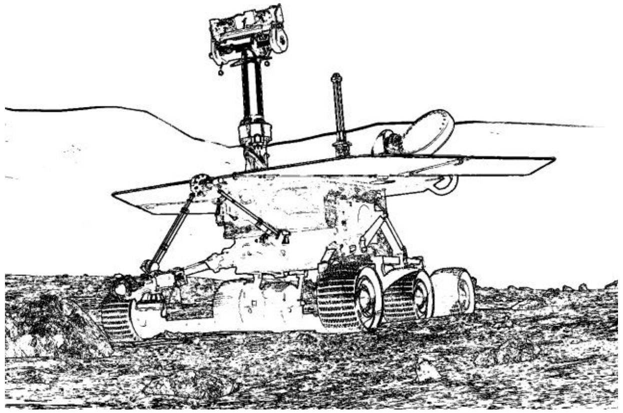


CSC477
Introduction to Mobile Robotics

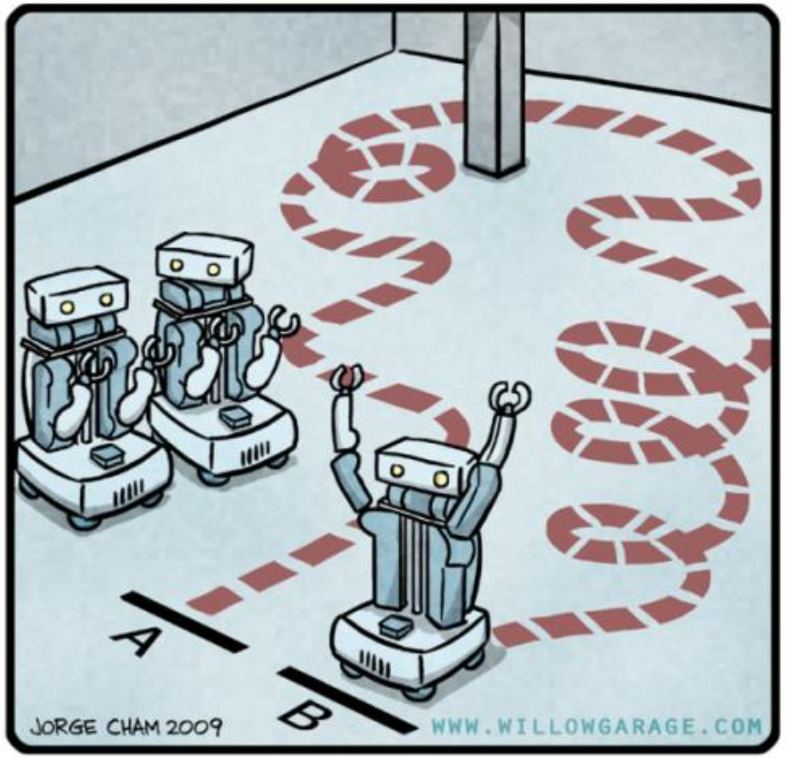
Florian Shkurti

Week #5: Discrete Planning in Known Environments

Today's agenda

- Dijkstra's Planning Algorithm
- A* Planning Algorithm
- Sampling Based Planners
  - Rapidly-exploring Random Trees (RRT)
  - Probabilistic Roadmaps (PRM)

R.O.B.O.T. Comics



"HIS PATH-PLANNING MAY BE SUB-OPTIMAL, BUT IT'S GOT FLAIR."

Planning

- So far we have been trying to compute state-dependent **feedback controllers u(x)=Kx**

Planning

- So far we have been trying to compute state-dependent **feedback controllers u(x)=Kx**

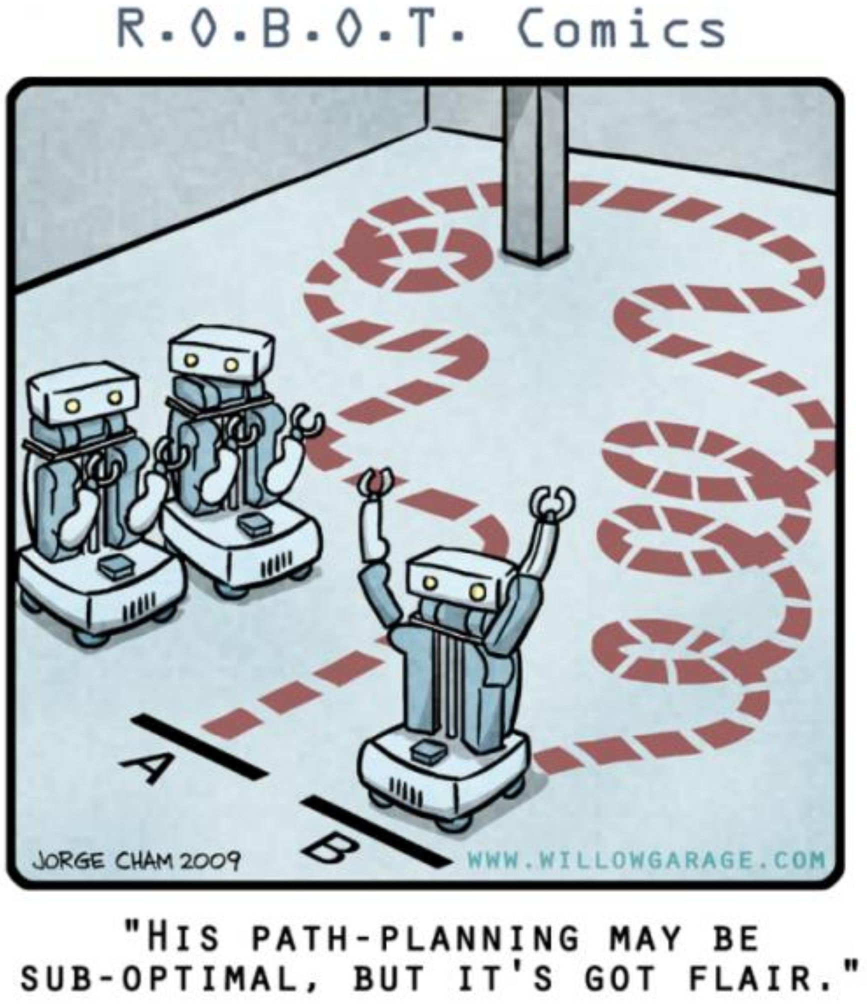
- A plan is usually "open-loop," in the sense that it is assumed that once computed you can execute it perfectly

- This is not realistic because: wind, drag, external forces, friction, unknown factors make the system diverge from the planned trajectory.

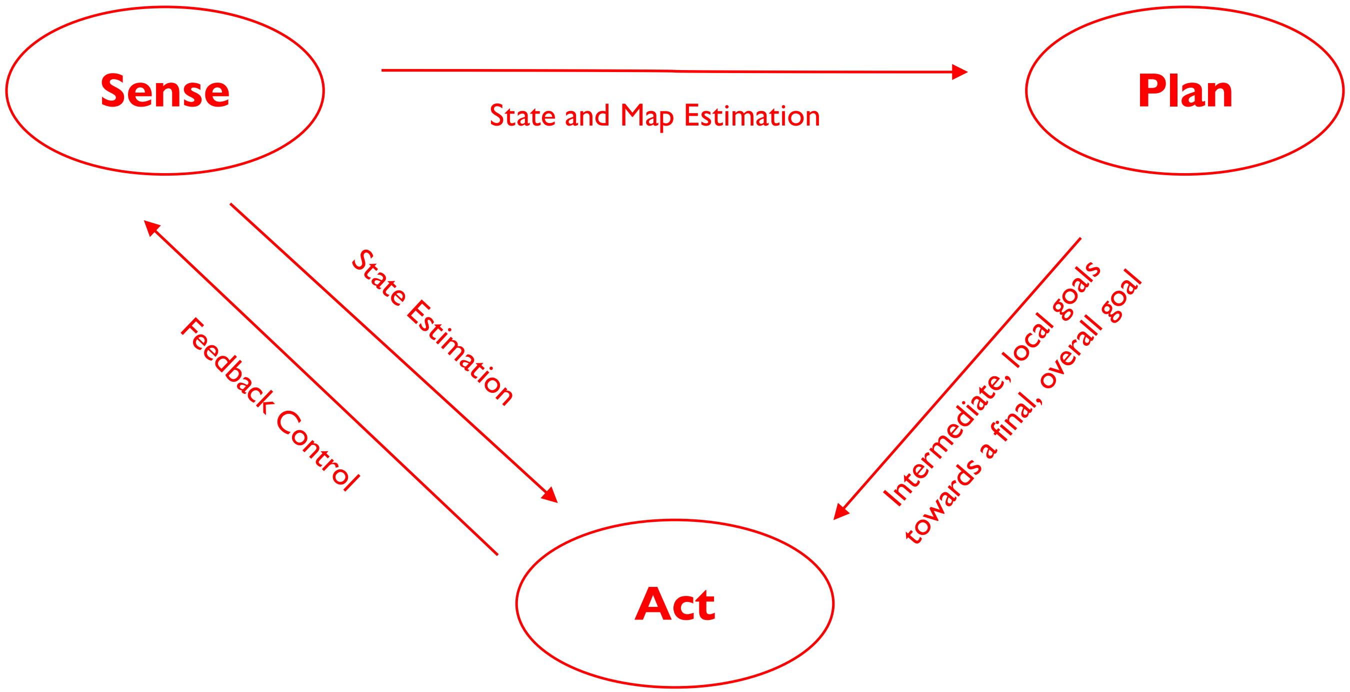
Planning

- So far we have been trying to compute state-dependent **feedback controllers $u(x)=Kx$**
- A plan is usually "open-loop," in the sense that it is assumed that once computed you can execute it perfectly
- This is not realistic because: wind, drag, external forces, friction, unknown factors make the system diverge from the planned trajectory.
- Planning does not usually take external disturbances into account.
  (External, independent feedback controllers have to make sure the robot is following the path closely)

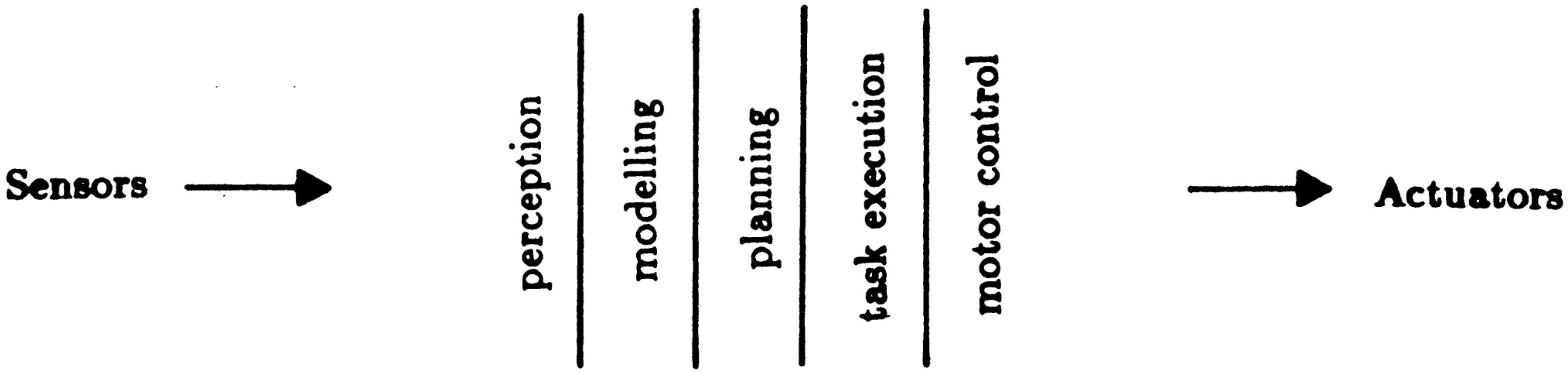
Why Bother Planning?



