$ are defined as follows:

$$p_{ij} = \begin{cases} d_{rj}^0 & if\ i = 0; 0 \le r \le m^0 - 1; and\ 0 \le j \le n \\ T_j^0 & if\ i = 1; and\ 0 \le j \le n \\ d_{(r+i-1)}^0 & if\ 2 \le i \le m^0 - r + 1; 0 \le r \le m^0 - 1; and\ 0 \le j \le n \\ d_{(i-m^0+r-1)j}^1 & if\ m^0 - r + 1 < i < m + 1;\ m^0 - r + 1 \le m \le m^0 + m^1 - r; and\ 0 \le j \le n \\ T_j^1 & if\ i = m + 1; and\ 0 \le j \le n \\ d_{\pi j}^1 & if\ i = m + 2;\ m^0 - r + 1 \le m \le m^0 + m^1 - r; and\ 0 \le j \le n \end{cases}$$

Consequently, the control polygon matrix $\mathbf{C}$ for $\emptyset$ is obtained.

1.2 Arranging non-periodic knot vectors and their orders along with the parameterizations of $\emptyset$:

1.2.2 Set k, the order along **v** direction, as: k = 2.

1.2.3 Set l, the order along **u** direction, as: l > 2.

u and **v** directions are illustrated in Figure 1-(a).

1.2.4 Set the knot vectors of $\emptyset$ along **u** and **v** directions having full multiplicities at two end knots, namely,

$$KV^u = \left[\underbrace{0 \cdots 0}_{l} \quad \mu_l \quad \cdots \quad \mu_{\theta-l} \quad \underbrace{1 \cdots 1}_{l}\right]$$

$$KV^v = \left[\underbrace{0 \cdots 0}_{k} \quad v_k \quad \cdots \quad v_{\theta'-k} \quad \underbrace{1 \cdots 1}_{k}\right]$$

Where$\theta = m + 2 + l, and \theta' = n + k$

1.2.5 Arrange the interior knot values uniformly.

1.2.6 Set $n^{\emptyset} = n = n^0 = n^1$.

1.3 Computing the data points $l(u, v)\left(u = \{0,1\}, v = \left\{0, \frac{1}{n^{\emptyset}}, \frac{2}{n^{\emptyset}} \cdots \frac{n^{\emptyset}-1}{n^{\emptyset}}, 1\right\}\right)$ for $\emptyset$.

After applying above three-step approach, the following B-spline properties are achieved in table 1.

Table 1: B-spline properties review

Property achieved	**Effects on the geometry**
Full multiplicity properties at two end knots	i. The boundary polygons of the control polygon mesh of B-spline blending surface are the control polygons of boundary curves of B-spline blending surface.
	ii. B-spline blending surface interpolates the four corner control points $p_{00}, p_{0n}, p_{(m+2)0},$ and $p_{(m+2)n}$
	iii. Four boundary curves of B-spline blending surface tangent to the first and last segments of their corresponding control polygons.
Order of the B-spline is equal to 2 (k = 2).	The parametric curve becomes its own control polygon.

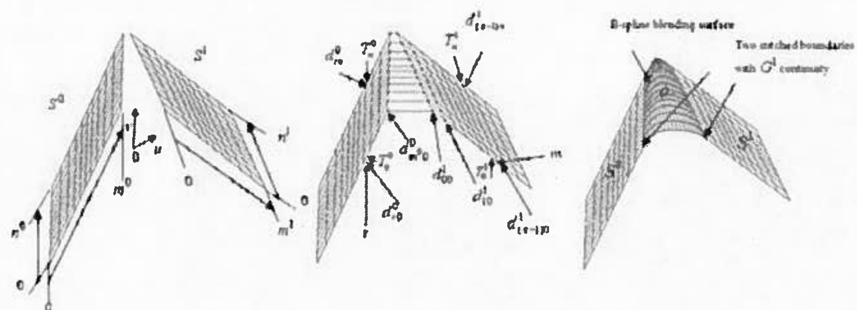
The two properties given in Table 1 and the condition discussed in *Step* 1.2.5 ensure that the boundaries along v direction are overlapped and that at the four end points of the control polygon, the G^1continuity is maintained.

As a result, the following conclusions are drawn:

1. By using $\emptyset$ to replace base surfaces' segments between two selected stitching boundaries, the G^1smooth transition task between given base surfaces is realized.
2. The proposed methodology gives greater flexibility by enabling the user to define the smoothing range on each base surface.

Case 2: Edge blending between two nonadjacent base surfaces

__**Objective**__: Given two nonadjacent base surfaces S^0 and S^1 as shown in Figure 2-(a), construct a $\emptyset$ that satisfies G^1continuity in the connecting boundaries as shown in Figure 2-(c).



