# A Really Catchy Title for My Technical Report

Technical Report

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**Abstract**

# 1 Introduction

Why this is important …

## 1.1 Motivation

## 1.2 Our Results

## 1.3 Organization

In Section 3 we give…, in Section 4…

# 2 Related Work

Akiba and Iwata [? ]…

# 3 Preliminaries

## 3.1 Something Central to Understanding Your Result

## 3.2 Should Probably Have Another Subsection Too

### 3.2.1 Even More Details on a Specific Method

# 4 Specific Contribution 1

In this section, we …

**Reduction 1.** *If graph $G = (V, E)$ contains a vertex $v \in V$ such that the closed neighborhood of $v$, denoted $N[v]$, is a clique, then $N[v]$ is in some minimum (vertex) clique cover and can be removed from $G$ to form reduced graph $G' = G[V \setminus N[v]]$, where $\theta(G) = 1 + \theta(G')$.*

*Proof.* Let $C = \{C_1, C_2, \ldots, C_{\theta(G)}\}$ be a given minimum clique cover for $G$. Since $C$ is a clique cover, there exists some clique in $C$ that covers $v$. Let $C_v \in C$ denote some clique such that $v \in C_v$. Let clique $N[v]$ be denoted by $X$.

If $X \in C$, then $X$ is in a minimum clique cover, as desired. If $X \notin C$, then we argue that we can swap $X$ with $C_v$ and maintain an optimal clique covering of $G$.

Observe that, by the definition of a clique, $C_v$ consits only of $v$ and vertices $u \in N(v)$. Therefore, $C_v \subset X$. Hence, all $v \in C_v$ are covered by $X$. Let $C' = (C \setminus \{C_v\}) \cup \{X\}$ and notice that the single removal of $C_v$ and single addition of clique $X$ ensures that $|C'| = |C| = k$. Thus, we have a minimum clique cover $C'$ such that $X \in C'$.

If $X = C_v$, then all $v \in X$ reside in a single clique $X$, as desired. If $X \neq C_v$, then there exists some $u \in N(v)$ such that $u \notin C_v$. Since $C$ is a clique cover, then there exists some clique in $C$ that covers $u$. Let $C_u \in C$ denote some clique such that $u \in C_u$. Then, by the definition of a clique, $C'_u = C_u \setminus X$ denotes some clique that covers all $u \in C_u$ and $u \notin X$. Since all $u \in X$ are covered by $X$ then we can swap $C_u$ with $C'_u$. Let $C'' = (C' \setminus \{C_u\}) \cup \{C'_u\}$ and notice that the single removal of $C_u$ and single addition of clique $C'_u$ ensures that $|C''| = |C'| = k$.

Observe that all $v \in X$ reside in a single clique $X$. Therefore, we can remove $X$ from $G$ to form reduced graph $G' = G[V \setminus N[v]]$, where $\theta(G) = 1 + \theta(G')$, as desired. □

**Reduction 2.** *If graph $G = (V, E)$ contains a maximal degree-two path $P = (v_1, v_2, \ldots, v_l)$ with $u \notin P$ and $w \notin P$ connected to $v_1$ and $v_l$ ($(u, v_1) \in E$ and $(w, v_l) \in E$) respectively then we can reduce $G$ in the following cases:*

*(1) If the length of P, denoted $|P|$, is odd and $u = w$, then we can remove $P \cup \{u\}$ from G to form $G' = G[V \setminus (P \cup \{u\})]$, where $\theta(G) = \frac{|P|+1}{2} + \theta(G')$.*

*(2) If $|P|$ is odd and $u \neq w$ such that $(u, w) \notin E$, contract P into a single vertex $v'$, giving us a new graph $G' = (V', E')$ with vertex set $V' = (V \setminus P) \cup \{v'\}$ and edge set $E' = (E \setminus \{\{x, y\} \in E | x \in P \vee y \in P\}) \cup \{\{x, v_1\} | x \in \{u, w\}\}$. Then $\theta(G) = \frac{|P|-1}{2} + \theta(G')$, and letting $C(G')$ denote a minimum clique cover for $G'$, we have the following two cases:*

   *(a) if $\{u, v'\} \in C(G')$, then $\{u, v_1\}$ is in a minimum clique cover of G.*

   *(b) Otherwise, $\{w, v_l\}$ is in a minimum clique cover of G.*

*(3)*

*Proof of Case 1.* Let $C = \{C_1, C_2, \ldots, C_{\theta(G)}\}$ be a given minimum clique cover for $G$.