Sense-Plan-Act Paradigm:
Planning Is Necessary



Sense-Plan-Act Paradigm: Planning Is Necessary



Subsumption Architecture: Planning Is Not Necessary

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
ARTIFICIAL INTELLIGENCE LABORATORY

Working Paper 303 September, 1987

PLANNING IS JUST A WAY OF AVOIDING FIGURING OUT WHAT TO DO NEXT

Rodney A. Brooks

Abstract. The idea of planning and plan execution is just an intuition based decomposition. There is no reason it *has to be* that way. Most likely in the long term, real empirical evidence from systems we know to be built that way (from designing them like that) will determine whether its a very good idea or not. Any particular planner is simply an abstraction barrier. Below that level we get a choice of whether to slot in another planner or to place a program which *does the right thing.* Why stop there? Maybe we can go up the hierarchy and eliminate the planners there too. To do this we must move from a state based way of reasoning to a process based way of acting.

Subsumption Architecture: Planning Is Not Necessary

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
ARTIFICIAL INTELLIGENCE LABORATORY

Working Paper 303 September, 1987

PLANNING IS JUST A WAY OF AVOIDING FIGURING OUT WHAT TO DO NEXT

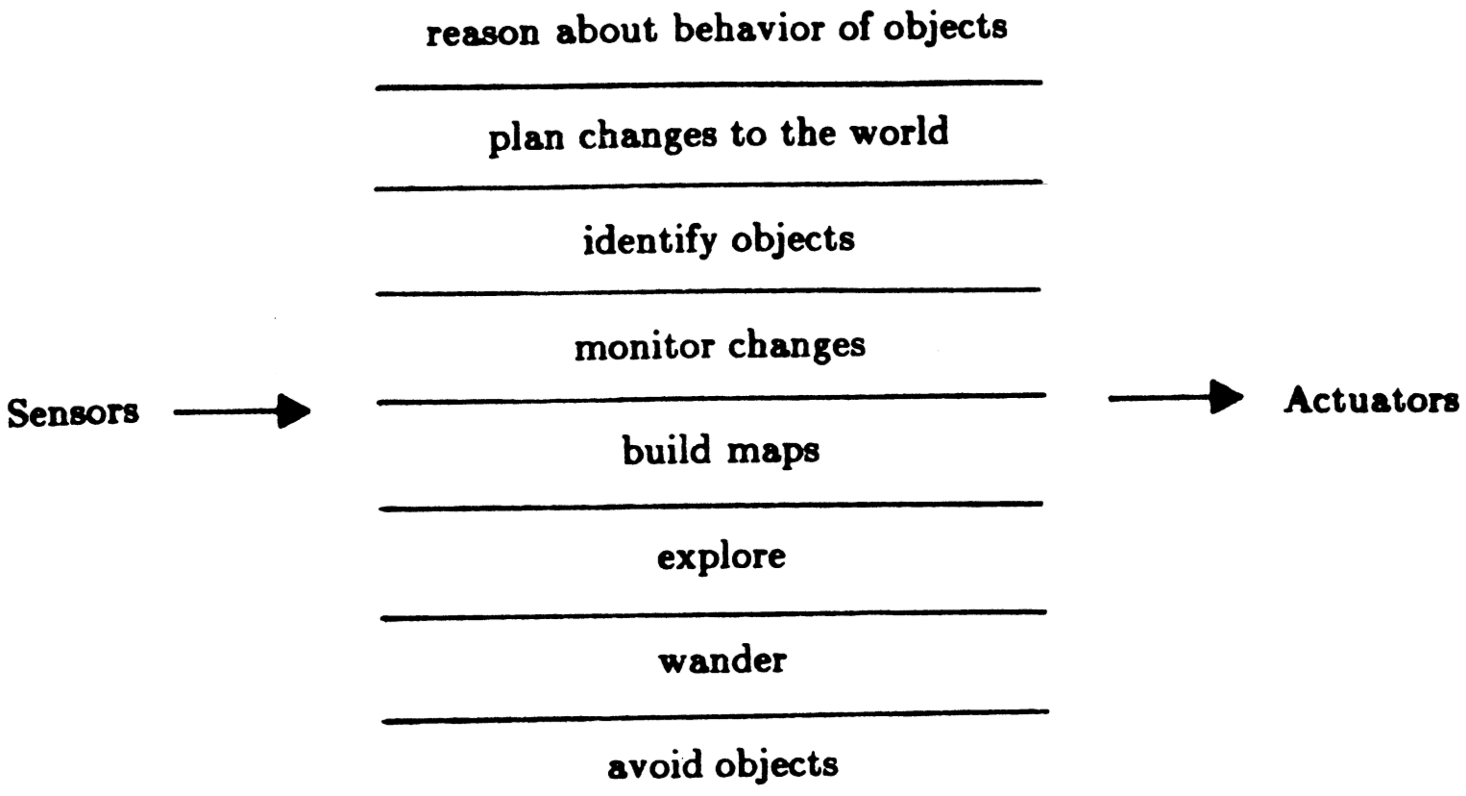
Rodney A. Brooks

Abstract. The idea of planning and plan execution is just an intuition based decomposition. There is no reason it *has to be* that way. Most likely in the long term, real empirical evidence from systems we know to be built that way (from designing them like that) will determine whether its a very good idea or not. Any particular planner is simply an abstraction barrier. Below that level we get a choice of whether to slot in another planner or to place a program which *does the right thing*. Why stop there? Maybe we can go up the hierarchy and eliminate the planners there too. To do this we must move from a state based way of reasoning to a process based way of acting.

He means: why bother estimating state and planning? It's too much work and could be error-prone. Why not only have a hierarchy of reactive behaviors/controllers?

One possibility:
instead of u(state)=…
use u(sensory observation)=…

Subsumption Architecture: Planning Is Not Necessary



Planning as graph search

- Graph nodes represent discrete states
- Edges represent transitions/actions
- Edges have weights

- Potential queries:
  - Shortest path from node a to node b, that does not hit obstacles
  - Is b reachable from a?

Planning as graph search

- Graph nodes represent discrete states
- Edges represent transitions/actions
- Edges have weights

- Potential queries:
  - Shortest path from node a to node b, that does not hit obstacles
  - Is b reachable from a?

- Typical assumptions:
  - Current state is known
  - Map is known
  - Map is mostly static

Dynamic Programming

$$D(v) = \min_{u \in N(v)} [d(v, u) + D(u)]$$

$$D(v_{\text{dest}}) = 0$$

Cost-to-go to destination
starting from node v

Instantaneous transition
cost needs to be non-negative

Dynamic Programming

$$D(v) = \min_{u \in N(v)} [d(v,u) + D(u)]$$

$$D(v_{\text{dest}}) = 0$$

Neighbors of v.
i.e. available actions

Cost-to-go to destination
starting from node v

Base case

Instantaneous transition
cost needs to be non-negative

Note: this should remind you
of the LQR cost-to-go update

$$J_{t+1}(\mathbf{x}) = \min_{\mathbf{u}} [g(\mathbf{x}_t, \mathbf{u}_t) + J_t(A\mathbf{x} + B\mathbf{u})]$$

$$J_0(\mathbf{x}) = \mathbf{x}^T Q \mathbf{x}$$

Dynamic Programming

Worst-Case Complexity:

$O(|V|^2)$

In 2D grid world

$O(|V|)$

$$D(v) = \min_{u \in N(v)} [d(v,u) + D(u)]$$

$$D(v_{\text{dest}}) = 0$$

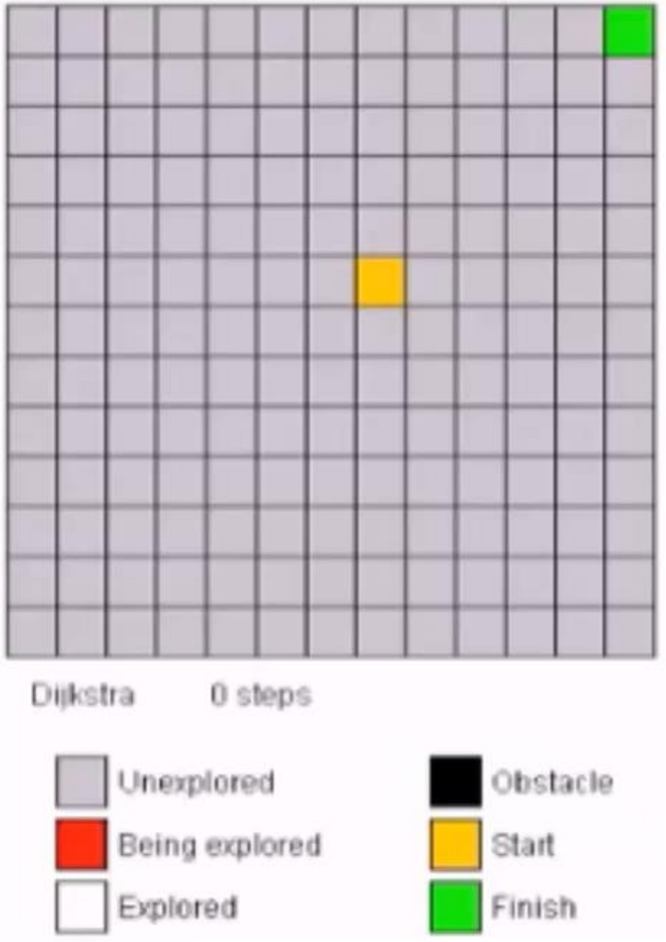
Cost-to-go to destination node starting from node v. Could also have denoted it

$D(v, v_{\text{dest}})$

Base case

Instantaneous transition cost for adjacent nodes needs to be non-negative

Dijkstra's algorithm: example runs



Dijkstra's algorithm

- Let $D(v)$ denote the length of the optimal path from the source node to node v (i.e. cost-to-come, not cost-to-go like before)
- Set $D(v) = \infty$ for all nodes except the source: $D(v_{\mathrm{src}}) = 0$
- Add all nodes to priority queue Q with cost-to-come as priority
- While Q is not empty:

Dijkstra's algorithm

- Let $D(v)$ denote the length of the optimal path from the source node to node v (i.e. cost-to-come, not cost-to-go like before)
- Set $D(v) = \infty$ for all nodes except the source: $D(v_{\text{src}}) = 0$
- Add all nodes to priority queue Q with cost-to-come as priority
- While Q is not empty:
  - Extract the node v with minimum cost-to-come from the queue Q
  - If found goal then done
  - Remove v from the queue

    The cost-to-come of v is final at this point.

    Need to check if we can reduce the cost-to-come of its neighbors.

