## CS 61A Fall 2021

# Structure and Interpretation of Computer Programs

FINAL SOLUTIONS

#### INSTRUCTIONS

This is your exam. Complete it either at exam.cs61a.org or, if that doesn't work, by emailing course staff with your solutions before the exam deadline.

This exam is intended for the student with email address <EMAILADDRESS>. If this is not your email address, notify course staff immediately, as each exam is different. Do not distribute this exam PDF even after the exam ends, as some students may be taking the exam in a different time zone.

For questions with circular bubbles, you should select exactly one choice.

You must choose either this option
Or this one, but not both!

For questions with square checkboxes, you may select multiple choices.

You could select this choice.

You could select this one too!

You may start your exam now. Your exam is due at <DEADLINE> Pacific Time. Go to the next page to begin.

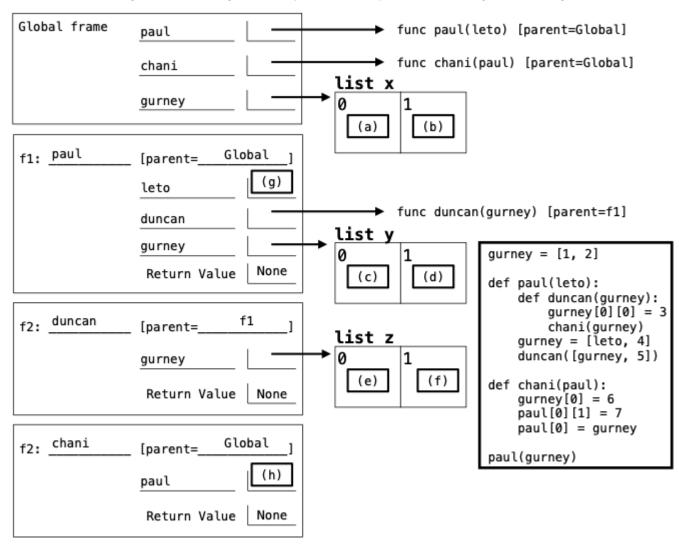
### Preliminaries

You can complete and submit these questions before the exam starts.		
(a)	What is your full name?	
(b)	What is your student ID number? A regex restricts inputs to numerical responses only.	

#### 1. (14.0 points) House Atreides

#### (a) (8.0 points)

The environment diagram below was generated by code that is provided to the right of the diagram.



- i. (1.0 pt) Which of these could fill in blank (a)?
  - $\bigcirc$  1
  - 3
  - 6
  - $\bigcirc$  7
- ii. (1.0 pt) Which of these could fill in blank (b)?
  - **2**
  - O 6
  - $\bigcirc$  7
  - O an arrow to list y (named gurney in the f1 frame)

