

Sequences

Class outline:

- Box+Pointer
- Slicing
- Recursive exercises
- Built-ins for iterables

Where to ask questions?

- **Zoom chat:** Good if you like getting responses from classmates or the lecture helper.
- **Zoom Q&A:** Good for asking questions that likely interest most students, and that should be answered in lecture.
- **Post-lecture OH:** Good for recapping a topic that went too fast. Or any questions!
- **Piazza thread:** Good for longer questions, tangential questions, or any unanswered questions.

Box + Pointer

Lists in environment diagrams

Lists are represented as a row of index-labeled adjacent boxes, one per element.

```
pair = [1, 2]
```

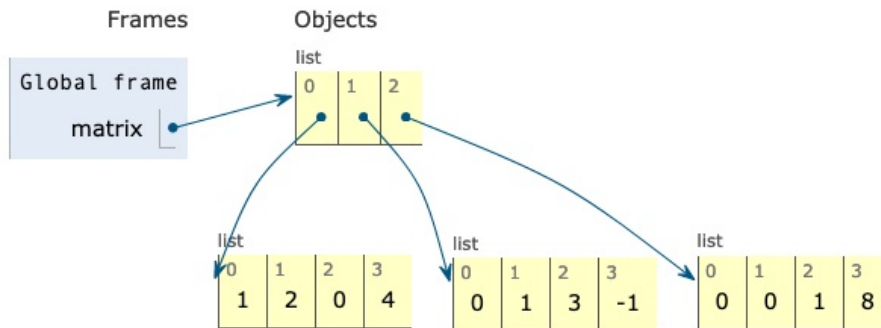


Try in PythonTutor.

Nested lists in environment diagrams

Each box either contains a primitive value or points to a compound value.

```
matrix = [ [1,2,0,4], [0,1,3,-1], [0,0,1,8] ]
```



Nested lists in environment diagrams

A very nested list:

```
worst_list = [ [1, 2],  
                [],  
                [ [3, False, None],  
                  [4, lambda: 5]]]
```



View in PythonTutor

Slicing

Slicing syntax

Slicing a list creates a new list with a subsequence of the original list.

```
letters = ["A", "B", "C", "D", "E", "F"]  
          #  0   1   2   3   4   5  
  
sublist1 = letters[1:]  
sublist2 = letters[1:4]
```

Slicing also works for strings.

```
compound_word = "cortaúñas"  
  
word1 = compound_word[:5]  
word2 = compound_word[5:]
```

Negatives indices and steps can also be specified.

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          #  0   1   2   3   4   5

sublist1 = letters[1:]    # ['B', 'C', 'D', 'E', 'F']
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```

Slicing also works for strings.

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compound_word = "cortaúñas"

word1 = compound_word[:5]    # "corta"
word2 = compound_word[5:]    # "úñas"
```

Negatives indices and steps can also be specified.

Copying whole lists

Slicing a whole list copies a list:

```
listA = [2, 3]
listB = listA

listC = listA[:]
listA[0] = 4
listB[1] = 5
```

`list()` creates a new list containing existing elements from any iterable:

```
listA = [2, 3]
listB = listA

listC = list(listA)
listA[0] = 4
listB[1] = 5
```



Try both in [PythonTutor](#).

Python3 provides more ways in the [copy module](#).

Recursion exercises

Recursively sum a list

Let's code this up recursively:

```
def sum_nums(nums):  
    """Returns the sum of the numbers in NUMS.  
    >>> sum_nums([6, 24, 1984])  
    2014  
    >>> sum_nums([-32, 0, 32])  
    0  
    """
```

Docstrings typically would not specify whether an approach was recursive or iterative, since that is an implementation detail.

However, we'll make it clear in assignments and exam questions.

Recursively sum a list (solution)

```
def sum_nums(nums):  
    """Returns the sum of the numbers in NUMS.  
    >>> sum_nums([6, 24, 1984])  
    2014  
    >>> sum_nums([-32, 0, 32])  
    0  
    """  
    if (nums == []):  
        return 0  
    else:  
        return nums[0] + sum_nums( nums[1:] )
```

When recursively processing lists, the base case is often the empty list and the recursive case is often all-but-the-first items.

Iteratively sum a range

Let's code this up iteratively:

