

COMP417  
Quiz on Kalman Filter  
Not for marks

Tuesday 7<sup>th</sup> March, 2017

**Notation and algebra**

1. What does  $x_{t+1|t}$  denote?  
(a)  $x_{t+1}|x_t$   
(b)  $x_{t+1}|z_{1:t}, u_{1:t}$
2. Suppose the 1D random variable  $x$  has variance  $\sigma_x^2 = 10$ . Then the random variable  $z = 3x + 1$  has variance:  
(a)  $\sigma_z^2 = 90$   
(b)  $\sigma_z^2 = 30$   
(c)  $\sigma_z^2 = 31$
3. Suppose the 1D random variable  $x$  has mean  $\mu_x = 10$ . Then the random variable  $z = 3x + 1$  has mean:  
(a)  $\mu_z = 90$   
(b)  $\mu_z = 30$   
(c)  $\mu_z = 31$
4. Suppose the 2D random variable  $\mathbf{x}$  has covariance matrix  $\Sigma_x$ . Then the covariance matrix of the random variable  $\mathbf{z} = \mathbf{A}\mathbf{x} + \mathbf{b}$ , where  $\mathbf{b}$  is a constant vector is:  
(a)  $\Sigma_z = \mathbf{A}\Sigma_x + \mathbf{b}$   
(b)  $\Sigma_z = \mathbf{A}\Sigma_x\mathbf{A}^T + \mathbf{b}$   
(c)  $\Sigma_z = \mathbf{A}\Sigma_x\mathbf{A}^T$   
(d) None of the above
5. Suppose the 2D random variable  $\mathbf{x}$  has mean  $\mu_x$ . Then the mean of the random variable  $\mathbf{z} = \mathbf{A}\mathbf{x} + \mathbf{b}$ , where  $\mathbf{b}$  is a constant vector is:  
(a)  $\mu_z = \mathbf{A}\mu_x + \mathbf{b}$   
(b)  $\mu_z = \mathbf{A}\mu_x\mathbf{A}^T + \mathbf{b}$   
(c)  $\mu_z = \mathbf{A}\mu_x\mathbf{A}^T$   
(d) None of the above

### Kalman Filter

6. Applying Bayes' rule with a Gaussian prior and a Gaussian measurement likelihood always results in a Gaussian posterior. **True** or **False**?
7. The propagation step of the Kalman Filter reduces the uncertainty of the state estimate. True or **False**?
8. The update step of the Kalman Filter increases the variance/covariance of the Gaussian state estimate. True or **False**?
9. Suppose your prior distribution for a 1D random variable is  $x \sim \mathcal{N}(0, 10^2)$ . Your measurement likelihood of  $x$  is  $z|x \sim \mathcal{N}(10, 1^2)$ , where  $z = x + w$ ,  $w \sim \mathcal{N}(0, 1^2)$ . In other words, initially you thought  $x$  was around 0 with variance  $10^2$ , but then you got a noisy measurement around 10 with variance 1. Which of the following statements are true about the posterior distribution  $x|z \sim \mathcal{N}(\mu, \sigma^2)$ , i.e your updated belief?
- (a)  $\mu$  is closer to 10 than to 1.
  - (b)  $\mu = 0.5$
  - (c)  $\sigma < \min(10, 1)$
  - (d)  $\sigma > \max(10, 1)$
  - (e)  $\sigma = 4.5$
10. Suppose the measurement model of a system is a linear function of the state plus noise:  $\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{n}$ . If the difference between the expected measurement  $\mathbf{H}\boldsymbol{\mu}_x$  and the actual measurement  $\bar{\mathbf{z}}$  is zero, then the update step of the Kalman Filter will:
- (a) Change the mean but not the variance
  - (b) Change the variance but not the mean
  - (c) Change neither the variance nor the mean
  - (d) Change both
11. The Kalman Filter is a good choice for problems where the distribution of your state estimate can be multimodal, i.e. in cases where there are multiple hypotheses that are equally likely, or more generally, multiple local maxima in the posterior distribution. True or **False**?