111.	(1.0 pt) Which of these could fill in blank (c)?
	<b>3</b>
	○ 6
	○ 7
	$\bigcirc$ an arrow to list x (named gurney in the Global frame)
iv.	(1.0 pt) Which of these could fill in blank (d)?
	<b>3</b>
	<b>O</b> 4
	<u> </u>
	• 7
v.	(1.0 pt) Which of these could fill in blank (e)?
	○ 3
	○ 6
	○ 7
	an arrow to list x (named gurney in the Global frame)
	O an arrow to list y (named gurney in the f1 frame)
vi.	(1.0 pt) Which of these could fill in blank (f)?
vi.	<ul><li>(1.0 pt) Which of these could fill in blank (f)?</li><li>3</li></ul>
vi.	
vi.	○ 3
vi.	<ul><li>○ 3</li><li>○ 4</li></ul>
vi.	<ul><li>○ 3</li><li>○ 4</li><li>○ 5</li></ul>
	<ul> <li>3</li> <li>4</li> <li>5</li> <li>6</li> </ul>
	<ul> <li>○ 3</li> <li>○ 4</li> <li>○ 5</li> <li>○ 6</li> <li>○ 7</li> </ul>
	<ul> <li>3</li> <li>4</li> <li>5</li> <li>6</li> <li>7</li> <li>(1.0 pt) Which of these could fill in blank (g)?</li> </ul>
	<ul> <li>3</li> <li>4</li> <li>5</li> <li>6</li> <li>7</li> <li>(1.0 pt) Which of these could fill in blank (g)?</li> <li>an arrow to list x (named gurney in the Global frame)</li> </ul>
	<ul> <li>3</li> <li>4</li> <li>5</li> <li>6</li> <li>7</li> <li>(1.0 pt) Which of these could fill in blank (g)?</li> <li>an arrow to list x (named gurney in the Global frame)</li> <li>an arrow to list y (named gurney in the f1 frame)</li> </ul>
vii.	<ul> <li>3</li> <li>4</li> <li>5</li> <li>6</li> <li>7</li> <li>(1.0 pt) Which of these could fill in blank (g)?</li> <li>an arrow to list x (named gurney in the Global frame)</li> <li>an arrow to list y (named gurney in the f1 frame)</li> <li>an arrow to list z (named gurney in the f2 frame)</li> <li>an arrow to another list that does not appear in the diagram</li> </ul>
vii.	<ul> <li>3</li> <li>4</li> <li>5</li> <li>6</li> <li>7</li> <li>(1.0 pt) Which of these could fill in blank (g)?</li> <li>an arrow to list x (named gurney in the Global frame)</li> <li>an arrow to list y (named gurney in the f1 frame)</li> <li>an arrow to list z (named gurney in the f2 frame)</li> <li>an arrow to another list that does not appear in the diagram</li> <li>(1.0 pt) Which of these could fill in blank (h)?</li> </ul>
vii.	<ul> <li>3</li> <li>4</li> <li>5</li> <li>6</li> <li>7</li> <li>(1.0 pt) Which of these could fill in blank (g)?</li> <li>an arrow to list x (named gurney in the Global frame)</li> <li>an arrow to list y (named gurney in the f1 frame)</li> <li>an arrow to list z (named gurney in the f2 frame)</li> <li>an arrow to another list that does not appear in the diagram</li> <li>(1.0 pt) Which of these could fill in blank (h)?</li> <li>an arrow to list x (named gurney in the Global frame)</li> </ul>
vii.	<ul> <li>3</li> <li>4</li> <li>5</li> <li>6</li> <li>7</li> <li>(1.0 pt) Which of these could fill in blank (g)?</li> <li>an arrow to list x (named gurney in the Global frame)</li> <li>an arrow to list y (named gurney in the f1 frame)</li> <li>an arrow to list z (named gurney in the f2 frame)</li> <li>an arrow to another list that does not appear in the diagram</li> <li>(1.0 pt) Which of these could fill in blank (h)?</li> </ul>

#### (b) (6.0 points)

Implement the Blink class. A Blink instance represents a linked list of numbers and can find the longest sublist starting with any particular value in constant time. A Blink instance b is constructed from a linked list s (a Link instance or Link.empty) and has the following attributes:

- b.link is s, the linked list from which b was constructed.
- b.rest is a Blink representing the rest of s. If b represents Link.empty, then it has no rest attribute.
- b.sublists is a dictionary with a key for each unique element in s. The value for a key k is the Link instance representing the longest sublist of s starting with k.

```
class Blink:
```

```
"""A Blink has link, rest, and sublists attributes for a linked list s.
>>> s = Link(3, Link(1, Link(4, Link(1, Link(5)))))
>>> b = Blink(s)
>>> b.link is s
>>> b.rest.rest.link is s.rest.rest
>>> b.rest.rest.rest.rest.link is Link.empty
>>> b.sublists[4]
Link(4, Link(1, Link(5)))
>>> b.sublists[1]
Link(1, Link(4, Link(1, Link(5))))
>>> b.rest.rest.sublists[1]
Link(1, Link(5))
>>> b.sublists[3] is s
True
11 11 11
def __init__(self, s):
    assert s is Link.empty or isinstance(s, Link)
    if s is not Link.empty:
        self.rest = _____
                       (a)
        # Copy the sublists dict of self.rest into a new dict.
        self.sublists = self.rest.sublists.copy()
          (b)
    else:
        self.sublists = _____
    self.link = _____
```

(d)

```
i. (1.0 pt) Fill in blank (a).
      Blink(s.rest)
ii. (3.0 pt) Fill in blank (b).
      self.sublists[s.first] = s
iii. (1.0 pt) Which of these could fill in blank (c)?
      self.rest.sublists
    \bigcirc self.rest.sublists.copy()
    • {}
    ○ {s: s}
    \bigcirc {s: self}
    ○ {s.first: s}
    ○ {s.first: self}
iv. (1.0 pt) Which of these could fill in blank (d)?
    s
    ○ s.rest
    \bigcirc s.link
    \bigcirc self
    O self.rest
    O self.rest.link
```

#### 2. (28.0 points) Arrakis

**Definition.** A worm is a non-negative integer in which the absolute difference between each pair of adjacent digits is 1. 4345 is a worm. 4334 and 4354 are not. All non-negative integers below 10 are worms.