(a) (b) (c)

Figure 2.(a) Base surfaces with a gap in between;**(b)** Control polygon of $\emptyset$;**(c)** Blended surface withG^1continuity for $r = 9; m = 3; m^0 = m^1 = 10; m^{\emptyset} = 20; and n^0 = n^1 = n^{\emptyset} = n = 10$

__Solution:__

In the problem defined in *Case 2*, $\emptyset$ is obtained by using one of the following two approaches.

i. Multi-row data-point-based method

Steps:

2.1 Constructing the control polygon for $\emptyset$ as follows:

Repeat *steps* 1.1.1 and 1.1.2 to obtain interpolation control points $T_j^0$and T_j^1.

Then the control points of $\emptyset$ are defined as follows:

$$p_{ij}=\begin{cases} d_{rj}^0 & if\ i = 0; 0 \le r \le m^0 - 1; and\ 0 \le j \le n \\ T_j^0 & if\ i = 1; and\ 0 \le j \le n \\ d_{(r+i-1)}^0 & if\ 2 \le i \le m^0 - r + 1; 0 \le r \le m^0 - 1; and\ 0 \le j \le n \\ d_{(i-m^0+r-2)j}^1 & if\ m^0 - r + 1 < i \le m;\ m^0 - r + 1 < m \le m^0 + m^1 - r + 1; and\ 0 \le j \le n \\ T_j^1 & if\ i = m + 1; m^0 - r + 1 < m \le m^0 + m^1 - r + 1; and\ 0 \le j \le n \\ d_{(\pi-1)j}^1 & if\ i = m + 2;\ m^0 - r + 1 < m \le m^0 + m^1 - r + 1; and\ 0 \le j \le n \end{cases}$$

Consequently, the control polygon matrix $\mathbf{C}$ of $\emptyset$ is obtained.

2.2 Repeat *Steps* 1.2. and 1.3.as discussed in *Case 1*, which leads to the result shown in Figure 2-(c).

ii. Single-boundary-line based method

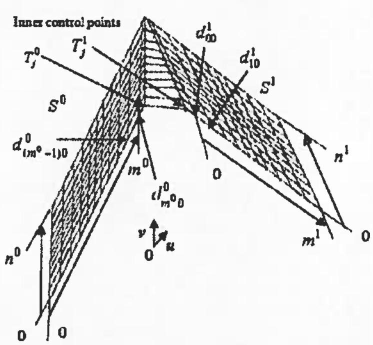


Figure 3. Inner control points of B-spline blending surface

Steps:

2.1 Repeat *steps* 1.1.1 and 1.1.2 to interpolate at least two rows of inner control points between two boundary lines $d^0_{m^0j}$andd^1_{0j}, $(0 \le j \le n)$ and construct control polygon (see Figure 3).

The control points of $\emptyset$ are defined as follows:

$$p_{ij} = \begin{cases} d^0_{m^0j} if\ i = 0; and 0 \le j \le n \\ T^0_j \ \ if\ i = 1; and\ 0 \le j \le n \\ T^1_j \ \ if\ i = 2; and\ 0 \le j \le n \\ d^1_{0j}\ if\ i = 3; and\ 0 \le j \le n \end{cases}$$

Where $\alpha^0_j$and $\alpha^1_j (0 \le j \le n)$ are determined empirically.

2.2 Repeat *Steps* 1.2. and 1.3. as discussed in *Case 1*.

It should be noted that the advantage of implementing the single-boundary-line based method is that all geometric features of the given base surfaces are maintained since no surface segment is being replaced.

Case 3: Edge blending with local overlap along the common boundary

<u>**Objective**</u>: Given two base surfaces $S^0$and S^1 that have some overlapping data points along their boundaries as shown in Figure 4-(a), obtain a $\emptyset$ between $S^0$and S^1 that ensures G^1continuity along the connecting boundary lines between S^0 and $\emptyset$, S^1 and $\emptyset$ as shown in Figure 4-(d).

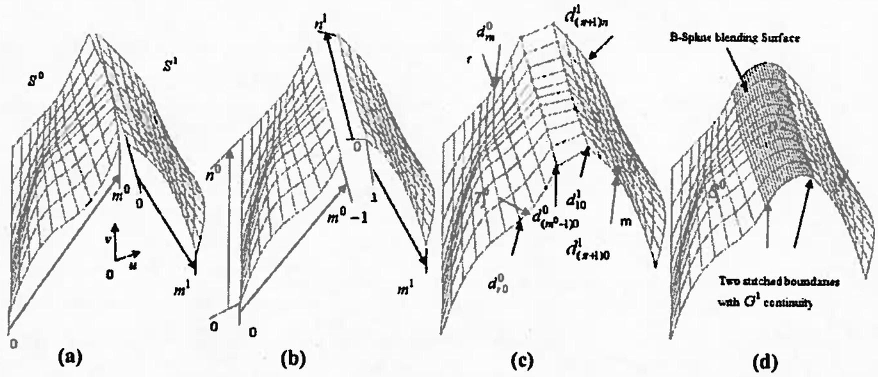


Figure 4.(a) Overlapping points;(b) After overlapping region is dropped; (c) Control polygon of $\emptyset$;(d)Final shape with G^1 continuity for $r = 8; m = 3;\ m^0 = m^1 = 10; m^\emptyset = 20;$ and $n^0 = n^1 = n^\emptyset = n = 10$

Solution:

Drop the overlapping segments from both base surfaces as shown in Figure 4-(b) when constructing the control polygon of B-spline blending surface $\emptyset$. The problem then turns out to be the same as discussed in *Case 2*.

