Projet4_StatsEnGrandeDim

December 12, 2018

1 Projet 4 - Statistiques Grandes Dimensions

```
In [1]: import numpy as np
        import pandas as pd
        import sklearn.linear_model as skl_linear_mdl
        import sklearn.model_selection as skl_mdl_selection
        import sklearn.metrics as skl_metrics
        import matplotlib.pyplot as plt
        from mpl_toolkits.mplot3d import Axes3D
        import random
```

```
In [2]: def print_plot(subtitle,ttle,xlab,ylab):
            plt.figure()
            plt.suptitle(subtitle,
                         fontsize=14)
            plt.title(ttle, fontsize=10)
            plt.ylabel(ylab)
            plt.xlabel(xlab)
```

```
In [3]: def print_plot3D(subtitle,ttle,xlab,ylab,zlab,alpha_rge,lambda_rge,quad_risk,toPivot):
            fig = plt.figure(figsize=(7,7))
            plt.suptitle(subtitle)
            ax = fig.add_subplot(111, projection='3d')
            alpha_axis, lambda_axis = np.meshgrid(alpha_rge, lambda_rge)
            if toPivot==0:
                ax.plot_surface(alpha_axis, lambda_axis, quad_risk.values)
            elif toPivot==1:
                ax.plot_surface(lambda_axis,alpha_axis,  all_quad_risk.values)
            ax.set_xlabel(xlab)
            ax.set_ylabel(ylab)
            ax.set_zlabel(zlab)
            plt.show()
```

2 Exercice 1 - Régularisation

Objectif du projet:

Dans ce projet, nous testerons différents estimateurs utilisés dans la prédiction de variables afin d'observer dans quel cas ceux-ci sont les plus appropriés. Les estimateurs sont les suivants: -Lasso -Ridge -Elastic Net

On rappelle: *Lasso:*

$$\hat{\beta^L} = argminL_\lambda(\beta)où\ L_\lambda(\beta) = \frac{1}{2n}\sum_{i=1}^{n}(Y_i - X_i * \beta) + \lambda||\beta||_1 avec\ (\lambda > 1)$$

Ridge

$$\hat{\beta^R} = argminR_\mu(\beta)\text{où}\ R_\mu(\beta) = \frac{1}{2n}\sum_{i=1}^{n}(Y_i - X_i * \beta) + \frac{\mu}{2}||\beta||_2^2\text{avec}\ (\mu > 0)$$

* Elastic Net

$$\beta^{\hat{E}N} = argminF_\lambda(\beta)\text{où}\ F_\lambda(\beta) = \frac{1}{2n}\sum_{i=1}^{n}(Y_i - X_i * \beta) + \lambda(\alpha||\beta||_1 + \frac{1-\alpha}{2}||\beta||_2^2)$$

3 Question 1

3.0.1 Simulation de X et Y

- On simule une matrice X (1000 * 5000) où chaque coordonnée suit une loi normale centrée réduite
- On simule un vecteur eta de (1000 * 1) où chaque coordonnée suit une loi normale centrée réduite
- On fixe un vecteur beta de (5000 * 1000) où chaque coordonnée est nulle, à l'exception des 15 premières qui valent 1
- On calcul finalement le Y simulé, qui est égale à X * beta + eta
- Enfin, on construit notre dataset, qui est composé du Y en première colonne (c'est la target) et des X sur les 5000 colonnes suivantes (ce sont les features simulées)

```
In [5]: # Simulating X
        xindex = ["X" + str(i) for i in range(1, 5001)]
        X = pd.DataFrame(np.reshape(np.random.normal(0, 1, (1000*5000)), (1000, 5000)),columns=
        # Simulating Eta
        eta = pd.DataFrame(np.random.normal(0, 1, 1000), columns=["Eta"])
        # Fixing Beta
        beta = pd.DataFrame(np.repeat(0, 5000), columns=["Beta"])
        for i in range(0, 15):
            beta.loc[i, ] = 1

        # Compute simulated Y
        Y = pd.DataFrame(np.dot(X, beta) + eta.values,columns=["Y"])
        dataset=pd.DataFrame()

        dataset = Y
        dataset[xindex] = X

        dataset.head(5)
```

Out[5]:
```
          Y        X1        X2        X3        X4        X5        X6  \
0 -9.495247  0.252529  0.978173 -1.648376 -0.745150 -0.549010  0.262922
1  1.057552  1.533244  2.006552 -0.190147  0.194154 -0.079179 -2.006603
2  5.028802 -0.166424  2.203721  0.553657 -0.116348  1.172426 -1.116846
3  2.892730 -0.163507  1.075256  0.880031 -0.866662  0.829735 -0.632193
4  9.268786  0.235276  0.227232  0.300307  0.607062  1.144957  0.806097

         X7        X8        X9    ...        X4991     X4992     X4993  \
0 -1.853830 -1.433932 -1.381969    ...     1.249191  0.614602  0.257282
1 -0.960380 -0.295381  0.505217    ...     0.870553  0.931574  1.179348
2  0.268590  0.096138  1.504544    ...     0.253599 -0.174195  1.164225
3  0.224687  2.140223 -0.254601    ...    -1.532971  0.734220 -2.213476
4  1.982821  2.302154 -0.770721    ...     0.209895  0.528178  1.088278

      X4994     X4995     X4996     X4997     X4998     X4999     X5000
0  2.753593  0.191534 -0.458562  0.659595 -1.540402 -1.696091 -0.216240
1 -1.245467  0.303608 -1.480956  2.010872 -0.051878  2.459523 -0.175188
2  1.068989  1.193183 -0.046513  1.621068 -0.521043  0.587093 -0.134657
3 -1.355496 -1.487412 -1.778325  1.236865  0.480267 -0.984885 -0.917793
4  0.738285 -1.541232  1.805350 -0.605179 -0.866061  0.621735 -0.598873

[5 rows x 5001 columns]
```

3.0.2 Séparation du dataset en Train / Test

- On sépare assez classiquement le dataset en 2 parties : le train set, 66% des données le test set, 34% des données

On utilise pour cela la fonction de sklearn : train_test_split

In [6]:
```python
# Splitting X in training and testing set
train_set, test_set = skl_mdl_selection.train_test_split(dataset,
                                                          train_size=0.66,
                                                          test_size=0.34)


X_train = train_set.iloc[:, 1:]
Y_train = train_set.iloc[:, 0]
X_test = test_set.iloc[:, 1:]
Y_test = test_set.iloc[:, 0]

X_train.head(5)
```

Out[6]:
```
           X1        X2        X3        X4        X5        X6        X7  \
241 -0.411098 -0.145020 -0.601383 -1.016331 -1.211025  0.230308  0.108508
96  -0.909034 -0.605538  0.102225  0.542965  1.159063 -1.391814 -0.927249
518 -1.109685  1.652172 -0.291129  0.912655  0.855644  0.826285 -0.448934
576 -0.837278  1.048874 -1.053501 -0.572636 -1.135570  0.940757  0.164049
35   0.880089 -0.184881  0.117213  0.231647 -1.480395  1.665049  1.736750
```

```
            X8        X9       X10    ...       X4991     X4992     X4993  \
241  1.038159 -0.035076  0.241963    ...   -0.216262  0.599253 -1.062285
96  -0.325360 -1.691028 -1.453019    ...    1.991209 -0.032368 -1.259970
518 -1.040584 -1.312625 -0.082198    ...    0.820791  2.303144  0.472122
576 -1.177026  0.284111 -0.776715    ...   -0.873612 -0.215861 -0.339654
35  -0.729738  1.122556 -1.219183    ...    0.592456  1.162374  0.742223

        X4994     X4995     X4996     X4997     X4998     X4999     X5000
241 -0.844311 -2.037630  1.039755  0.369866  0.073692  2.185047 -0.961506
96   0.302735  1.830717 -0.267040  1.010104  1.145018  1.549268  1.234028
518  0.361112 -0.138547 -0.385430 -0.720588  1.084893 -0.397409 -0.706167
576  1.017378 -0.062421 -2.025236 -1.283571 -1.305765  0.573330 -0.256140
35  -0.217050 -1.052846  0.237344 -1.439791 -0.915768 -0.643530  0.887570

[5 rows x 5000 columns]
```

3.0.3 A) On estime la Régression Elastic Net pour alpha variant de 0 à 1

On estime les 5000 coefficients pour alpha variant entre 0 et 1 (lambda est égal à 1): * on crée un range de alpha de taille 11 * on itère sur ce range, en calculant à chaque itération le vecteur beta estimé de (5000*1)* on remplit un dictionnaire qui contiendra les 5000 * 11 betas issus de nos simulations

```
In [7]: def elastic_net_return_coeff(iterateOnL1 ,alpha_value, l1ratio_value, X_train, Y_train)
            all_beta_chap = dict()
            toIterate = np.linspace(0, 1, 11)
            for i_toIterate in toIterate:

                if iterateOnL1==True:
                    lasso = skl_linear_mdl.ElasticNet(alpha=alpha_value, l1_ratio=i_toIterate,
                                                      fit_intercept=False)#, max_iter = 10000)

                if iterateOnL1==False:
                    lasso = skl_linear_mdl.ElasticNet(alpha=i_toIterate, l1_ratio=l1ratio_valu
                                                      fit_intercept=False)#, max_iter = 10000)

                lasso.fit(X_train, Y_train)
                beta_chap = lasso.coef_

                all_beta_chap.update({i_toIterate: beta_chap})

            return all_beta_chap
```

- Affichage des coefficients pour chaque valeur de alpha

On affiche, pour chacun des 5000 betas, sa valeur en fonction de alpha On peut donc observer dans quelle mesure le modèle shrinke les coefficients en fonction de la valeur de alpha. Dans notre cas, on sais que 15 coefficients sont égaux à 1, tandis que les 4985 autres sont nuls : * on observe sur le graphique que pour un alpha entre 0 et 0.2, nos 15 coefficients valent à peu près 0.5, et que leur variance diminue à mesure que alpha augmente. Cependant, ils sont en moyenne de plus en plus éloigné de leurs vrais valeurs (à savoir, 1), et sont tous nuls lorsque alpha vaut 1. * En ce qui concerne les 4985 coefficients nuls, leur variance est élevé pour un alpha proche de zéro (et ils ne sont donc pas tous nuls). En revanche, dès que alpha augmente, ils deviennts tous nuls (leurs vraie valeur)

```
La bonne valeur de alpha est donc un peu éloigné de 0 (afin que les 4985 coefficents nuls
```

In [8]:
```python
all_beta_chap = dict()

all_beta_chap = elastic_net_return_coeff(iterateOnL1=True,alpha_value=1,l1ratio_value=
                                         X_train=X_train,Y_train=Y_train)

beta_vs_alpha = pd.DataFrame(all_beta_chap)
print_plot("Regularization Path for Elastic Net over Alpha Parameter","Lambda = 1","Al
alpha_range=np.linspace(0,1,11)
for row in beta_vs_alpha.iterrows():
    row = row[1]
    plt.plot(alpha_range, row)

beta_vs_alpha.head(20)
```

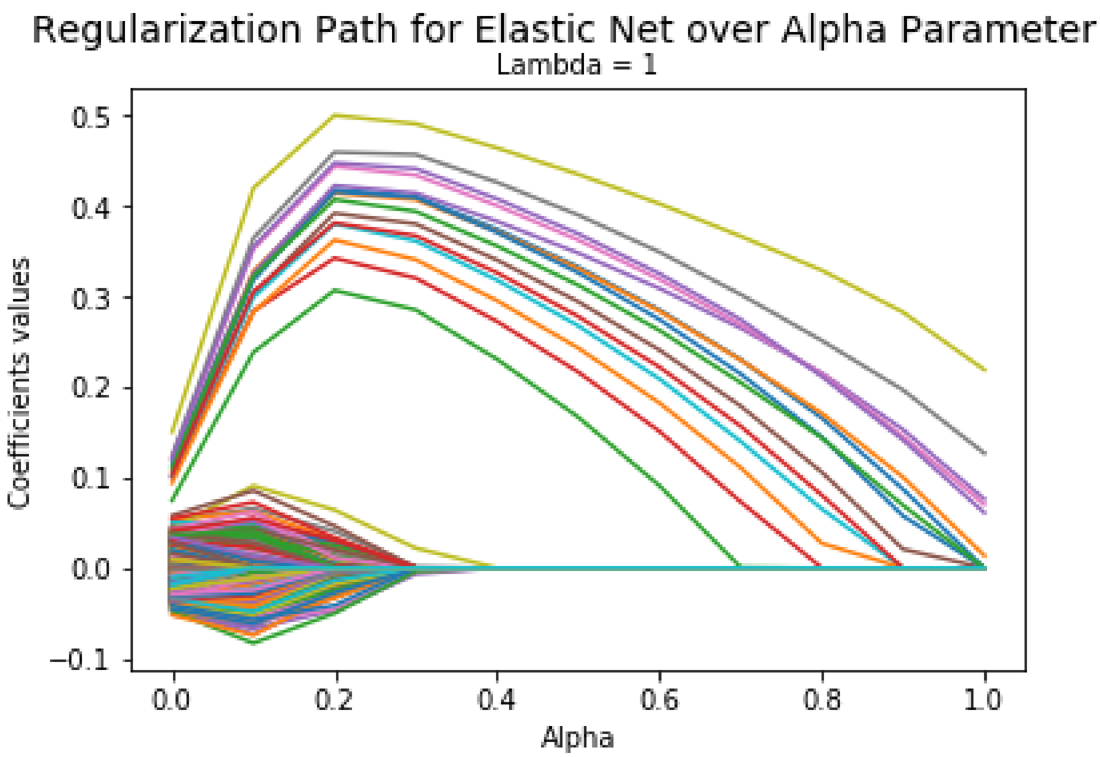
```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  ConvergenceWarning)
```

Out[8]:

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	\
0	0.100806	0.323203	0.417339	0.409784	0.373449	0.331859	0.284457	
1	0.106871	0.327132	0.413363	0.406178	0.370956	0.329974	0.283242	
2	0.074613	0.237693	0.306034	0.284478	0.230360	0.166249	0.090825	
3	0.100188	0.283142	0.341606	0.319563	0.272003	0.216055	0.150631	
4	0.120269	0.324353	0.421593	0.413410	0.382287	0.347236	0.308203	
5	0.104607	0.301891	0.391174	0.379363	0.339577	0.293497	0.240428	
6	0.119875	0.351485	0.442831	0.432993	0.399694	0.361618	0.318276	
7	0.123641	0.363620	0.458348	0.455737	0.425183	0.389203	0.348364	
8	0.149943	0.418535	0.499025	0.489648	0.463071	0.433925	0.401651	
9	0.101699	0.298290	0.379534	0.360586	0.317404	0.267062	0.208529	
10	0.114183	0.316986	0.414996	0.408583	0.370037	0.325454	0.274041	
11	0.093782	0.281892	0.361690	0.339886	0.294738	0.242542	0.181977	
12	0.111085	0.321473	0.406366	0.393105	0.355100	0.311764	0.262063	
13	0.102499	0.304409	0.380443	0.366291	0.325590	0.277355	0.221610	
14	0.121217	0.354160	0.446415	0.440141	0.407051	0.368502	0.324577	
15	-0.013283	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000	
16	0.020846	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	
17	0.013522	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	
18	-0.000957	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	
19	0.032096	0.029276	0.000000	0.000000	0.000000	0.000000	0.000000	

	0.7	0.8	0.9	1.0

0	0.229974	0.165231	0.086854	0.000000
1	0.229451	0.170432	0.100176	0.013836
2	0.000873	0.000000	0.000000	0.000000
3	0.073064	0.000000	0.000000	0.000000
4	0.264556	0.214905	0.152503	0.076177
5	0.178629	0.105620	0.020721	0.000000
6	0.268459	0.213713	0.144600	0.070305
7	0.301591	0.250769	0.196183	0.126612
8	0.365799	0.328105	0.281959	0.218631
9	0.139578	0.065072	0.000000	0.000000
10	0.214092	0.144151	0.057395	0.000000
11	0.110845	0.027127	0.000000	0.000000
12	0.204448	0.143290	0.069195	0.000000
13	0.156431	0.080252	0.000000	0.000000
14	0.274067	0.211117	0.140981	0.061083
15	-0.000000	-0.000000	-0.000000	-0.000000
16	0.000000	0.000000	0.000000	0.000000
17	0.000000	0.000000	0.000000	0.000000
18	0.000000	0.000000	-0.000000	-0.000000
19	0.000000	0.000000	0.000000	0.000000



3.0.4 B) Chemin de Régularisation (Regularization Path) de l'estimateur Lasso

On estime les 5000 coefficients pour lambda variant entre 0 et 1 (alpha est égal à 1): * on crée un range de lambda de taille 11 * on itère sur ce range, en calculant à chaque itération le vecteur beta estimé de (5000*1)* on remplit un dictionnaire qui contiendra les 5000 * 11 betas issus de nos simulations

De la même manière que pour le coefficient alpha, on affiche, pour chacun des 5000 betas, sa valeur en fonction de lambda : * On voit qu'entre 0 et 0.2, les 15 coefficients sont très proches de leurs vrai valeurs (1), puis descendent linéairement jusqu'à 0, lorsque lambdba vaut 1 * les 4985 nuls sont un peu dispersés autour de 0, pour alpha entre 0 et 0.2. Après 0.2, les 4985 coefficients estimés sont tous nuls (égaux à leurs vrai valeur, donc)