Dijkstra's algorithm

- Let $D(v)$ denote the length of the optimal path from the source node to node v (i.e. cost-to-come, not cost-to-go like before)
- Set $D(v) = \infty$ for all nodes except the source: $D(v_{\text{src}}) = 0$
- Add all nodes to priority queue Q with cost-to-come as priority
- While Q is not empty:
  - Extract the node v with minimum cost-to-come from the queue Q
  - If found goal then done
  - Remove v from the queue
    The cost-to-come of v is final at this point.
    Need to check if we can reduce the cost-to-come of its neighbors.
  - For u in neighborhood of v:
    - If $d(u,v) + D(v) < D(u)$ then
      - Update priority of u in Q to be $d(u,v) + D(v)$

Dijkstra's algorithm

- Let $D(v)$ denote the length of the optimal path from the source node to node v (i.e. cost-to-come, not cost-to-go like before)
- Set $D(v) = \infty$ for all nodes except the source: $D(v_{\text{src}}) = 0$
- Add all nodes to priority queue Q with cost-to-come as priority
- While Q is not empty:
  - Extract the node v with minimum cost-to-come from the queue Q
  - If found goal then done
  - Remove v from the queue

    The cost-to-come of v is final at this point.

    Need to check if we can reduce the cost-to-come of its neighbors.
  - For u in neighborhood of v:
    - If $d(u,v) + D(v) < D(u)$ then
      - Update priority of u in Q to be $d(u,v) + D(v)$

$O(1)$

$O(\log|V|)$

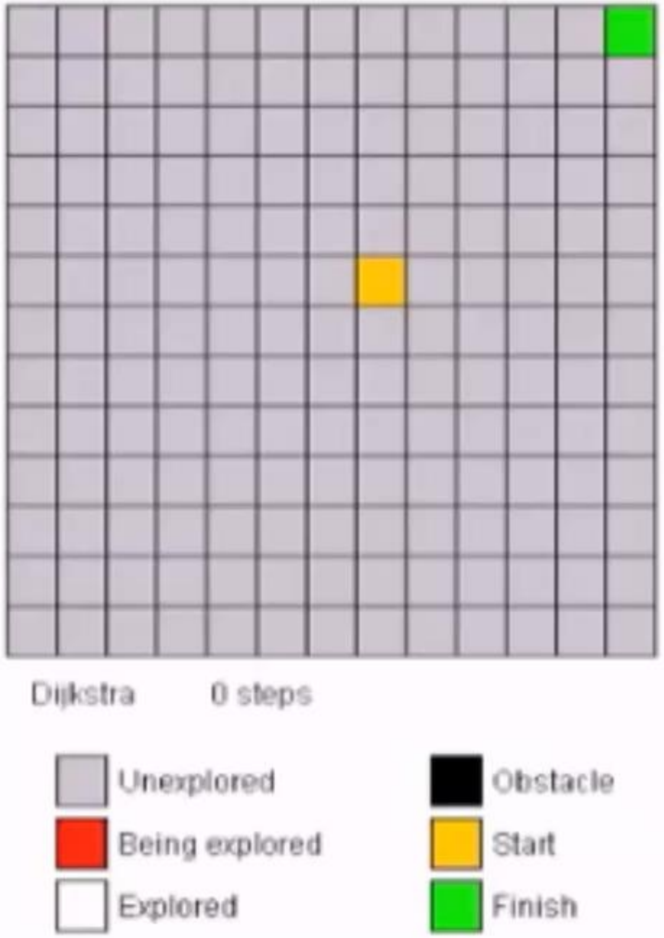
$O(1)$

For Fibonacci heaps

$$O(|E|T_{\text{update priority}} + |V|T_{\text{remove min}}) = O(|E| + |V|\log|V|)$$

Dijkstra’s algorithm: example runs

Many nodes are explored unnecessarily. We are sure that they are not going to be part of the solution.



A* Search: Main Idea

- Modifies Dijkstra's algorithm to be more efficient
- Expands fewer nodes than Dijkstra's by using a heuristic

- While Dijkstra prioritizes nodes based on cost-to-come
- A* prioritizes them based on:

cost-to-come to v + lower bound on cost-to-go from v to v_{dest}

$$f(v) = g(v) + h(v)$$

Lower bound on cost of path from source to destination that passes through v

Optimistic search: explore node with smallest f(v) next

A* Search: Main Idea

- Modifies Dijkstra's algorithm to be more efficient
- Expands fewer nodes than Dijkstra's by using a heuristic

- While Dijkstra prioritizes nodes based on cost-to-come
- A* prioritizes them based on:

cost-to-come to v + lower bound on cost-to-go from v to v_{dest}

$$f(v) = g(v) + h(v)$$

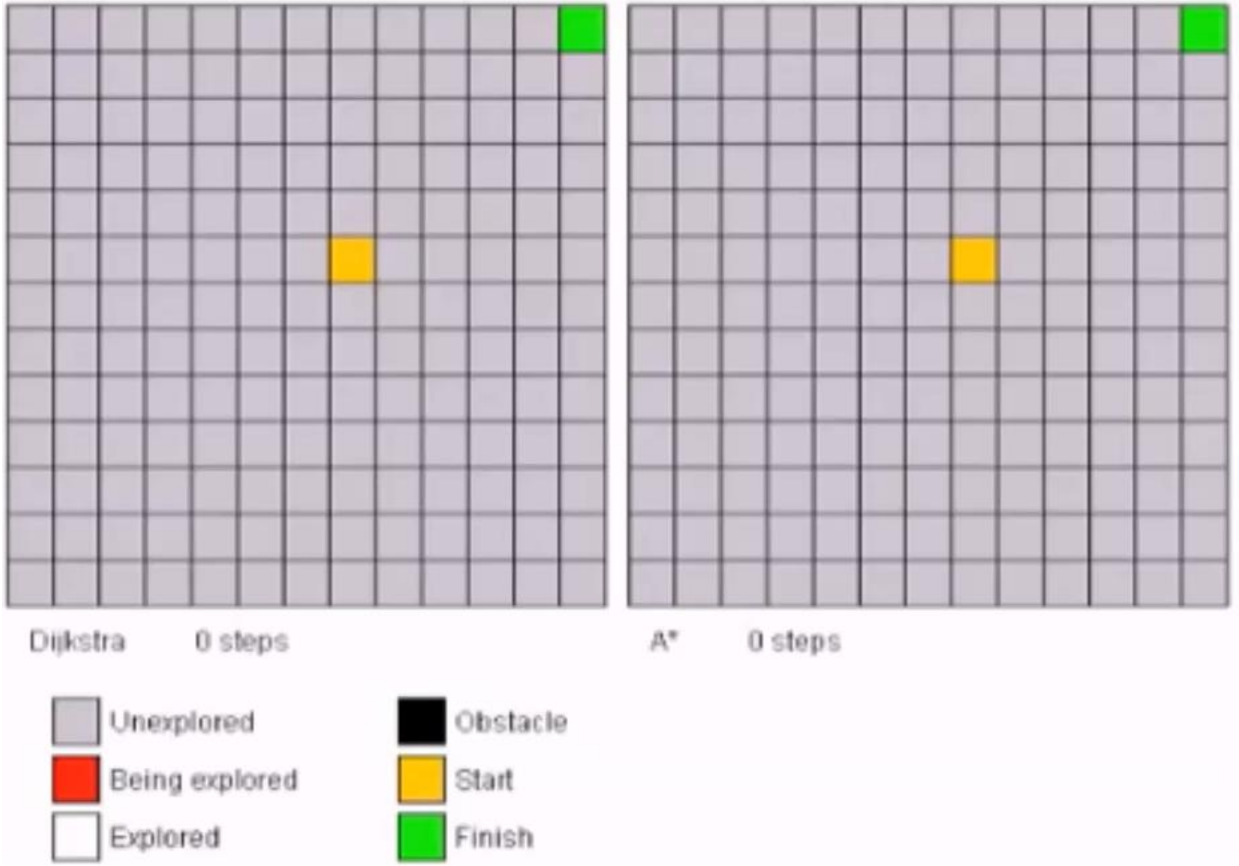
Lower bound on cost of path from source to destination that passes through v

h() is called a heuristic. h() must be **admissible,** i.e. underestimate the cost-to-go from v to destination. h() must also be **monotonic,** i.e. satisfy the triangle inequality.

A* Search

- Set $g(v) = \infty$ for all nodes except the source: $g(v_{\text{src}}) = 0$
- Set $f(v) = \infty$ for all nodes except the source: $f(v_{\text{src}}) = h(v_{\text{src}})$
- Add v_{src} to priority queue Q with priority $f(v_{\text{src}})$
- While Q is not empty:
  - Extract the node v with minimum $f(v)$ from the queue Q
  - If found goal then done. Follow the parent pointers from v to get the path.
  - Remove v from the queue Q
  - explored(v) = true
  - For u in neighborhood of v if not explored(u):
    - If u not in Q then
      - Add u in Q with cost-to-come $g(u) = g(v) + d(v, u)$ and priority $f(u) = g(u) + h(u)$
      - Set the parent of u to be v
    - Else if $g(v) + d(v, u) < g(u)$
      - Update the cost-to-come and the priority of u in Q
      - Set the parent of u to be v

Dijkstra vs A*



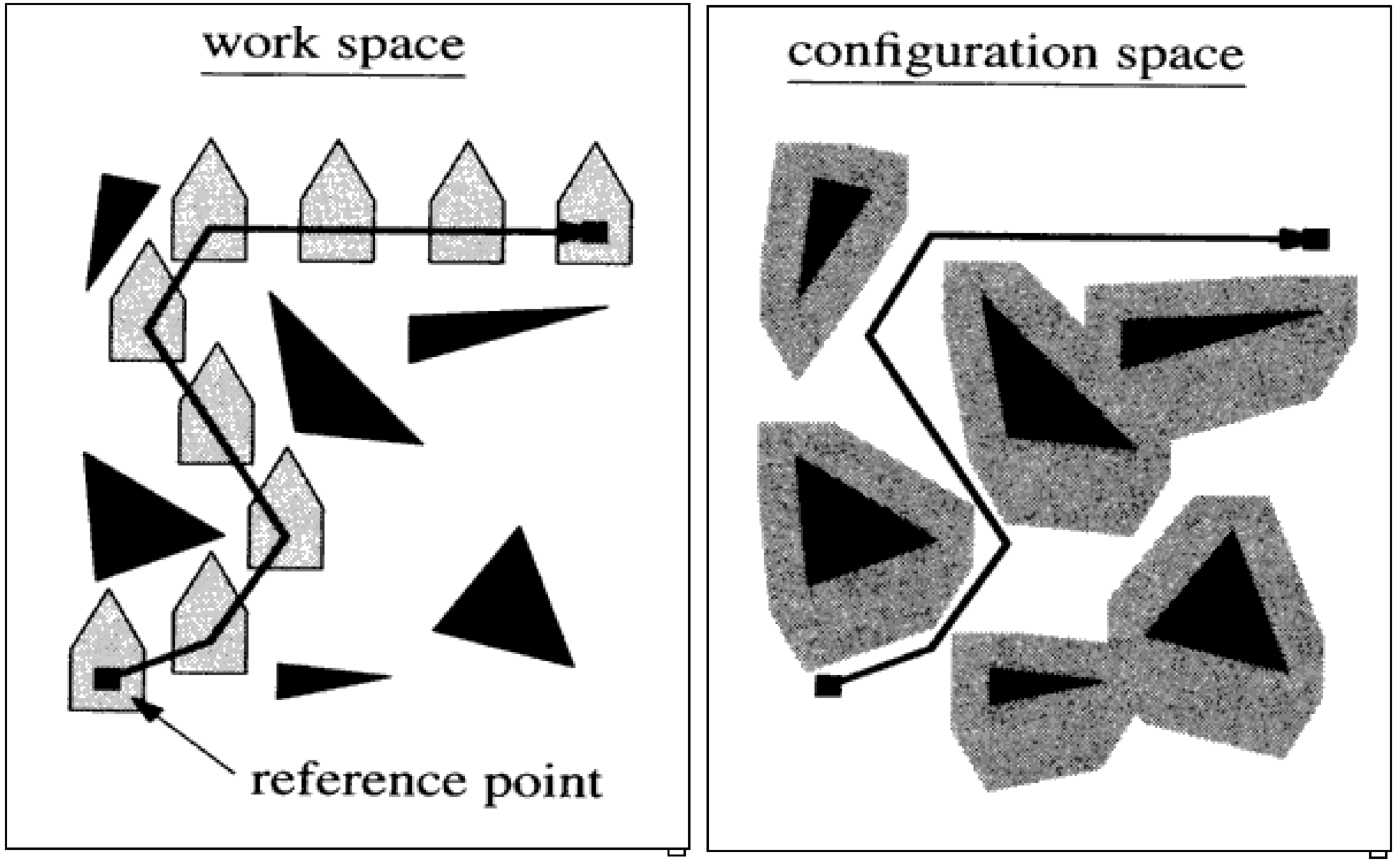
A* for cars

Configuration Space

Idea: dilate obstacles to account for the ways the robot can collide with them.