You may use near in the problems below.

```
def near(i, j): """Return whether digits i and j have absolute difference equal to 1.""" assert i >= 0 and i < 10 and j >= 0 and j < 10 return abs(i - j) == 1
```

#### (a) (4.0 points)

Implement  $is\_worm$ , which takes a non-negative integer n and returns True if n is a worm and False otherwise.

i. (1.0 pt) Which of these could fill in blank (a)?

```
\bigcirc n == 0
```

 $\bigcirc$  n <= 0

 $\bigcirc$  n > 0 and n < 10

ii. (1.0 pt) Fill in blank (b).

```
return True
```

- iii. (1.0 pt) Which of these could fill in blank (c)?
  - O n % 10
  - O n // 10
  - O n % 100
  - O n // 100
  - O (n % 10) // 10
  - (n // 10) % 10
  - O (n % 10) // 100
  - O (n // 100) % 10
- iv. (1.0 pt) Which of these could fill in blank (d)?
  - O n % 10
  - on // 10
  - O n % 100
  - O n // 100
  - (n % 10) // 10
  - O (n // 10) % 10
  - O (n % 10) // 100
  - (n // 100) % 10

#### (b) (7.0 points)

Implement sandworm, which takes a non-negative integer n and returns the largest worm that appears among the digits of n in order (but not necessarily formed of adjacent digits).

A worm a is larger than a worm b if a > b. Worms are integers.

```
def sandworm(n):
    """Return the largest worm formed by selecting some digits of non-negative n.
                                  # 13[5]31
   >>> sandworm(13531)
   5
   >>> sandworm(152)
                          # [1]5[2]
   12
                              # [3]1[4]1[5]92[6]53[5]
   >>> sandworm(31415926535)
   34565
   >>> sandworm(314159265358973) # [3]1[4]1[5]92[6]53589[7]3
   34567
    11 11 11
    if n == 0:
        return 0
   def use_last(n):
        "Return the largest worm in n that includes n \% 10"
       return tooth(n // 10, n % 10)
   def tooth(n, d):
        "Return the largest worm formed by some digits of n followed by digit d."
        if n == 0:
           return _____
       skip = _____
(b)
        if near(n % 10, d):
           return max(skip, _____)
(c)
        else:
            return skip
   return max(_____, use_last(n))
 i. (2.0 pt) Fill in blank (a).
     d
```

ii. (2.0 pt) Fill in blank (b).

```
tooth(n // 10, d)
```

- iii. (2.0 pt) Which of these could fill in blank (c)?
  - 10 \* use\_last(n) + n % 10
  - 10 \* use\_last(n // 10) + n % 10
  - 10 \* sandworm(n) + n % 10
  - 10 \* sandworm(n // 10) + n % 10
  - 10 \* use\_last(n) + d
  - 10 \* use\_last(n // 10) + d
  - $\bigcirc$  10 \* sandworm(n) + d
  - 10 \* sandworm(n // 10) + d
- iv. (1.0 pt) Which of these could fill in blank (d)?
  - sandworm(n // 10)
  - O use\_last(n // 10)
  - tooth(n // 10, 0)
  - 10 \* sandworm(n // 10) + n % 10
  - 10 \* use\_last(n // 10) + n % 10
  - 10 \* tooth(n // 10, 0) + n % 10

#### (c) (7.0 points)

Implement thumper, a generator function that takes a positive integer k and a digit m. It yields all k-digit worms with digits that are all less than or equal to m, and it yields these results in increasing order. Assume the number 0 has 0 digits.

```
def thumper(k, m):
    """Yield all k-digit worms with digits that are at most m, in increasing order.
   >>> list(thumper(1, 7)) # Note: O has no digits, so it is not a 1-digit worm.
    [1, 2, 3, 4, 5, 6, 7]
   >>> list(thumper(2, 3))
    [10, 12, 21, 23, 32]
   >>> list(thumper(3, 3))
    [101, 121, 123, 210, 212, 232, 321, 323]
   if k == 1:
           (a)
    else:
        for w in _____:
            if ____:
                yield 10 * w + (w \% 10 - 1)
            if ____:
                yield 10 * w + (w \% 10 + 1)
 i. (3.0 pt) Fill in blank (a).
     yield from range(1, m + 1)
ii. (2.0 pt) Fill in blank (b).
     thumper(k-1, m)
```