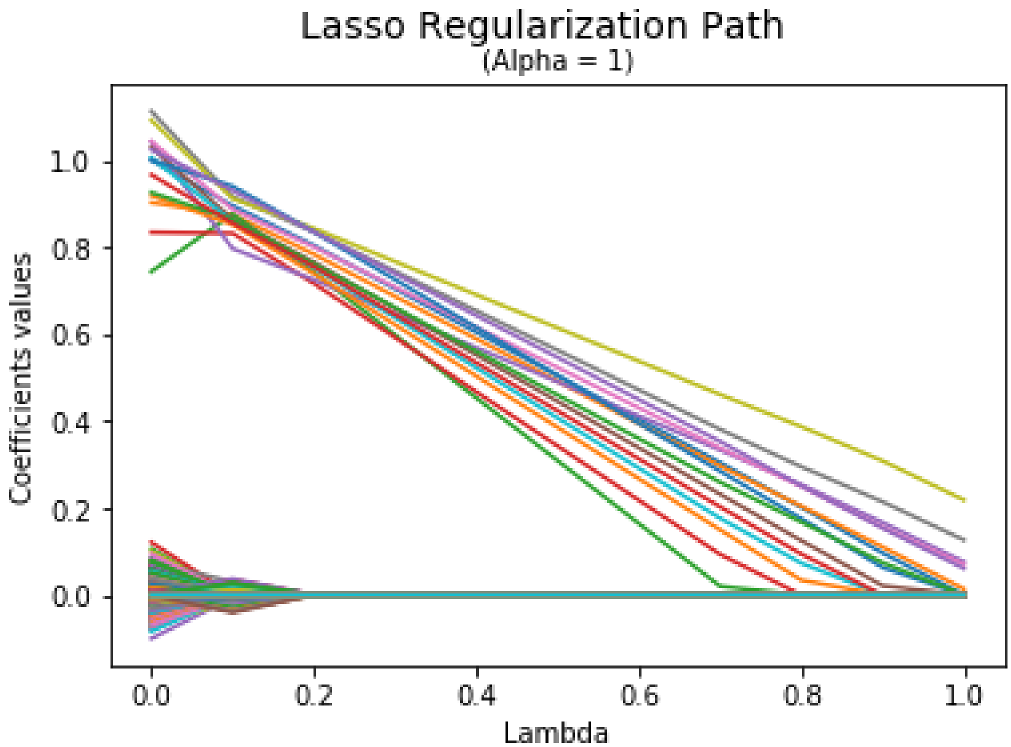
```
Graphiquement, on poserais un lambda égal à 0.1 ou 0.2
```

- Méthode 1 : En calculant nous mêmes

```
In [141]: def print_regularization_path(X_train,Y_train,isLasso=False):
              all_quad_risk = dict()
              all_beta_chap = dict()
              lambda_range = np.linspace(0, 1, 11)
              if isLasso==True:
                  all_beta_chap = elastic_net_return_coeff(False,1,1,X_train,Y_train)
              if isLasso==False:
                  all_beta_chap = elastic_net_return_coeff(False,1,0,X_train,Y_train)

              beta_vs_lambda = pd.DataFrame(all_beta_chap)
              if isLasso==True:
                  print_plot("Lasso Regularization Path","(Alpha = 1)","Lambda","Coefficients v
              if isLasso==False:
                  print_plot("Ridge Regularization Path","(Alpha = 0)","Lambda","Coefficients v
              for row in beta_vs_lambda.iterrows():
                  row = row[1]
                  plt.plot(lambda_range, row)
```

```
In [10]: print_regularization_path(X_train,Y_train,isLasso=True)
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:14: UserWarnin

/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  positive)
```



- Méthode 2 : Avec la méthode Path

```
In [11]: #lasso = skl_linear_mdl.ElasticNet(l1_ratio=1)
         #lasso_path = lasso.path(X_train, Y_train, l1_ratio=1)
         #lasso_path_result = lasso_path[1]
         #print_plot("","Lasso Regularization Path","Lambda","Coefficients values")

         #for i in range(0, len(lasso_path_result)):
         #    plt.plot(lasso_path[0], lasso_path_result[i])
```

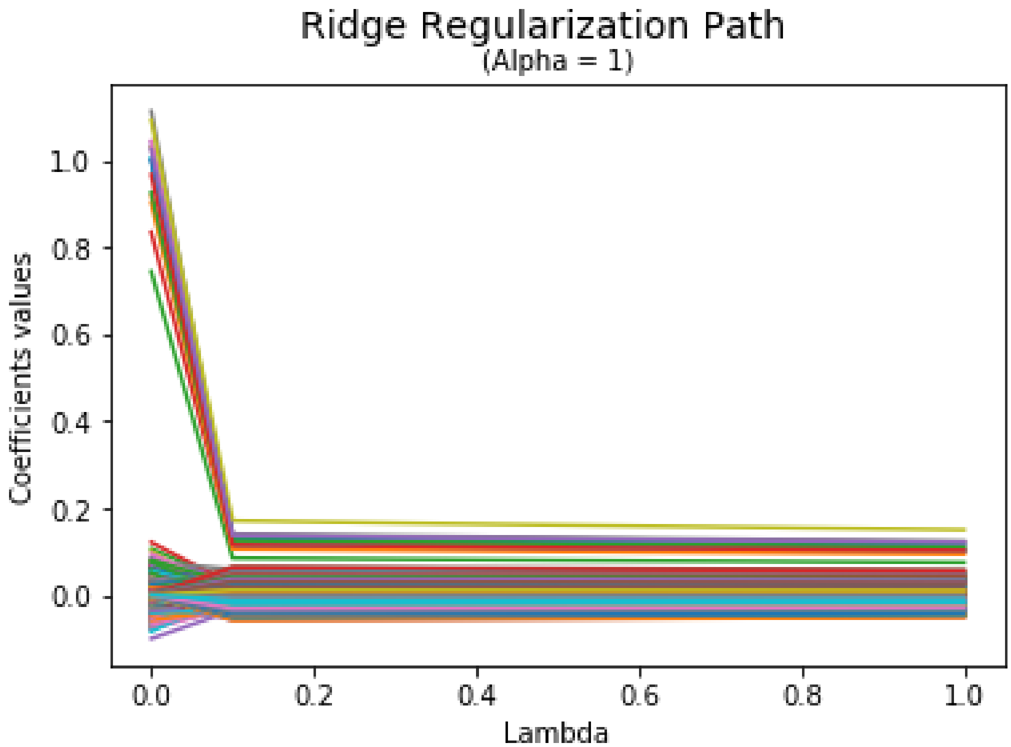
3.0.5 B.1) Chemin de Régularisation de l'estimateur Ridge

De la même manière que pour l'estimateur Lasso, on affiche, pour chacun des 5000 coefficients du modèle Ridge, leur valeur en fonction de Lambda (entre 0 et 1), pour un alpha égal à 0 (par définition du modèle Ridge) On peut observer que les 15 coefficients unitaires tendent beaucoup plus vite vers 0 (à 0.1, ils sont déjà presque nuls)

```
In [12]: print_regularization_path(X_train,Y_train,False)
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:14: UserWarning

/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  positive)
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  ConvergenceWarning)
```



3.0.6 C) Optimisation du / des paramètres de régularisation (lambda et alpha)

Nous allons maintenant, pour chacun des 3 modèles, optimiser leurs paramètres de régularisation : * Pour le Lasso, on optimise donc pour lambda entre 0 et 1, sachant qu'alpha vaut 1 (par défintion du modèle Lasso) * Pour le Ridge, on optimise donc pour lambda entre 0 et 1, sachant qu'alpha vaut 0 (par défintion du Ridge) * Pour l'Elastic Net, qui est une pondération des modèles Lasso et Ridge, on optimise donc sur lambda et sur alpha, variant tout deux entre 0 et 1

Nous optimiserons ces paramètres par validation croisée : * estimation du modèle sur le training set * calcul du MSE par rapport au test set

```
In [13]: def elastic_net_return_error(X_train, Y_train, X_test, Y_test, alpha_range, lambda_ra
             all_quad_risk = pd.DataFrame(index=alpha_range,
                                          columns=lambda_range)

             for i_alpha in alpha_range:
                 for i_lambda in lambda_range:
                     en = skl_linear_mdl.ElasticNet(alpha=i_lambda, l1_ratio=i_alpha,
                                                    fit_intercept = False, max_iter = 10000)
                     en.fit(X_train, Y_train)
```

```
                beta_chap = en.coef_
                Y_predict = en.predict(X_test)
                quad_risk = skl_metrics.mean_squared_error(Y_test, Y_predict)
                all_quad_risk.loc[i_alpha, i_lambda] = quad_risk

        min_mse = all_quad_risk.min().min()
        x, y = np.where(all_quad_risk.values == min_mse)

        alpha_opt_elastic_net = all_quad_risk.index[x].tolist()[0]
        #lambda_opt_elastic_net = all_quad_risk.index[x].tolist()[0]
        lambda_opt_elastic_net = all_quad_risk.columns[y].tolist()[0]

        return lambda_opt_elastic_net, alpha_opt_elastic_net, all_quad_risk
```

```
In [14]: def optimise_params(X_train, Y_train, X_test, Y_test, model_num):
             all_quad_risk = dict()

             if model_num == 1:
                 alpha_opt = 1
                 #lambda_opt, all_quad_risk=Elastic_Net_return_error(1, X_train, Y_train, X_te
                 lambda_opt, _, all_quad_risk = elastic_net_return_error(X_train, Y_train, X_te
                                                                         [alpha_opt], np.linspa
             elif model_num == 2:
                 alpha_opt = 0
                  #lambda_opt, all_quad_risk=Elastic_Net_return_error(0, X_train, Y_train, X_t
                 lambda_opt, _, all_quad_risk = elastic_net_return_error(X_train, Y_train, X_te
                                                                         [alpha_opt], np.linspa
             elif model_num == 3:
                  lambda_opt, alpha_opt, all_quad_risk = elastic_net_return_error(X_train, Y_tr
                                                                                  np.linspace((

             return lambda_opt, alpha_opt, all_quad_risk

         def print_opt_model(alpha_opt, lambda_opt, model_num):
             if model_num == 1:
                 print_plot("Lasso Regression","Lambda Optimal : " + str(lambda_opt),"Lambda","
             elif model_num == 2:
                 print_plot("Ridge Regression","Lambda Optimal : " + str(lambda_opt),"Lambda","
             elif model_num == 3:
                 pass
```

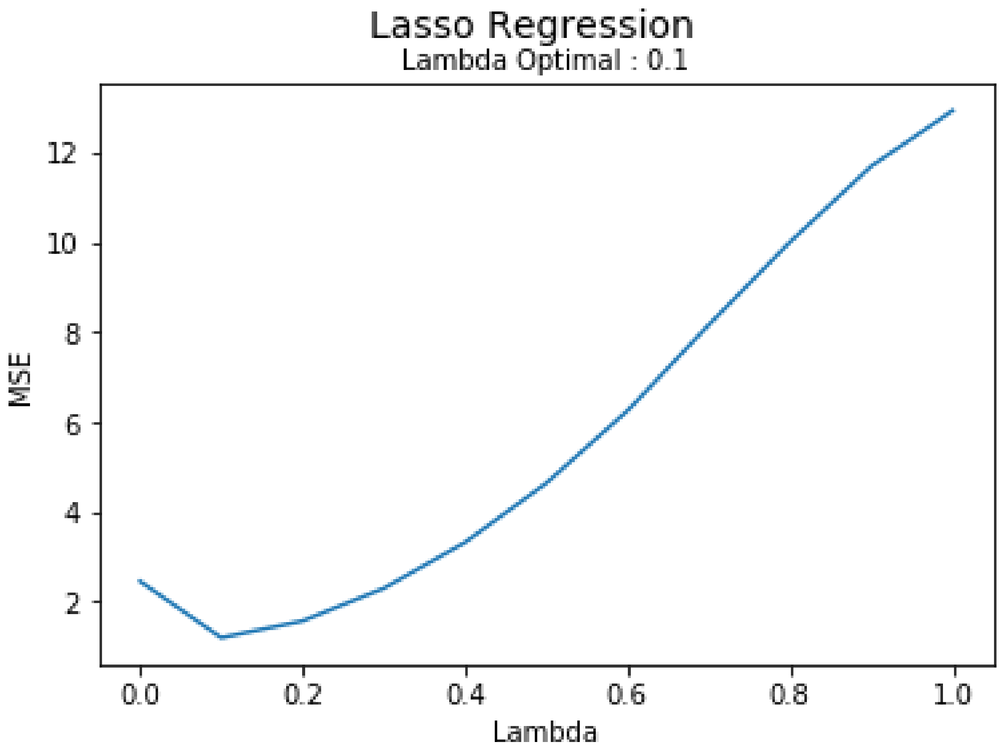
- Lasso

```
In [15]: lambda_opt_lasso, _, all_quad_risk = optimise_params(X_train, Y_train, X_test, Y_test
         print_plot("Lasso Regression","Lambda Optimal : " + str(lambda_opt_lasso),"Lambda","MS
         plt.plot(all_quad_risk.columns,
                  all_quad_risk.values.reshape(11, 1))
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:9: UserWarning
  if __name__ == '__main__':
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  positive)
```

```
Out[15]: [<matplotlib.lines.Line2D at 0x1a14aaacc0>]
```

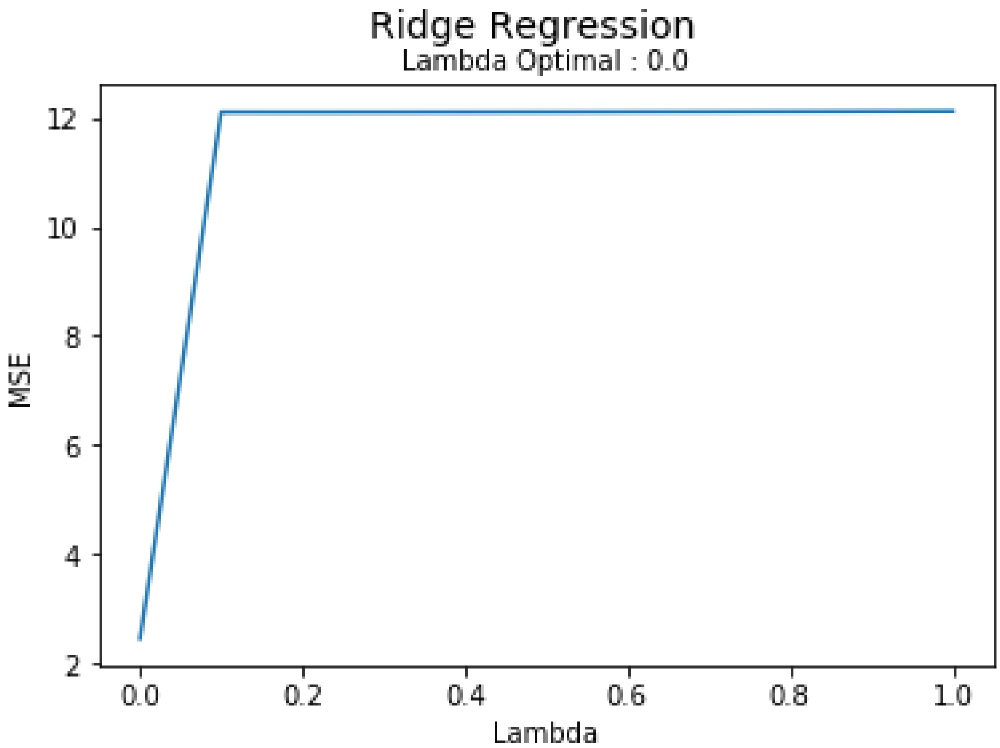


- Ridge

```
In [16]: lambda_opt_ridge, _, all_quad_risk = optimise_params(X_train, Y_train, X_test, Y_test
         print_plot("Ridge Regression","Lambda Optimal : " + str(lambda_opt_ridge),"Lambda","MS
         plt.plot(all_quad_risk.columns,
                  all_quad_risk.values.reshape(11, 1))
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:9: UserWarning
  if __name__ == '__main__':
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  positive)
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  ConvergenceWarning)
```