```
def sum_up_to(n):  
    """Returns the sum of positive numbers from 1 up to N (inclusive)  
>>> sum_up_to(5)  
15  
"""
```

Iteratively sum a range (solution)

Using the `range` type:

```
def sum_up_to(n):  
    """Returns the sum of positive numbers from 1 up to N (inclusive)  
    >>> sum_up_to(5)  
    15  
    """  
    sum = 0  
    for n in range(0, n+1):  
        sum += n  
    return sum
```

Remember that `range(start, end)` always ends right before `end`.

Recursively sum a range

Now try it recursively:

```
def sum_up_to(n):  
    """Returns the sum of positive numbers from 1 up to N (inclusiv  
>>> sum_up_to(5)  
15  
"""
```

Recursively sum a range (solution)

Now try it recursively:

```
def sum_up_to(n):  
    """Returns the sum of positive numbers from 1 up to N (inclusive)  
    >>> sum_up_to(5)  
    15  
    """  
    if n == 1:  
        return 1  
    else:  
        return n + sum_up_to(n-1)
```

Reversing a string

Recursively reversing a string

```
def reverse(s):  
    """Returns a string with the letters of S  
    in the inverse order.  
    >>> reverse('ward')  
    'draw'  
    """
```

Breaking it down into subproblems:

```
reverse("ward") =  
reverse("ard") =  
reverse("rd") =  
reverse("d") =
```

Recursively reversing a string

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    """Returns a string with the letters of S  
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Breaking it down into subproblems:

```
reverse("ward") = reverse("ard") + "w"  
reverse("ard") = reverse("rd") + "a"  
reverse("rd") = reverse("d") + "r"  
reverse("d") =
```


Recursively reversing a string

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def reverse(s):  
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Breaking it down into subproblems:

```
reverse("ward") = reverse("ard") + "w"  
reverse("ard") = reverse("rd") + "a"  
reverse("rd") = reverse("d") + "r"  
reverse("d") = "d"
```

Recursively reversing a string (solution)

```
def reverse(s):  
    """Returns a string with the letters of S  
    in the inverse order.  
    >>> reverse('ward')  
    'draw'  
    """  
    if len(s) == 1:  
        return s  
    else:  
        return reverse(s[1:]) + s[0]
```

When recursively processing strings, the base case is typically an empty string or single-character string, and the recursive case is often all-but-the-first characters.

Recursively reversing a string (visual)

reverse(ward)
ret: draw



reverse(ard)
ret: dra



reverse(rd)
ret: dr



reverse(d)
ret: d

Exercise: Reversing a number

```
def reverse(n):  
    """Returns N with the digits reversed.  
    >>> reverse_digits(123)  
    321  
    """
```

See walkthrough video [here](#)

Helper functions

If a recursive function needs to keep track of more state than the arguments of the original function, you may need a helper function.

```
def fUnKyCaSe(text):  
    """Returns TEXT in fUnKyCaSe  
>>> fUnKyCaSe("wats up")  
    'wAtS Up'  
    """
```

Helper functions

If a recursive function needs to keep track of more state than the arguments of the original function, you may need a helper function.

```
def fUnKyCaSe(text):  
    """Returns TEXT in fUnKyCaSe  
    >>> fUnKyCaSe("wats up")  
    'wAtS Up'  
    """  
  
    def toggle_case(letter, should_up_case):  
        return letter.upper() if should_up_case else letter.lower()  
  
    def up_down(text, should_up_case):  
        if len(text) == 1:  
            return toggle_case(text, should_up_case)  
        else:  
            return toggle_case(text[0], should_up_case) + up_down(text[1:], not should_up_case)  
  