iii.	(1.0 pt) Which of these could fill in blank (c)?
	○ True
	○ False
	○ w > 0
	○ w >= 0
	○ w < m
	○ w <= m
	● w % 10 > 0
	○ w % 10 >= 0
	○ w % 10 < m
	○ w % 10 <= m
iv.	(1.0 pt) Which of these could fill in blank (d)?
iv.	(1.0 pt) Which of these could fill in blank (d)?
iv.	
iv.	○ True
iv.	<pre>True False</pre>
iv.	<pre></pre>
iv.	<pre>True   False   w &gt; 0   w &gt;= 0</pre>
iv.	<pre>True False w &gt; 0 w &gt;= 0 w &lt; m</pre>
iv.	<pre>True False w &gt; 0 w &gt;= 0 w &lt; m w &lt;= m</pre>
iv.	<pre>True   False   w &gt; 0   w &gt;= 0   w &lt; m   w &lt;= m   w % 10 &gt; 0</pre>

#### (d) (8.0 points)

Implement segment, which takes a positive integer n (such as 3456) and a two-argument function grouped. It returns a linked list s containing linked lists of digits (such as <<3 4> <5 6>>). Together, the elements of s contain all digits of n in order. Two adjacent digits a and b (with a to the left of b) appear in the same element of s if grouped(a, b) returns a true value.

The Link class appears on page 2 (left column) of the midterm 2 study guide.

```
def segment(n, grouped):
    """Return a linked list of linked lists of the digits of positive n.
   Adjacent digits a and b appear in the same linked list if grouped(a, b).
   >>> print(segment(3233344, lambda a, b: a == b))
    <<3> <2> <3 3 3> <4 4>>
   >>> print(segment(314159, lambda a, b: a == 1))
    <<3> <1 4> <1 5> <9>>
    .....
   part = Link.empty
   parts = Link.empty
   while n:
        if part is Link.empty or ____:
               (b)
        else:
               (c)
            part = _____
           (e)
   return _____
              (f)
 i. (2.0 pt) Fill in blank (a).
```

```
grouped(n % 10, part.first)
```

```
ii. (2.0 pt) Which of these could fill in blank (b)?
   part = Link(n, part)
   part = Link(n % 10, part)

    part = Link(n // 10 % 10, part)
   part.rest = Link(n)
   part.rest = Link(n % 10)
   part.rest = Link(n // 10 % 10)
iii. (1.0 pt) Which of these could fill in blank (c)?
   parts.append(part)
   parts.rest = Link(part)
   parts = Link(part, parts)
   parts = part + parts
   O parts += part
   parts += Link(part)
iv. (1.0 pt) Which of these could fill in blank (d)?
   O part.first
   O part.rest
   O Link.empty
   Link(n % 10)
   O Link(n, part)
   Link(n % 10, part)
v. (1.0 pt) Fill in blank (e).
     n = n // 10
vi. (1.0 pt) Fill in blank (f).
     Link(part, parts)
```

#### (e) (2.0 points)

Implement desert, which takes a positive integer n. It returns a linked list s containing linked lists of digits. Together, the elements of s contain all digits of n in order, and s is the shortest linked list for which each element contains the digits of a worm. Assume that segment is implemented correctly.

## 

i. (2.0 pt) Fill in blank (a).

```
segment(n, near)
```

#### 3. (8.0 points) Caladan

**Definition.** A fruit is a **leaf node** that has a parent but no siblings. That is, its parent has no other children.

The Tree class appears on page 2 (left column) of the midterm 2 study guide.

You may use fruited\_branch in the problems below.

```
def fruited_branch(t):
```

"""Return whether Tree t has exactly one child that is a fruit (a leaf with no siblings).

```
>>> fruited_branch(Tree(4))
False
>>> fruited_branch(Tree(4, [Tree(5)]))
True
>>> fruited_branch(Tree(4, [Tree(5, [Tree(6)])]))
False
"""
return len(t.branches) == 1 and t.branches[0].is_leaf()
```

#### (a) (4.0 points)

Implement sum\_fruit\_labels, which takes a Tree instance t. It returns the sum of the labels of the fruits in t. If t has no fruits, 0 is returned.

```
def sum_fruit_labels(t):
```

"""Return the sum of the labels of the fruits of Tree t.

i. (2.0 pt) Fill in blank (a).

```
t.branches[0].label
```

ii. (2.0 pt) Fill in blank (b).

```
sum([sum_fruit_labels(b) for b in t.branches])
```

