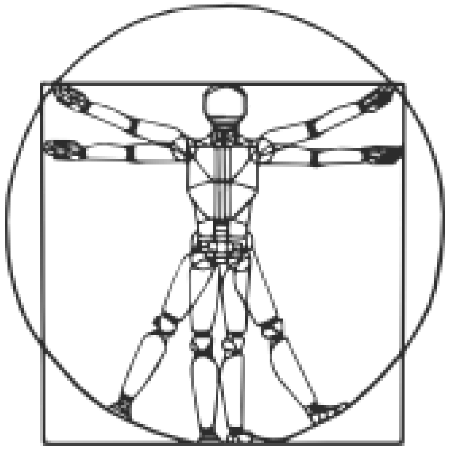

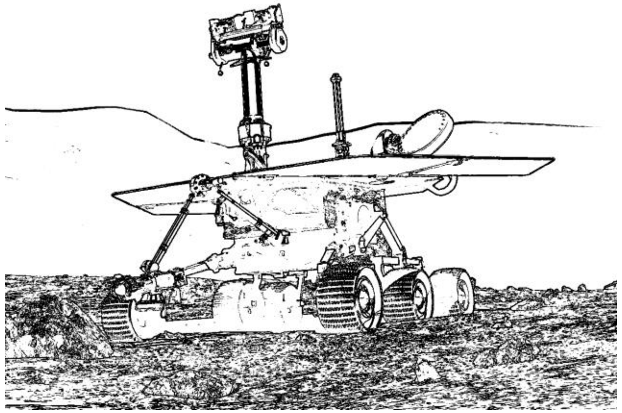

COMP417
Introduction to Robotics and Intelligent Systems

Lecture 9: Sampling-Based Path Planning

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Announcements

- NSERC USRA deadline
- A1 submission instructions

Drawbacks of grid-based planners

- Grid-based planning works well for grids of up to 3-4 dimensions

- State-space discretization suffers from combinatorial explosion:

- If the state is $\mathbf{x} = [x_1, ..., x_D]$ and we split each dimension into N bins then we will have N^D nodes in the graph.

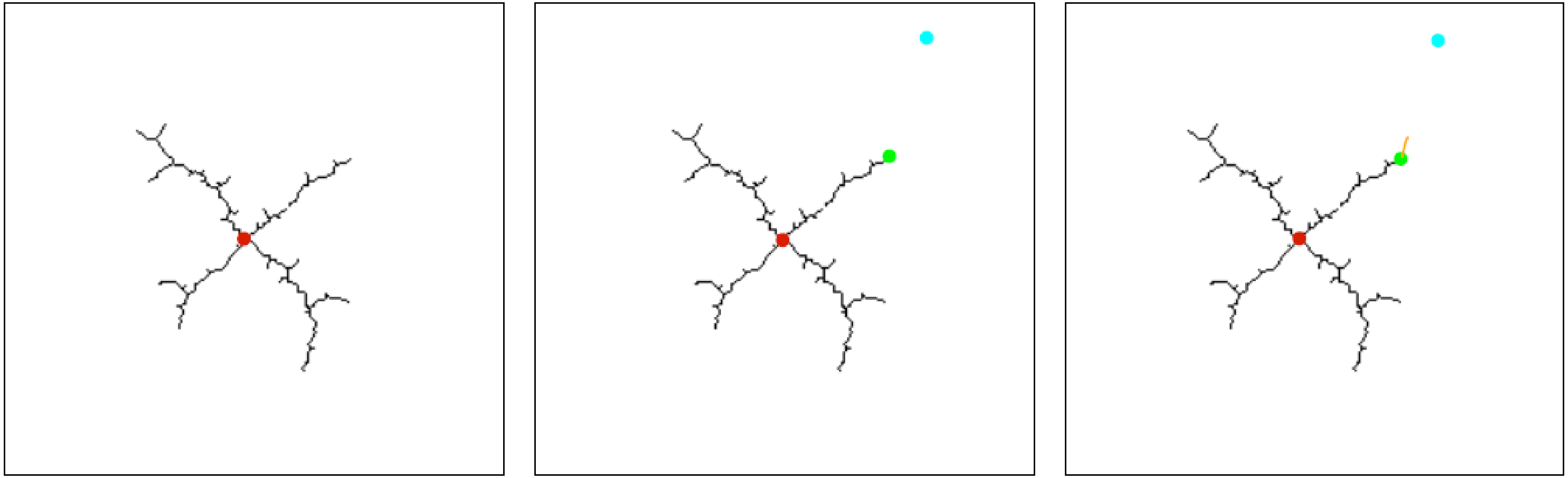
- This is not practical for planning paths for robot arms with multiple joints, or other high-dimensional systems.

(Sub)Sampling the state-space

- Need to find ways to reduce the continuous domain into a sparse representation: graphs, trees etc.

- Today:
- Rapidly-exploring Random Tree (RRT),
- Probabilistic RoadMap (PRM)
- Visibility Planning
- Smoothing Planned Paths

RRT



Main idea: maintain a tree of reachable configurations from the root
Main steps:

- Sample random state
- Find the closest state (node) already in the tree
- Steer the closest node towards the random state

RRT

$V \leftarrow \{x_{\text{init}}\};\ E \leftarrow \emptyset;$

for $i = 1, \ldots, n$ **do**

  $x_{\text{rand}} \leftarrow \texttt{SampleFree}_i;$

  $x_{\text{nearest}} \leftarrow \texttt{Nearest}(G = (V, E), x_{\text{rand}});$

  $x_{\text{new}} \leftarrow \texttt{Steer}(x_{\text{nearest}}, x_{\text{rand}})\ ;$

  if $\texttt{ObtacleFree}(x_{\text{nearest}}, x_{\text{new}})$ **then**

    $V \leftarrow V \cup \{x_{\text{new}}\};\ E \leftarrow E \cup \{(x_{\text{nearest}}, x_{\text{new}})\}\ ;$

return $G = (V, E);$

RRT

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return $G = (V, E);$

Things to pay attention to:

SampleFree() needs to sample a random state from the uniform distribution. How do you sample rotations uniformly?

RRT

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return $G = (V, E);$

Nearest() searches for the nearest neighbor of a given vector. Brute force search examines |V| nodes (increasing). Is there a more efficient method?

RRT

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return $G = (V, E);$

Steer() finds the controls that take the nearest state to the new state. Easy for omnidirectional robots. What about non-holonomic systems?

RRT

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ObstacleFree() checks the path from the nearest state to the new state for collisions. How do you do collision checks?

RRT

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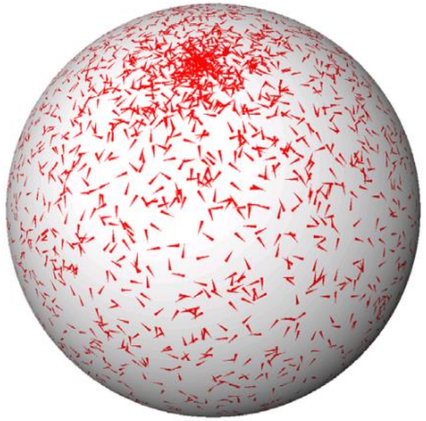
return $G = (V, E);$

Upside of using ObstacleFree(): you don’t need to model obstacles in Steer(). For example, if Steer() computes LQR controllers you don’t need to model obstacles in the control computation.

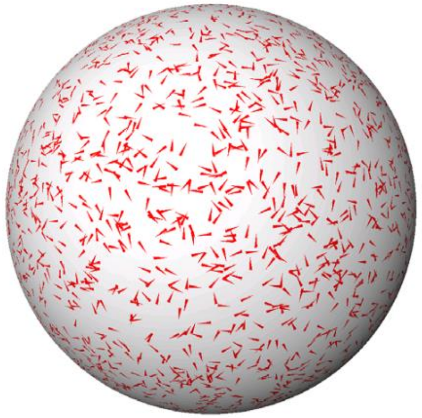
RRT: uniform sampling

- Only tricky case is when the state contains rotation components
- For example: $\mathbf{x} = [{}^W_B\mathbf{q} \; {}^W\mathbf{p}_{WB}]$
- State involving both rotation and translation components is often called **the pose** of the system.
- Idea #1: Uniformly sample 3 Euler angles (roll, pitch, yaw)

3D rotation visualization:
rotation axis is a point on a sphere,
rotation angle is the direction
of the red arrow



Idea #1
Not uniform after all



Correct, uniform

RRT: uniform sampling

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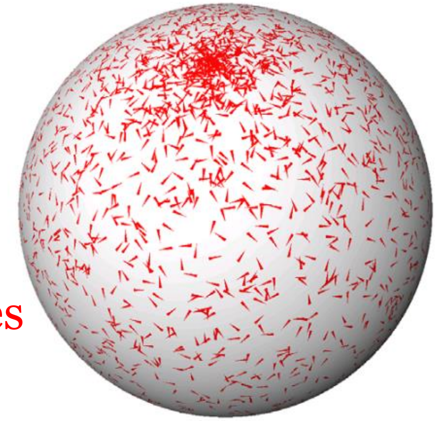
Nonuniformity at the north pole
caused by Gimbal Lock: same rotation
parameterized by different Euler angles



Idea #1
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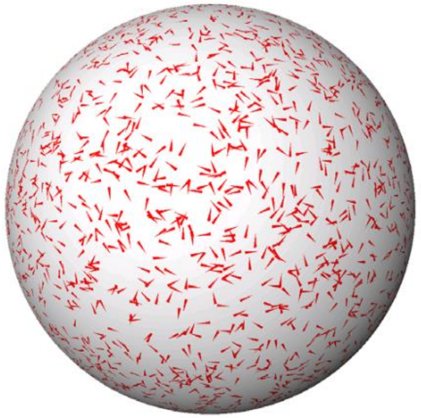


Correct, uniform

RRT: uniform sampling

- Idea #2: Uniformly sample a quaternion
- First, uniformly sample $u_1, u_2, u_3 \in [0, 1]$
- Then output the unit quaternion

$$\mathbf{q} = [\sqrt{1-u_1}\sin(2\pi u_2),\ \sqrt{1-u_1}\cos(2\pi u_2),\ \sqrt{u_1}\sin(2\pi u_3),\ \sqrt{u_1}\cos(2\pi u_3)]$$

- Idea #3: Uniformly sample rotation matrices.
- It's possible but we won't discuss it here.