    return up_down(text, False)
```

Recursion on different data types

Data type	Base case condition	Current item	Recursive case argument
Numbers	<code>== 0</code> <code>== 1</code>	<code>n % 10</code>	<code>n // 10</code>
Lists	<code>== []</code>	<code>L[0]</code>	<code>L[1:]</code> <code>L[:-1]</code>
Strings	<code>== ''</code> <code>len(S) == 1</code>	<code>S[0]</code>	<code>S[1:]</code> <code>S[:-1]</code>

Built-in functions for iterables

Functions that process iterables

The following built-in functions work for sequence types (lists, strings, etc) and any other **iterable** data type.

Function	Description
<code>sum(iterable, start)</code>	Returns the sum of values in <code>iterable</code> , initializing sum to <code>start</code>
<code>all(iterable)</code>	Return <code>True</code> if all elements of <code>iterable</code> are true (or if <code>iterable</code> is empty)
<code>any(iterable)</code>	Return <code>True</code> if any element of <code>iterable</code> is true. Return <code>False</code> if <code>iterable</code> is empty.
<code>max(iterable, key=None)</code>	Return the max value in <code>iterable</code>
<code>min(iterable, key=None)</code>	Return the min value in <code>iterable</code>

Examples with sum/any/all

```
sum([73, 89, 74, 95], 0) # 331
```

```
all([True, True, True, True])  
any([False, False, False, True])
```

```
all([x < 5 for x in range(5)])
```

```
perfect_square = lambda x: x == round(x ** 0.5) ** 2  
any([perfect_square(x) for x in range(50, 60)])
```

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```
sum([73, 89, 74, 95], 0) # 331
```

```
all([True, True, True, True]) # True  
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perfect_square = lambda x: x == round(x ** 0.5) ** 2
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```
any([perfect_square(x) for x in range(50, 60)]) # False
```

Examples with max/min

```
max([73, 89, 74, 95])      # 95  
max(["C+", "B+", "C", "A"])  
max(range(10))
```

Examples with max/min

```
max([73, 89, 74, 95])      # 95  
max(["C+", "B+", "C", "A"]) # C+  
max(range(10))
```


Examples with max/min

```
max([73, 89, 74, 95])      # 95  
max(["C+", "B+", "C", "A"]) # C+  
max(range(10))             # 9
```

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```
max([73, 89, 74, 95])      # 95
max(["C+", "B+", "C", "A"]) # C+
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```

A key function can decide how to compare each value:

```
coords = [ [37, -144], [-22, -115], [56, -163] ]
max(coords, key=lambda coord: coord[0])
min(coords, key=lambda coord: coord[0])
```

```
gymnasts = [ ["Brittany", 9.15, 9.4, 9.3, 9.2],
              ["Lea", 9, 8.8, 9.1, 9.5],
              ["Maya", 9.2, 8.7, 9.2, 8.8] ]
min(gymnasts, key=lambda scores: min(scores[1:]))
max(gymnasts, key=lambda scores: sum(scores[1:], 0))
```

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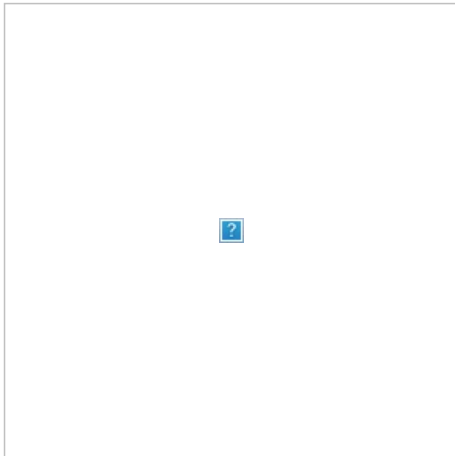
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min(gymnasts, key=lambda scores: min(scores[1:])) # ["Maya", ..
max(gymnasts, key=lambda scores: sum(scores[1:], 0)) # ["Brittany",
```

Python Project of The Day!

Sea Level Rise

[Sea Level Rise](#), by Douwe Osinga: Visualize sea levels and population density on interactive maps.



Technologies used: Python (notebook) with PIL/numpy/Rasterio,
HTML/CSS/JS with PanZoom
([Github repository](#))