Why? Instead of planning in the work space and checking whether the robot's body collides with obstacles, plan in configuration space where you can treat the robot as a point because the obstacles are dilated.

This idea is typically not used for robots with high-dimensional states.

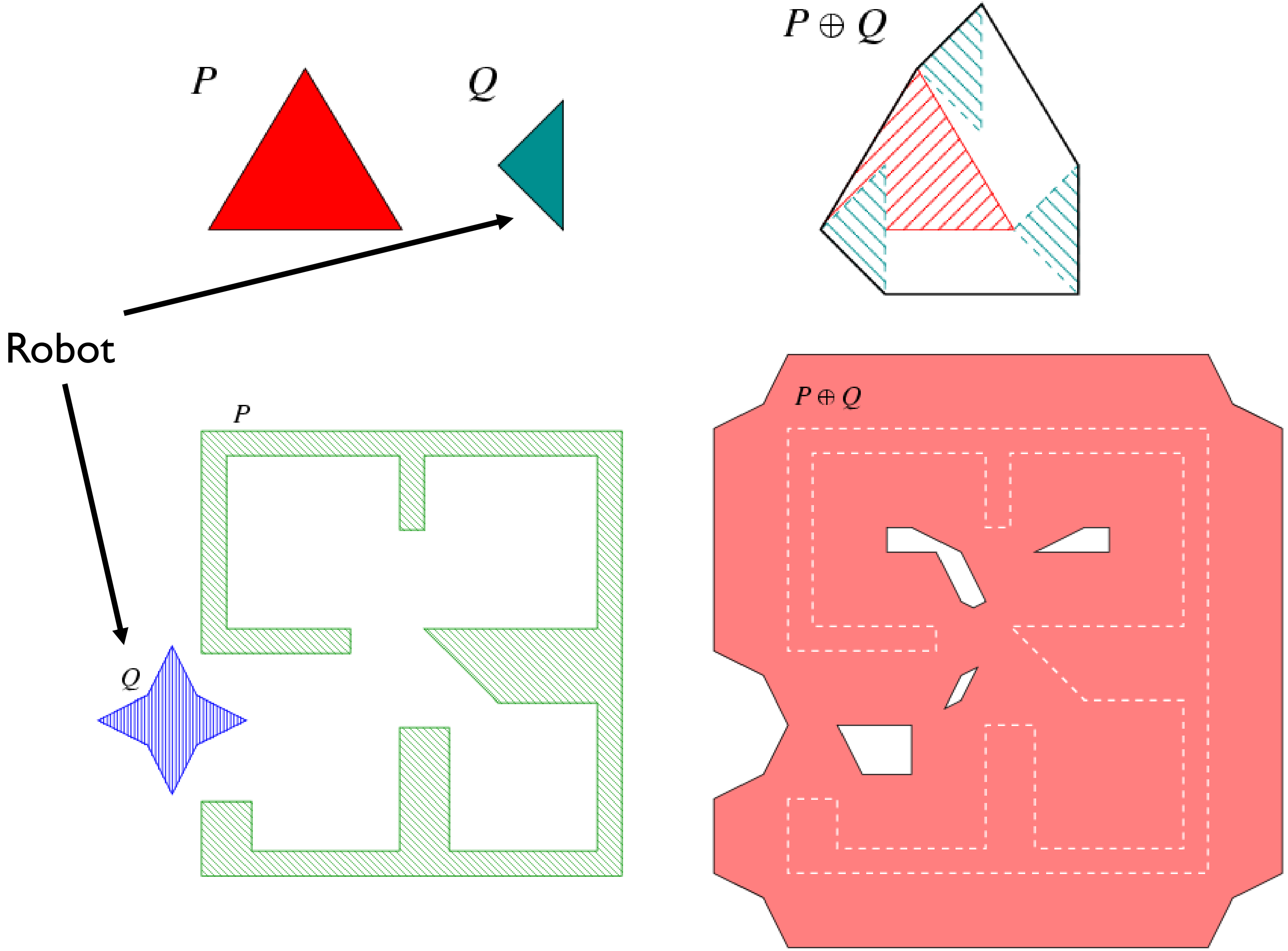


Configuration Space

How do we dilate obstacles?

Minkowski Sum

$$P \oplus Q = \{p + q \mid p \in P,\ q \in Q\}$$



Drawbacks of grid-based planners

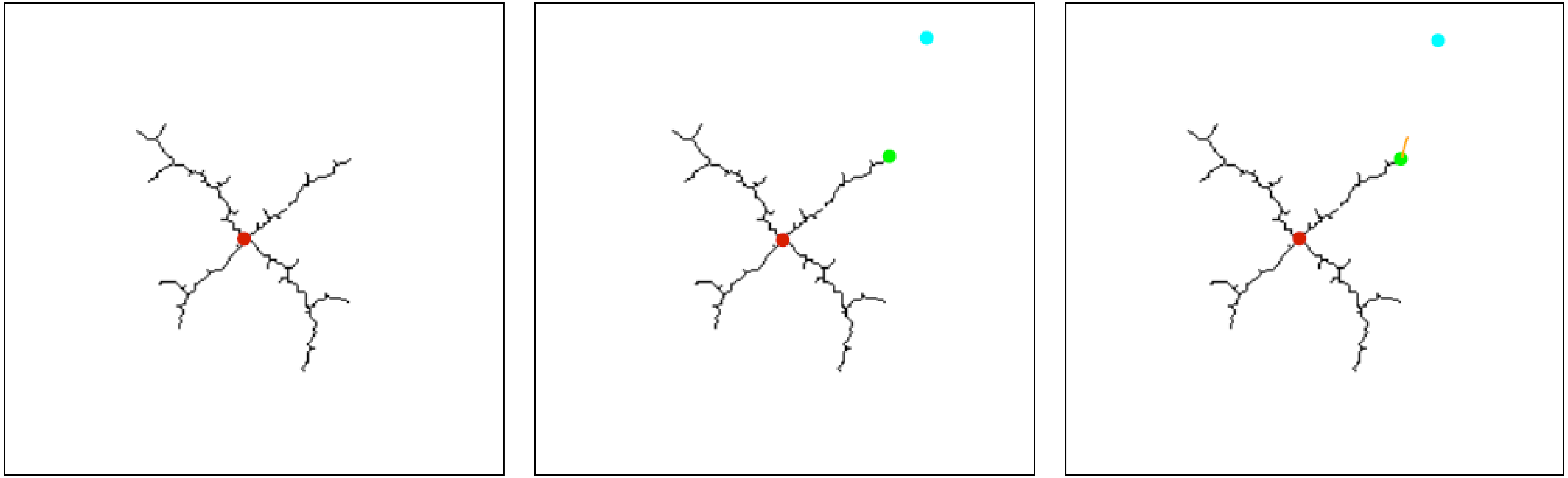
- Grid-based planning works well for grids of up to 3-4 dimensions
- State-space discretization suffers from combinatorial explosion:
- If the state is $\mathbf{x} = [x_1, ..., x_D]$ and we split each dimension into N bins then we will have N^D nodes in the graph.
- This is not practical for planning paths for robot arms with multiple joints, or other high-dimensional systems.

Sampling the state-space

- Need to find ways to reduce the continuous domain into a sparse representation: graphs, trees etc.

- Today:
- Rapidly-exploring Random Tree (RRT),
- Probabilistic RoadMap (PRM)
- Visibility Planning
- Smoothing Planned Paths

RRT



Main idea: maintain a tree of reachable configurations from the root
Main steps:

- Sample random state
- Find the closest state (node) already in the tree
- Steer the closest node towards the random state

RRT

$V \leftarrow \{x_{\mathrm{init}}\}$; $E \leftarrow \emptyset$;

for $i = 1, \ldots, n$ **do**

- $x_{\mathrm{rand}} \leftarrow \texttt{SampleFree}_i$;
- $x_{\mathrm{nearest}} \leftarrow \texttt{Nearest}(G = (V, E), x_{\mathrm{rand}})$;
- $x_{\mathrm{new}} \leftarrow \texttt{Steer}(x_{\mathrm{nearest}}, x_{\mathrm{rand}})$;
- **if** $\texttt{ObtacleFree}(x_{\mathrm{nearest}}, x_{\mathrm{new}})$ **then**
    - $V \leftarrow V \cup \{x_{\mathrm{new}}\}$; $E \leftarrow E \cup \{(x_{\mathrm{nearest}}, x_{\mathrm{new}})\}$;

return $G = (V, E)$;

RRT

$V \leftarrow \{x_{\text{init}}\};\ E \leftarrow \emptyset;$

for $i = 1, \ldots, n$ **do**

$\quad x_{\text{rand}} \leftarrow \texttt{SampleFree}_i;$

$\quad x_{\text{nearest}} \leftarrow \texttt{Nearest}(G = (V, E), x_{\text{rand}});$

$\quad x_{\text{new}} \leftarrow \texttt{Steer}(x_{\text{nearest}}, x_{\text{rand}})\ ;$

$\quad$ **if** $\texttt{ObtacleFree}(x_{\text{nearest}}, x_{\text{new}})$ **then**

$\quad\quad V \leftarrow V \cup \{x_{\text{new}}\};\ E \leftarrow E \cup \{(x_{\text{nearest}}, x_{\text{new}})\}\ ;$

return $G = (V, E);$

Things to pay attention to:

SampleFree() needs to sample a random state from the uniform distribution. How do you sample rotations uniformly?