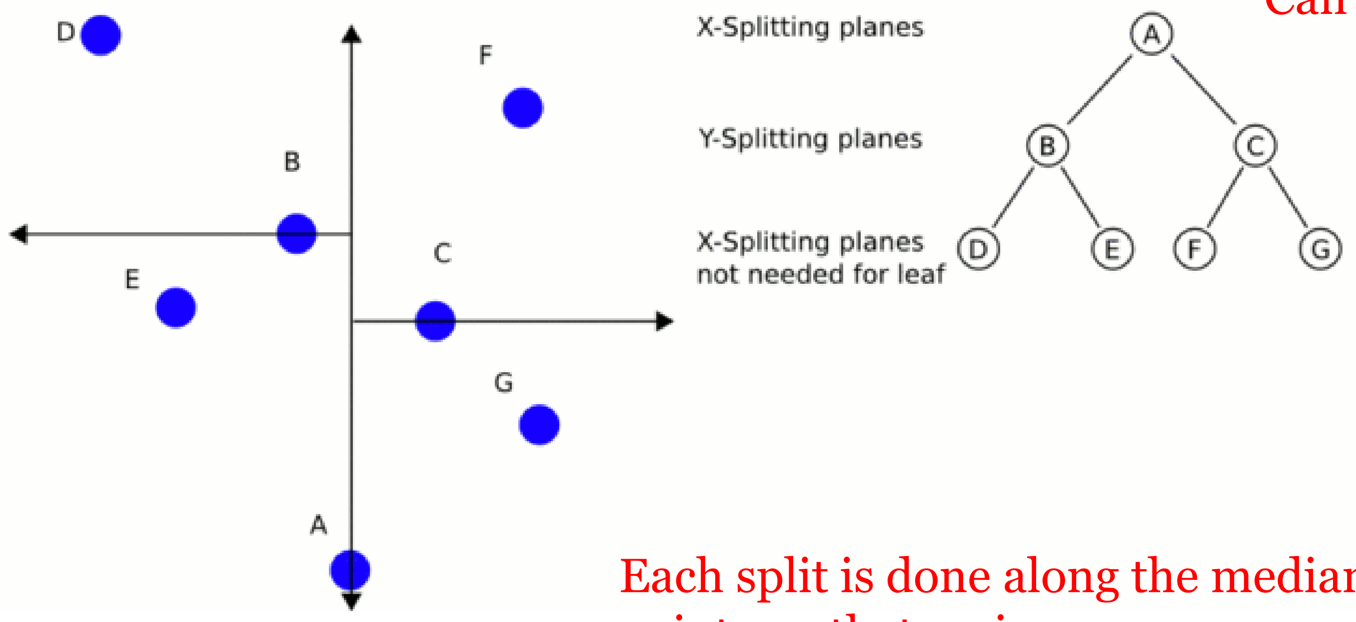
RRT: finding the nearest neighbor

- Any alternatives to linear (brute force) search?

RRT: finding the nearest neighbor

- Any alternatives to linear (brute force) search?
- Idea #1: space partitioning, e.g. kd-trees

Balanced kd-tree:
Can query in O(logn)



Each split is done along the median of the points on that region

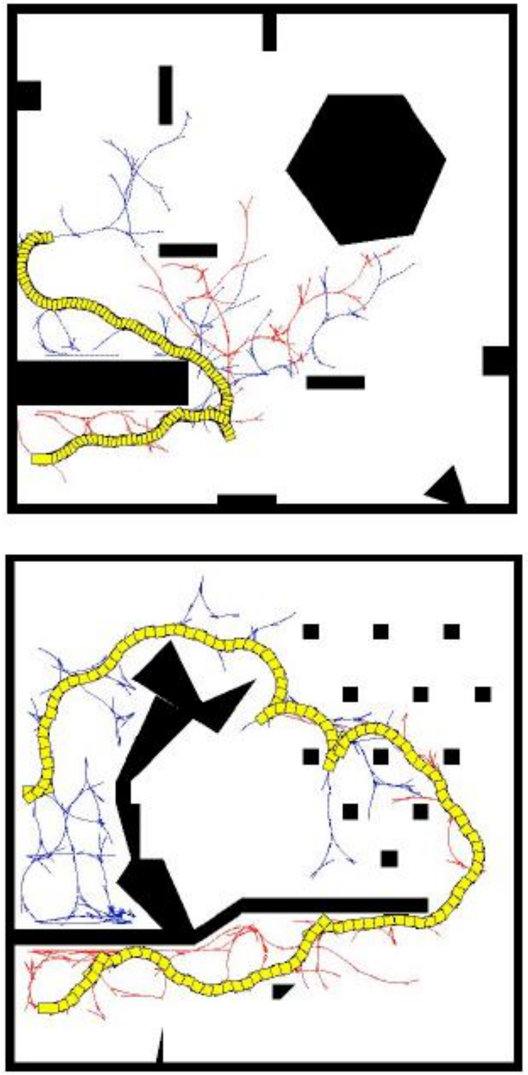
RRT: finding the nearest neighbor

- Any alternatives to linear (brute force) search?
- Idea #1: space partitioning, e.g. kd-trees
- Idea #2: locality-sensitive hashing
  - Maintains buckets
  - Similar points are placed on the same bucket
  - When searching consider only points that map to the same bucket

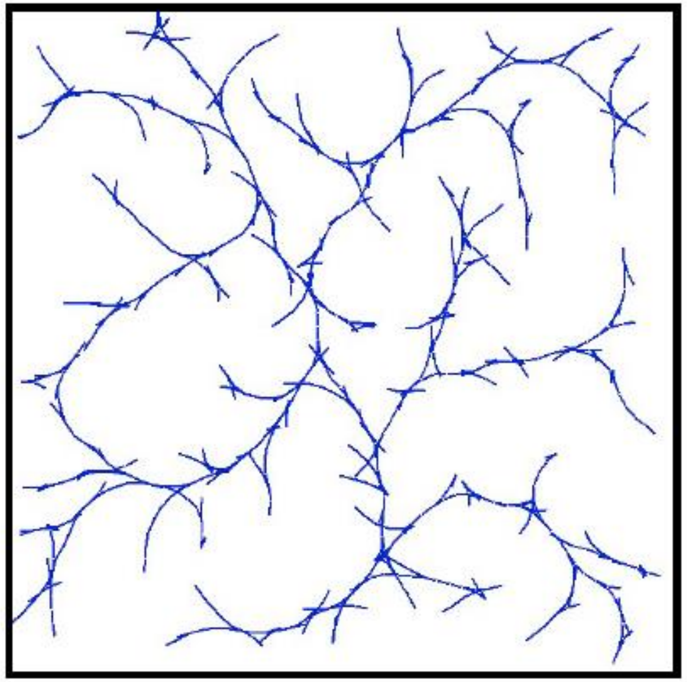
RRT: steering to a given state

- This is an optimal control problem, but without a specified time constraint
- For omnidirectional systems we can connect states by a straight line.
- For more complicated systems you could use LQR.
- You could also use a large set of predefined controls, one of which could be able to take the system close to the given state