3. CONCLUSION

In this paper, one of the challenging problems in computer graphics, the surface blending problem is discussed. The paper introduced a technique to handle two-surface blending cases. The proposed surface blending method achieves G^1 blending of two arbitrary parametric surfaces and handles the geometric drawbacks such as sharp edges, gaps, and overlaps. The final surface after blending is a smooth surface which consists of a B-spline blending surface and part of base surfaces. The surface blending method proposed in this paper fully utilizes the properties of B-splines, thus ensuring the G^1 continuity to be satisfied along the stitching boundaries of base surfaces and B-spline blending surface. In the proposed method, the control polygon of the B-spline blending surface is formed by two base-surface data pairs and interpolation points, and the blending region is user-defined. This leads to two advantages: i) blending can be achieved locally without deforming the main geometric features of base surfaces; ii) blending can be achieved between two arbitrary parametric surfaces. Finally, it is worth to note that the method proposed in this paper is also suitable for G^1blending of two arbitrary parametric curves.

4. REFERENCES

Barsky, B.A., DeRose, T.D., 1990, Geometric Continuity of Parametric Curves: Constructions of Geometrically Continuous Splines. *IEEE Computer Graphics &Applications*. **10**(1), 60-68.

Barsky, B.A., DeRose, T.D., 1989, Geometric Continuity of Parametric Curves: Three Equivalent Characterizations. *IEEE Computer Graphics & Applications*. **9**(6), 60-69.

Cheng, J., Gao, X.S., 2003, Constructing Blending Surfaces for Two Arbitrary parametric Surfaces. *MMRC, AMSS, Academic, Sinica, Beijing*. **22**, 14-28.

Deng, C.Y., Ma, W., 2011, Constructing an Interpolating Subdivision Scheme from Doo-Sabin Subdivision. *12th International Conference on Computer-Aided Design and Computer Graphics*. 215-222.

Huang, L., Zhu, X., 2000, Construction of Blending Surface. *Technical report, Beijing University of Aeronautics & Astronautics*.

Lai, M-J, Jian, J., A.L., Patrick F.C., 2005, Removal of Gaps among Compound Bi-Cubic Parametric B-spline Surface. *Wavelets and Splines: Athen*. 287-313.

Li, G.Q., Li, H., 2002, Blending Parametric Patches with Subdivision Surfaces. *Journal of Computer Science and Technology*. **17**(4), 498-506.

Lukacs, G., 1998, Differential Geometry of Variable Radius Rolling Ball Blending Surfaces.

Computer Aided Geometric Design. **15**(6), 585-613.

Pungotra, H., Knopf, G.K., Canas, R., 2010, Merging Multiple B-Spline Patches in a Virtual Reality Environment. *Computer-Aided Design*. **42**(10), 847-859.

Shi, X., Wang, T., Yu, P., 2004, A Practical Construction of Smooth Biquintic B-spline Surfaces over Arbitrary Parametric topology. *Computer-Aided Design*. **36**(5), 413-424.

Wan Din, W.R., Rambely, A.S., Jemain, A.A., 2013, Smoothing of GRF Data Using Functional Data Analysis Technique. *International Journal of Applied Mathematics and Statistics*. **47**(17).

Wallner, J., Dyn, N., 2005, Convergence and C1-analysis of Subdivision Schemes on Manifolds by Proximity. *Computer Aided Geometric Design*. **22**(7), 593-622.

Xu, G., 2008, Finite Element Methods for Geometric Modeling and Processing Using General Fourth Order Geometric Flows.*GMP 2008, LNCS 4975*. 164-177.

Xu, G., 2009, Mixed Finite Element Methods for Geometric Modeling Using General Fourth Order Geometric Flows. *Computer Aided Geometric Design*. **26**(4), 378-395.

Yang, L., Ong, S.K., Nee, A.Y.C., 2011, Surface Blending Approach for Displacement Features on Freeform Surfaces. *Computer-Aided Design*. **43**(1), 57-66.

Zakaria R., Wahab, A.F., 2013, Type-2 Fuzzy Bezier Surface. *International Journal of Applied Mathematics and Statistics*. **40**(10).

Zhang, J.M., Wang, H.L., 2013, Remove and Redistribute the Control Points for Bezier Curve Approximation. *International Journal of Applied Mathematics and Statistics*. **48**(18).