Since $C$ is a clique cover, there exists some clique in $C$ that covers $v_1$. Let $C_v \in C$ denote some clique such that $v_1 \in C_v$. Notice that since $(u, v_1) \in E$, and $v_1$ is degree-two, there exists a clique containing only $u$ and $v_1$. Let clique $\{u, v_1\}$ be denoted by $X$.

If $X \in C$, then $X$ is in a minimum clique cover, as desired. If $X \notin C$, then we argue that we can swap $X$ with $C_v$ and maintain an optimal clique covering of $G$.

Notice that since $|P|$ is odd and all $v \in P$ have degree-two, then the minumum clique covering contains $\frac{|P|-1}{2} + 1$ cliques. Hence there exists a clique covering some $v \in P$ that conly contains $v$.

Since $P$ contains only degree-two verticies, then we can swap cliques covering $v \in P$ such that the $C_v = \{v_1\}$.

Observe that since $C_v$ consits only of $v_1$, $C_v \subset X$. Hence, all $v \in C_v$ are covered by $X$. Let $C' = (C \setminus \{C_v\}) \cup \{X\}$ and notice that the single removal of $C_v$ and single addition of clique $X$ ensures that $|C'| = |C| = k$. Thus, we have a minimum clique cover $C'$ such that $X \in C'$.

If $X = C_v$, then all $v \in X$ reside in a single clique $X$, as desired. If $X \neq C_v$, then $u \notin C_v$. Since $C$ is a clique cover, then there exists some clique in $C$ that covers $u$. Let $C_u \in C$ denote some clique such that $u \in C_u$. Then, by the definition of a clique, $C'_u = C_u \setminus \{u\}$ denotes some clique that covers all $y \in C_u$ such that $y \neq u$. Since $u$ is covered by $X$ then we can swap $C_u$ with $C'_u$. Let $C'' = (C' \setminus \{C_u\}) \cup \{C'_u\}$ and notice that the single removal of $C_u$ and single addition of clique $C'_u$ ensures that $|C''| = |C'| = k$.

Observe that all $v \in (P \cup \{v\})$ reside in $\frac{|P|+1}{2}$ cliques with no overlap. Therefore, we can remove $(P \cup \{v\})$ from $G$ to form reduced graph $G' = G[V \setminus (P \cup \{v\})]$, where $\theta(G) = \frac{|P|+1}{2} + \theta(G')$, as desired. □

*Proof of Case 2.* We perform a proof in three parts. First, we show that for any clique cover $C'$ of $G'$, there is a clique cover $C$ of $G$ of size $|C| = |C'| + \frac{|P|-1}{2}$. Then we show that $\theta(G) = \theta(G') + \frac{|P|-1}{2}$. Then this implies that if $|C'| = \theta(G')$ (i.e., $C'$ is a minimum clique cover of $G'$), then $C = \theta(G)$ (i.e., $C$ is a minimum clique cover of $G$). We then conclude by constructing a minimum clique cover of $G$ from a minimum clique cover of $G'$.

**Claim 1.** *Let $C'$ be a clique cover of $G'$, we show that there exists a clique cover $C$ of $G$ with size* $|C| = |C'| + \frac{|P|-1}{2}$.

There are two cases.