RRT: steering to a given state



nonholonomic constraints

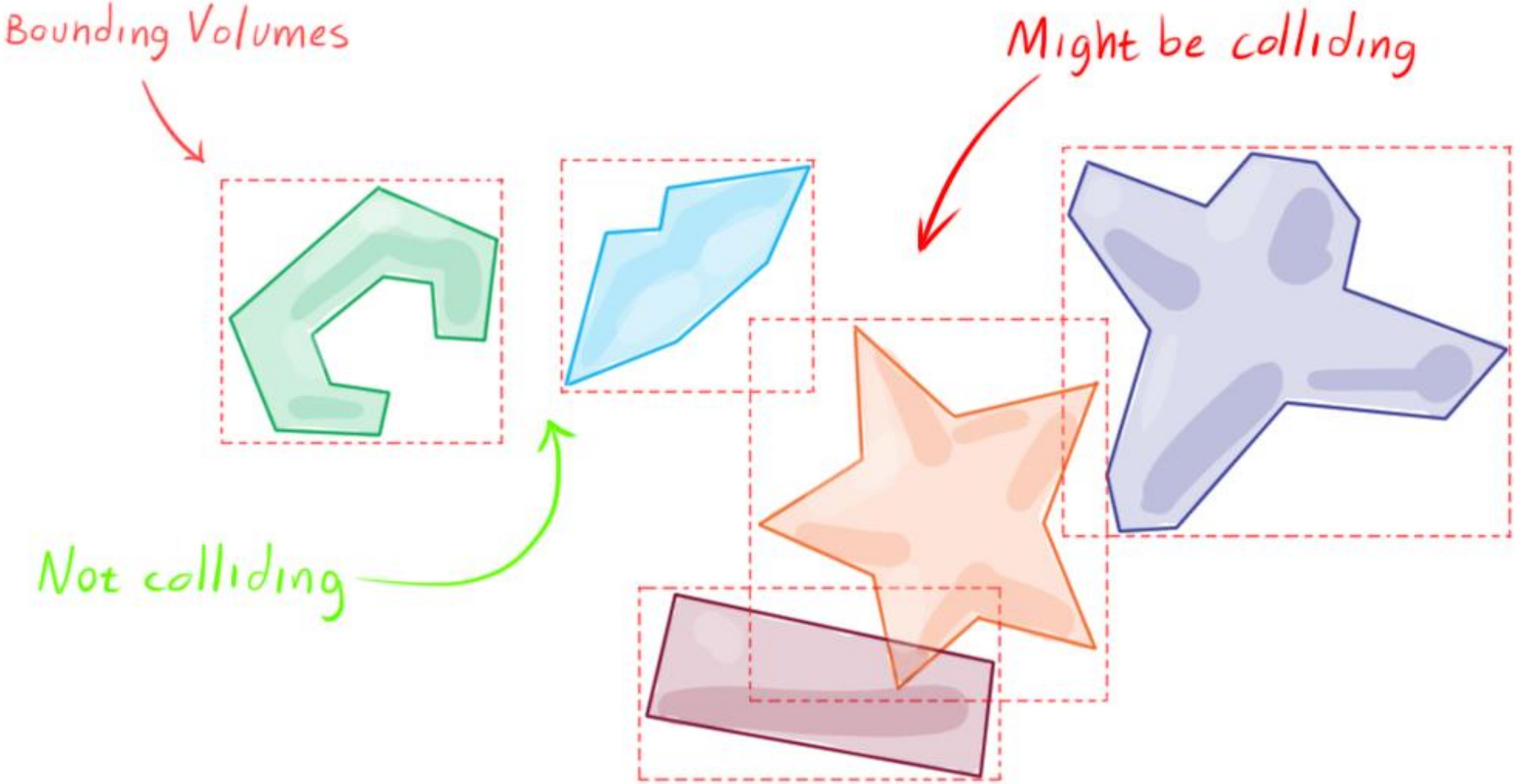


RRT for a robot with car-like kinematics

can the control problem get more difficult?

RRT: collision detection

- Main idea: bounding volume collision detection

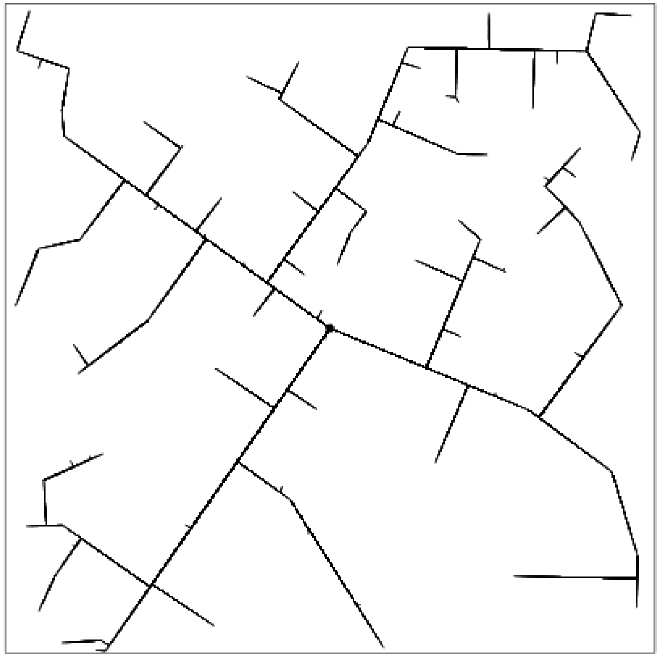


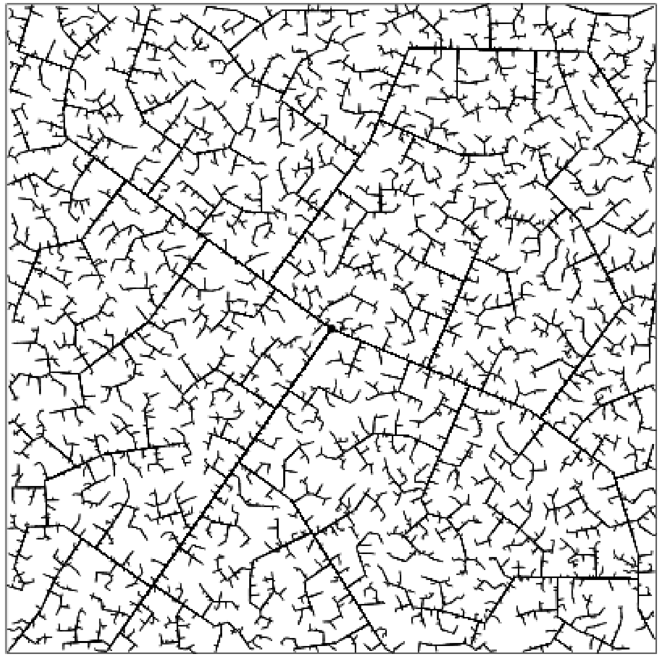
Source: https://www.toptal.com/game/video-game-physics-part-ii-collision-detection-for-solid-objects

RRT example: moving a piano

RRT: properties of the planning algorithm

#1: The RRT will eventually cover the space, i.e. it is a space-filling tree


45 iterations


2345 iterations

Source: Planning Algorithms, Lavalle

RRT: properties of the planning algorithm

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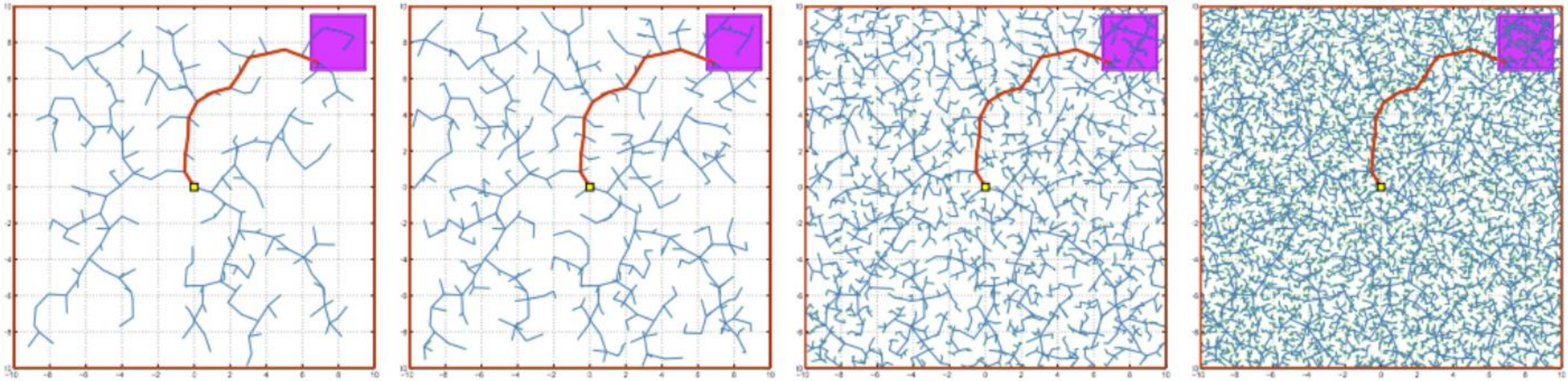
#2: The RRT will NOT compute the optimal path asymptotically



