

# Mining for Substructure: Inferring subhalo population properties from strong lenses with machine learning

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## ABSTRACT

We develop methods to analyze a statistical sample of strong lenses in a principled way to look for dark matter substructure with likelihood-free inference techniques.

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### 1. INTRODUCTION

### 2. STRONG LENSING FORMALISM

### 3. SIMULATING A POPULATION OF LENSES

In strong lensing systems, the background light emission source can in general be a point-like quasar or supernova, or a faint, extended “blue” galaxy. The former results in multiple localized images on the lens plane rather than extended arc-like images, providing the ability to probe substructure over a limited region on the lens plane. For this reason, we focus our method towards galaxy-galaxy lenses—systems producing images with extended arcs—since we aim to disentangle the collective, sub-threshold effect of a population of subhalo perturbers over multiple images. Young, blue galaxies are ubiquitous in the redshift regime  $z \gtrsim 1$  and dominate the faint end of the galaxy luminosity function, resulting in a much larger deliverable sample of galaxy-galaxy strong lenses compared to quadruply- and doubly-imaged quasars/supernovae.

The fact that the strong lens population is expected to be dominated by higher-redshift ( $z \gtrsim 1$ ) blue source galaxies lensed by intermediate-redshift ( $z \sim 0.5$ –1) elliptical galaxies presents significant challenges for quantifying the lens population obtainable with future observations. Specifically, planned ground-based surveys like LSST and space telescopes like *Euclid* present complementary challenges for delivering images of strong lensing systems suitable for substructure studies. LSST is expected to image in six bands, allowing efficient source

selection and distinguishing source and lens emission, but at the cost of lower resolution by virtue of being a ground-based instrument. *Euclid* imaging is expected to be much higher in resolution but with a single optical pass-band (VIS). Near-IR imaging from WFIRST may deliver a high-resolution, multi-wavelength dataset that is more suitable for substructure studies, although the lens and source populations may differ from those probed by optical telescopes.

In light of these uncertainties, we limit the scope of the present study to developing a class of methods that can be adapted to the specifications of a galaxy-galaxy strong lensing population obtained with the next generation of optical, near-IR and radio telescopes. In particular, we confine ourselves to a setting where the main methodological points can be made without detailed modeling of the detector capabilities and the deliverable lensing dataset, which is outside of the scope of the current paper.

We now describe our models for the background source, lensing galaxy and population parameters of the lens systems used in this study.

#### 3.1. Background source

We model the emission from background source galaxies using a Sérsic profile, with the surface brightness given by

$$\Sigma(r) = \Sigma_e \exp \left\{ -b_n \left[ \left( \frac{r}{r_e} \right)^{1/n} - 1 \right] \right\}, \quad (1)$$

where  $r_e$  is the effective circular half-light radius,  $n$  is the Sérsic index, and  $b_n$  is a factor depending on  $n$  that

ensures that  $r_e$  contains half the total intensity from the source galaxy, given by (Ciotti & Bertin 1999)

$$b_n \approx 2n - \frac{1}{3} + \frac{4}{405n} + \frac{46}{25515n^2} + \frac{131}{1148175n^3} - \frac{2194697}{30690717750n^4}.$$

We assume  $n = 1$  for the source galaxies, corresponding to a flattened exponential profile and consistent with expectation for blue-type galaxies at the relevant redshifts.  $\Sigma_e$  encodes the flux at half-light radius, which can be mapped onto the total flux (or magnitude) associated with a given galaxy. Treatment of the other Sérsic parameters, in particular the total emission and half-light radius, in the context of population studies is described in Secs. 3.4 and 3.5 below.

### 3.2. Lensing host galaxy

Cosmological  $N$ -body simulations suggest that the dark matter distribution in structures at galactic scales can be well-described by a universal, spherically symmetric NFW profile. However, strong lensing probes a region of the host galaxy much smaller than the typical virial radii of galaxy-scale dark matter halo, and the mass budget here is dominated by the baryonic bulge component of the galaxy. Taking this into account, the total mass budget of the lensing host galaxy, being early-type, can be well describe by a singular isothermal ellipsoid (SIE) profile, known as the bulge-halo conspiracy since neither the dark matter nor the baryonic components are individually isothermal. The host profile is thus described as

$$\rho(x, y) = \frac{\sigma_v^2}{2\pi G (x^2/q + qy^2)} \quad (2)$$

where  $\sigma_v$  is the velocity dispersion of the lens galaxy  $q$  is the ellipsoid axis ratio, with  $q = 1$  corresponding to a spherical profile. The Einstein radius for this profile, giving the characteristic lensing scale, is given by

$$\theta_E = 4\pi \left( \frac{\sigma_v}{c} \right)^2 \frac{D_{ls}(z_l, z_s)}{D_s(z_s)} \quad (3)$$

where  $D_{ls}$  and  $D_s$  are respectively the angular diameter distances from the source to the lens planes and from the source plane to the observer respectively.