RRT

Things to pay attention to:

$V \leftarrow \{x_{\text{init}}\};\ E \leftarrow \emptyset;$

for $i = 1, \ldots, n$ **do**

  $x_{\text{rand}} \leftarrow \texttt{SampleFree}_i;$

  $x_{\text{nearest}} \leftarrow \texttt{Nearest}(G = (V, E), x_{\text{rand}});$

  $x_{\text{new}} \leftarrow \texttt{Steer}(x_{\text{nearest}}, x_{\text{rand}})\ ;$

  if $\texttt{ObtacleFree}(x_{\text{nearest}}, x_{\text{new}})$ **then**

    $V \leftarrow V \cup \{x_{\text{new}}\};\ E \leftarrow E \cup \{(x_{\text{nearest}}, x_{\text{new}})\}\ ;$

return $G = (V, E);$

Nearest() searches for the nearest neighbor of a given vector. Brute force search examines |V| nodes (increasing). Is there a more efficient method?

RRT

Things to pay attention to:

$V \leftarrow \{x_{\text{init}}\};\ E \leftarrow \emptyset;$

for $i = 1, \ldots, n$ **do**

  $x_{\text{rand}} \leftarrow \texttt{SampleFree}_i;$

  $x_{\text{nearest}} \leftarrow \texttt{Nearest}(G = (V, E), x_{\text{rand}});$

  $x_{\text{new}} \leftarrow \texttt{Steer}(x_{\text{nearest}}, x_{\text{rand}})\ ;$

  if $\texttt{ObtacleFree}(x_{\text{nearest}}, x_{\text{new}})$ **then**

    $V \leftarrow V \cup \{x_{\text{new}}\};\ E \leftarrow E \cup \{(x_{\text{nearest}}, x_{\text{new}})\}\ ;$

return $G = (V, E);$

Steer() finds the controls that take the nearest state to the new state. Easy for omnidirectional robots. What about non-holonomic systems?

RRT

Things to pay attention to:

$V \leftarrow \{x_{\text{init}}\};\ E \leftarrow \emptyset;$

for $i = 1, \ldots, n$ **do**

  $x_{\text{rand}} \leftarrow \texttt{SampleFree}_i;$

  $x_{\text{nearest}} \leftarrow \texttt{Nearest}(G = (V, E), x_{\text{rand}});$

  $x_{\text{new}} \leftarrow \texttt{Steer}(x_{\text{nearest}}, x_{\text{rand}})\ ;$

  if $\texttt{ObtacleFree}(x_{\text{nearest}}, x_{\text{new}})$ **then**

    $V \leftarrow V \cup \{x_{\text{new}}\};\ E \leftarrow E \cup \{(x_{\text{nearest}}, x_{\text{new}})\}\ ;$

return $G = (V, E);$

ObstacleFree() checks the path from the nearest state to the new state for collisions. How do you do collision checks?

RRT

Things to pay attention to:

$V \leftarrow \{x_{\mathrm{init}}\};\ E \leftarrow \emptyset;$

for $i = 1, \ldots, n$ **do**

$\quad x_{\mathrm{rand}} \leftarrow \mathtt{SampleFree}_i;$

$\quad x_{\mathrm{nearest}} \leftarrow \mathtt{Nearest}(G = (V, E), x_{\mathrm{rand}});$

$\quad x_{\mathrm{new}} \leftarrow \mathtt{Steer}(x_{\mathrm{nearest}}, x_{\mathrm{rand}})\ ;$

$\quad$ **if** $\mathtt{ObtacleFree}(x_{\mathrm{nearest}}, x_{\mathrm{new}})$ **then**

$\quad\quad V \leftarrow V \cup \{x_{\mathrm{new}}\};\ E \leftarrow E \cup \{(x_{\mathrm{nearest}}, x_{\mathrm{new}})\}\ ;$

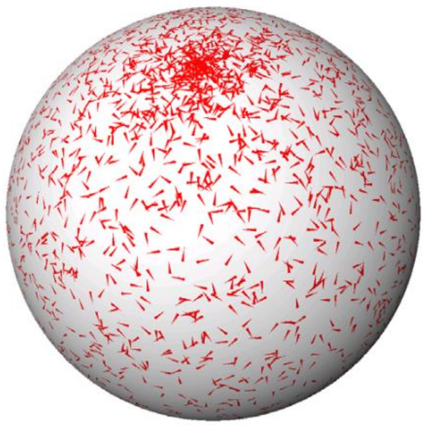
return $G = (V, E);$

Upside of using ObstacleFree(): you don't need to model obstacles in Steer(). For example, if Steer() computes LQR controllers you don't need to model obstacles in the control computation.

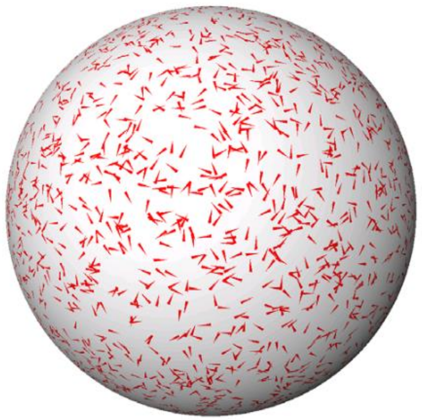
RRT: uniform sampling

- Only tricky case is when the state contains rotation components
- For example: $\mathbf{x} = [{}^{W}_{B}\mathbf{q} \;\; {}^{W}\mathbf{p}_{WB}]$
- State involving both rotation and translation components is often called **the pose** of the system.
- Idea #1: Uniformly sample 3 Euler angles (roll, pitch, yaw)

3D rotation visualization:
rotation axis is a point on a sphere,
rotation angle is the direction
of the red arrow



Idea #1
Not uniform after all

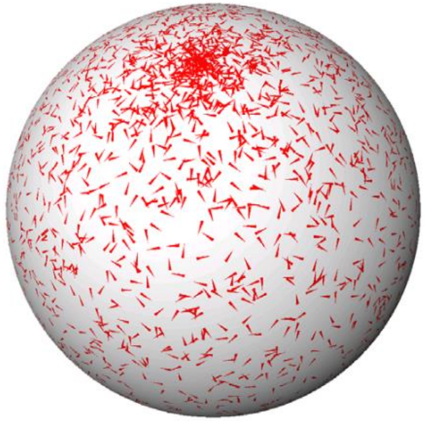


Correct, uniform

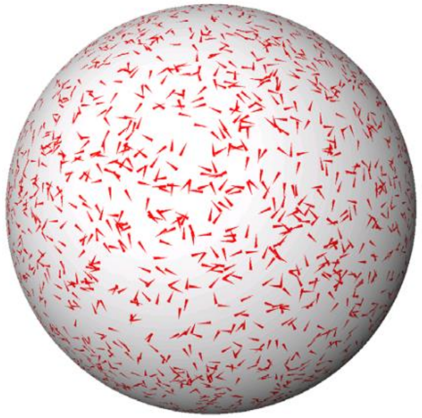
RRT: uniform sampling

- Only tricky case is when the state contains rotation components
- For example: $\mathbf{x} = [{}^{W}_{B}\mathbf{q} \;\; {}^{W}\mathbf{p}_{WB}]$
- State involving both rotation and translation components is often called **the pose** of the system.
- Idea #1: Uniformly sample 3 Euler angles (roll, pitch, yaw)

Nonuniformity at the north pole caused by Gimbal Lock: same rotation parameterized by different Euler angles



Idea #1
Not uniform after all



Correct, uniform

RRT: uniform sampling

- Idea #2: Uniformly sample a quaternion
- First, uniformly sample $u_1, u_2, u_3 \in [0, 1]$
- Then output the unit quaternion

$$\mathbf{q} = [\sqrt{1-u_1}\sin(2\pi u_2),\ \sqrt{1-u_1}\cos(2\pi u_2),\ \sqrt{u_1}\sin(2\pi u_3),\ \sqrt{u_1}\cos(2\pi u_3)]$$

- Idea #3: Uniformly sample rotation matrices.
- It's possible but we won't discuss it here.

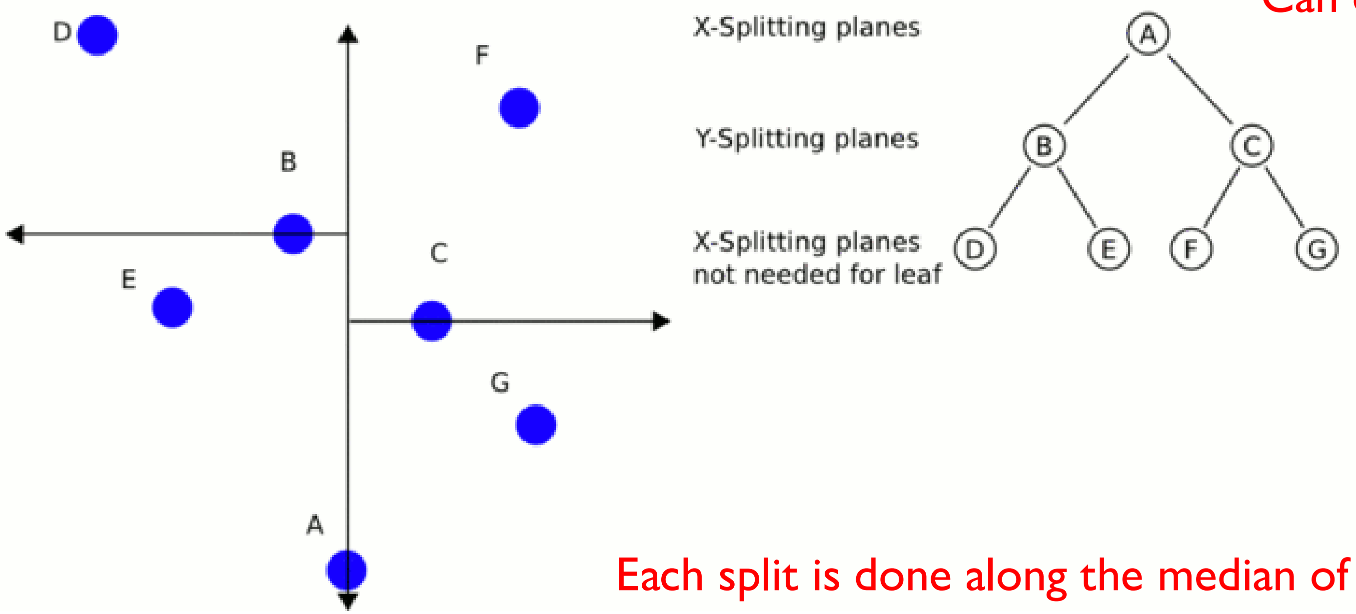
RRT: finding the nearest neighbor

- Any alternatives to linear (brute force) search?