Source: Karaman, Frazzoli, 2010

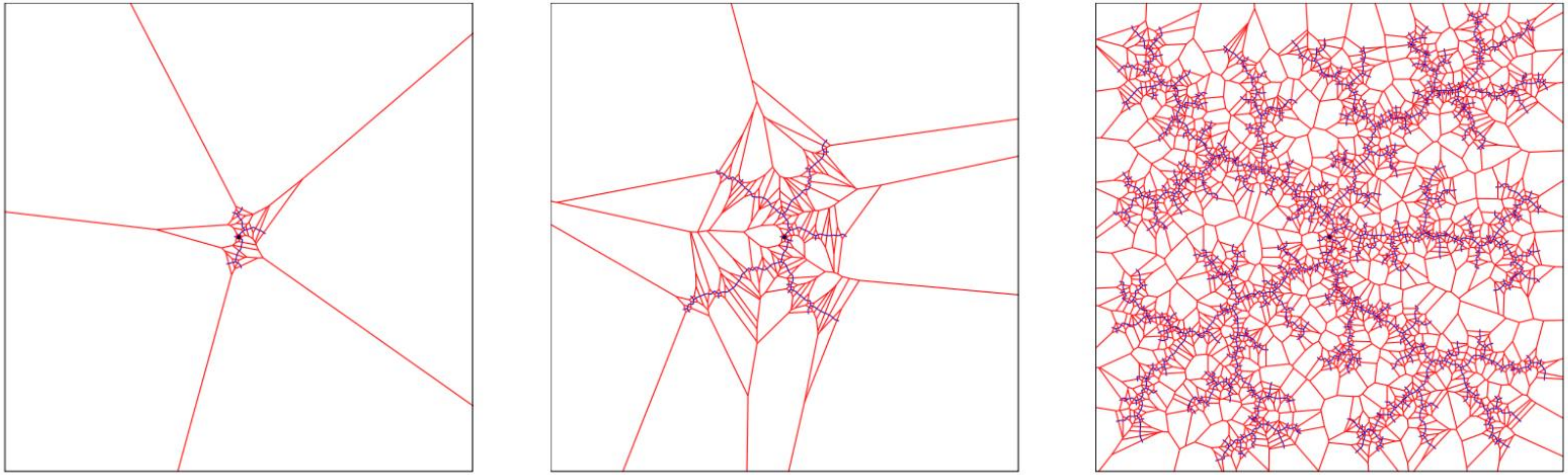
This problem has been addressed in recent years by RRT*, BIT*, Fast-Marching Trees

RRT: properties of the planning algorithm

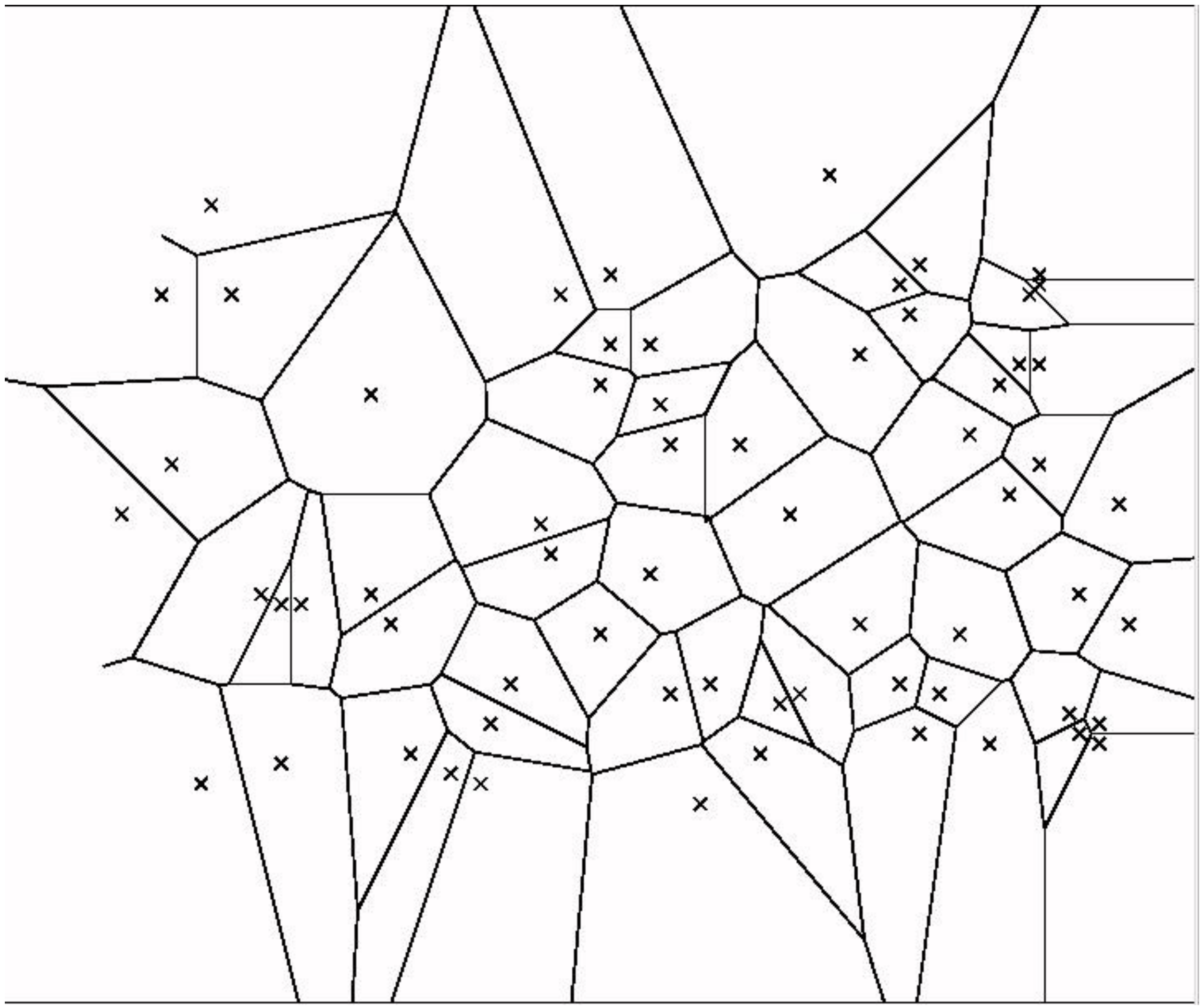
#1: The RRT will eventually cover the space, i.e. it is a space-filling tree

#2: The RRT will NOT compute the optimal path asymptotically

#3: The RRT will exhibit "Voronoi bias," i.e. new nodes will fall in free regions of Voronoi diagram (cells consist of points that are closest to a node)



Voronoi diagram



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RRT: properties of the planning algorithm

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#3: The RRT will exhibit “Voronoi bias,” i.e. new nodes will fall in free regions of Voronoi diagram

#4: The probability of RRT finding a path increases exponentially in the number of iterations

#5: The distribution of RRT’s nodes is the same as the distribution used in SampleFree()

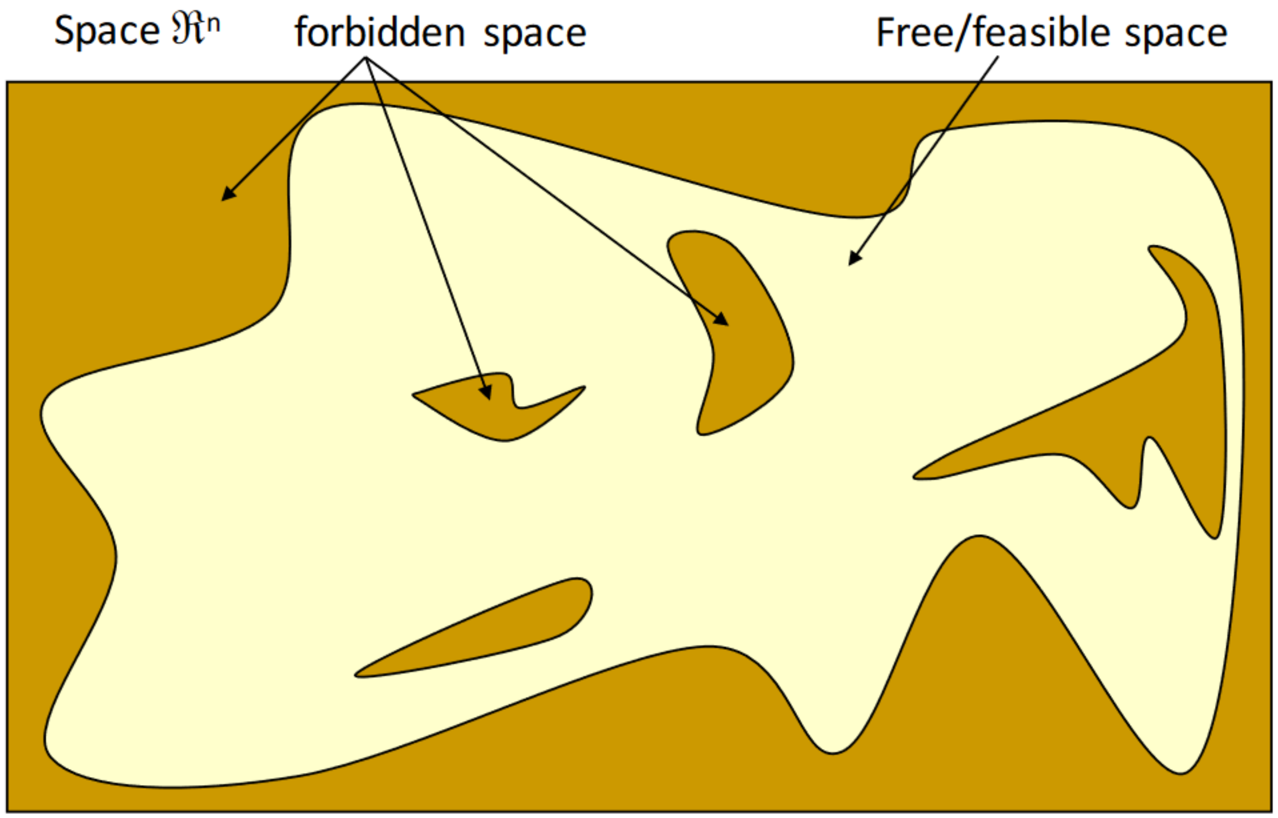
RRT variants: bidirectional search

Probabilistic RoadMaps (PRMs)

