Calculator

Class outline:

- Programming languages
- Parsing a language
- The Calculator language
- Evaluating a language
- Interactive interpreters

Programming languages

Levels of languages

High-level programming language
(Python, C++, JavaScript)

⬇

Assembly language
(Hardware-specific)

⬇

Machine language
(Hardware-specific)

Machine language

The language of the machine is all 1s and 0s, often specifying the action and the memory address to act on:

```
00000100 10000010  # Load data in 10000010
00000001 10000001  # Subtract data at 10000001
00000101 10000100  # Store result in 10000100
00001011 10000100  # Etc..
00001101 00010000
00010100 00000010
00000101 10000011
00001111 00000000
00010100 00000011
00000101 10000011
```

Code is executed directly by the hardware.

Assembly language

Assembly language was introduced for (slightly) easier programming.

Machine code	Assembly code
`00000100 10000010`	`LOD Y`
`00000001 10000001`	`SUB X`
`00000101 10000100`	`STO T1`
`00001011 10000100`	`CPL T1`
`00001101 00010000`	`JMZ 16`
`00010100 00000010`	`LOD #2`
`00000101 10000011`	`STO Z`
`00001111 00000000`	`HLT`
`00010100 00000011`	`LOD #3`
`00000101 10000011`	`STO Z`

Assembly still has a 1:1 mapping with machine language, however.

Higher-level languages

Higher level languages:

- provide means of abstraction such as naming, function definition, and objects
- abstract away system details to be independent of hardware and operating system

```
if x > y:
    z = 2
else:
    z = 3
```

Statements & expressions are either **interpreted** by another program or **compiled** (translated) into a lower-level language.

Compiled vs. interpreted

When a program is **compiled**, the source code is translated into machine code, and that code can be distributed and run repeatedly.

`Source code → Compiler → Machine code → Output`

When a program is **interpreted**, an interpreter runs the source code directly (without compiling it first).

`Source code → Interpreter → Output`

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In its most popular implementation (CPython), Python programs are interpreted but have a compile step:

`Source code → Compiler → Bytecode → Virtual Machine → Output`

Phases of an interpreter/compiler

In order to either interpret or compile source code, a program must be written that understands that source code.

Typical phases of understanding:

`Source code → Lexing → Parsing → Abstract Syntax Tree`

Lexing & Parsing

Reading Scheme Lists

A Scheme list is written as elements in parentheses:

```
(<element_0> <element_1> ... <element_n>)
```

Each <element> can be a combination or primitive.

```
(+ (* 3 (+ (* 2 4) (+ 3 5))) (+ (- 10 7) 6))
```

The task of parsing a language involves turning a string representation of an expression into a structured object representing the expression.

Parsing

A parser takes text and returns an expression object.

Text	Lexical Analysis	Tokens	Syntactic Analysis	Expression
`'(+ 1'`	→	`'('`, `'+'`, `1`	→	`Pair('+', Pair(1, ...))` printed as (+ 1 (- 23) (* 4 5.6))
`'   (- 23)'`	→	`'('`, `'-'`, `23`, `')'`		
`'   (* 4 5.6))'`	→	`'('`, `'*'`, `4`, `5.6`, `')'`, `')'`		

Lexical analysis

`' (* 4 5.6))'` → `'('`, `'*'`, `4`, `5.6`, `')', ')'`

- Iterative process
- Checks for malformed tokens
- Determines types of tokens
- Processes one line at a time

Syntactic analysis

`'('`, `'+'`, `1`, ... → `Pair('+', Pair(1, ...))`

- Tree-recursive process
- Balances parentheses
- Returns tree structure
- Processes multiple lines

In scheme_reader.py, each call to `scheme_read` consumes the input tokens for exactly one expression.

- Base case:
- Recursive case:

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In scheme_reader.py, each call to `scheme_read` consumes the input tokens for exactly one expression.