#### (b) (4.0 points)

Implement pruned, which takes a Tree instance t. If t contains at least one fruit, it returns a Tree instance with only the nodes of t that appear on a path from the root to a fruit. If t contains no fruit, pruned(t) returns None. Calling pruned(t) should not modify t.

```
def pruned(t):
    """Return a Tree with only the nodes of t that are on a path to a fruit.
   >>> t = Tree(5, [Tree(6, [Tree(7)]), Tree(8), Tree(9, [Tree(10)])])
   >>> pruned(t)
   Tree(5, [Tree(6, [Tree(7)]), Tree(9, [Tree(10)])])
   >>> t # t is not modified by calling pruned(t)
   Tree(5, [Tree(6, [Tree(7)]), Tree(8), Tree(9, [Tree(10)])])
   >>> pruned(Tree(2, [Tree(3), Tree(4)])) is None # No fruit!
   True
    .....
    if fruited_branch(t):
        return _____
                  (a)
    cut = [pruned(b) for b in t.branches] # Some items in cut might be None
    if ____:
          (b)
        return _____
                  (c)
 i. (1.0 pt) Which of these could fill in blank (a)?
    t.branches[0]
   Tree(t.label)
   Tree(t.label, t.branches[0])
   Tree(t.label, [b for b in t.branches if fruited_branch(b)])
ii. (1.0 pt) Which of these could fill in blank (b)?
   O cut
   O cut is not None
   O None in cut
   any(cut)
    all(cut)
iii. (2.0 pt) Fill in blank (c).
     Tree(t.label, [b for b in cut if b])
```

#### 4. (15.0 points) Spice

**Definition.** A repeated call is a nested call expression in which each subexpression is either a number, a symbol, or a call with exactly one operand. For example, (((f 2) 3) 4) is a repeated call.

Reminder. In Scheme, the call expression (f 2) is a 2-element list containing the symbol f and the number 2. Therefore, one expression can evaluate to another expression. For example, the expression (list 'f 2) evaluates to (f 2).

#### (a) (4.0 points)

Implement repeated-call, a procedure that takes an operator expression and a list of operand expressions. It returns a repeated call for the operator and operands. If operands is nil, the result is the operator expression.

```
;;; Construct a repeated call expression from an operator and a list of operands.
;;; scm> (repeated-call 'f '(2 3 4))
;;; (((f 2) 3) 4)
;;; scm> (repeated-call '(f 2) '(3 4))
;;; (((f 2) 3) 4)
;;; scm> (repeated-call 'f nil)
(define (repeated-call operator operands)
    (if (null? operands)
        operator
            (a)
                     (b)
 i. (1.0 pt) Which of these could fill in blank (a)?
   O cons
   O cdr
   O list
   append
   \bigcirc map
   repeated-call
ii. (2.0 pt) Fill in blank (b).
     (list operator (car operands))
iii. (1.0 pt) Which of these could fill in blank (c)?
   O operands
   (cdr operands)
   (cons operator operands)
   (cons operator (cdr operands))
   (repeated-call operator operands)
```

(repeated-call operator (cdr operands))

#### (b) (4.0 points)

Complete the implementation of curry, a higher-order procedure that is called repeatedly on a non-negative integer num-args and then a procedure f. It returns a curried version of f that, when called repeatedly num-args times, returns the result of applying f to those arguments. Assume that f can take num-args arguments.

As a special case, ((curry 0) f) calls f on no arguments, which is equivalent to evaluating (f).

*Hint*: The built-in apply procedure takes a procedure f and a list of arguments s and applies f to the elements of s. For example,

```
• (apply + '(1 2 3)) is equivalent to (+ 1 2 3) and evaluates to 6.
 • (apply + '()) is equivalent to (+) and evaluates to 0.
;;; Return a curried version of f that can be called repeatedly num-args times.
;;; scm> (((((curry 3) +) 4) 5) 6) ; (+ 4 5 6) evaluates to 15
;;; 15
;;; scm> ((curry 0) +)
                                       ; (+) evaluates to 0
;;; scm> (((curry 1) +) 3)
                                       ; (+ 3) evaluates to 3
;;; scm> (((((curry 3) list) 4) 5) 6) ; (list 4 5 6) evaluates to (4 5 6)
;;; (4 5 6)
(define (curry num-args)
    (lambda (f) (curry-helper num-args (lambda (s) (apply f s)))))
;;; curry-helper's argument g is a one-argument procedure that takes a list.
;;; scm> ((((curry-helper 3 cdr) 5) 6) 7) ; (cdr '(5 6 7)) => (6 7)
;;; (6 7)
(define (curry-helper num-args g)
    (if (= num-args 0)
           (a)
        (lambda (x) (curry-helper (- num-args 1) _____))))
 i. (2.0 pt) Fill in blank (a).
     (g nil)
ii. (2.0 pt) Fill in blank (b).
```