- RRTs were good for single-query path planning
- You need to re-plan from scratch for every query A → B

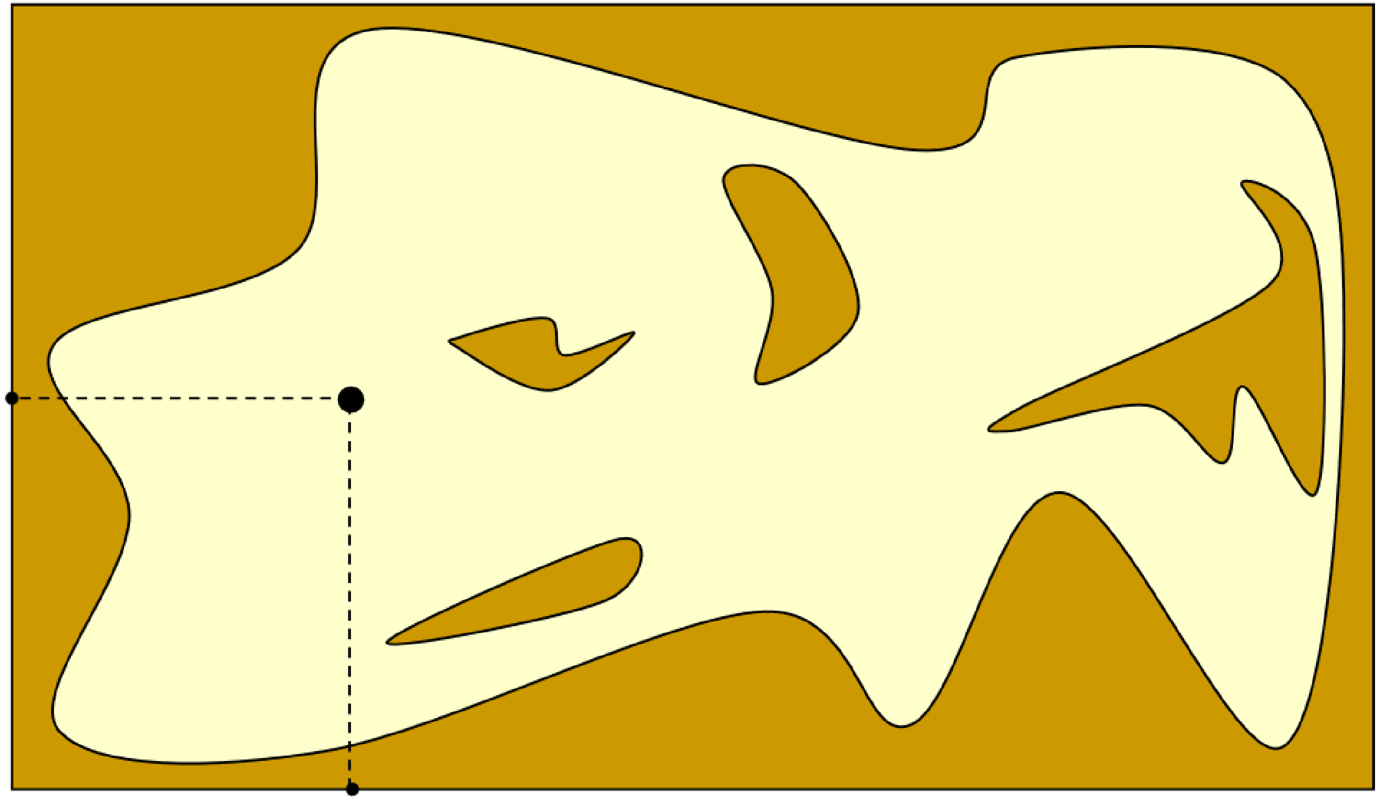
- PRM addresses this problem
- It is good for multi-query path planning