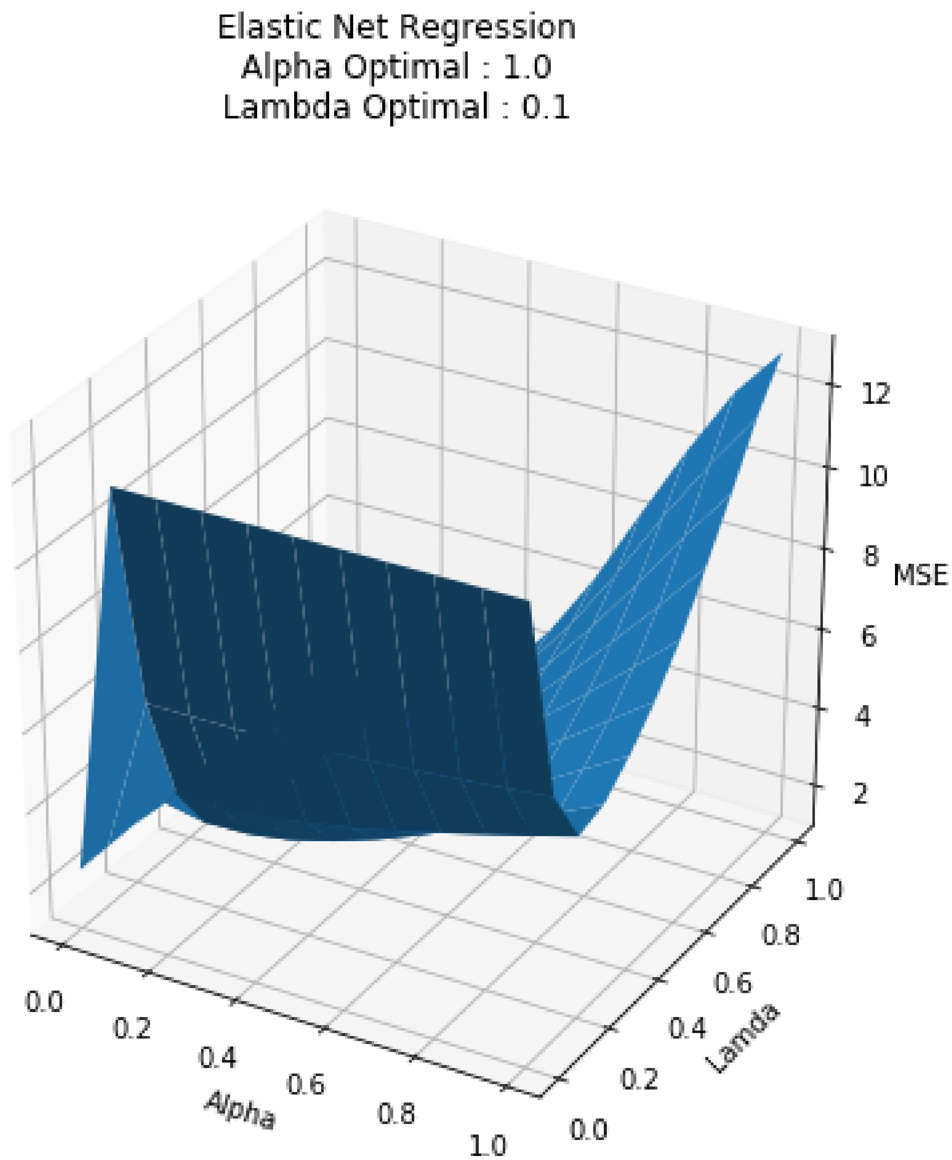
```
Out[16]: [<matplotlib.lines.Line2D at 0x1a14732978>]
```

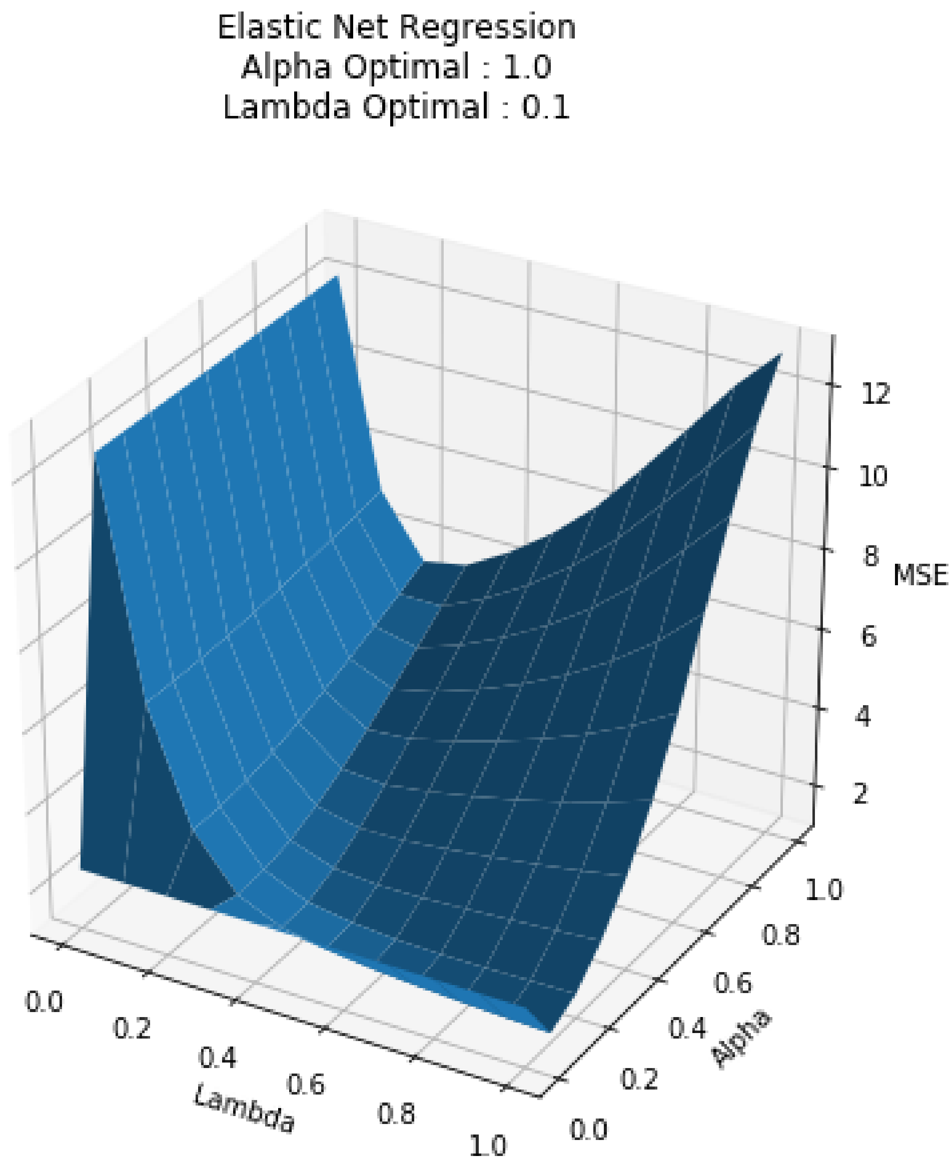


- Elastic Net

```
In [17]: lambda_opt_elastic_net, alpha_opt_elastic_net, all_quad_risk = optimise_params(X_trai

         print_plot3D("Elastic Net Regression\nAlpha Optimal : " + str(alpha_opt_elastic_net) +
                   "\nLambda Optimal : " + str(lambda_opt_elastic_net),"", "Alpha", "Lamda", "M
                      np.linspace(0, 1, 11), np.linspace(0, 1, 11), all_quad_risk,0)

         print_plot3D("Elastic Net Regression\nAlpha Optimal : " + str(alpha_opt_elastic_net) +
                   "\nLambda Optimal : " + str(lambda_opt_elastic_net),"", "Lambda", "Alpha", "

                      np.linspace(0, 1, 11), np.linspace(0, 1, 11), all_quad_risk,1)

         opt_elastic_net=[]
         opt_elastic_net.append(alpha_opt_elastic_net)
         opt_elastic_net.append(lambda_opt_elastic_net)
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:9: UserWarning
  if __name__ == '__main__':
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  positive)
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  ConvergenceWarning)
```


Elastic Net Regression
Alpha Optimal : 1.0
Lambda Optimal : 0.1
MSE
2
4
6
8
10
12
Alpha
0.0
0.2
0.4
0.6
0.8
1.0
Lamda
0.0
0.2
0.4
0.6
0.8
1.0



3.0.7 D) Score des estimateurs

Enfin, on sélectionne le meilleurs des 3 modèles, en sélectionnant celui qui affiche le MSE le plus faible. On prend également soin de calculer le coefficient de détermination R2 du modèle.

```
In [18]: def score(X_train,Y_train, lambda_opt,isLasso):
             if isLasso==True:
                 model = skl_linear_mdl.ElasticNet(alpha=lambda_opt, l1_ratio=1,
                                                    fit_intercept=False, max_iter = 10000)
             if isLasso==False:
                 model = skl_linear_mdl.ElasticNet(alpha=lambda_opt, l1_ratio=0,
```

```
                                            fit_intercept=False, max_iter = 10000)
        model.fit(X_train, Y_train)
        score_model = model.score(X_test, Y_test)
        Y_predict = model.predict(X_test)
        mse_model = skl_metrics.mean_squared_error(Y_test, Y_predict)

        return[score_model,mse_model]
```

```
In [19]: def score_elastic_net(X_train,Y_train, opt_elastic_net):
             en = skl_linear_mdl.ElasticNet(alpha=opt_elastic_net[1], l1_ratio=opt_elastic_net
                                        fit_intercept=False, max_iter = 10000)
             en.fit(X_train, Y_train)
             score_en = en.score(X_test, Y_test)
             Y_predict_en = en.predict(X_test)
             mse_en = skl_metrics.mean_squared_error(Y_test, Y_predict_en)

             return([score_en,mse_en])
```

```
In [20]: def print_score(X_train, Y_train, lambda_opt_lasso, lambda_opt_ridge, opt_elastic_net)
             lasso=score(X_train,Y_train,lambda_opt=lambda_opt_lasso,isLasso=True)
             score_lasso=lasso[0]
             mse_lasso=lasso[1]
             ridge=score(X_train,Y_train,lambda_opt=lambda_opt_ridge,isLasso=False)
             score_ridge=ridge[0]
             mse_ridge=ridge[1]
             en=score_elastic_net(X_train,Y_train,opt_elastic_net=opt_elastic_net)
             score_en=en[0]
             mse_en=en[1]

             df_score = pd.DataFrame([score_lasso, score_ridge, score_en],
                                    columns=["Score (R2)"], index=["Lasso", "Ridge", "Elastic Net"]
             df_score["MSE"] = [mse_lasso, mse_ridge, mse_en]

             best_model_R2 = df_score.index[np.argmax(df_score["Score (R2)"].values)]
             best_model_mse = df_score.index[np.argmin(df_score["MSE"].values)]
             df_score.loc["BEST MODEL", "Score (R2)"] = best_model_R2
             df_score.loc["BEST MODEL", "MSE"] = best_model_mse
             print(df_score.head())
```

```
In [21]: print_score(X_train,Y_train,lambda_opt_lasso,lambda_opt_ridge,opt_elastic_net)
```

```
            Score (R2)      MSE
Lasso         0.912584  1.19948
Ridge         0.821044  2.45554
Elastic Net   0.912584  1.19948
BEST MODEL       Lasso    Lasso
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:8: UserWarning
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  positive)
```

4 Question 2

On reproduit les étapes de la Question 1, sur un Dataset différent. Pour chacun des 3 modèles : * On affiche le regularization Path * On optimise le/les paramètre(s) de régularisation
Enfin, on sélectionne le meilleur modèle

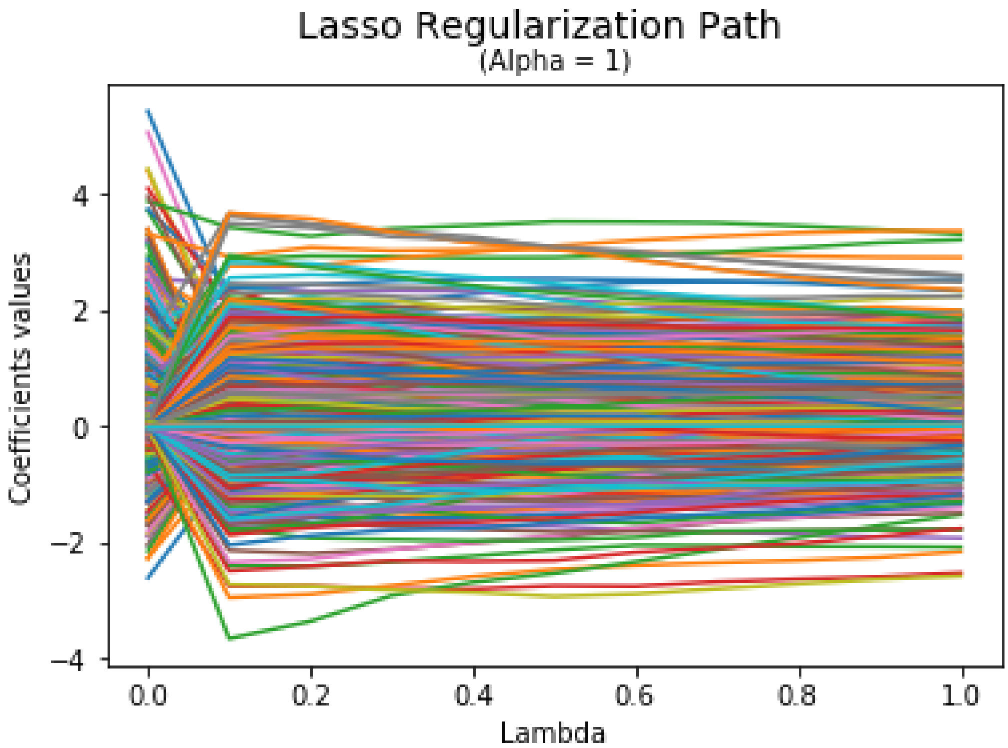
```
In [22]: # Simulating X
         xindex = ["X" + str(i) for i in range(1, 5001)]
         X = pd.DataFrame(np.reshape(np.random.normal(0, 1, (1000*5000)), (1000, 5000)),
                             columns=xindex)
         # Simulating Eta
         eta = pd.DataFrame(np.random.normal(0, 1, 1000),
                               columns=["Eta"])

         # Simulating Beta
         beta = pd.DataFrame(np.repeat(0, 5000),
                                columns=["Beta"])
         for i in range(0, 1500):
             beta.loc[i, ] = 1

         # Compute simulated Y
         Y = pd.DataFrame(np.dot(X, beta) + eta.values,
                           columns=["Y"])

         dataset=pd.DataFrame()
         dataset = Y
         dataset[xindex] = X

         # Splitting X in training and testing set
         train_set, test_set = skl_mdl_selection.train_test_split(dataset,
                                                                   train_size=0.66,
                                                                   test_size=0.34)

         X_train = train_set.iloc[:, 1:]
         Y_train = train_set.iloc[:, 0]
         X_test = test_set.iloc[:, 1:]
         Y_test = test_set.iloc[:, 0]
```

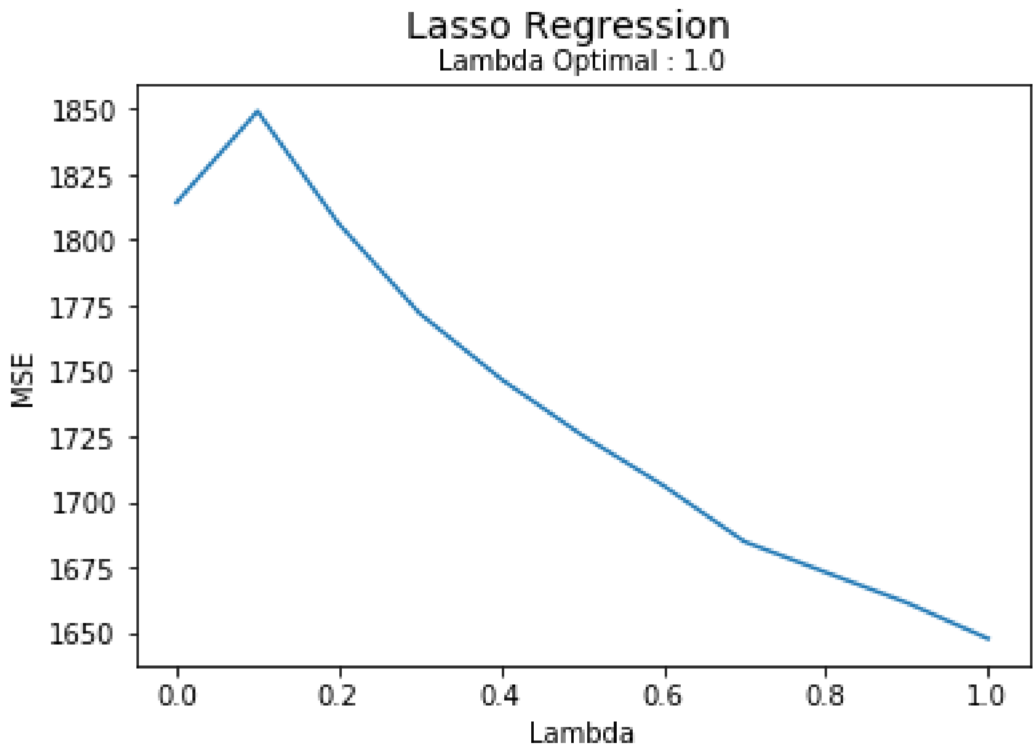
- Lasso

```
In [23]: print_regularization_path(X_train,Y_train,isLasso=True)
         lambda_opt_lasso, _, all_quad_risk = optimise_params(X_train, Y_train, X_test, Y_test
         print_plot("Lasso Regression","Lambda Optimal : " + str(lambda_opt_lasso),"Lambda","MS
         plt.plot(all_quad_risk.columns,
                  all_quad_risk.values.reshape(11, 1))
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:14: UserWarning

/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  positive)
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:9: UserWarning
  if __name__ == '__main__':
```

Out[23]: [<matplotlib.lines.Line2D at 0x1a148ee898>]



