

# Non linear model.

$$- \operatorname{div}(\lambda(u) \nabla u) + w \cdot \nabla u = f \quad \text{in } \Omega$$

$$- \lambda(u) \partial_n u = \begin{cases} cu & \Gamma_{\text{out}} \\ 0 & \Gamma_{\text{wall}} \end{cases}$$

$$u = u_{\text{in}} \quad \text{on } \Gamma_{\text{in}}$$

◦ Weak form.  $V_0 = \{v \in H^1(\Omega), v = 0 \text{ on } \Gamma_{\text{in}}\}$   
 $u \in V_\varepsilon$ .  $V_\varepsilon = V \oplus u_{\text{in}}|_{\Gamma_{\text{in}}}$

$$\int_{\Omega} \lambda(u) \nabla u \cdot \nabla v \, dx + \int_{\Omega} w \cdot \nabla u \, v \, dx = \int_{\Omega} f v \, dx + \int_{\Gamma_{\text{out}}} cu v \, ds$$

$$a(u, v) = \ell(v)$$

◦ Newton-Raphson algo. Trouver  $\delta u \in V_0$  tq

$$\partial_u a(u^{(k)}, v) \cdot \delta u = - (a(u^{(k)}, v) - \ell(v))$$

$$\forall v \in V_0.$$

On  $a$ :

$$\begin{aligned} \circ \partial_u a(u^{(k)}, v) \cdot \delta u &= \int_{\Omega} \lambda'(u^{(k)}) \cdot \delta u \nabla u^{(k)} \nabla v dx \\ &+ \int_{\Omega} \lambda(u^{(k)}) \nabla(\delta u) \nabla v dx + \int_{\Omega} w \nabla(\delta u) v dx \\ &+ \int_{\Gamma_{out}} c \delta u v ds \end{aligned}$$

NB. c.n.d'existence - on note  $\lambda'(u^{(k)}) > 0$   
 $\forall k, \forall x$ .