PRM

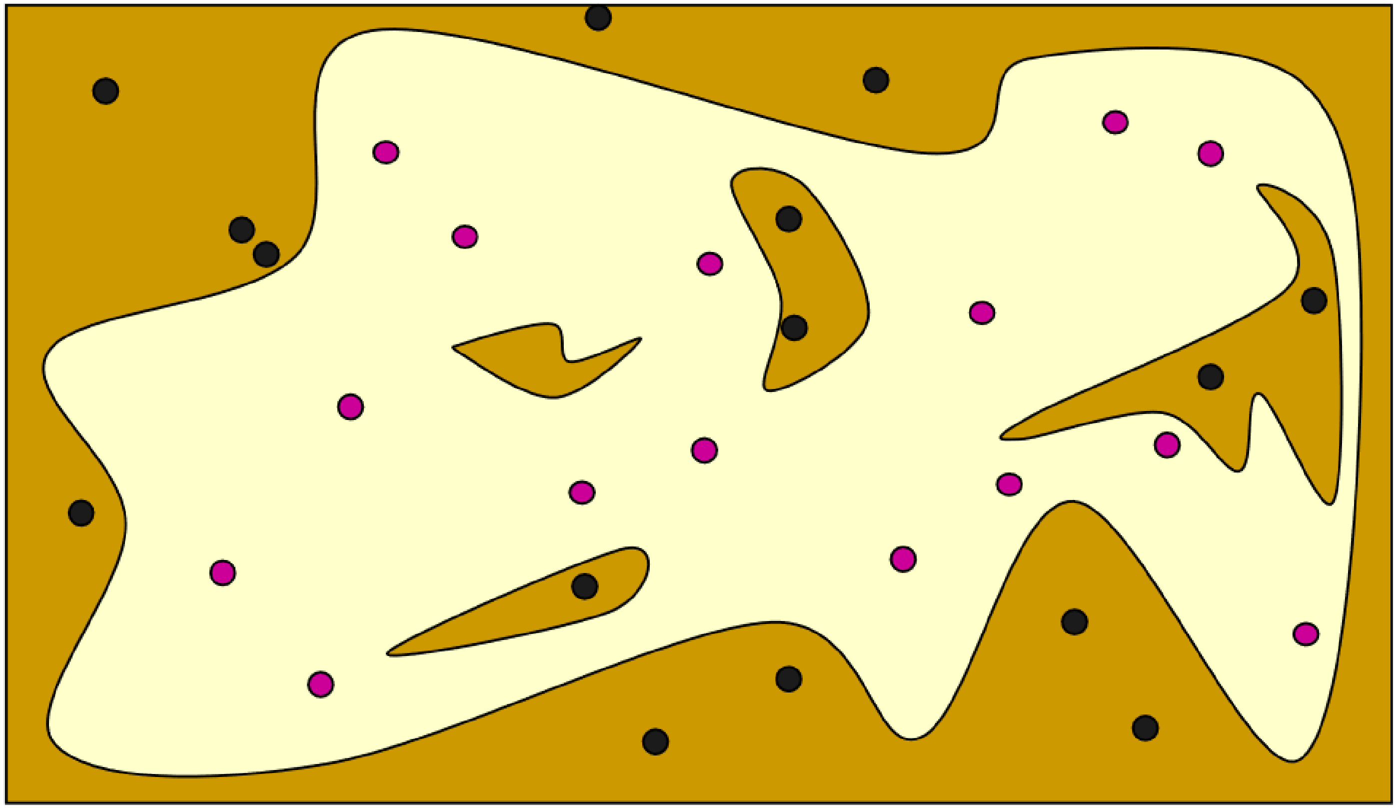


PRM

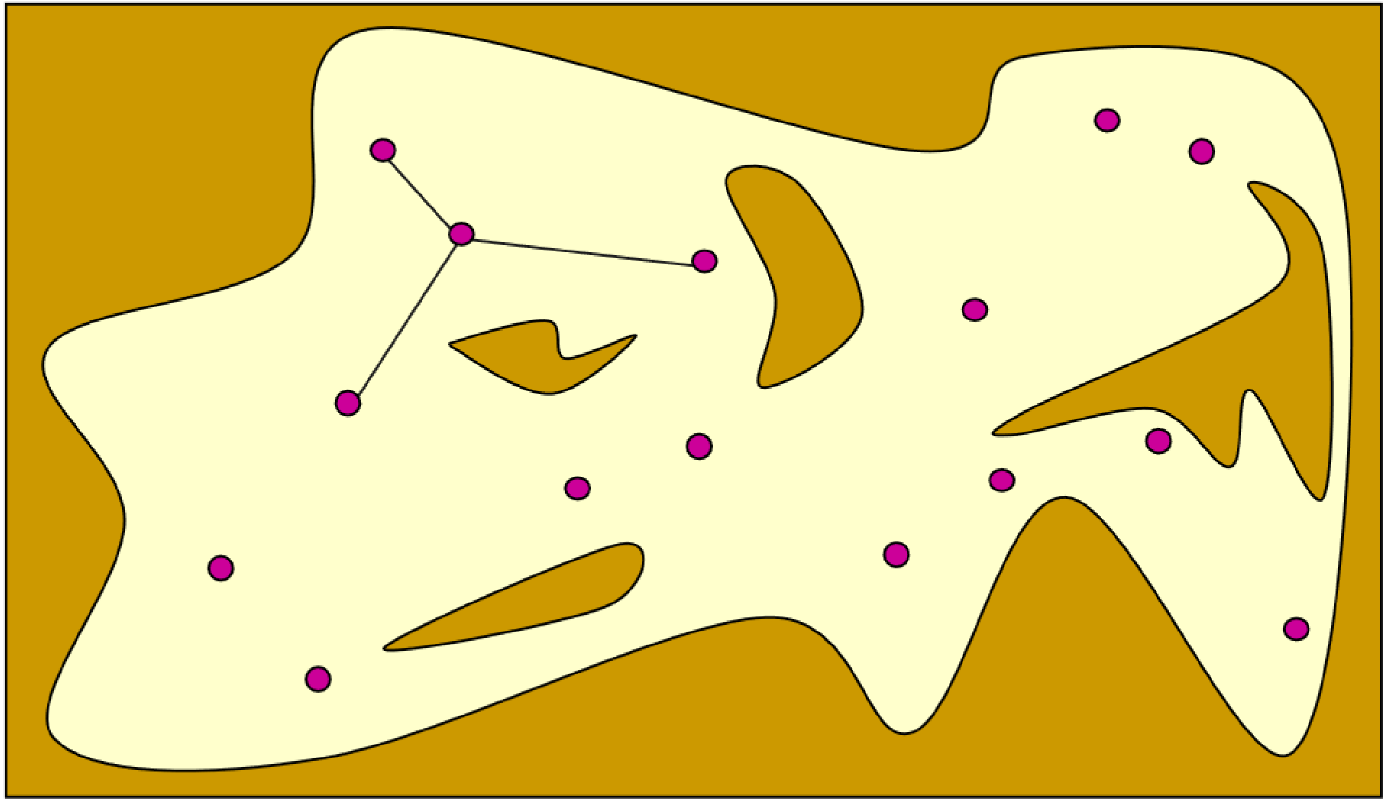
Configurations are sampled by picking coordinates at random



PRM

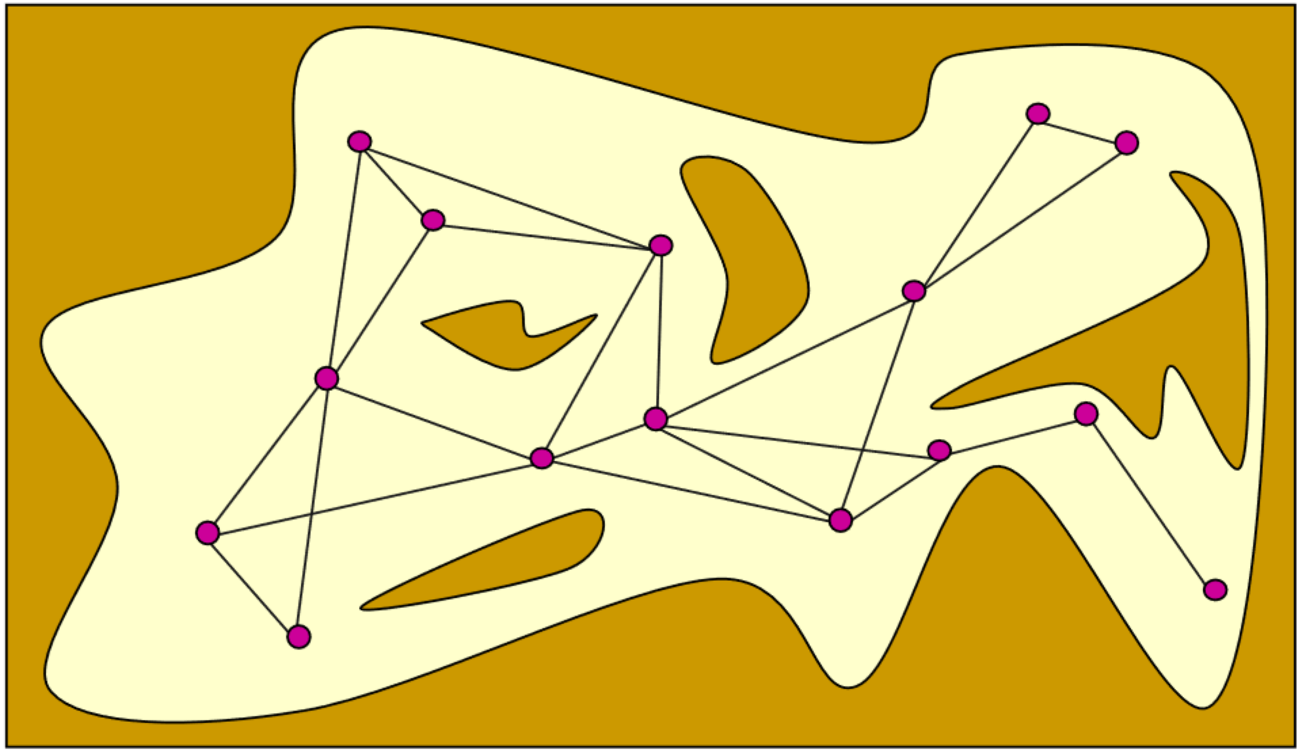


PRM



Each node is connected to its neighbors (e.g. within a radius)

PRM



PRM

$V \leftarrow \{x_{\text{init}}\} \cup \{\texttt{SampleFree}_i\}_{i=1,\dots,n}$; $E \leftarrow \emptyset$;

foreach $v \in V$ **do**

  $U \leftarrow \texttt{Near}(G = (V, E), v, r) \setminus \{v\}$;

  foreach $u \in U$ **do**

    if $\texttt{CollisionFree}(v, u)$ **then** $E \leftarrow E \cup \{(v, u), (u, v)\}$

return $G = (V, E)$;

To perform a query (A->B) we need to connect A and B to the PRM.
We can do this by nearest neighbor search (kd-trees, hashing etc.)

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Range search can be done efficiently using a kd-tree

To perform a query (A->B) we need to connect A and B to the PRM.
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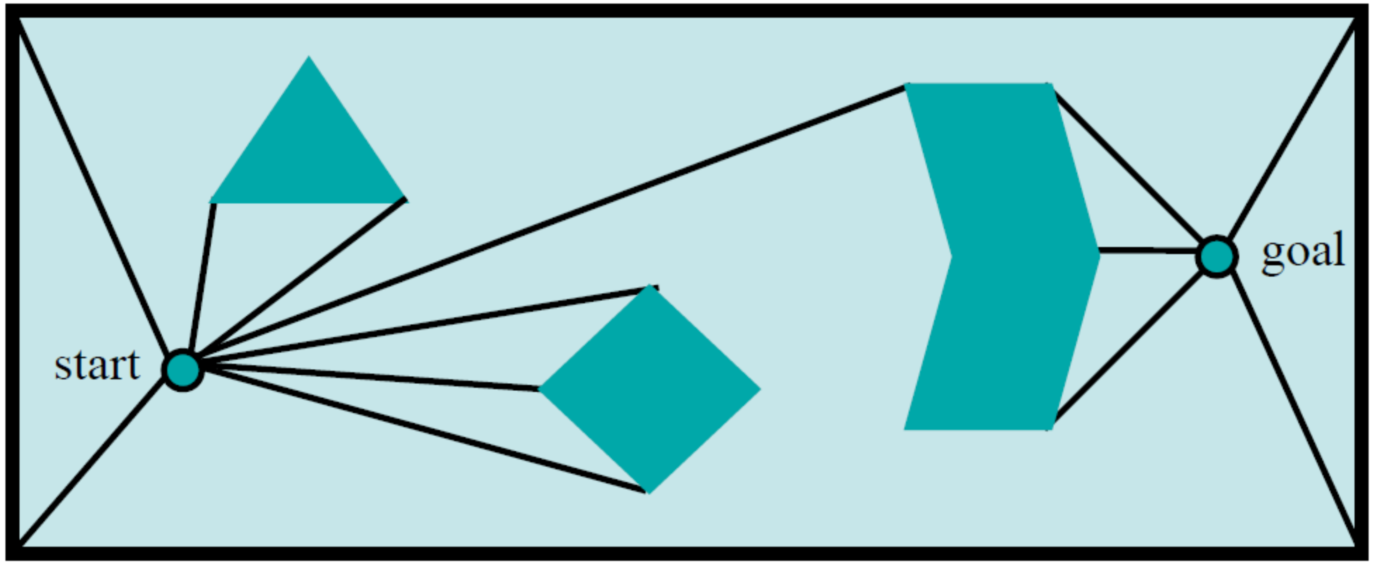
Visibility Graph Path Planning

- First, draw lines of sight from the start and goal to all "visible" vertices and corners of the world.