The deflection field for the SIE profile is given by (Keeton 2001)

$$\phi_x = \frac{\theta_E q}{\sqrt{1 - q^2}} \tan^{-1} \left[ \frac{\sqrt{1 - q^2} x}{\psi} \right] \quad (4)$$

$$\phi_y = \frac{\theta_E q}{\sqrt{1 - q^2}} \tanh^{-1} \left[ \frac{\sqrt{1 - q^2} y}{\psi + q^2} \right] \quad (5)$$

with  $\psi \equiv \sqrt{x^2 q^2 + y^2}$ .

Although the total galaxy mass (baryons + dark matter) describe the macro lensing field, for the purposes of describing substructure we require being able to map the measure properties of an SIE lens onto the properties of the host dark matter halo. To do this, we relate the central stellar velocity dispersion  $\sigma_v$  to the mass  $M_{200}$  of the host dark matter halo. Zahid et al. (2018) derived a tight correlation between  $\sigma_v$  and  $M_{200}$ , modeled as

$$\log \left( \frac{M_{200}}{10^{12} \text{ M}_\odot} \right) = \alpha + \beta \left( \frac{\sigma_v}{100 \text{ km s}^{-1}} \right) \quad (6)$$

with  $\alpha = 0.09$  and  $\beta = 3.48$  with a mean log-normal scatter of 0.13 dex. We model the host dark matter halo with a Navarro-Frenk-White (NFW) profile (Navarro et al. 1996, 1997)

$$\rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2} \quad (7)$$

where  $\rho_s$  and  $r_s$  are the scale density and scale radius, respectively. The halo virial mass  $M_{200}$  describes the total mass contained within the virial radius  $r_{200}$ , defined as the radius within which the mean density is 200 times the critical density of the universe and related to the scale radius through the concentration parameter  $c_{200} \equiv r_{200}/r_s$ . Thus, an NFW halo is completely described by the parameters  $\{M_{200}, c_{200}\}$ . We use the concentration-mass relation from Sánchez-Conde & Prada (2014) assuming a log-normal distribution for  $c_{200}$  around the median inferred value given by the relation with scatter 0.15 dex.

The spherically symmetric deflection for an NFW perturber is given by

$$\phi_r = 4\kappa_s r_s \frac{\ln(x/2) + \mathcal{F}(x)}{x} \quad (8)$$

where  $x = r/r_s$ ,  $\kappa_s = \rho_s r_s / \Sigma_{\text{cr}}$  with  $\Sigma_{\text{cr}}$  the critical surface density, and

$$\mathcal{F}(x) = \begin{cases} \frac{1}{\sqrt{x^2 - 1}} \tan^{-1} \sqrt{x^2 - 1} & (x > 1) \\ \frac{1}{\sqrt{1 - x^2}} \tanh^{-1} \sqrt{1 - x^2} & (x < 1) \\ 1 & (x = 1). \end{cases} \quad (9)$$

We described the population parameters we use to model the host ellipticity and velocity dispersion (and thus its Einstein radius and halo mass) in Secs. 3.4 and 3.5 below.

### 3.3. Lensing substructure

The ultimate goal of our method is to characterize the substructure population in strong lenses. Here we

describe our procedure to model the substructure contribution to the lensing signal. Understanding the expected abundance of substructure in galaxies over a large range of epochs is a complex task and an active area of research. Properties of individual subhalos (such as their density profiles) as well as those that describe their population (such as the mass and spatial distribution) are strongly affected by their host environment, and accurately modeling all aspects of subhalo evolution and environment is beyond the scope of this paper. Instead, we use simple physically justifiable assumptions to model the substructure contributions in order to highlight the broad methodological points associated with the application of our method.

$\Lambda$ CDM, often called the standard model of cosmology, predicts a scale-invariant power spectrum of primordial fluctuations and the existence of substructure over a broad range of masses with equal contribution per logarithmic mass interval. We parameterize the distribution of subhalo masses  $m_{200}$  in a given host halo of mass  $M_{200}$ —the subhalo mass function—as power law distribution:

$$\frac{M_{200,0}}{M_{200}} \frac{dn}{d \ln m_{200}} = \alpha \left( \frac{m_{200}}{m_{200,0}} \right)^\beta \quad (10)$$

where  $\alpha$  encodes the overall substructure abundance, with larger  $\alpha$  corresponding to more substructure, and the slope  $\beta$  encodes the relative contribution of subhalos at different masses, with more negative  $\beta$  corresponding to a steeper slope with more low-mass subhalos. The normalization factors  $m_{200,0}$  and  $M_{200,0}$  are arbitrarily set to  $10^8 M_\odot$  and the Milky Way mass  $M_{\text{MW}} \simeq 1.1 \times 10^{12} M_\odot$ , respectively.