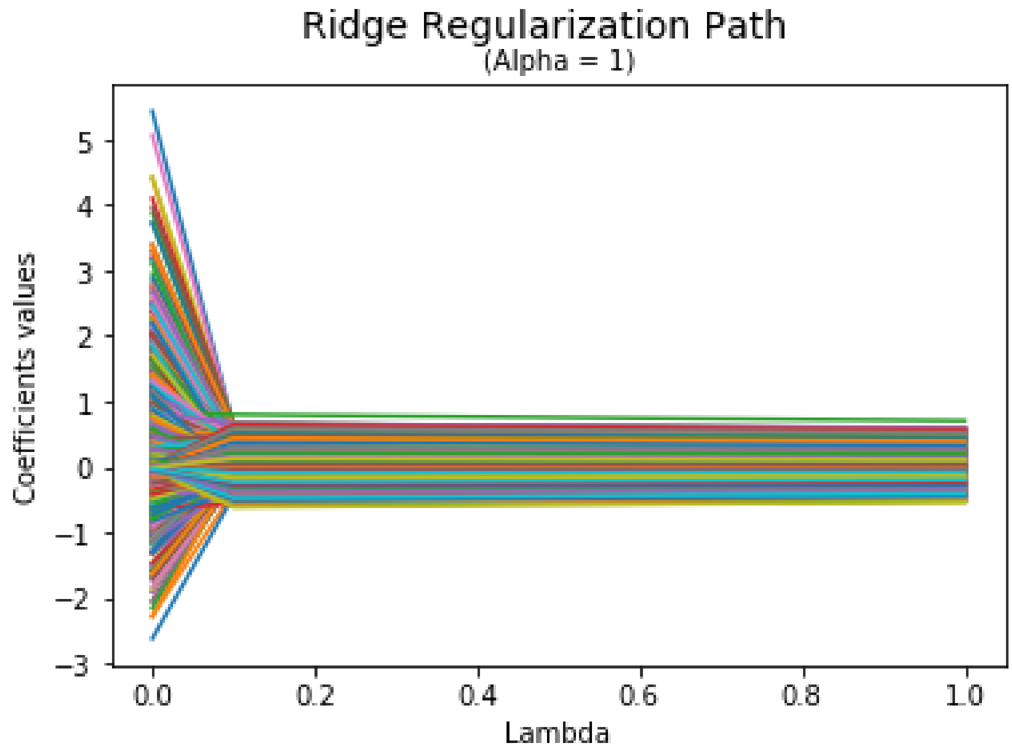


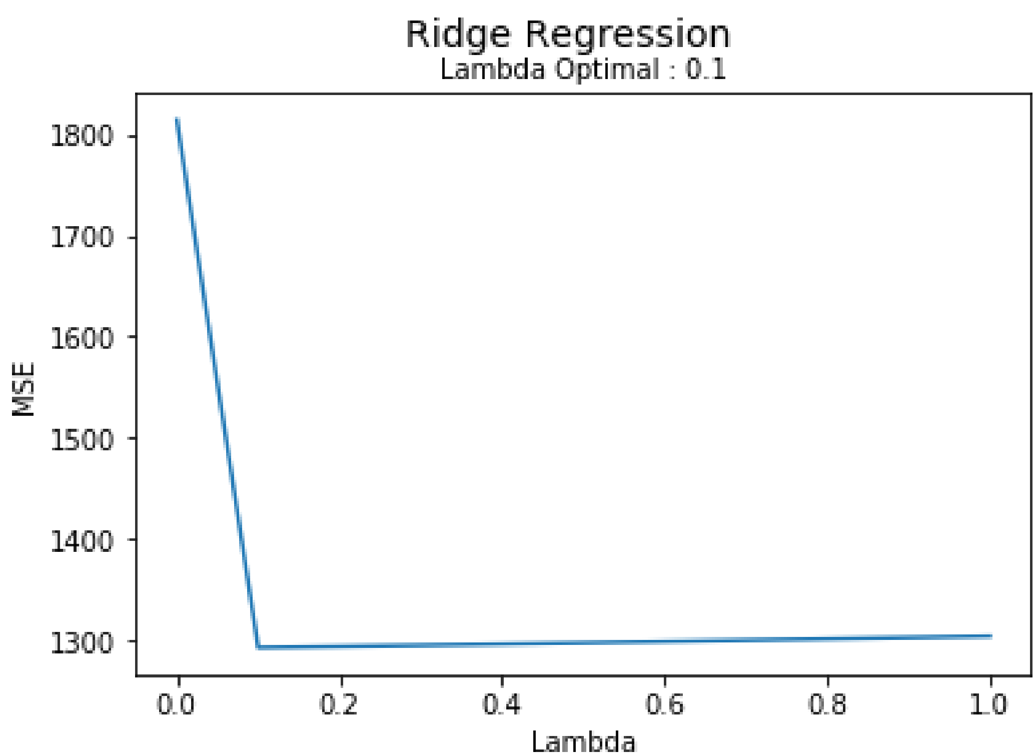
- Ridge

```
In [24]: print_regularization_path(X_train,Y_train,isLasso=False)
         lambda_opt_ridge, _, all_quad_risk = optimise_params(X_train, Y_train, X_test, Y_test
         print_plot("Ridge Regression","Lambda Optimal : " + str(lambda_opt_ridge),"Lambda","MS
         plt.plot(all_quad_risk.columns,
                  all_quad_risk.values.reshape(11, 1))
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:14: UserWarning

/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  positive)
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  ConvergenceWarning)
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:9: UserWarning
  if __name__ == '__main__':
```

```
Out[24]: [<matplotlib.lines.Line2D at 0x1a15e68860>]
```

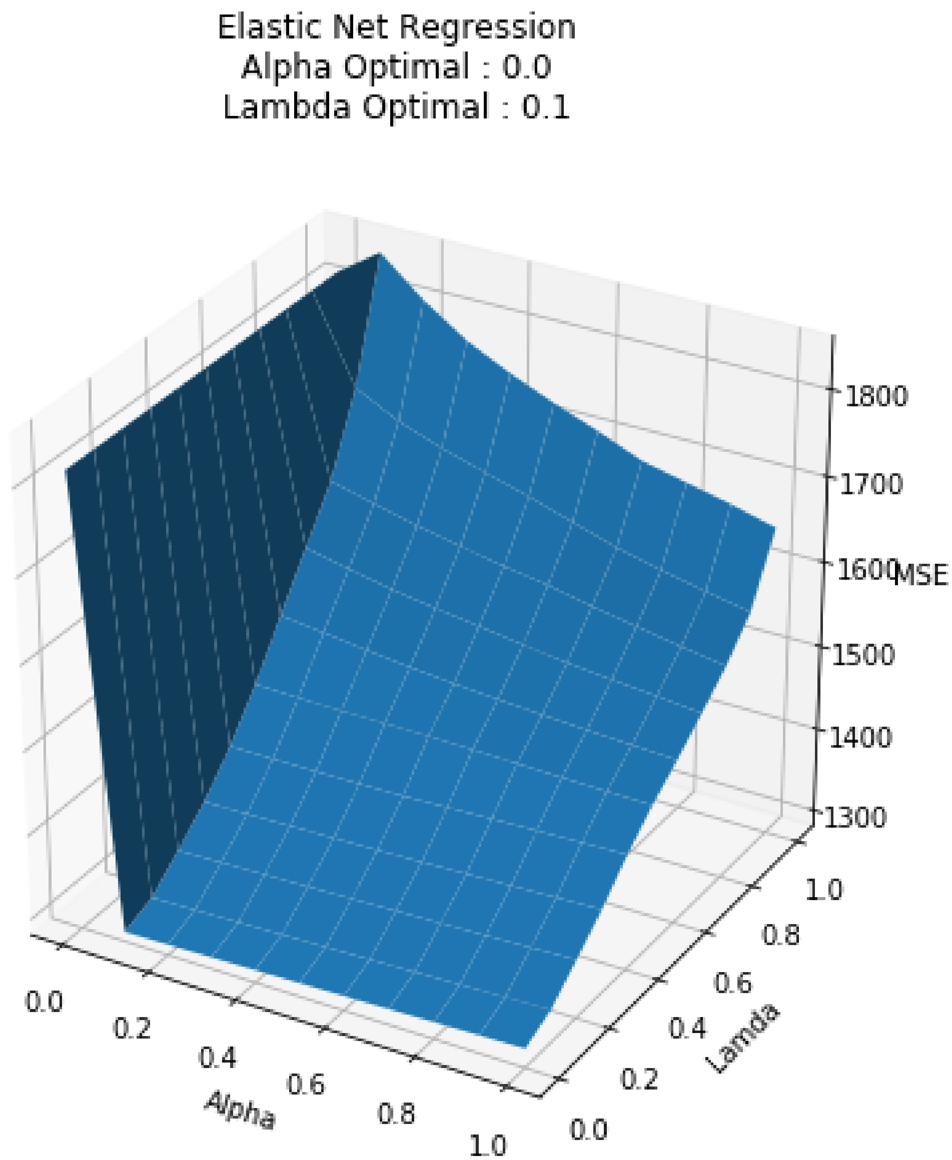

Ridge Regularization Path
(Alpha = 1)
Coefficients values
5
4
3
2
1
0
−1
−2
−3
0.0
0.2
0.4
0.6
0.8
1.0
Lambda

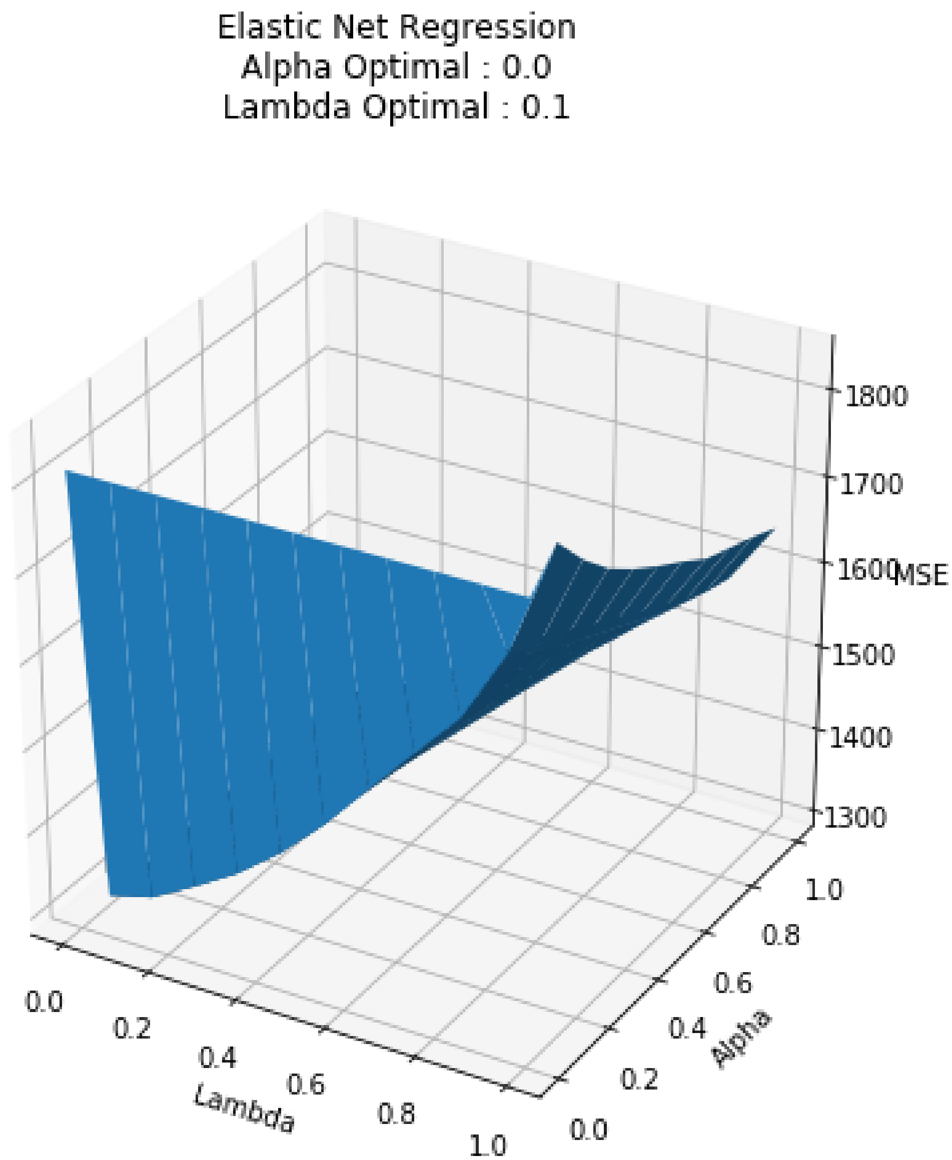

Ridge Regression
Lambda Optimal : 0.1
MSE
1800
1700
1600
1500
1400
1300
0.0
0.2
0.4
0.6
0.8
1.0
Lambda

- Elastic Net

In [25]:
```
lambda_opt_elastic_net, alpha_opt_elastic_net, all_quad_risk = optimise_params(X_trai

print_plot3D("Elastic Net Regression\nAlpha Optimal : " + str(alpha_opt_elastic_net) 
          "\nLambda Optimal : " + str(lambda_opt_elastic_net),"", "Alpha", "Lamda", "
             np.linspace(0, 1, 11), np.linspace(0, 1, 11), all_quad_risk,0)

print_plot3D("Elastic Net Regression\nAlpha Optimal : " + str(alpha_opt_elastic_net) 
          "\nLambda Optimal : " + str(lambda_opt_elastic_net),"", "Lambda", "Alpha", 
             np.linspace(0, 1, 11), np.linspace(0, 1, 11), all_quad_risk,1)

opt_elastic_net=[]
opt_elastic_net.append(alpha_opt_elastic_net)
opt_elastic_net.append(lambda_opt_elastic_net)
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:9: UserWarning
  if __name__ == '__main__':
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  positive)
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  ConvergenceWarning)
```


Elastic Net Regression
Alpha Optimal : 0.0
Lambda Optimal : 0.1
1800
1700
1600
MSE
1500
1400
1300
0.0
0.2
0.4
0.6
0.8
1.0
Alpha
0.0
0.2
0.4
0.6
0.8
1.0
Lamda



- Score

```
In [26]: print_score(X_train,Y_train,lambda_opt_lasso,lambda_opt_ridge,opt_elastic_net)
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  ConvergenceWarning)


              Score (R2)       MSE
Lasso          -0.069466   1647.65
Ridge           0.160783   1292.92
```

```
Elastic Net   0.160783  1292.92
BEST MODEL       Ridge     Ridge
```

5 Question 3

On reproduit les étapes de la Question 1, sur un Dataset différent. Pour chacun des 3 modèles : * On affiche le regularization Path * On optimise le/les paramètre(s) de régularisation

Enfin, on sélectionne le meilleur modèle

In [33]:
```
# Simulating X
xindex = ["X" + str(i) for i in range(1, 51)]
cov_mat = np.repeat(0.7, 50*50).reshape(50, 50)
for i in range(0, 50):
    for j in range(i, 50):
        cov_mat[i, j] = np.power(cov_mat[i, j], abs(i-j))
        cov_mat[j, i] = np.power(cov_mat[j, i], abs(i-j))

X = pd.DataFrame(np.reshape(np.random.multivariate_normal([0] * 50, cov_mat, 100),
                            (100, 50)),
                 columns=xindex)

# Simulating Eta
eta = pd.DataFrame(np.random.normal(0, 1, 100),
                   columns=["Eta"])
# Simulating Beta
beta = pd.DataFrame(np.repeat(0, 50),
                    columns=["Beta"])

beta.loc[0, ] = 10
beta.loc[1, ] = 10
beta.loc[2, ] = 5
beta.loc[3, ] = 5
for i in range(4, 14):
    beta.loc[i, ] = 1

# Compute simulated Y
Y = pd.DataFrame(np.dot(X, beta) + eta.values,
                 columns=["Y"])

dataset = pd.DataFrame()


dataset = Y
dataset[xindex] = X

# Splitting X in training and testing set
```

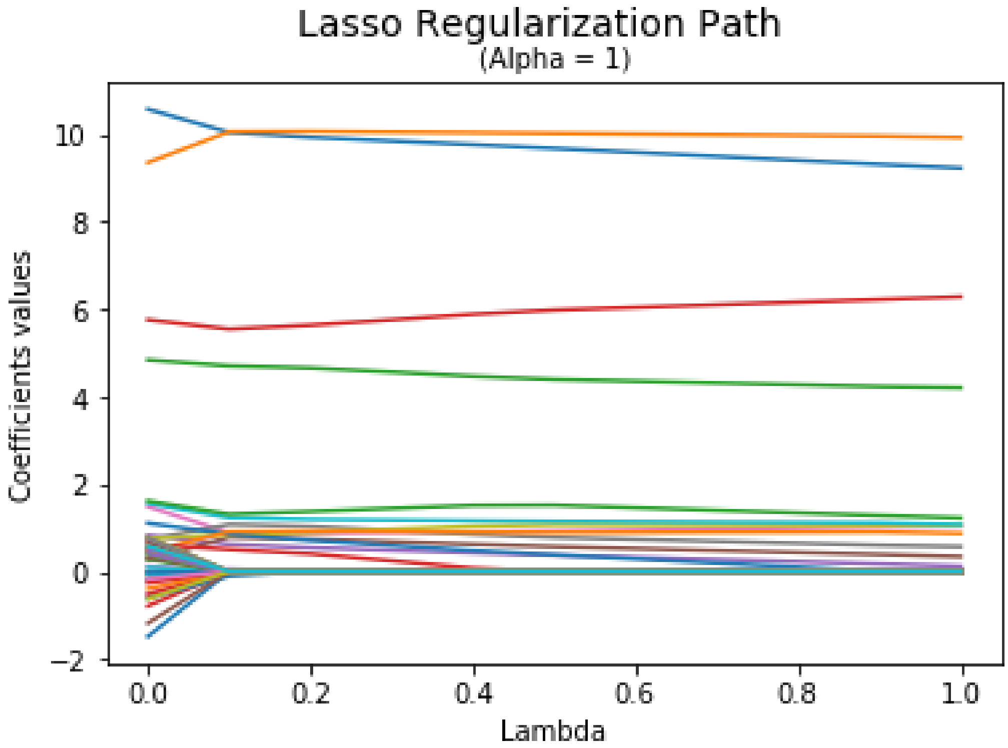
```
train_set, test_set = skl_mdl_selection.train_test_split(dataset,
                                                         train_size=0.66,
                                                         test_size=0.34)

X_train = train_set.iloc[:, 1:]
Y_train = train_set.iloc[:, 0]
X_test = test_set.iloc[:, 1:]
Y_test = test_set.iloc[:, 0]
```