RRT: finding the nearest neighbor

- Any alternatives to linear (brute force) search?
- Idea #1: space partitioning, e.g. kd-trees

Balanced kd-tree:
Can query in O(logn)



Each split is done along the median of the points on that region

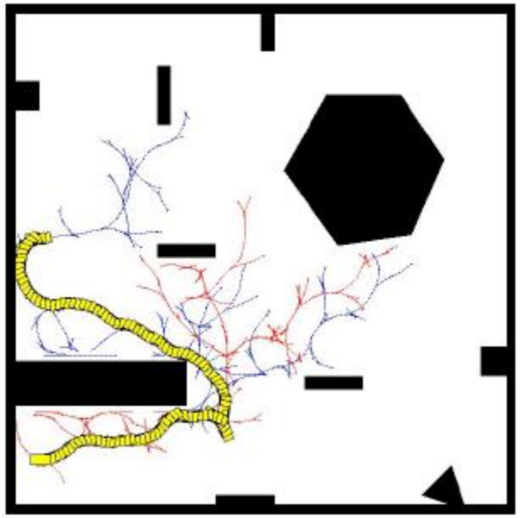
RRT: finding the nearest neighbor

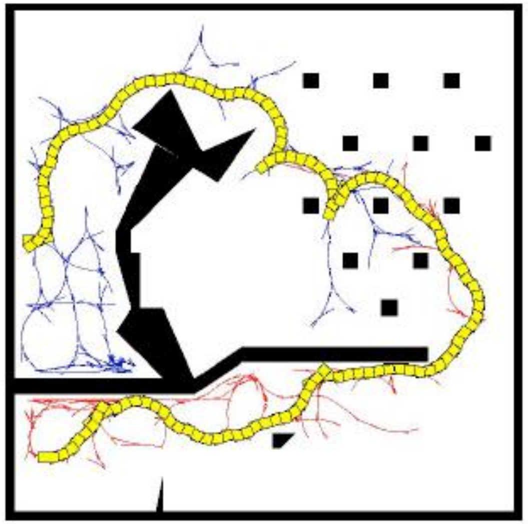
- Any alternatives to linear (brute force) search?
- Idea #1: space partitioning, e.g. kd-trees
- Idea #2: locality-sensitive hashing
  - Maintains buckets
  - Similar points are placed on the same bucket
  - When searching consider only points that map to the same bucket

RRT: steering to a given state

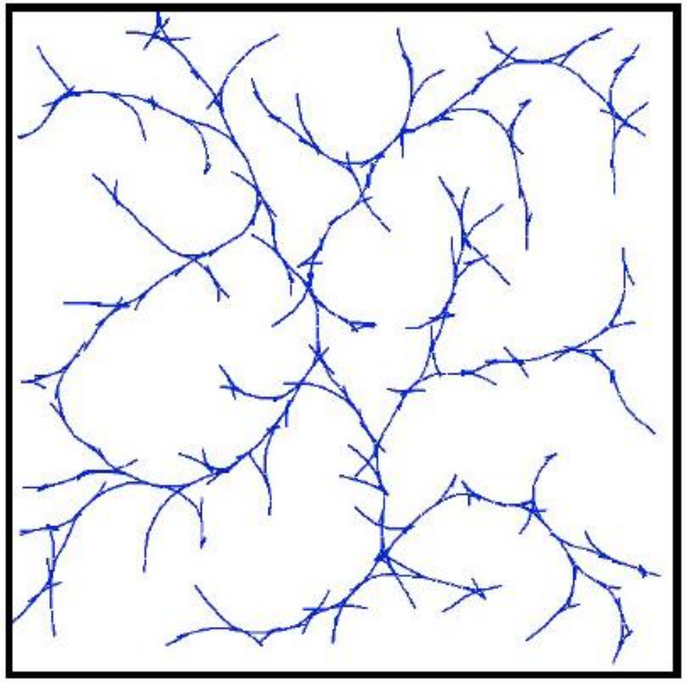
- This is an optimal control problem, but without a specified time constraint
- For omnidirectional systems we can connect states by a straight line.
- For more complicated systems you could use LQR.
- You could also use a large set of predefined controls, one of which could be able to take the system close to the given state

RRT: steering to a given state





nonholonomic constraints

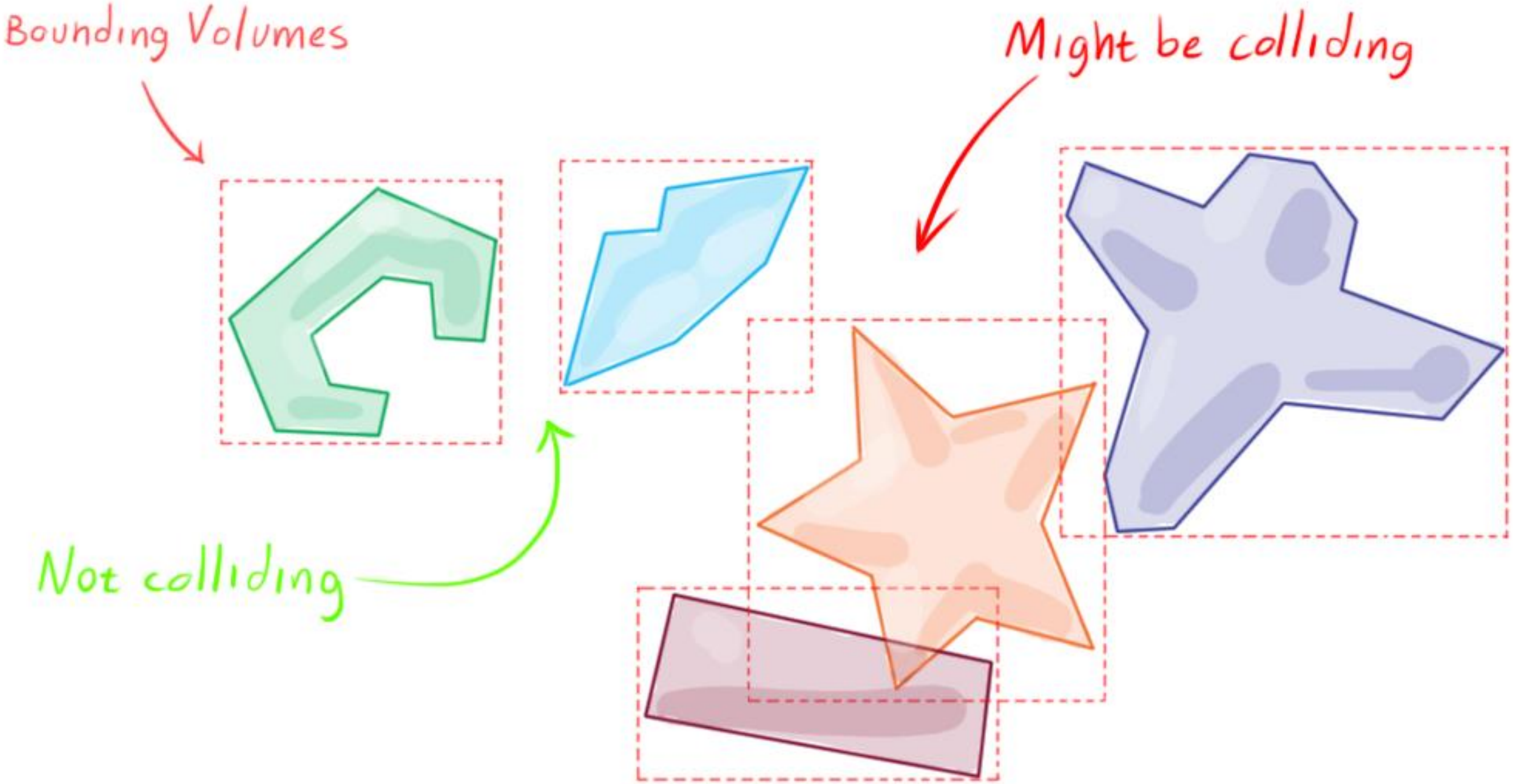


RRT for a robot with car-like kinematics

can the control problem get more difficult?

RRT: collision detection

- Main idea: bounding volume collision detection

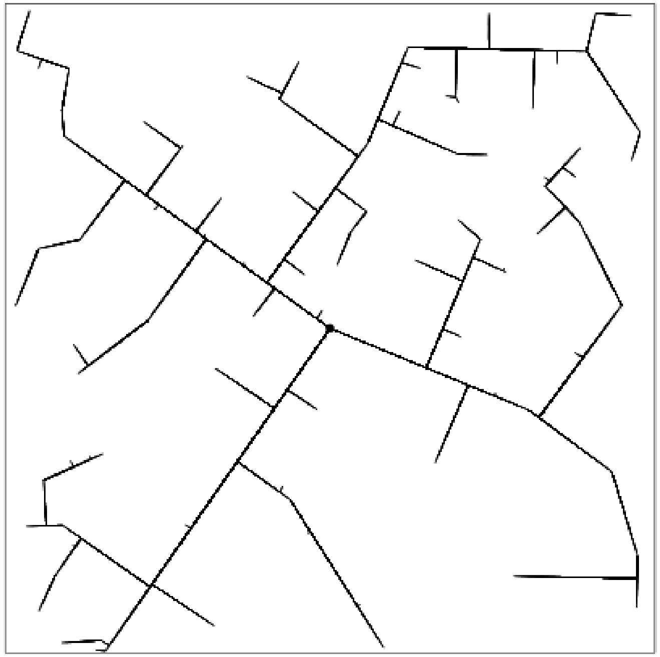


Source: https://www.toptal.com/game/video-game-physics-part-ii-collision-detection-for-solid-objects