Theory and simulations within the framework of  $\Lambda$ CDM predict a slope  $\beta \sim -0.9$ , giving a nearly scale-invariant spectrum of subhalos, which we assume in our fiducial setup. We follow the specifications in [Hütten et al. \(2016\)](#) in order to set the overall fiducial abundance of subhalos, normalizing  $\alpha$  to give 150 subhalos in expectation between  $10^8 M_\odot$  and  $10^{10} M_\odot$  for a Milky Way-sized host halo. The linear scaling of the halo mass function with the host halo mass  $M_{200}$  is additionally described in [Han et al. \(2016\)](#); [Despali & Vegetti \(2017\)](#).

This procedure tells us the total number  $n_{\text{tot}}$  of subhalos within the virial radius  $R_{200}$  of the host halo. Strong lensing probes a region much smaller this scale—the typical Einstein radii for the host deflector are much smaller than the virial radius of the host dark matter halos. In order to obtain the expected number of subhalos within the lensing observations region of interest, we scale the total number of subhalos obtained from the above pro-

cedure by the ratio of projected mass within our region of interest  $\theta_{\text{ROI}}$  and within the angular virial radius  $\theta_{200}$ .

We assume the subhalos to be distributed in number density following the host NFW dark matter profile. In this case, the NFW enclosed mass function is  $M_{\text{enc}}(x) \propto \ln(x/2) + \mathcal{F}(x)$ , where  $x$  is the angular radius in units of the virial radius,  $x \equiv \theta/\theta_{200}$  and  $\mathcal{F}$  is given by Eq. 9 above. The number of subhalos within our ROI is thus obtained as  $n_{\text{ROI}} = n_{\text{tot}} \times M_{\text{enc}}(\theta_{\text{ROI}})/M_{\text{enc}}(\theta_{200})$ .

Since strong lensing probes the line-of-sight distribution of subhalos within the host, their projected spatial distribution is approximately uniform within the lensing ROI ([Despali & Vegetti 2017](#)). We thus distribute subhalos isotropically within our ROI. The density profile of subhalos is assumed to be NFW and given by Eq. 7, with associated lensing properties as described and the concentration inferred from the relation modeled in [Sánchez-Conde & Prada \(2014\)](#).

We finally emphasize that we do not intent to capture all of the intricacies of the subhalo distribution, such as the effects of baryonic physics, tidal disruption of subhalos in proximity to the center of the host and redshift evolution of host as well as substructure properties. Although our description can be extended to take these into account, their precise characterization and effect is still subject to large uncertainties, and our simple model above captures the essential physics for demonstration purposes.

### 3.4. Observational considerations

As noted above, our method is best-suited to analyzing a statistical sample of strong lenses to search for substructure, such as those that are expected to be obtained in the near future with optical telescopes like *Euclid* and LSST. Given the challenges associated with the precise characterization of such a sample at the present time, we describe here the observational characteristics we assume in order to build up training and testing samples to validate our inference techniques.

We largely follow the description in [Collett \(2015\)](#), and use the associated *LensPop* package, to characterize our mock observations. In particular, we use the detector configuration for *Euclid*, assuming a zero-point magnitude  $m_{\text{AB}} = 25.5$  in the single optical VIS passband, pixel size 0.1 arcsec, a Gaussian point spread function (PSF) with FWHM 0.18 arcsec, individual exposures with exposure time 565 s, an isotropic sky background with magnitude  $22.8 \text{ arcsec}^{-2}$  in the detector passband. [\[SM: Read-out noise and gain?\]](#).

These properties, in particular the exposure, sky background and PSF shape are expected to vary somewhat across the lens sample. Additionally, a given region may

be imaged by multiple exposures over a range of color bands. Although these variations can easily be incorporated into our analysis, modeling this is beyond the scope of this study. We briefly comment on how this information can be taken into account later.

### 3.5. Population statistics of the lens sample

*Software:* *Astropy* (Astropy Collaboration et al. 2013, 2018), *IPython* (Pérez & Granger 2007), *LensPop* (Collett 2015), *matplotlib* (Hunter 2007), *NumPy* (Van Der Walt et al. 2011), *PyTorch* (Paszke et al. 2017), *SciPy* (Jones et al. 2001–).

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