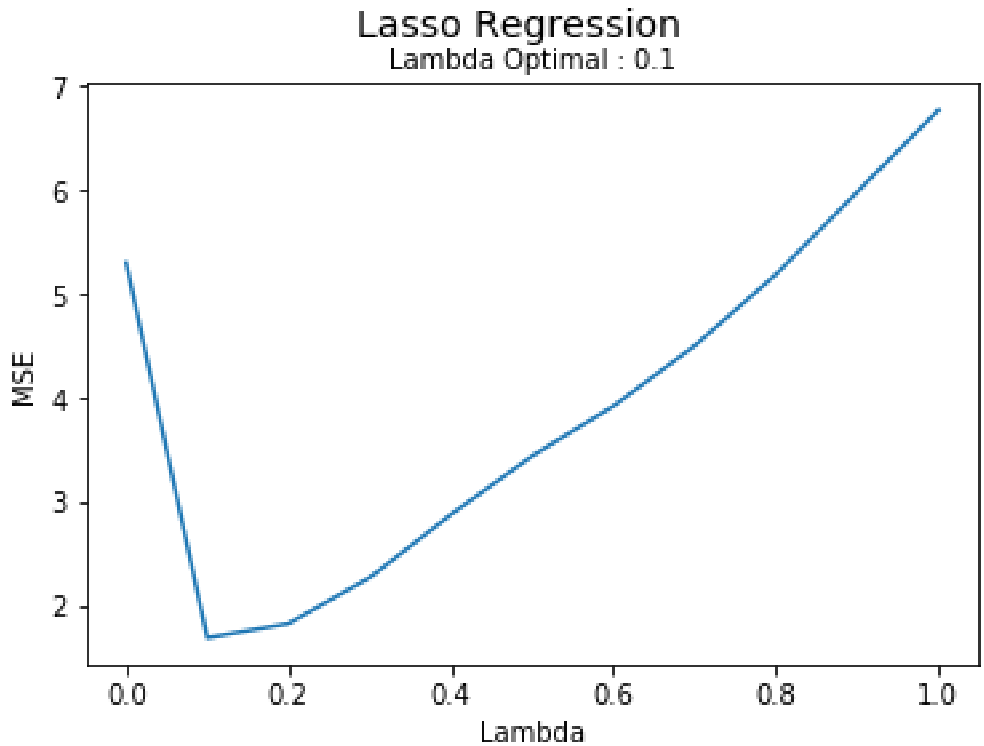
- Lasso

```
In [34]: print_regularization_path(X_train,Y_train,isLasso=True)
         lambda_opt_lasso, _, all_quad_risk = optimise_params(X_train, Y_train, X_test, Y_test
         print_plot("Lasso Regression","Lambda Optimal : " + str(lambda_opt_lasso),"Lambda","MS
         plt.plot(all_quad_risk.columns,
                  all_quad_risk.values.reshape(11, 1))
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:14: UserWarnin

/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  positive)
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  ConvergenceWarning)
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:9: UserWarning
  if __name__ == '__main__':
```

```
Out[34]: [<matplotlib.lines.Line2D at 0x1a1393eeb8>]
```



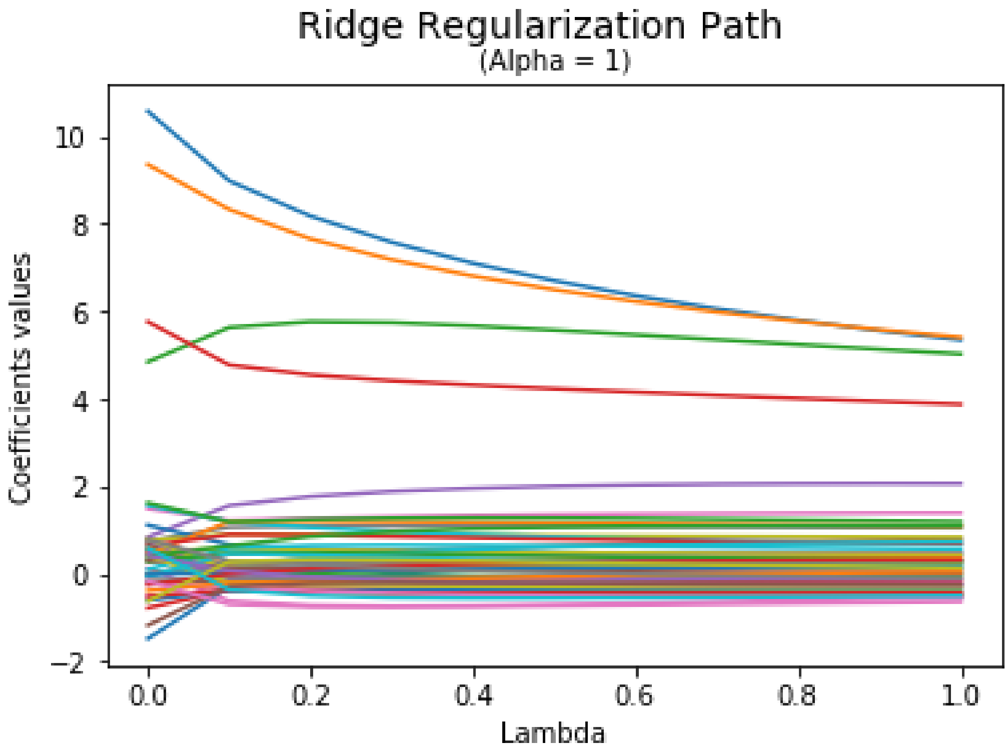


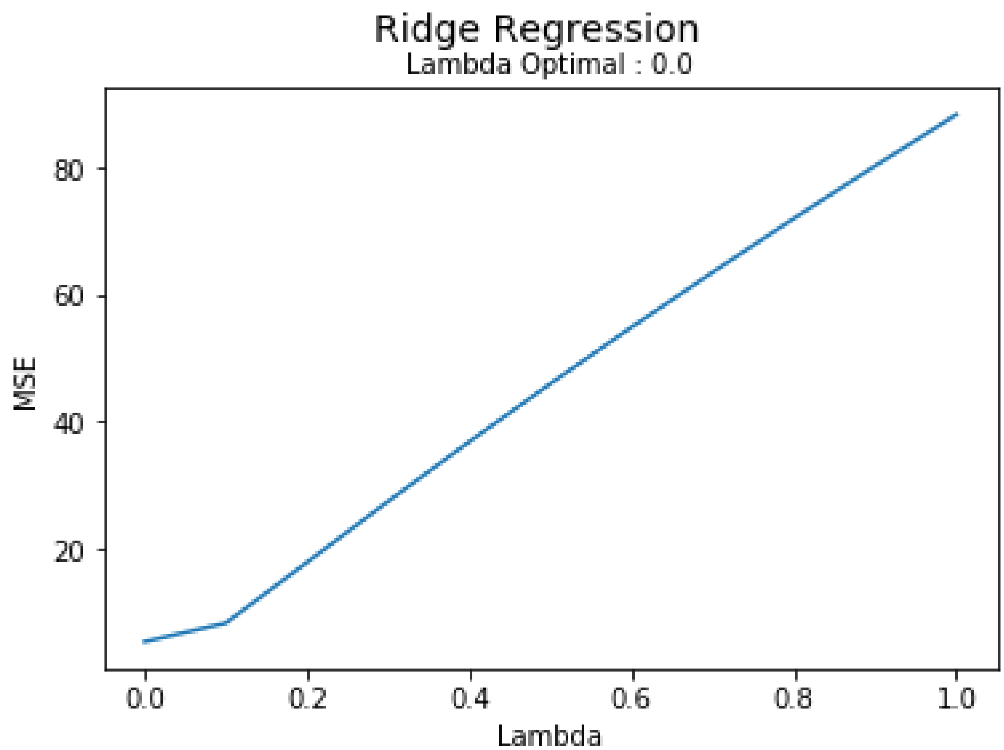
- Ridge

```
In [35]: print_regularization_path(X_train,Y_train,isLasso=False)
         lambda_opt_ridge, _, all_quad_risk = optimise_params(X_train, Y_train, X_test, Y_test
         print_plot("Ridge Regression","Lambda Optimal : " + str(lambda_opt_ridge),"Lambda","MS
         plt.plot(all_quad_risk.columns,
                  all_quad_risk.values.reshape(11, 1))
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:14: UserWarning

/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desce
  positive)
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desce
  ConvergenceWarning)
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:9: UserWarning
  if __name__ == '__main__':
```

```
Out[35]: [<matplotlib.lines.Line2D at 0x1a2049c780>]
```

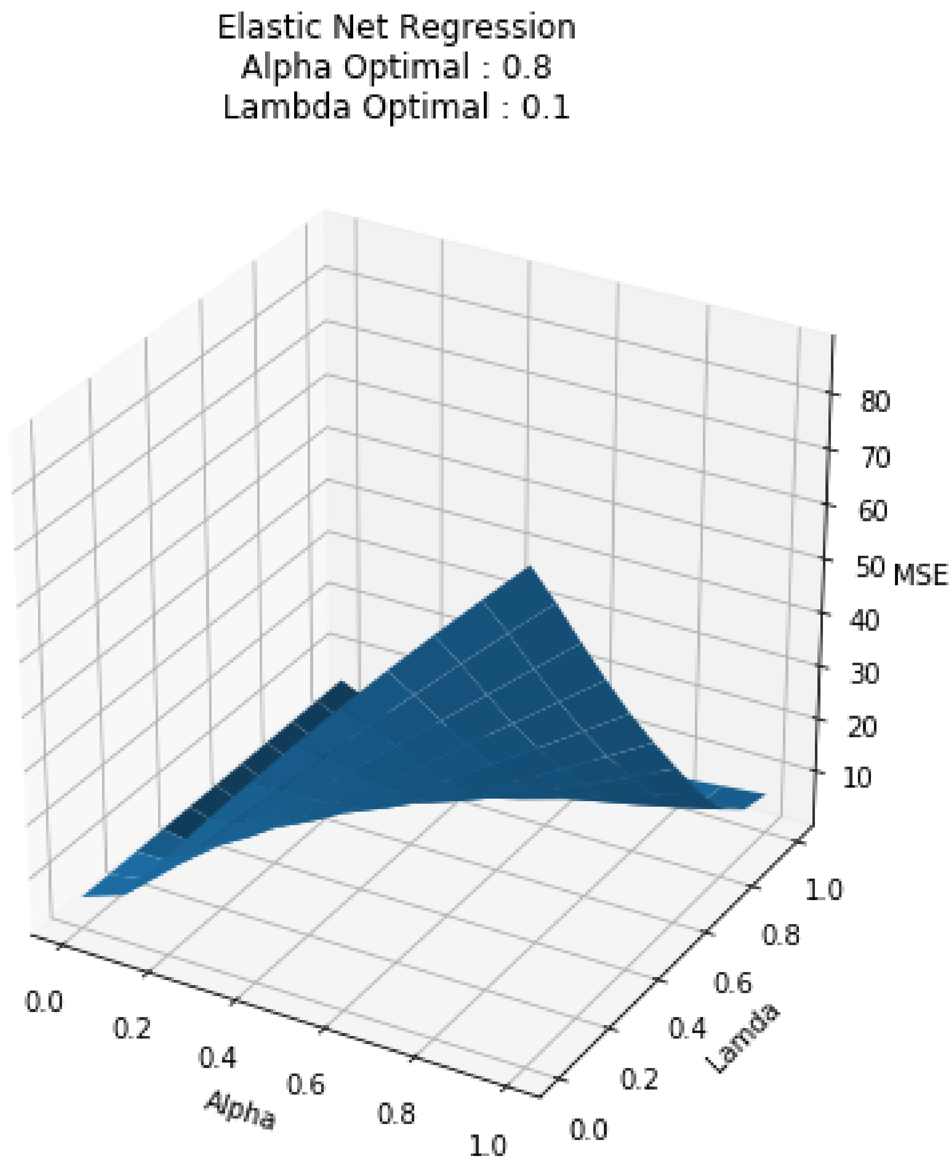

Ridge Regularization Path
(Alpha = 1)
Coefficients values
Lambda
10
8
6
4
2
0
−2
0.0
0.2
0.4
0.6
0.8
1.0

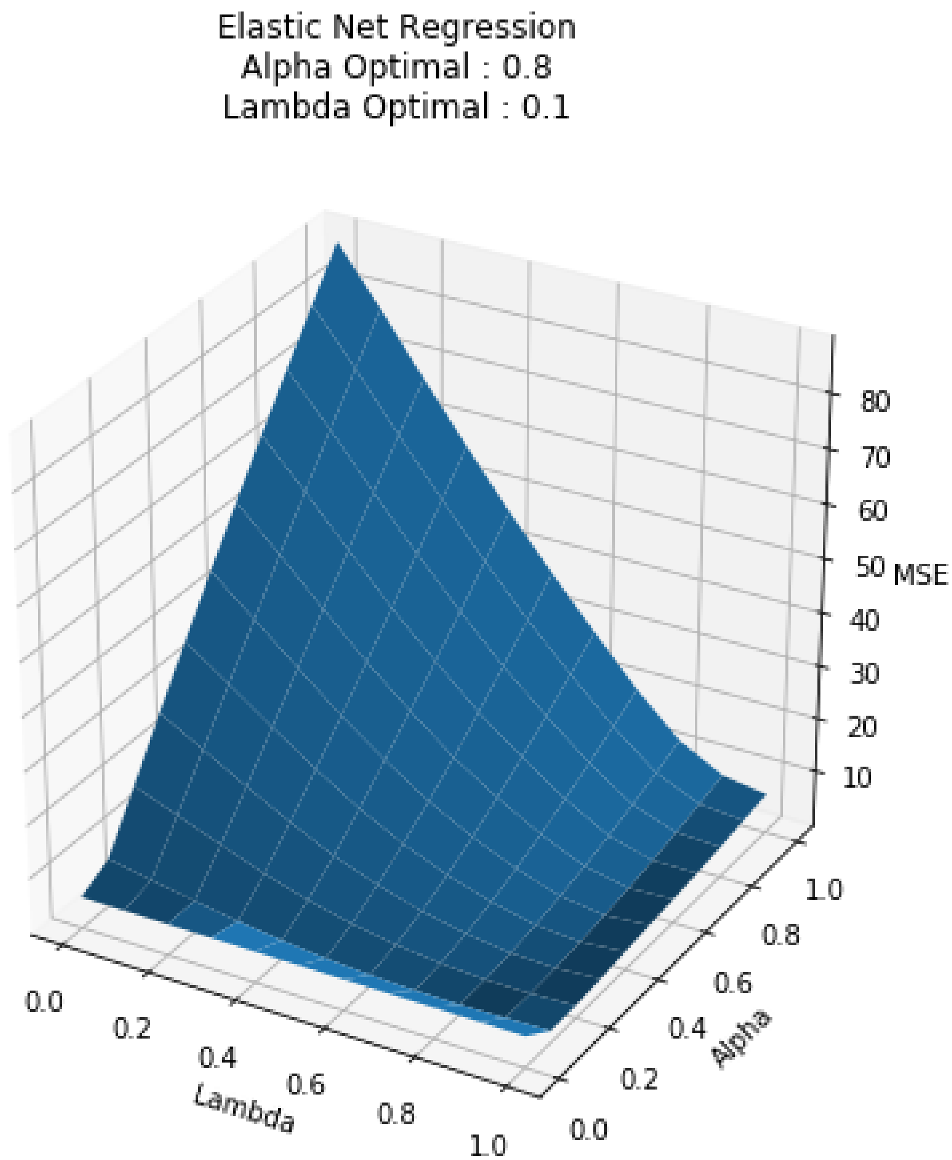

Ridge Regression
Lambda Optimal : 0.0
MSE
Lambda
80
60
40
20
0.0
0.2
0.4
0.6
0.8
1.0

- Elastic Net

```
In [36]: lambda_opt_elastic_net, alpha_opt_elastic_net, all_quad_risk = optimise_params(X_trai

         print_plot3D("Elastic Net Regression\nAlpha Optimal : " + str(alpha_opt_elastic_net) 
                  "\nLambda Optimal : " + str(lambda_opt_elastic_net),"", "Alpha", "Lamda", "
                     np.linspace(0, 1, 11), np.linspace(0, 1, 11), all_quad_risk,0)

         print_plot3D("Elastic Net Regression\nAlpha Optimal : " + str(alpha_opt_elastic_net) 
                  "\nLambda Optimal : " + str(lambda_opt_elastic_net),"", "Lambda", "Alpha", 
                     np.linspace(0, 1, 11), np.linspace(0, 1, 11), all_quad_risk,1)

         opt_elastic_net=[]
         opt_elastic_net.append(alpha_opt_elastic_net)
         opt_elastic_net.append(lambda_opt_elastic_net)
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:9: UserWarning
  if __name__ == '__main__':
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  positive)
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  ConvergenceWarning)
```


Elastic Net Regression
Alpha Optimal : 0.8
Lambda Optimal : 0.1
MSE
80
70
60
50
40
30
20
10
Alpha
0.0
0.2
0.4
0.6
0.8
1.0
Lamda
0.0
0.2
0.4
0.6
0.8
1.0



- Score

```
In [37]: print_score(X_train,Y_train,lambda_opt_lasso,lambda_opt_ridge,opt_elastic_net)
```

```
               Score (R2)          MSE
Lasso            0.997821      1.68367
Ridge            0.993142      5.30039
Elastic Net      0.998388      1.24564
BEST MODEL    Elastic Net  Elastic Net
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:8: UserWarning
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  positive)
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  ConvergenceWarning)
```

6 Exercice 2

Pour l'application à des données réelles, on choisi de partir d'un dataset sur le Bitcoin d'une quizaine de features, puis de faire du feature engineering sur ces features afin d'avoir une matrice de features de taille très importante (comme pour l'exercice 1).