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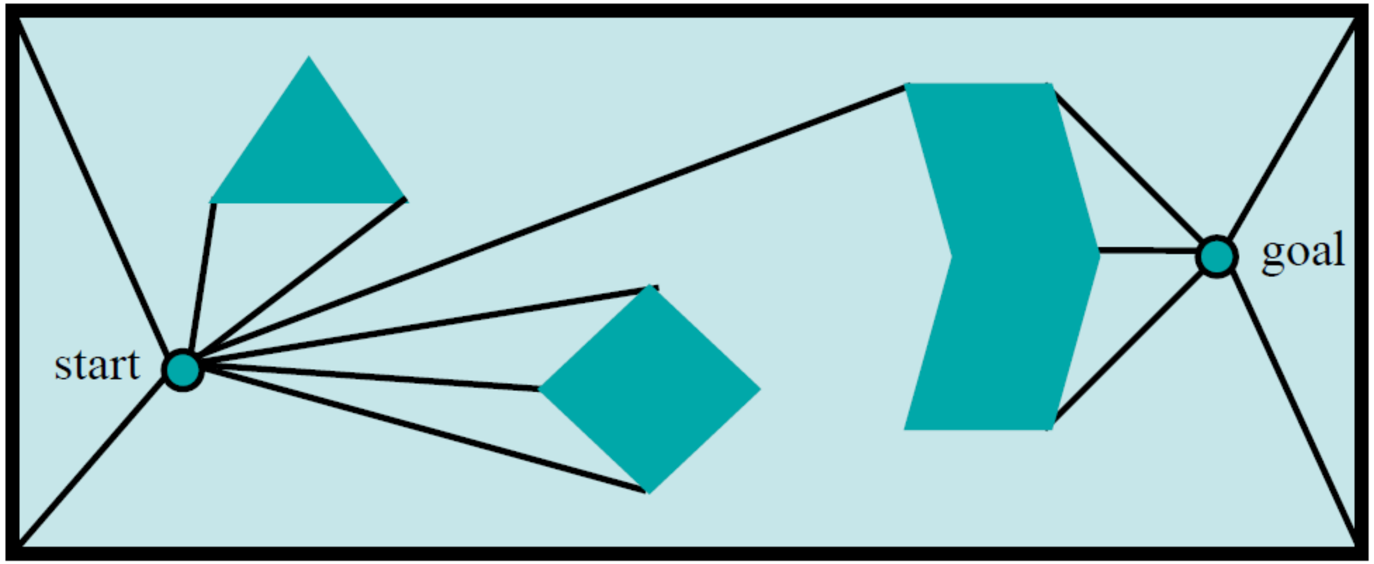
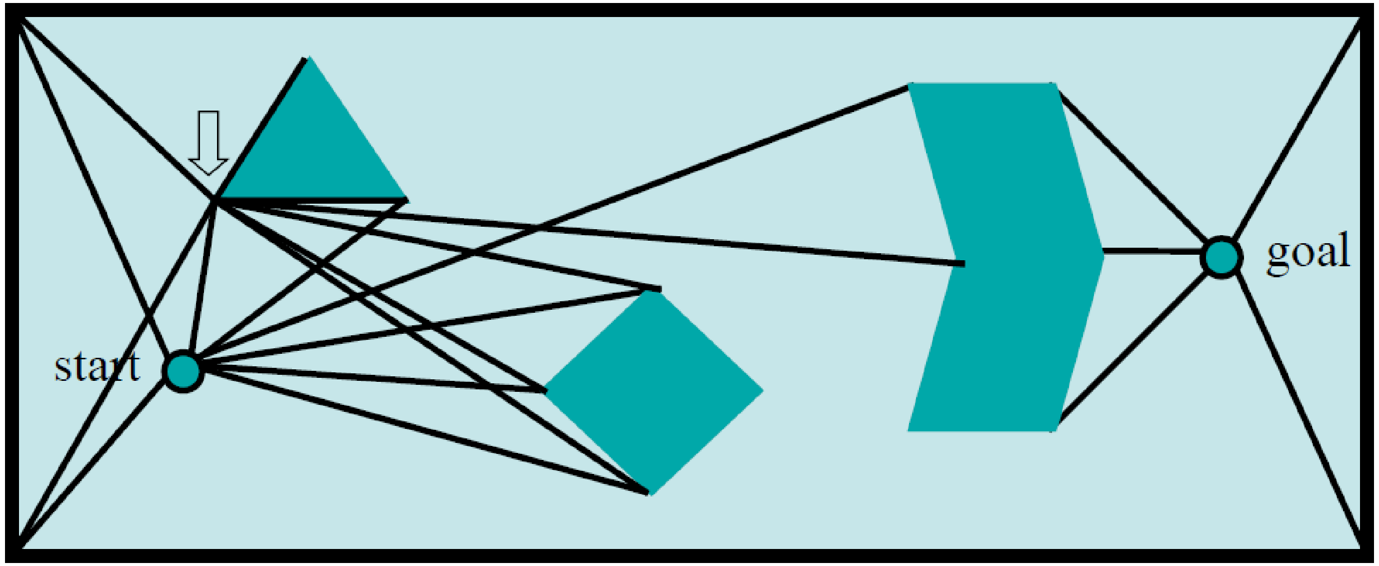


Visibility Graph Path Planning

- Second, draw lines of sight from every vertex of every obstacle like before. Remember lines along edges are also lines of sight.



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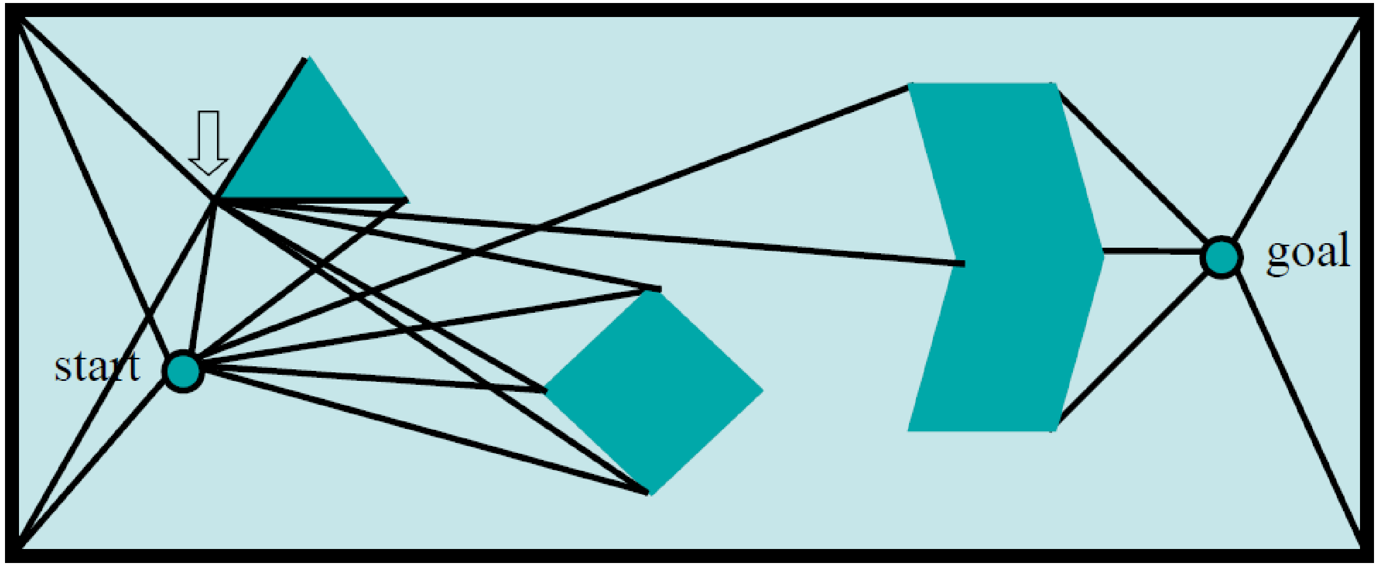