*Case a ($\{u, v'\} \in C'$).* Since in $G$, $u$ is an endpoint of $P$, such that $(u, v_1) \in E$, then if $\{u, v'\} \in C'$, $C = (C' \setminus \{u, v'\}) \cup \{u, v_1\} \cup C(P \setminus \{v_1\})$. Since $P \setminus \{v_1\}$ consists of $|P| - 1$ elements, which is even, and $P$ is degree-two, then $C(P \setminus \{v_1\}) = \{\{v_2, v_3\}, \{v_4, v_5\}, \ldots, \{v_{l-1}, v_l\}\}$. Observe that $|C(P \setminus \{v_1\})| = \frac{|P|-1}{2}$. Thus $|C| = |C'| + |C(P \setminus \{v_1\})|$, which is equivalent to $|C'| + \frac{|P|-1}{2}$, as desired.

*Case b ($\{w, v'\} \in C'$).* Since $w$ is also an endpoint of $P$ in $G$, then we can make a symmetric argument to case a.

**Claim 2.** $\theta(G) = \theta(G') + \frac{|P|-1}{2}$

We show that $\theta(G) \leq \theta(G') + \frac{|P|-1}{2}$ and $\theta(G) \geq \theta(G') + \frac{|P|-1}{2}$.

*Case 1 ($\theta(G) \leq \theta(G') + \frac{|P|-1}{2}$).* Let $\theta(G')$ be a minimum clique cover of $G'$. Then by Claim 1, there is a clique cover $C$ of $G$ of size $|C| = |C(G')| + \frac{|P|-1}{2} = \theta(G') + \frac{|P|-1}{2}$. Since any clique cover of $G$ has size at least $\theta(G)$, we have that $\theta(G) \leq |C| = \theta(G') + \frac{|P|-1}{2}$.

*Case 2 ($\theta(G) \geq \theta(G') + \frac{|P|-1}{2}$).* Let $CG$ be a minimum clique cover of $G$ that contains either $\{u, v_1\}$ or $\{w, v_l\}$.

- *Case 2a.* Let $\{u, v_1\} \in C(G)$. Observe that since $(P \setminus \{v_1\})$ is degree-two, a minimum clique coverage consists of $\frac{|P|-1}{2}$ elements. Therefore, $\theta(G) - \frac{|P|-1}{2} \leq \theta(G')$, which is equivalent to $\theta(G) \geq \theta(G') + \frac{|P|-1}{2}$.

- *Case 2b.* Since $(P \setminus \{v_l\})$ is also degree-two and a minimum clique coverage consists of $\frac{|P|-1}{2}$ elements, we can make a symmetrical argument to case 2a.

We now show how to construct a minimum clique cover of $G$ from a minimum clique cover of $G'$. Let $C(G')$ be a minimum independent set of $G'$. Applying Claim 1 with $C' = C(G')$ gives a minimum clique cover $C$ of $G$ of size $|C| = |C'| + \frac{|P|-1}{2} = \theta(G') + \frac{|P|-1}{2} = \theta(G)$, where the last equality is by Claim 2 and thus $C$ is a minimum clique cover of $G$. Then by the proof of Claim 1, we have that

(a) if $\{u, v'\} \in C'$, then $((C'(G') \setminus \{u, v'\}) \cup \{u, v_1\} \cup \{\{v_2, v_3\}, \{v_4, v_5\}, \ldots, \{v_{l-1}, v_l\}\})$ is a minimum vertex cover of $G$.

(b) otherwise, $\{w, v'\} \in C'$, then $((C'(G') \setminus \{w, v'\}) \cup \{w, v_l\} \cup \{\{v_1, v_2\}, \{v_3, v_4\}, \ldots, \{v_{l-2}, v_{l-1}\}\})$ is a minimum vertex cover of $G$.

□

# 5 Specific Contribution 2

In this section, we …

## 5.1 A Subsection Title

# 6 Experimental Results

In this section…

- item 1
- item 2
- item 3

# 7 Conclusion

## 7.1 Contributions

## 7.2 Future Work

# Acknowledgments