On décide également d'introduire des features qui,vraisemblablement, n'auront pas d'effet sur la target : leurs coefficients de régression seront à priori nuls ! (et c'est ce qu'on souhaite).

On se retrouve finalement avec une matrice de (1337 * 127), qui parait être un bon candidat pour une régression pénalisée.

6.1 Initialisation

- Import des données

```
In [123]: import datetime

          df = pd.read_csv("bitcoin_dataset.csv")
          df_2 = pd.read_csv("bitcoin_price.csv")
          df_3 = pd.read_csv("litecoin_price.csv")
          df_4 = pd.read_csv("monero_price.csv")

          df_2 = df_2[::-1]
          df_2 = df_2.reset_index(drop=True)

          df_3 = df_3[::-1]
          df_3 = df_3.reset_index(drop=True)

          df_4 = df_4[::-1]
          df_4 = df_4.reset_index(drop=True)

          df["Date"] = df["Date"].apply(lambda x: datetime.datetime.strptime(x, "%Y-%m-%d %H:%S
          df_2["Date"] = df_2["Date"].apply(lambda x: datetime.datetime.strptime(x, "%b %d, %Y
          df_3["Date"] = df_3["Date"].apply(lambda x: datetime.datetime.strptime(x, "%b %d, %Y
          df_4["Date"] = df_4["Date"].apply(lambda x: datetime.datetime.strptime(x, "%b %d, %Y

          df_3.columns = ["Date", "Open_Litecoin", "High_Litecoin", "Low_Litecoin",
                          "Close_Litecoin", "Volume_Litecoin", "Market Cap_Litecoin"]

          df_4.columns = ["Date", "Open_Monero", "High_Monero", "Low_Monero",
                          "Close_Monero", "Volume_Monero", "Market Cap_Monero"]

          df = df.merge(df_2)
```

```
df = df.merge(df_3)
df = df.merge(df_4)
df.head(5)
```

Out[123]:

```
         Date  btc_market_price  btc_total_bitcoins  btc_market_cap  \
0  2014-05-21            494.87          12798675.0    6.333680e+09
1  2014-05-22            523.84          12802850.0    6.706645e+09
2  2014-05-23            527.47          12806800.0    6.755203e+09
3  2014-05-24            521.52          12811000.0    6.681193e+09
4  2014-05-25            575.00          12814775.0    7.368496e+09

   btc_trade_volume  btc_blocks_size  btc_avg_block_size  \
0      1.185312e+07          18419.0            0.250119
1      2.358140e+07          18459.0            0.241909
2      2.826269e+07          18501.0            0.271977
3      9.312733e+06          18533.0            0.195074
4      3.730564e+07          18566.0            0.219971

   btc_n_orphaned_blocks  btc_n_transactions_per_block  \
0                    0.0                         389.0
1                    2.0                         519.0
2                    0.0                         408.0
3                    0.0                         503.0
4                    1.0                         410.0

   btc_median_confirmation_time      ...        Low_Litecoin  \
0                      6.533333      ...               10.53
1                      7.450000      ...               10.48
2                      6.933333      ...               10.65
3                      7.350000      ...               10.99
4                      8.466667      ...               11.12

   Close_Litecoin  Volume_Litecoin  Market Cap_Litecoin  Open_Monero  \
0           10.59        2,653,860          302,609,000         2.47
1           10.67        5,186,880          302,201,000         1.59
2           11.23        9,134,640          304,423,000         2.05
3           11.10        2,574,380          320,801,000         2.92
4           11.53        9,072,890          318,703,000         4.04

   High_Monero  Low_Monero  Close_Monero  Volume_Monero  Market Cap_Monero
0         2.65        1.23          1.60        246,540          2,079,640
1         2.19        1.36          2.10        132,918          1,371,470
2         3.43        2.05          2.96        266,852          1,816,200
3         4.01        2.62          3.70        248,028          2,653,720
4         4.04        2.80          3.14        283,545          3,774,890

[5 rows x 42 columns]
```

- Feature Engineering

```
In [124]: df["Open_Return"] = df["Open"].pct_change()
          df["High_Return"] = df["High"].pct_change()
          df["Low_Return"] = df["Low"].pct_change()
          df["Close_Return"] = df["Close"].pct_change()

          df["Open_Return_Litecoin"] = df["Open_Litecoin"].pct_change()
          df["High_Return_Litecoin"] = df["High_Litecoin"].pct_change()
          df["Low_Return_Litecoin"] = df["Low_Litecoin"].pct_change()
          df["Close_Return_Litecoin"] = df["Close_Litecoin"].pct_change()

          df["Open_Return_Monero"] = df["Open_Monero"].pct_change()
          df["High_Return_Monero"] = df["High_Monero"].pct_change()
          df["Low_Return_Monero"] = df["Low_Monero"].pct_change()
          df["Close_Return_Monero"] = df["Close_Monero"].pct_change()

          df["Volume"] = df["Volume"].apply(lambda x: float(x.replace(",", "")) if x != "-" els
          df["Market Cap"] = df["Market Cap"].apply(lambda x: float(x.replace(",", "")) if x !=

          df["Volume_Litecoin"] = df["Volume_Litecoin"].apply(lambda x: float(x.replace(",", "
          df["Market Cap_Litecoin"] = df["Market Cap_Litecoin"].apply(lambda x: float(x.replace

          df["Volume_Monero"] = df["Volume_Monero"].apply(lambda x: float(x.replace(",", "")) i
          df["Market Cap_Monero"] = df["Market Cap_Monero"].apply(lambda x: float(x.replace(",

          # Moyenne Mobile
          for i_lag in [3, 6, 9]:
              df["Open_Return_MA" + str(i_lag) + "D"] = df["Open_Return"].rolling(i_lag).mean()
              df["High_Return_MA" + str(i_lag) + "D"] = df["High_Return"].rolling(i_lag).mean()
              df["Low_Return_MA" + str(i_lag) + "D"] = df["Low_Return"].rolling(i_lag).mean()
              df["Close_Return_MA" + str(i_lag) + "D"] = df["Close_Return"].rolling(i_lag).mean

              df["Open_Return_Litecoin_MA" + str(i_lag) + "D"] = df["Open_Return_Litecoin"].rol
              df["High_Return_Litecoin_MA" + str(i_lag) + "D"] = df["High_Return_Litecoin"].rol
              df["Low_Return_Litecoin_MA" + str(i_lag) + "D"] = df["Low_Return_Litecoin"].rolli
              df["Close_Return_Litecoin_MA" + str(i_lag) + "D"] = df["Close_Return_Litecoin"].r

              df["Open_Return_Monero_MA" + str(i_lag) + "D"] = df["Open_Return_Monero"].rolling
              df["High_Return_Monero_MA" + str(i_lag) + "D"] = df["High_Return_Monero"].rolling
              df["Low_Return_Monero_MA" + str(i_lag) + "D"] = df["Low_Return_Monero"].rolling(i
              df["Close_Return_Monero_MA" + str(i_lag) + "D"] = df["Close_Return_Monero"].rolli

          # Ecart Type Mobile
          for i_lag in [3, 6, 9]:
              df["Open_Return_MA" + str(i_lag) + "D"] = df["Open_Return"].rolling(i_lag).std()
              df["High_Return_MA" + str(i_lag) + "D"] = df["High_Return"].rolling(i_lag).std()
              df["Low_Return_MA" + str(i_lag) + "D"] = df["Low_Return"].rolling(i_lag).std()
              df["Close_Return_MA" + str(i_lag) + "D"] = df["Close_Return"].rolling(i_lag).std
```

```
    df["Open_Return_Litecoin_MA" + str(i_lag) + "D"] = df["Open_Return_Litecoin"].ro
    df["High_Return_Litecoin_MA" + str(i_lag) + "D"] = df["High_Return_Litecoin"].ro
    df["Low_Return_Litecoin_MA" + str(i_lag) + "D"] = df["Low_Return_Litecoin"].roll
    df["Close_Return_Litecoin_MA" + str(i_lag) + "D"] = df["Close_Return_Litecoin"].

    df["Open_Return_Monero_MA" + str(i_lag) + "D"] = df["Open_Return_Monero"].rolling
    df["High_Return_Monero_MA" + str(i_lag) + "D"] = df["High_Return_Monero"].rolling
    df["Low_Return_Monero_MA" + str(i_lag) + "D"] = df["Low_Return_Monero"].rolling(
    df["Close_Return_Monero_MA" + str(i_lag) + "D"] = df["Close_Return_Monero"].roll

# Skew Mobile
for i_lag in [3, 6, 9]:
    df["Open_Return_MA" + str(i_lag) + "D"] = df["Open_Return"].rolling(i_lag).skew(
    df["High_Return_MA" + str(i_lag) + "D"] = df["High_Return"].rolling(i_lag).skew(
    df["Low_Return_MA" + str(i_lag) + "D"] = df["Low_Return"].rolling(i_lag).skew()
    df["Close_Return_MA" + str(i_lag) + "D"] = df["Close_Return"].rolling(i_lag).ske

    df["Open_Return_Litecoin_MA" + str(i_lag) + "D"] = df["Open_Return_Litecoin"].ro
    df["High_Return_Litecoin_MA" + str(i_lag) + "D"] = df["High_Return_Litecoin"].ro
    df["Low_Return_Litecoin_MA" + str(i_lag) + "D"] = df["Low_Return_Litecoin"].roll
    df["Close_Return_Litecoin_MA" + str(i_lag) + "D"] = df["Close_Return_Litecoin"].

    df["Open_Return_Monero_MA" + str(i_lag) + "D"] = df["Open_Return_Monero"].rolling
    df["High_Return_Monero_MA" + str(i_lag) + "D"] = df["High_Return_Monero"].rolling
    df["Low_Return_Monero_MA" + str(i_lag) + "D"] = df["Low_Return_Monero"].rolling(
    df["Close_Return_Monero_MA" + str(i_lag) + "D"] = df["Close_Return_Monero"].roll

# Cosinus sur Moyenne Mobile
for i_lag in [3, 6, 9]:
    df["Open_Return_Cosinus_MA_" + str(i_lag) + "D"] = \
        df["Open_Return_MA" + str(i_lag) + "D"].apply(lambda x: np.cos(x))
    df["High_Return_Cosinus_MA_" + str(i_lag) + "D"] = \
        df["High_Return_MA" + str(i_lag) + "D"].apply(lambda x: np.cos(x))
    df["Low_Return_Cosinus_MA_" + str(i_lag) + "D"] = \
        df["Low_Return_MA" + str(i_lag) + "D"].apply(lambda x: np.cos(x))
    df["Close_Return_Cosinus_MA_" + str(i_lag) + "D"] = \
        df["Close_Return_MA" + str(i_lag) + "D"].apply(lambda x: np.cos(x))

# Sinus sur Moyenne Mobile
for i_lag in [3, 6, 9]:
    df["Open_Return_Sinus_MA_" + str(i_lag) + "D"] = \
        df["Open_Return_MA" + str(i_lag) + "D"].apply(lambda x: np.sin(x))
    df["High_Return_Sinus_MA_" + str(i_lag) + "D"] = \
        df["High_Return_MA" + str(i_lag) + "D"].apply(lambda x: np.sin(x))
    df["Low_Return_Sinus_MA_" + str(i_lag) + "D"] = \
        df["Low_Return_MA" + str(i_lag) + "D"].apply(lambda x: np.sin(x))
    df["Close_Return_Sinus_MA_" + str(i_lag) + "D"] = \
        df["Close_Return_MA" + str(i_lag) + "D"].apply(lambda x: np.sin(x))
```

```
# Exp sur Moyenne Mobile
for i_lag in [3, 6, 9]:
    df["Open_Return_Exp_MA_" + str(i_lag) + "D"] = \
        df["Open_Return_MA" + str(i_lag) + "D"].apply(lambda x: np.exp(x))
    df["High_Return_Exp_MA_" + str(i_lag) + "D"] = \
        df["High_Return_MA" + str(i_lag) + "D"].apply(lambda x: np.exp(x))
    df["Low_Return_Exp_MA_" + str(i_lag) + "D"] = \
        df["Low_Return_MA" + str(i_lag) + "D"].apply(lambda x: np.exp(x))
    df["Close_Return_Exp_MA_" + str(i_lag) + "D"] = \
        df["Close_Return_MA" + str(i_lag) + "D"].apply(lambda x: np.exp(x))

# Classification
# Création de la Target : signal d'achat ou de vente
#dataset["target"] = \
#     [1 if dataset.loc[row + 1, "btc_return"] > 0 else 0
#      for row in range(0, len(dataset.loc[:, "btc_return"]))]

# Regression
# Création de la target : rentabilité du jour d'après
df["Target"] = df["Close_Return"].shift(-1)

df = df.reindex(
    np.concatenate((["Date", "Target"], df.columns[1:-1].tolist())),
    axis=1)

df = df.dropna().reset_index(drop=True)
df.head(5)
```

Out[124]:

```
         Date    Target  btc_market_price  btc_total_bitcoins  btc_market_cap  \
0  2014-05-30  0.013570            609.03          12834000.0    7.816291e+09
1  2014-05-31  0.010502            620.45          12837800.0    7.965213e+09
2  2014-06-01  0.048220            674.98          12842275.0    8.668279e+09
3  2014-06-04  0.018843            644.66          12856025.0    8.287765e+09
4  2014-06-06  0.001943            655.75          12863350.0    8.435142e+09

   btc_trade_volume  btc_blocks_size  btc_avg_block_size  \
0      2.175838e+07          18761.0            0.234589
1      1.080980e+07          18793.0            0.211037
2      3.394311e+07          18827.0            0.192052
3      2.831485e+07          18943.0            0.207971
4      1.286732e+07          19021.0            0.264822

   btc_n_orphaned_blocks  btc_n_transactions_per_block  \
0                    1.0                          363.0
1                    0.0                          459.0
2                    1.0                          438.0
3                    0.0                          411.0
```

```
4                    2.0                          421.0

            ...            Low_Return_Exp_MA_3D  Close_Return_Exp_MA_3D  \
0           ...                        0.288141                3.510433
1           ...                        3.837640                3.049477
2           ...                        3.437376                5.590915
3           ...                        1.586938                0.667321
4           ...                        0.182540                0.202531

   Open_Return_Exp_MA_6D  High_Return_Exp_MA_6D  Low_Return_Exp_MA_6D  \
0               5.463303               2.221119              3.947754
1               1.842962               4.549867              1.681664
2               4.711021               2.076485              2.190026
3               2.717718               2.081822              0.627698
4               2.955675               6.960447              0.379141

   Close_Return_Exp_MA_6D  Open_Return_Exp_MA_9D  High_Return_Exp_MA_9D  \
0                1.714983               3.271087               1.476899
1                3.991242               1.869539               2.403294
2                4.937080               3.260566               1.719723
3                1.169969               3.384024               3.633603
4                0.408644               3.902769               3.674279

   Low_Return_Exp_MA_9D  Close_Return_Exp_MA_9D
0              2.654409                1.773699
1              1.579691                3.007898
2              2.268653                2.886790
3              1.148574                2.199385
4              0.667117                1.764899

[5 rows x 127 columns]
```

- Normalisation des données

In [125]:
```python
dataset = df
# Min-Max Scalings
dataset.iloc[:, 1:] = dataset.iloc[:, 1:].apply(lambda x: (x - np.min(x))/(np.max(x)

dataset.head(10)
```

Out[125]:
```
         Date    Target  btc_market_price  btc_total_bitcoins  btc_market_cap  \
0 2014-05-30  0.485048          0.022385            0.000000        0.016643
1 2014-05-31  0.478435          0.022976            0.000940        0.017103
2 2014-06-01  0.559737          0.025798            0.002048        0.019272
3 2014-06-04  0.496414          0.024229            0.005450        0.018098
4 2014-06-06  0.459986          0.024803            0.007263        0.018552
5 2014-06-07  0.459648          0.024728            0.008209        0.018503
6 2014-06-08  0.432868          0.024609            0.009137        0.018419
```

```
7 2014-06-09  0.469047          0.024554            0.009979        0.018383
8 2014-06-10  0.389366          0.024500            0.011074        0.018351
9 2014-06-11  0.298925          0.023987            0.011977        0.017965

   btc_trade_volume  btc_blocks_size  btc_avg_block_size  \
0          0.003575         0.000000            0.073250
1          0.001528         0.000231            0.048326
2          0.005853         0.000476            0.028235
3          0.004801         0.001312            0.045082
4          0.001913         0.001874            0.105245
5          0.001767         0.002090            0.037778
6          0.000632         0.002299            0.034673
7          0.002094         0.002544            0.094944
8          0.001982         0.002832            0.066179
9          0.002705         0.003092            0.091972

   btc_n_orphaned_blocks  btc_n_transactions_per_block  \
0               0.142857                      0.041436
1               0.000000                      0.080435
2               0.142857                      0.071904
3               0.000000                      0.060935
4               0.285714                      0.064998
5               0.285714                      0.055248
6               0.285714                      0.051186
7               0.142857                      0.030061
8               0.285714                      0.011375
9               0.142857                      0.024374

            ...           Low_Return_Exp_MA_3D  Close_Return_Exp_MA_3D  \
0           ...                       0.020313                0.608826
1           ...                       0.668587                0.524638
2           ...                       0.595484                0.988801
3           ...                       0.257523                0.089566
4           ...                       0.001026                0.004677
5           ...                       0.408491                0.027083
6           ...                       0.236051                0.999488
7           ...                       0.942956                0.000030
8           ...                       0.051654                0.019669
9           ...                       0.486011                0.106510

   Open_Return_Exp_MA_6D  High_Return_Exp_MA_6D  Low_Return_Exp_MA_6D  \
0               0.500525               0.191485              0.368380
1               0.162842               0.400767              0.151991
2               0.430357               0.178487              0.200535
3               0.244434               0.178967              0.051348
4               0.266629               0.617403              0.027614
5               0.318354               0.741408              0.060061
6               0.445188               0.339706              0.077338
```

```
7              0.568597             0.247656             0.095129
8              0.141806             0.091603             0.130587
9              0.089555             0.059537             0.306438

   Close_Return_Exp_MA_6D  Open_Return_Exp_MA_9D  High_Return_Exp_MA_9D  \
0                0.152270               0.219382               0.088035
1                0.365816               0.123296               0.145602
2                0.454549               0.218661               0.103124
3                0.101140               0.227125               0.222055
4                0.029717               0.262688               0.224583
5                0.043875               0.297693               0.244211
6                0.057650               0.308350               0.239730
7                0.017751               0.346795               0.722995
8                0.017846               0.389390               0.809245
9                0.030135               0.494066               0.167831

   Low_Return_Exp_MA_9D  Close_Return_Exp_MA_9D
0              0.164014                0.118860
1              0.096204                0.204594
2              0.139674                0.196181
3              0.069002                0.148430
4              0.038625                0.118249
5              0.053619                0.128334
6              0.043217                0.124655
7              0.055810                0.049512
8              0.062254                0.056779
9              0.104012                0.077423

[10 rows x 127 columns]
```