- Base case: symbols and numbers
- Recursive case: read subexpressions and combine them

Pair class

The Pair class represents Scheme pairs and lists. A list is a pair whose second element is either pair or a list.

```
class Pair:
```

```
s = Pair(1, Pair(2, Pair(3, nil)))
print(s)
len(s)
```

Improper lists:

```
print(Pair(1, 2))
print(Pair(1, Pair(2, 3)))
len(Pair(1, Pair(2, 3)))
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Improper lists:

```
print(Pair(1, 2))   # (1 . 2)
print(Pair(1, Pair(2, 3))) # (1 2 . 3)
len(Pair(1, Pair(2, 3))) Error!
```

The Calculator Language

What's in a language?

A programming language has:

- **Syntax**: The legal statements and expressions in the language
- **Semantics**: The execution/evaluation rule for those statements and expressions

To create a new programming language, you either need a:

- **Specification**: A document describe the precise syntax and semantics of the language
- **Canonical Implementation**: An interpreter or compiler for the language

Calculator language syntax

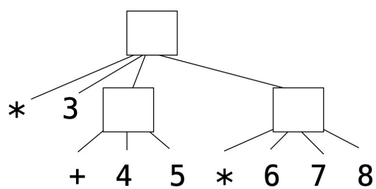
The Calculator language has primitive expressions and call expressions. (That's it!)

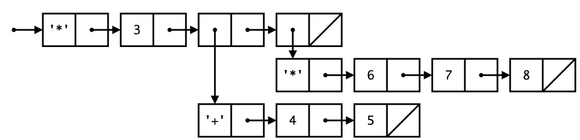
A **primitive expression** is a number: `2` `-4` `5.6`

A **call expression** is a combination that begins with an operator (`+`, `-`, `*`, `/`) followed by 0 or more expressions: `(+ 1 2 3)` `(/ 3 (+ 4 5))`

Expression **Expression tree** **Representation as pairs**

```
(* 3
    (+ 4 5)
    (* 6 7 8))
```





Calculator language semantics

The value of a calculator expression is defined recursively.

- **Primitive**: A number evaluates to itself.
- **Call**: A call expression evaluates to its argument values combined by an operator.
  - `+`: Sum of the arguments
  - `*`: Product of the arguments
  - `-`: If one argument, negate it. If more than one, subtract the rest from the first.
  - `/`: If one argument, invert it. If more than one, divide the rest from the first.

Expression	Expression Tree
`(+ 5 (* 2 3) (* 2 5 5))`	

```
(+ 5
    (* 2 3)
    (* 2 5 5))
```

Evaluation

The eval function

The eval function computes the value of an expression.

It is a generic function that behaves according to the type of the expression (primitive or call).

Implementation	**Language semantics**

```
def calc_eval(exp):
    if isinstance(exp, (int, float)):
        return exp
    elif isinstance(exp, Pair):
        arguments = exp.rest.map(calc_eval)
        return calc_apply(exp.first, argumen
    else:
        raise TypeError
```

A **number** evaluates...
to itself

A **call expression** evaluates...
to its argument values combined by an operator

Applying built-in operators

The apply function applies some operation to a (Scheme) list of argument values

In calculator, all operations are named by built-in operators: +, -, *, /

Implementation	**Language semantics**

```
def calc_apply(operator, args):
    if operator == '+':
        return reduce(add, args, 0)
    elif operator == '-':
        ...
    elif operator == '*':
        ...
    elif operator == '/':
        ...
    else:
    raise TypeError
```

`+`
Sum of the arguments

`-`
...

`*`
...

`/`
...

Interactive interpreters

REPL: Read-Eval-Print Loop

The user interface for many programming languages is an interactive interpreter

- Print a prompt
- Read text input from the user
- Parse the text input into an expression
- Evaluate the expression
- If any errors occur, report those errors, otherwise
- Print the value of the expression and repeat

Raising exceptions

Exceptions can be raised during lexical analysis, syntactic analysis, eval, and apply.

Example exceptions

- **Lexical analysis**: The token 2.3.4 raises `ValueError("invalid numeral")`
- **Syntactic analysis**: An extra) raises `SyntaxError("unexpected token")`
- **Eval**: An empty combination raises `TypeError("() is not a number or call expression")`
- **Apply**: No arguments to - raises TypeError("- requires at least 1 argument")

Handling exceptions

An interactive interpreter prints information about each error.

A well-designed interactive interpreter should not halt completely on an error, so that the user has an opportunity to try again in the current environment.