(lambda (s) (g (cons x s)))

#### (c) (7.0 points)

Implement one-arg, which takes a Scheme expression s. It returns a call expression that would evaluate to the same value as s (calling the same procedures), but which uses curry to ensure that all call expressions have exactly one operand. Call expressions that already have one operand are unchanged.

- Assume s contains only numbers, symbols, and call expressions; no special forms.
- Assume that each operator (first sub-expression) of a call expression in s is a symbol (such as +).
- Assume that each operand of a call expression in s is either a number or another call expression.

```
;;; Take a (possibly nested) call expression s and return
   an equivalent expression in which all calls have one argument.
;;;
;;; scm> (one-arg '(abs 3)) ; (abs 3) already takes just 1 argument
   (abs 3)
;;;
;;; scm> (+ 4 5 6)
;;; 15
;;; scm> (one-arg '(+ 4 5 6))
   (((((curry 3) +) 4) 5) 6)
   scm> (eval (one-arg '(+ 4 5 6))); Same value as (+ 4 5 6)
   15
;;;
;;;
;;; scm> (one-arg '(+ (- 4) (*) (* 5 6)))
;;; (((((curry 3) +) (- 4)) ((curry 0) *)) ((((curry 2) *) 5) 6))
(define (one-arg s)
   (if (number? s) s
       (let ((num-args (- (length s) 1)))
           (if (= num-args 1)
              (_____)))
(a) (b) (c)
              (map _____ (cdr s)))))))
i. (1.0 pt) Which of these could fill in blank (a)?
   O cons
   O car
   O cdr
   list
   append
   O length
```

ii.	(1.0 pt) Which of these could fill in blank (b)?
	○ s
	(car s)
	○ (cdr s)
	○ (car (cdr s))
	(car (cdr (cdr s)))
iii.	(1.0 pt) Which of these could fill in blank (c)?
	○ <b>s</b>
	○ (car s)
	(cdr s)
	(car (cdr s))
	(car (cdr (cdr s)))
iv.	(2.0 pt) Fill in blank (d).
	(list 'curry num-args) or (cons 'curry (cons num-args nil)) or \((curry ,num-args)')
v.	(1.0 pt) Which of these could fill in blank (e)?
	○ s
	○ 's
	(car s)
	(car s)
	○ (cdr s)
	○ '(cdr s)
vi.	(1.0 pt) Which of these could fill in blank (f)?
	• one-arg
	• one-arg
	car
	○ car
	car cdr
	car cdr (lambda (x) (car (cdr x)))

#### 5. (10.0 points) Gom Jabbar

(a) (4.0 pt) Which of the following strings is entirely matched (from beginning to end) by the regular expression below? Check all that apply.

```
(([cs][61]|[abc])?(cs)+)|([cs]+61+)

cs61a

cs61acscs

cs61

cscs

cscs611

cccsss61

s1cs
```

(b) (2.0 pt) Write a short string (fewer than 10 characters) that matches the BNF grammar below, but is guaranteed to cause an error when evaluated by Scheme, regardless of how any symbols such as f are defined in the environment.

Notes: The %ignore /\s+/ directive ignores whitespace in the string. The INT terminal matches integers.

```
?start: expr
expr: INT | "(" operator expr+ ")"
operator: PROCEDURE | expr
PROCEDURE: "f"
%ignore /\s+/
%import common.INT
```

- (2 3) or any call expression with an integer literal operator.
- (c) (4.0 pt) Write a SQL query that generates a one-column table of the names of all dogs with exactly one child that has short fur. (The dog may have multiple children, but only one can have short fur.)

Assume the parents and dogs tables from page 1 (right column) of the final study guide have been created.

The result of your query should contain two rows: "Abraham" and "Fillmore". Your query should select the rows described even if the contents of parents and dogs were different; no credit for SELECT "Abraham" UNION SELECT "Fillmore".

```
SELECT parent FROM parents, dogs WHERE child=name AND fur="short" GROUP BY parent HAVING COUNT(*)=1;
```

## 6. (0.0 points) Just for Fun

Draw a picture of something you enjoyed about CS 61A.

No more questions.