6.2 Estimation du modèle

- Séparation du dataset en train / test set

In [126]:
```
# Splitting X in training and testing set
train_set, test_set = skl_mdl_selection.train_test_split(dataset,
                                                          train_size=0.66,
                                                          test_size=0.34)


X_train = train_set.iloc[:, 2:]
Y_train = train_set.iloc[:, 1]
X_test = test_set.iloc[:, 2:]
Y_test = test_set.iloc[:, 1]

X_train.head(5)
```

Out[126]:
```
     btc_market_price  btc_total_bitcoins  btc_market_cap  btc_trade_volume  \
417          0.004602            0.406774        0.004383          0.002842
```

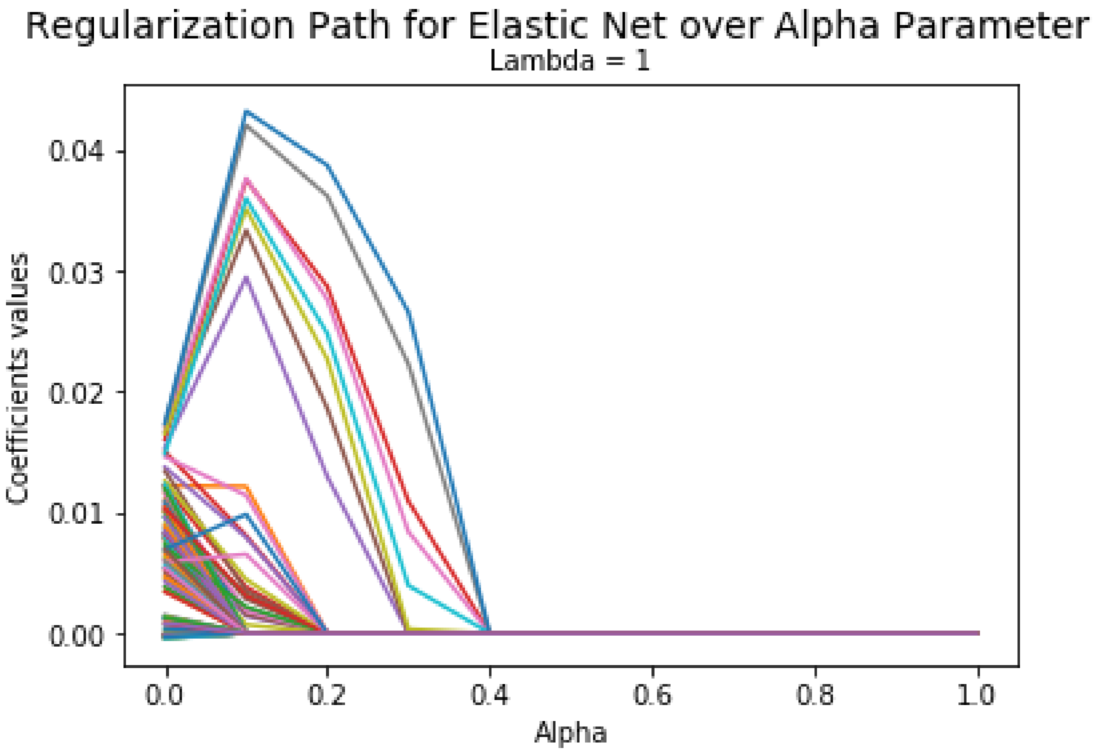
```
625         0.011619            0.605905        0.011435         0.009023
31          0.024197            0.035176        0.018312         0.001447
949         0.037084            0.814424        0.036931         0.005050
595         0.010794            0.577429        0.010546         0.007294

     btc_blocks_size  btc_avg_block_size  btc_n_orphaned_blocks  \
417         0.164505            0.218889               0.428571
625         0.302944            0.643204               0.000000
31          0.008806            0.124665               0.142857
949         0.582884            0.870451               0.142857
595         0.278031            0.494704               0.000000

     btc_n_transactions_per_block  btc_median_confirmation_time  \
417                      0.220992                      0.105634
625                      0.379830                      0.116197
31                       0.069466                      0.109155
949                      0.722588                      0.295070
595                      0.448484                      0.054930

     btc_hash_rate           ...             Low_Return_Exp_MA_3D  \
417       0.010771           ...                         0.002852
625       0.047577           ...                         0.031828
31        0.001262           ...                         0.928016
949       0.111808           ...                         0.110487
595       0.040191           ...                         0.031696

     Close_Return_Exp_MA_3D  Open_Return_Exp_MA_6D  High_Return_Exp_MA_6D  \
417                0.004956               0.005697               0.002987
625                0.999579               0.062438               0.065590
31                 0.255520               0.335948               0.564966
949                0.042182               0.146475               0.208309
595                0.653416               0.036335               0.937034

     Low_Return_Exp_MA_6D  Close_Return_Exp_MA_6D  Open_Return_Exp_MA_9D  \
417              0.002543                0.009349               0.004433
625              0.104093                0.167555               0.043817
31               0.527932                0.576896               0.118522
949              0.025065                0.068369               0.417839
595              0.104486                0.324257               0.015288

     High_Return_Exp_MA_9D  Low_Return_Exp_MA_9D  Close_Return_Exp_MA_9D
417               0.005271              0.003627                0.005421
625               0.020711              0.041174                0.061209
31                0.231817              0.159569                0.118191
949               0.433807              0.029055                0.334613
595               0.372724              0.062221                0.161905

[5 rows x 125 columns]
```

A) On estime la Régression Elastic Net pour alpha variant de 0 à 1

In [127]:
```
# ATTENTION : dans la fonction ElasticNet de sklearn, le paramètre
# l1_ratio correspond au alpha de la formule du projet,
# tandis que le paramètre alpha correspond au lambda de la formule du projet ...

all_beta_chap = dict()
#models = list()
alpha_range = [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1]
for i_alpha in alpha_range:
    en = skl_linear_mdl.ElasticNet(l1_ratio=i_alpha, fit_intercept=False, max_iter =
    en.fit(X_train, Y_train)
    beta_chap = en.coef_
    all_beta_chap.update({i_alpha: beta_chap})
    #models.append(en)

beta_vs_alpha = pd.DataFrame(all_beta_chap)

plt.figure()
plt.suptitle("Regularization Path for Elastic Net over Alpha Parameter",
            fontsize=14)
plt.title("Lambda = 1", fontsize=10)
for row in beta_vs_alpha.iterrows():
    row = row[1]
    plt.plot(alpha_range, row)
plt.ylabel("Coefficients values")
plt.xlabel("Alpha")
```

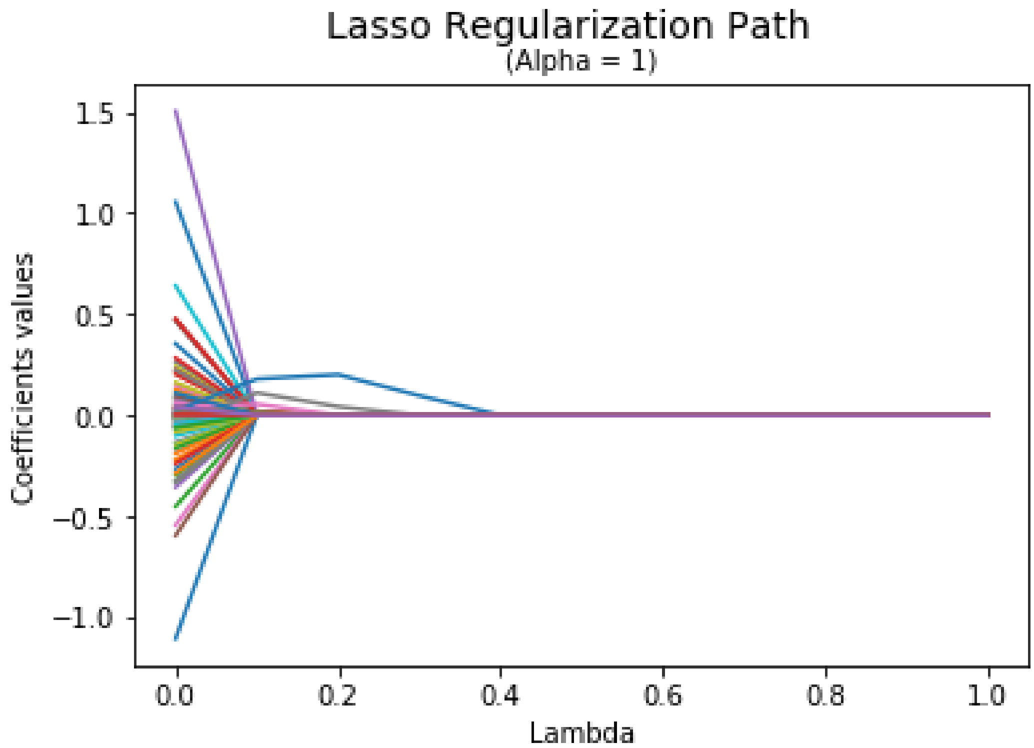
```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  ConvergenceWarning)
```

Out[127]:
```
Text(0.5,0,'Alpha')
```

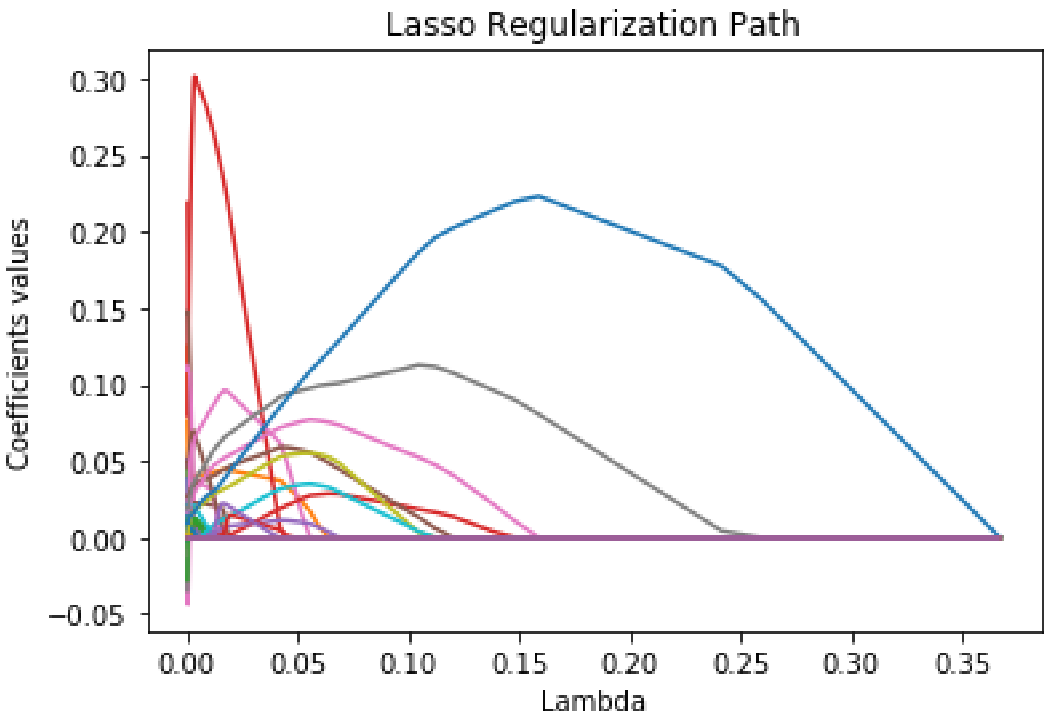


B) Lasso Regularization Path

```
In [142]: print_regularization_path(X_train,Y_train,isLasso=True)

/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:14: UserWarning

/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desce
  positive)
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desce
  ConvergenceWarning)
```



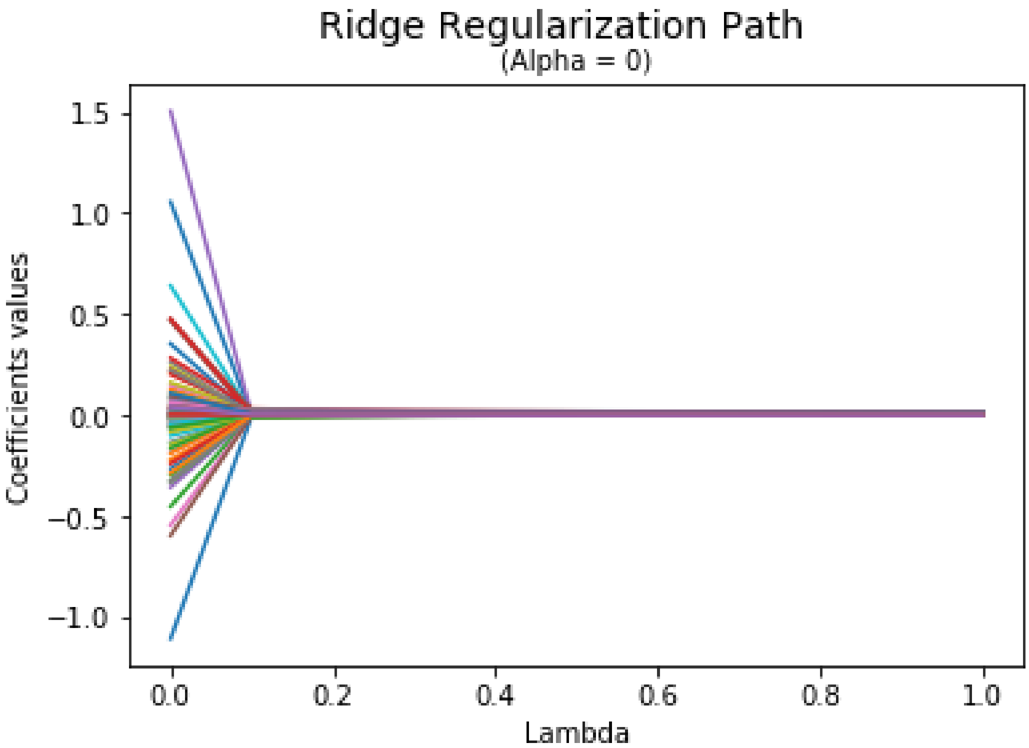

```
In [143]: lasso = skl_linear_mdl.ElasticNet(l1_ratio=1, fit_intercept= False, max_iter = 10000)
          lasso_path = lasso.path(X_train, Y_train, l1_ratio=1)
          lasso_path_result = lasso_path[1]
          plt.figure()
          plt.title("Lasso Regularization Path")
          for i in range(0, len(lasso_path_result)):
              plt.plot(lasso_path[0], lasso_path_result[i])
          plt.ylabel("Coefficients values")
          plt.xlabel("Lambda")

Out[143]: Text(0.5,0,'Lambda')
```



B.1) Ridge Regularization Path

```
In [144]: print_regularization_path(X_train,Y_train,isLasso=False)
```