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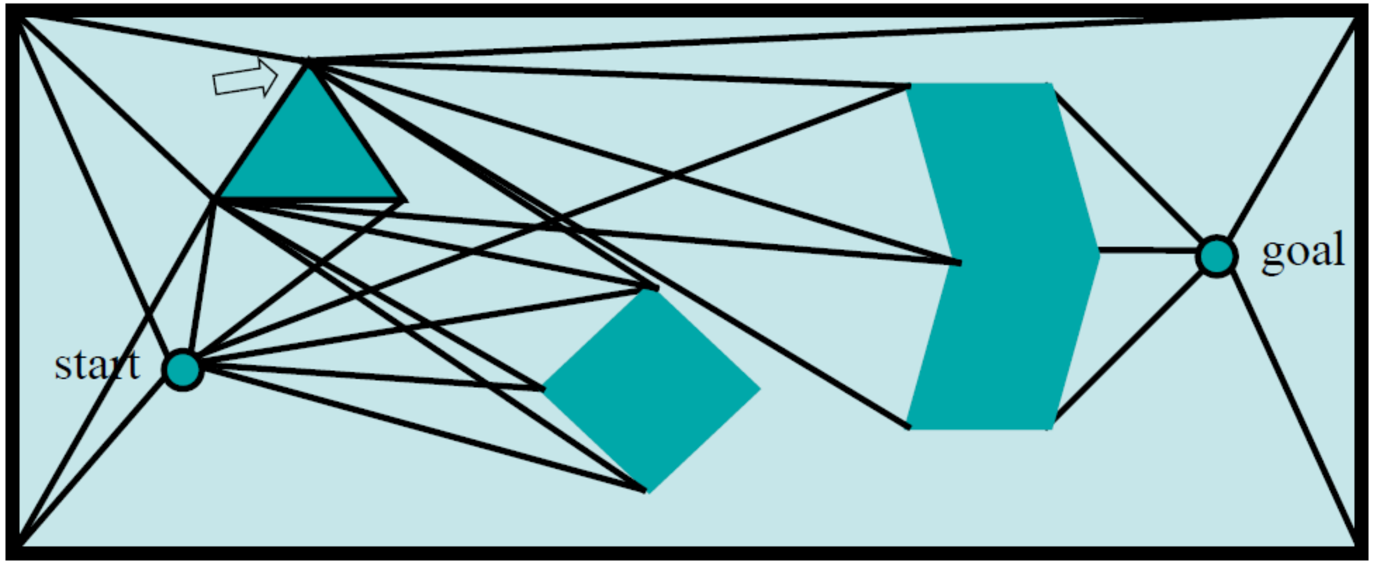
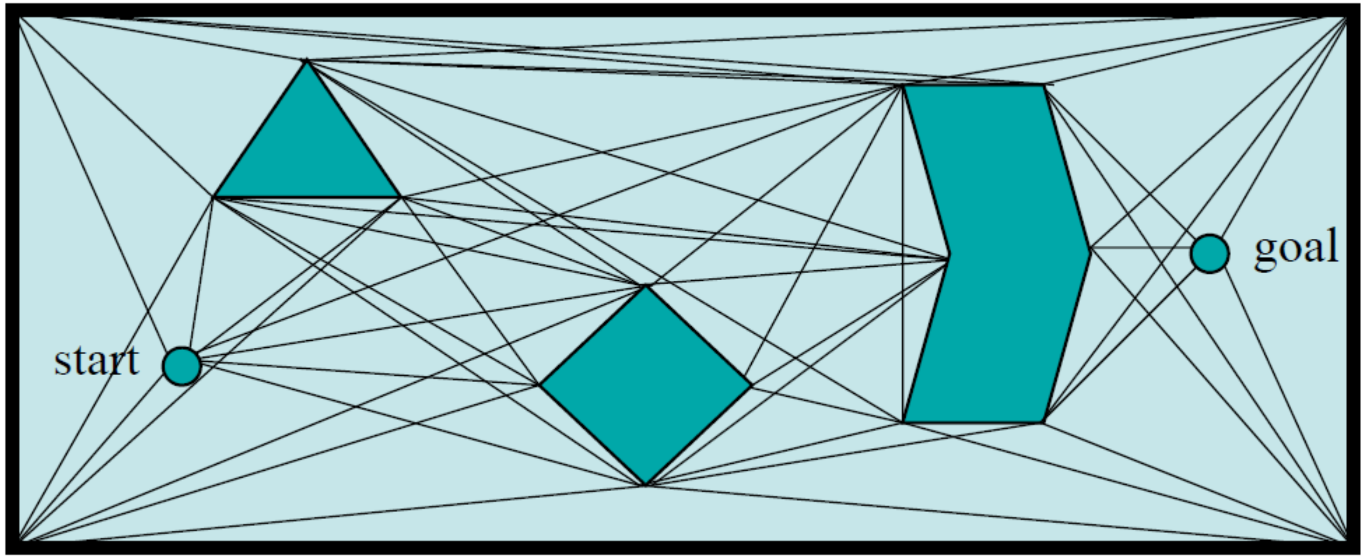


Visibility Graph Path Planning

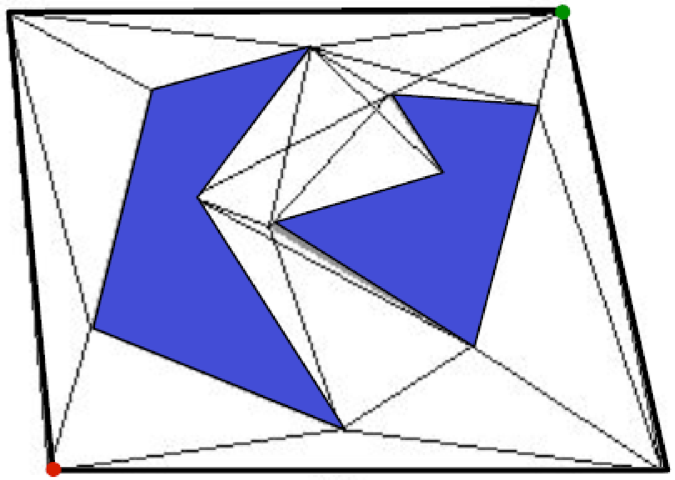
- Repeat until you're done.

Visibility graph

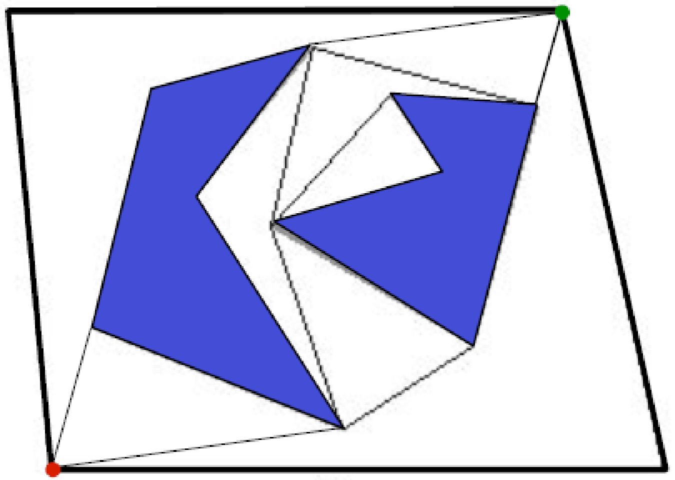


Can use graph search on visibility graph to find shortest path

Visibility Graph Path Planning



Full visibility graph



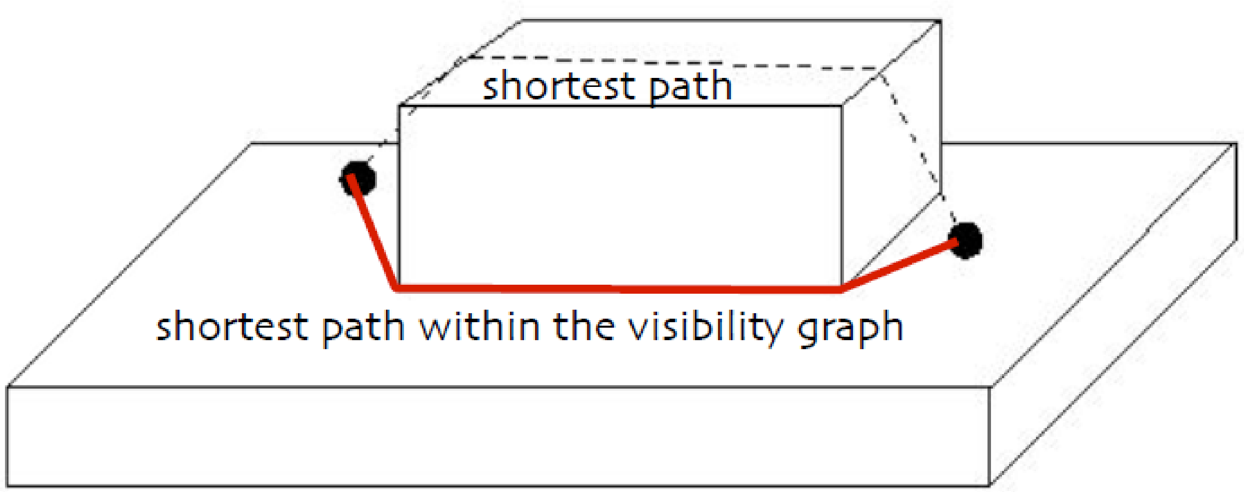
Reduced visibility graph, i.e., not including segments that extend into obstacles on either side.

(but keeping endpoints' roads)

Potential problem: shortest path touches obstacle corners. Need to dilate obstacles.

Visibility Graph Path Planning

Visibility graphs do not preserve their optimality in higher dimensions:



Path smoothing

- Plans obtained from any of these planners are not going to be smooth
- A plan is a sequence of states: $\pi = (\mathbf{x}_{\text{src}}, \mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N, \mathbf{x}_{\text{dest}})$

- We can get a smoother path $\text{smooth}(\pi) = (\mathbf{x}_{\text{src}}, \mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_N, \mathbf{x}_{\text{dest}})$ by minimizing the following cost function

$$f(\mathbf{y}_1, ..., \mathbf{y}_N) = \sum_{t=1}^{N} ||\mathbf{y}_t - \mathbf{x}_t||^2 + \alpha \sum_{t=1}^{N} ||\mathbf{y}_t - \mathbf{y}_{t-1}||^2$$

Stay close to the old path

Penalize squared length

- May need to stop smoothing when smooth path comes close to obstacles.