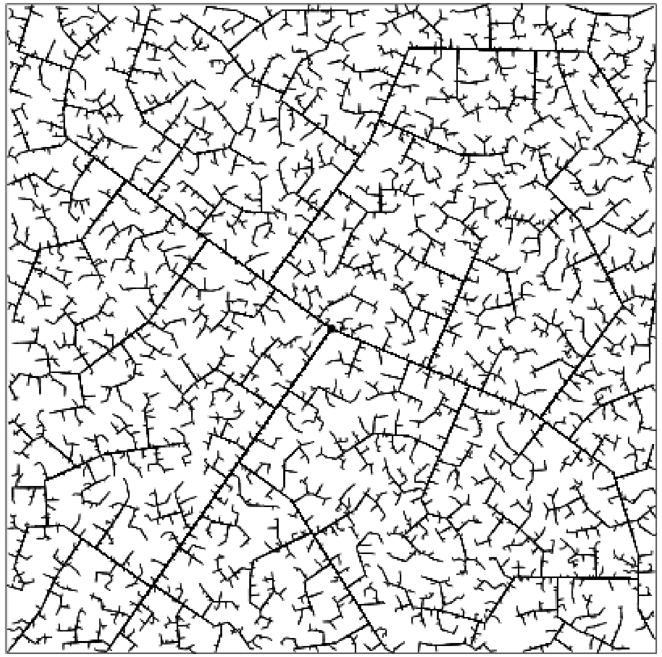
RRT example: moving a piano

RRT: properties of the planning algorithm

#1: The RRT will eventually cover the space, i.e. it is a space-filling tree



45 iterations



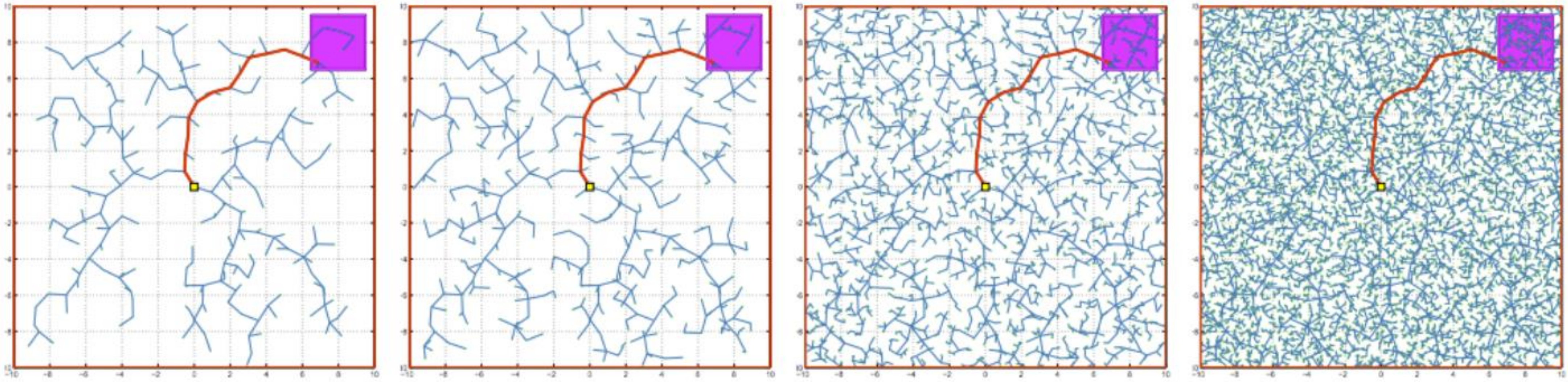
2345 iterations

Source: Planning Algorithms, Lavalle

RRT: properties of the planning algorithm

#1: The RRT will eventually cover the space, i.e. it is a space-filling tree

#2: The RRT will NOT compute the optimal path asymptotically



Source: Karaman, Frazzoli, 2010

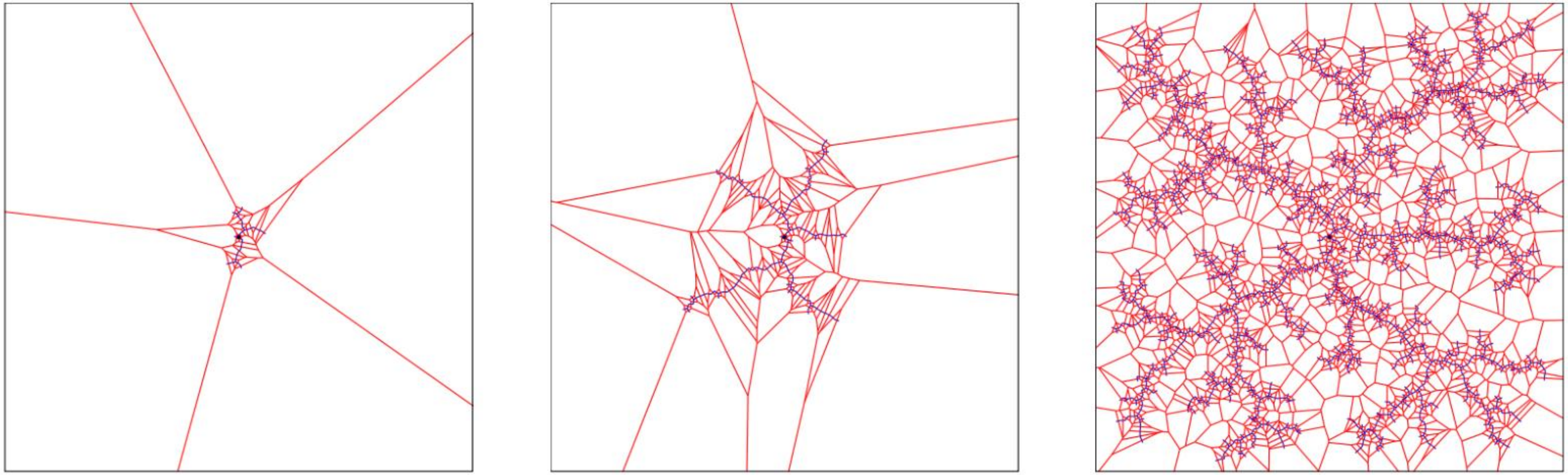
This problem has been addressed in recent years by RRT*, BIT*, Fast-Marching Trees

RRT: properties of the planning algorithm

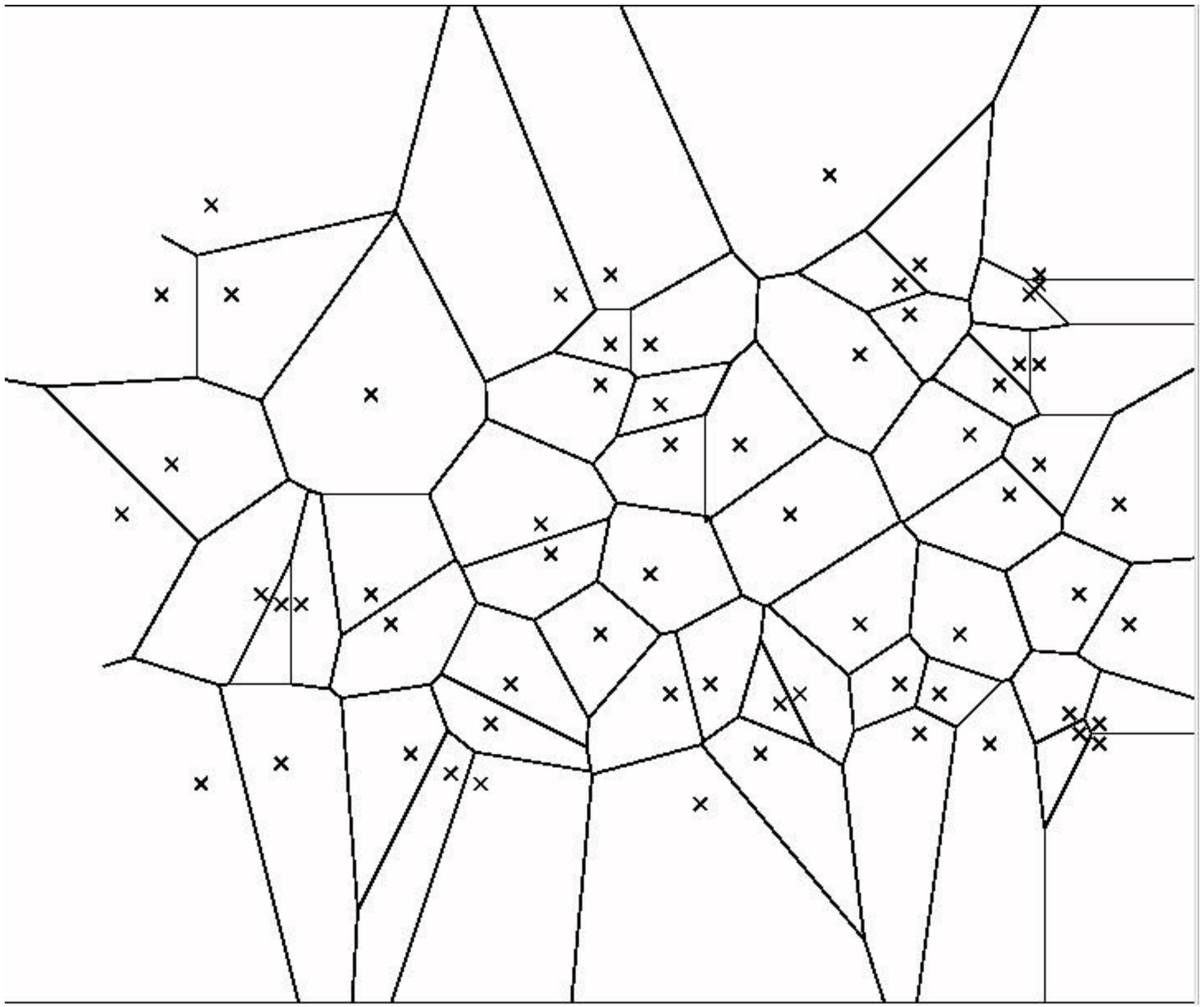
#1: The RRT will eventually cover the space, i.e. it is a space-filling tree

#2: The RRT will NOT compute the optimal path asymptotically

#3: The RRT will exhibit "Voronoi bias," i.e. new nodes will fall in free regions of Voronoi diagram (cells consist of points that are closest to a node)



Voronoi diagram



RRT: properties of the planning algorithm

#1: The RRT will eventually cover the space, i.e. it is a space-filling tree

#2: The RRT will NOT compute the optimal path asymptotically

#3: The RRT will exhibit "Voronoi bias," i.e. new nodes will fall in free regions of Voronoi diagram

#4: The probability of RRT finding a path increases exponentially in the number of iterations

RRT: properties of the planning algorithm

#1: The RRT will eventually cover the space, i.e. it is a space-filling tree

#2: The RRT will NOT compute the optimal path asymptotically

#3: The RRT will exhibit "Voronoi bias," i.e. new nodes will fall in free regions of Voronoi diagram

#4: The probability of RRT finding a path increases exponentially in the number of iterations

#5: The distribution of RRT's nodes is the same as the distribution used in SampleFree()

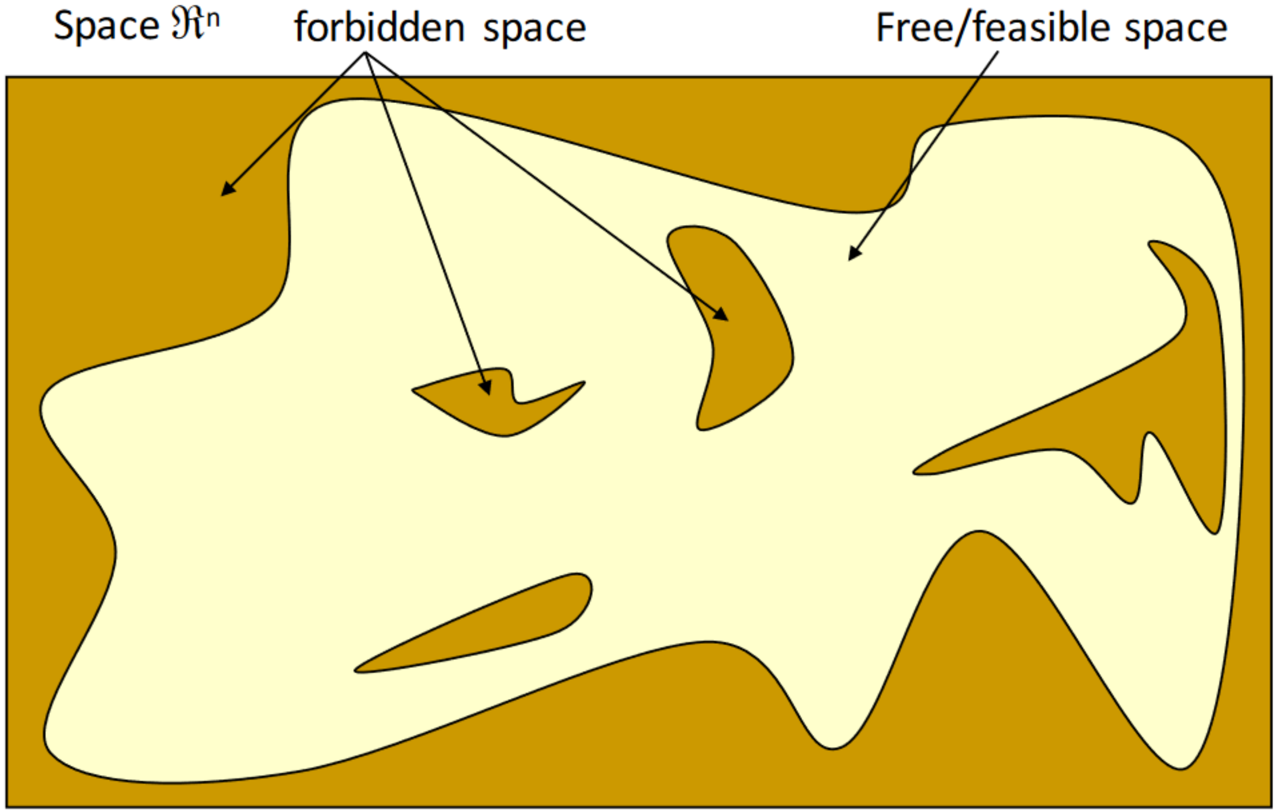
RRT variants: bidirectional search

Probabilistic RoadMaps (PRMs)