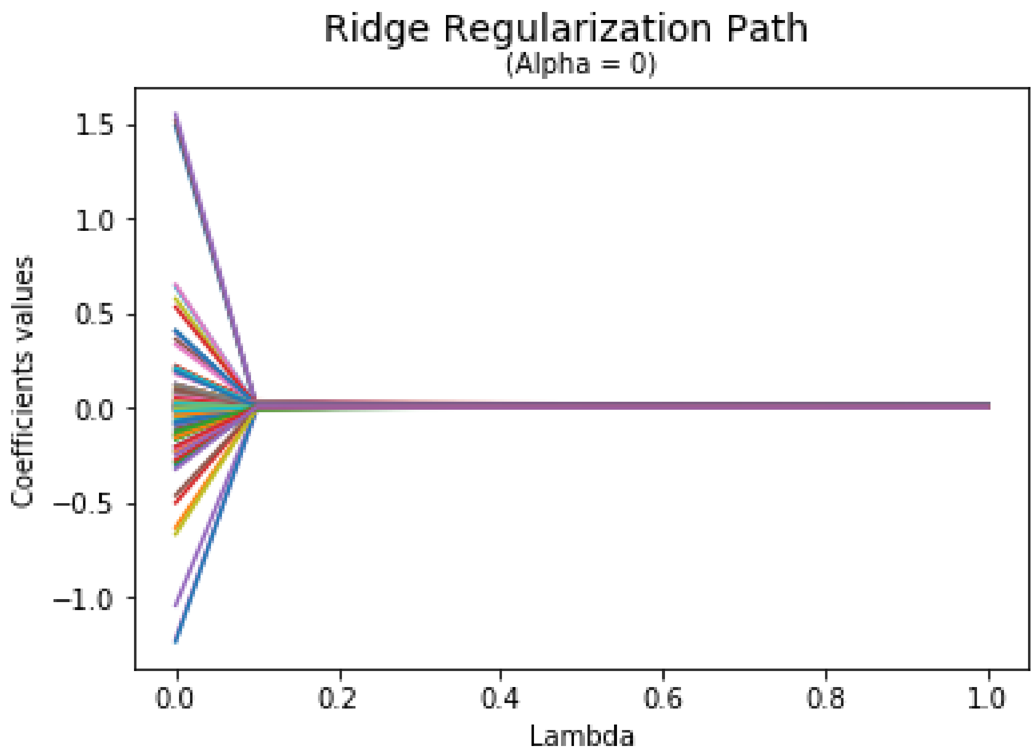
```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:14: UserWarning

/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  positive)
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  ConvergenceWarning)
```



In [145]:
```
# Méthode 1.1) Avec une boucle
all_beta_chap = dict()
lambda_range = [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1]
for i_lambda in lambda_range:
    ridge = skl_linear_mdl.ElasticNet(alpha=i_lambda, l1_ratio=0, fit_intercept=False
    ridge.fit(X_train, Y_train)
    beta_chap = ridge.coef_
    all_beta_chap.update({i_lambda: beta_chap})

beta_vs_lambda = pd.DataFrame(all_beta_chap)

# Méthode 1.2) Avec la méthode Path
ridge = skl_linear_mdl.ElasticNet(l1_ratio=1, fit_intercept=False, max_iter = 10000)
lasso_path = ridge.path(X_train, Y_train, l1_ratio=1)

plt.figure()
plt.suptitle("Ridge Regularization Path", fontsize=14)
plt.title("(Alpha = 0)", fontsize=10)
for row in beta_vs_lambda.iterrows():
    row = row[1]
    plt.plot(lambda_range, row)
plt.ylabel("Coefficients values")
plt.xlabel("Lambda")
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:6: UserWarning

/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  positive)
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  ConvergenceWarning)
```

```
Out[145]: Text(0.5,0,'Lambda')
```



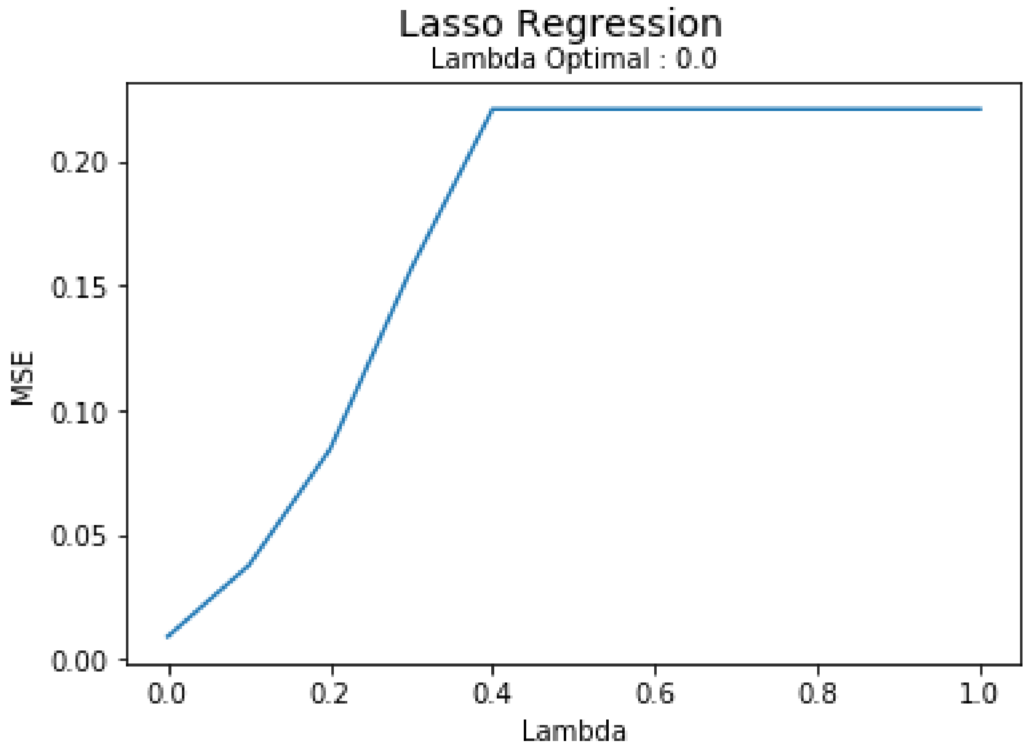
Optimisation du / des paramètres de régularisation (lambda et alpha)

- Lasso

```
In [159]: lambda_opt_lasso, _, all_quad_risk = optimise_params(X_train, Y_train, X_test, Y_tes
          print_plot("Lasso Regression","Lambda Optimal : " + str(lambda_opt_lasso),"Lambda","M
          plt.plot(all_quad_risk.columns,
                   all_quad_risk.values.reshape(11, 1))
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:9: UserWarning
  if __name__ == '__main__':
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  positive)
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  ConvergenceWarning)
```

Out[159]: [<matplotlib.lines.Line2D at 0x1a12ca7470>]

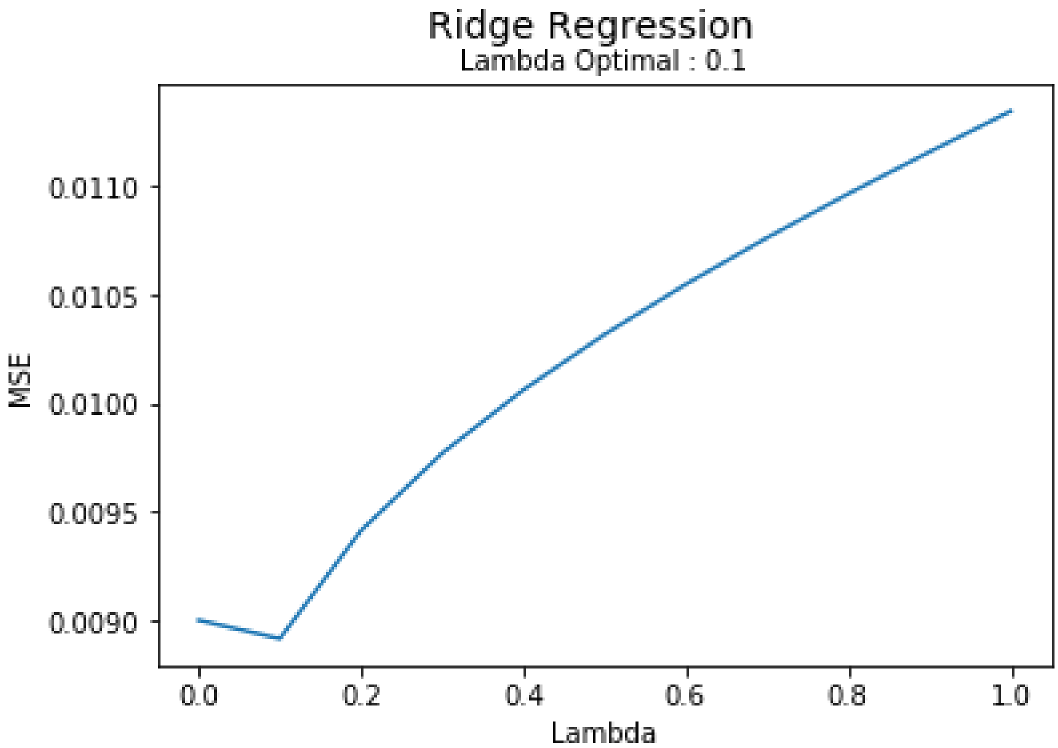


- Ridge

```
In [160]: lambda_opt_ridge, _, all_quad_risk = optimise_params(X_train, Y_train, X_test, Y_tes
          print_plot("Ridge Regression","Lambda Optimal : " + str(lambda_opt_ridge),"Lambda","M
          plt.plot(all_quad_risk.columns,
                   all_quad_risk.values.reshape(11, 1))
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:9: UserWarning
  if __name__ == '__main__':
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  positive)
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  ConvergenceWarning)
```

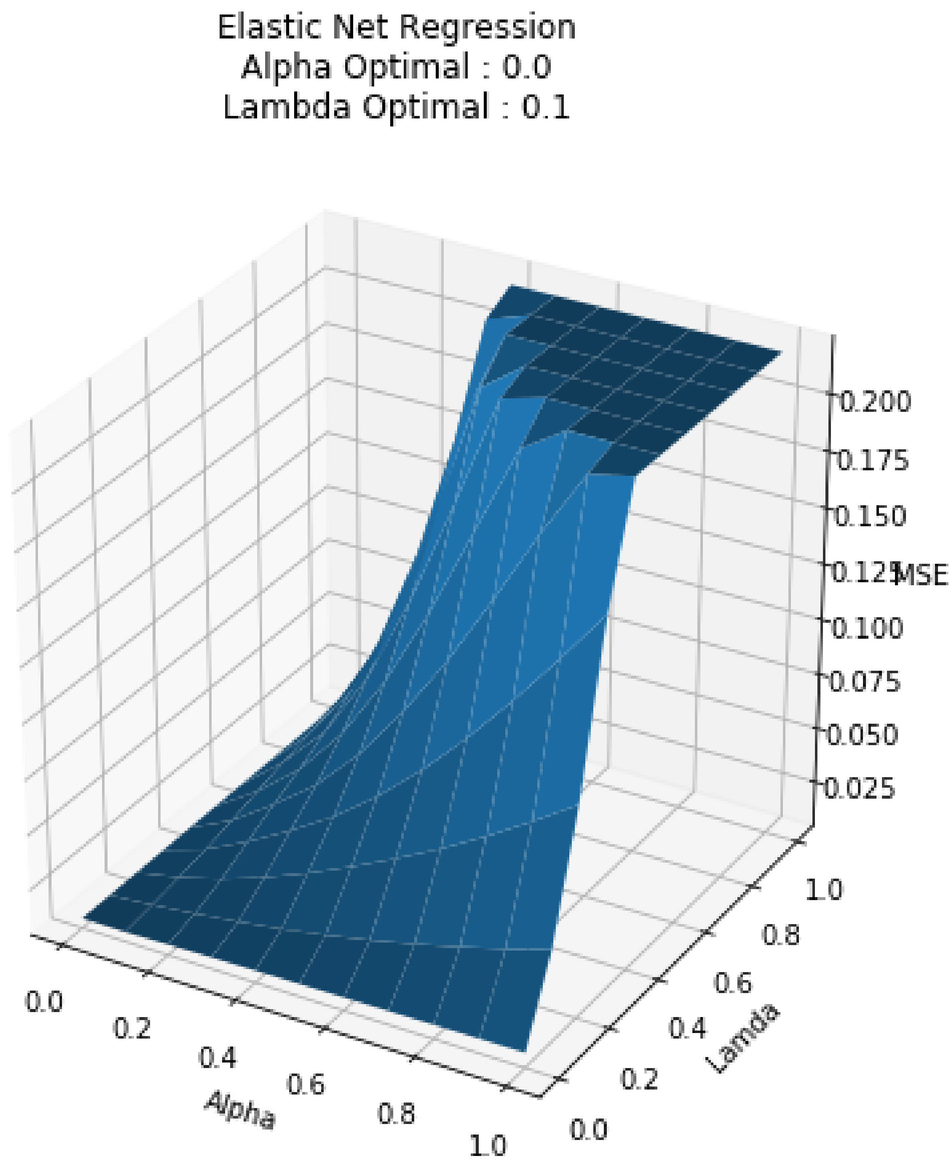
Out[160]: [<matplotlib.lines.Line2D at 0x1a12b8c9e8>]

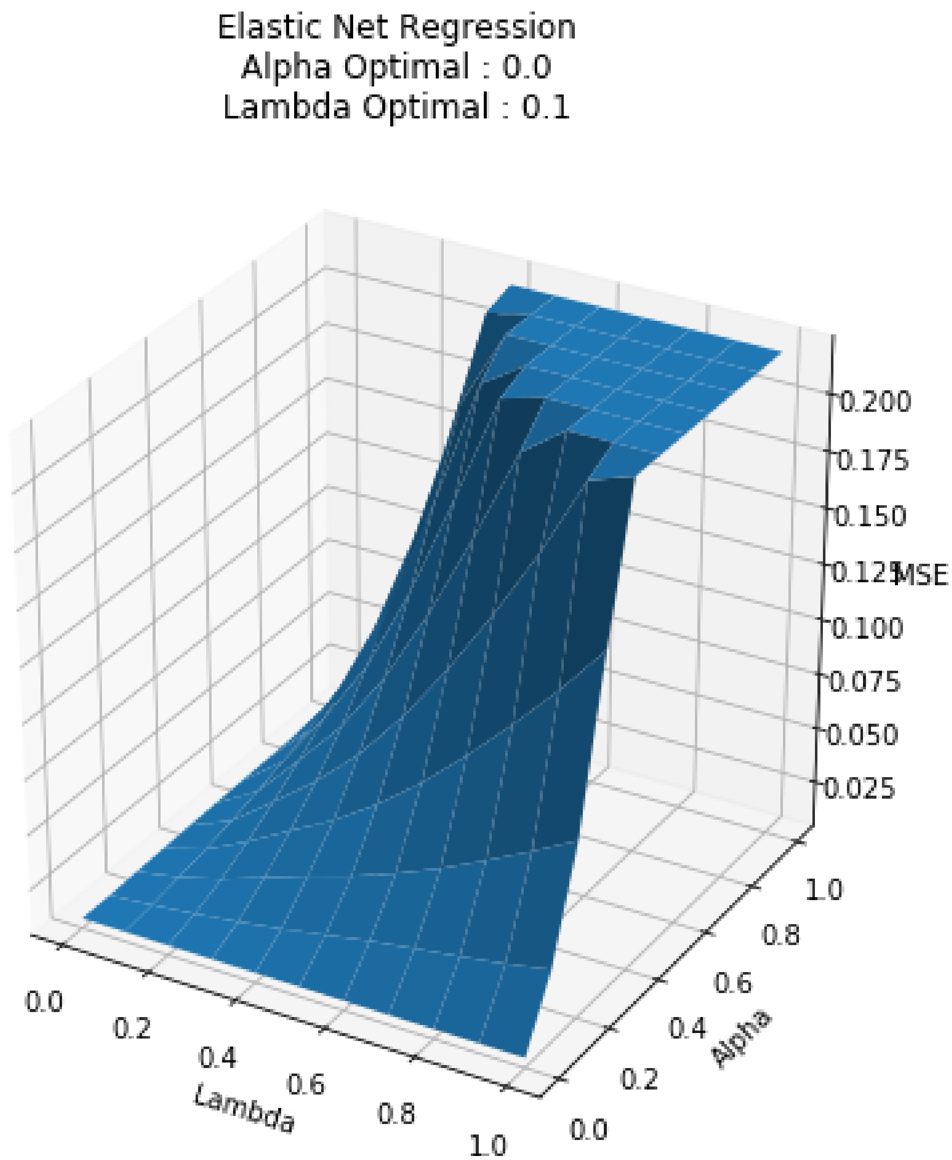


- Elastic Net

```
In [161]: lambda_opt_elastic_net, alpha_opt_elastic_net, all_quad_risk = optimise_params(X_trai

          print_plot3D("Elastic Net Regression\nAlpha Optimal : " + str(alpha_opt_elastic_net)
                    "\nLambda Optimal : " + str(lambda_opt_elastic_net),"", "Alpha", "Lamda", "
                       np.linspace(0, 1, 11), np.linspace(0, 1, 11), all_quad_risk,0)

          print_plot3D("Elastic Net Regression\nAlpha Optimal : " + str(alpha_opt_elastic_net)
                    "\nLambda Optimal : " + str(lambda_opt_elastic_net),"", "Lambda", "Alpha",
                       np.linspace(0, 1, 11), np.linspace(0, 1, 11), all_quad_risk,1)

          opt_elastic_net=[]
          opt_elastic_net.append(alpha_opt_elastic_net)
          opt_elastic_net.append(lambda_opt_elastic_net)
```

```
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:9: UserWarning
  if __name__ == '__main__':
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  positive)
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  ConvergenceWarning)
```


Elastic Net Regression
Alpha Optimal : 0.0
Lambda Optimal : 0.1
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
MSE
0.0
0.2
0.4
0.6
0.8
1.0
Alpha
0.0
0.2
0.4
0.6
0.8
1.0
Lamda



- Score des estimateurs

```
In [162]: print_score(X_train,Y_train,lambda_opt_lasso,lambda_opt_ridge,opt_elastic_net)

/Users/virgileamato/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:8: UserWarning

/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  positive)
/Users/virgileamato/anaconda3/lib/python3.6/site-packages/sklearn/linear_model/coordinate_desc
  ConvergenceWarning)
```

	Score (R2)	MSE
Lasso	-0.356518	0.00899991
Ridge	-0.343664	0.00891463
Elastic Net	-0.343664	0.00891463
BEST MODEL	Ridge	Ridge