- RRTs were good for single-query path planning
- You need to re-plan from scratch for every query A $\rightarrow$ B

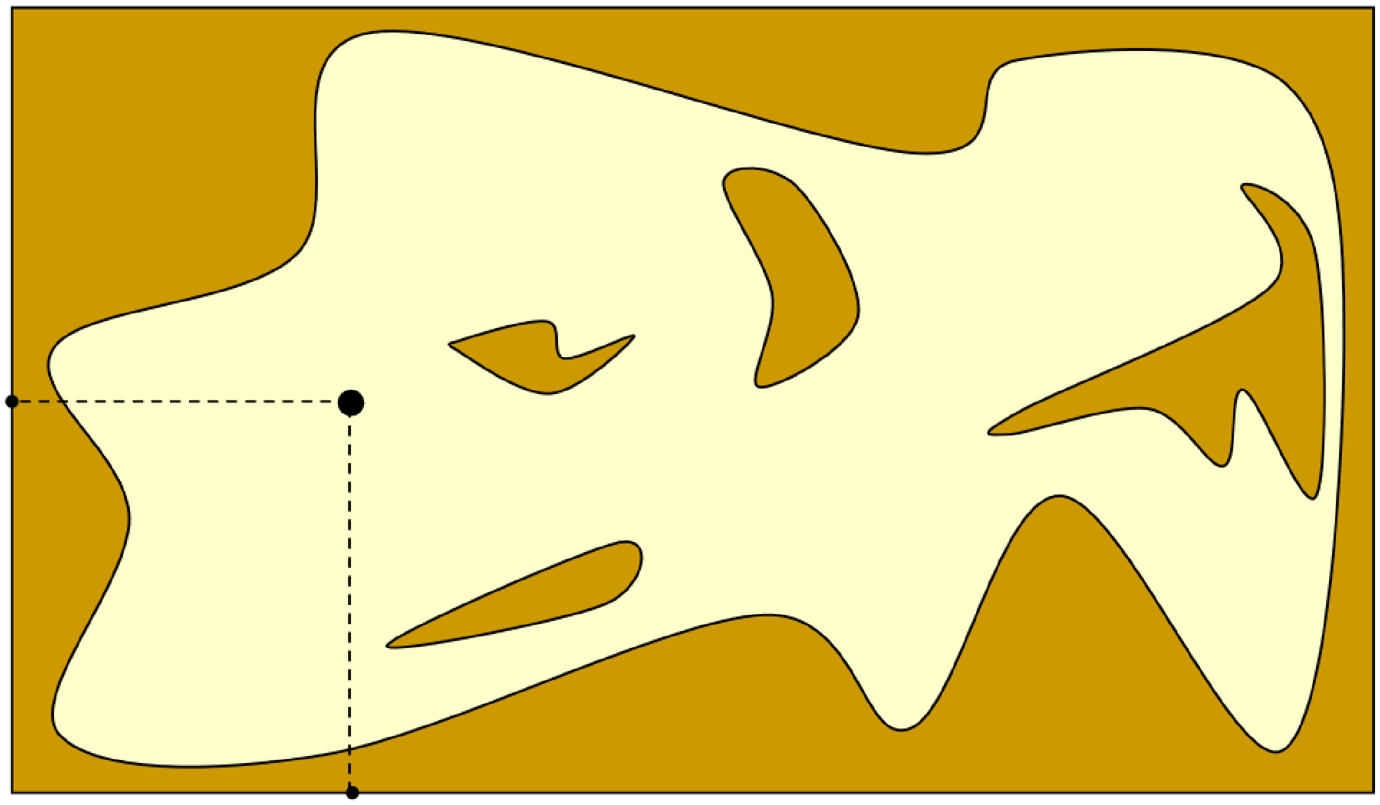
- PRM addresses this problem
- It is good for multi-query path planning

PRM

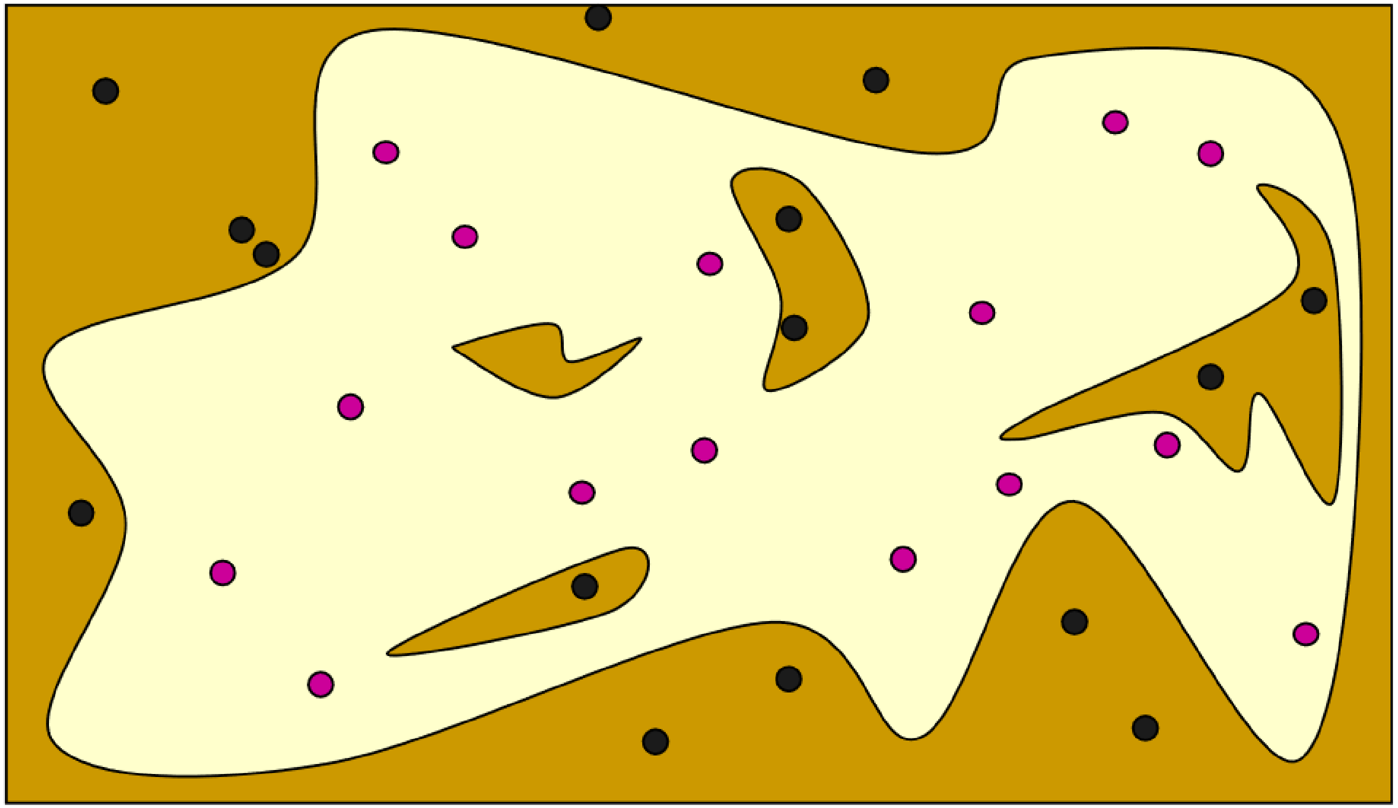


PRM

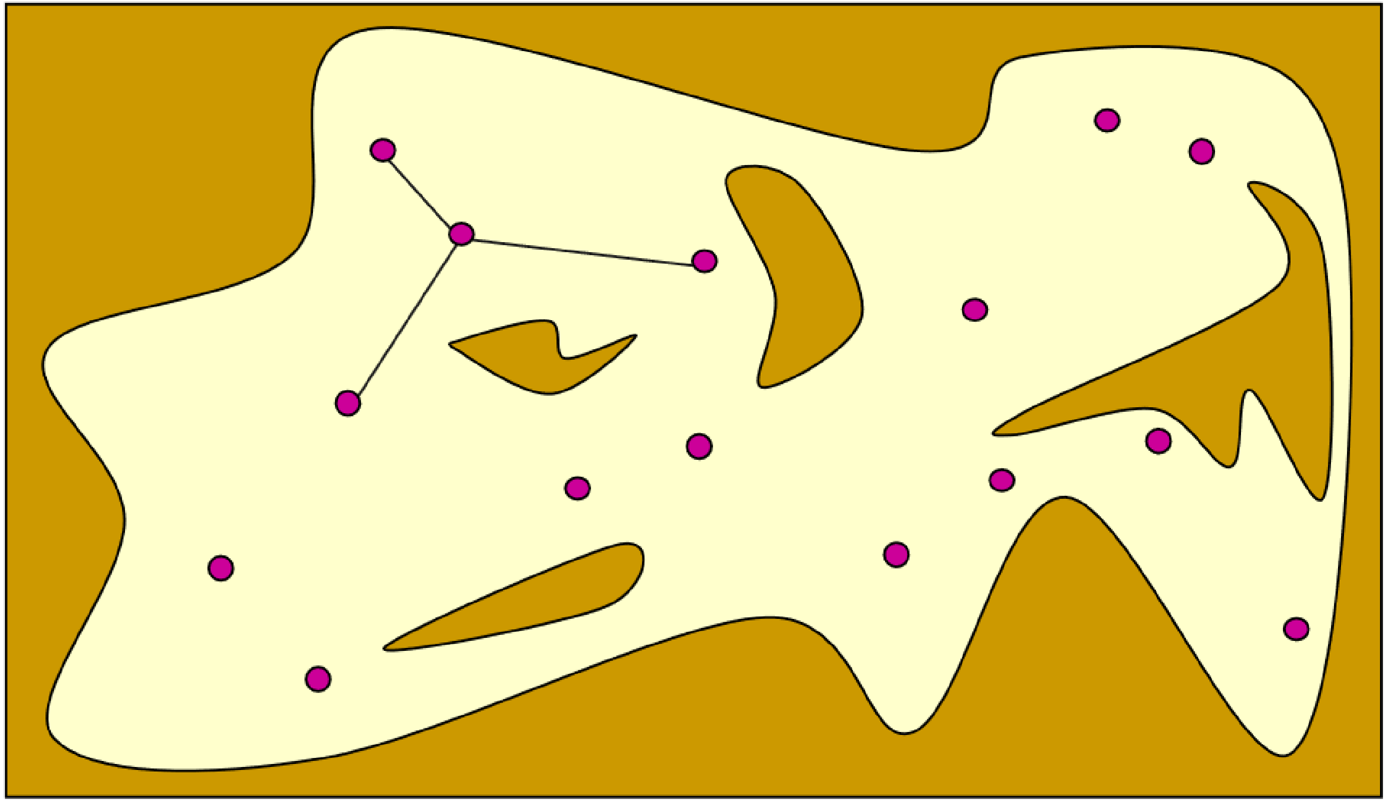
Configurations are sampled by picking coordinates at random



PRM



PRM

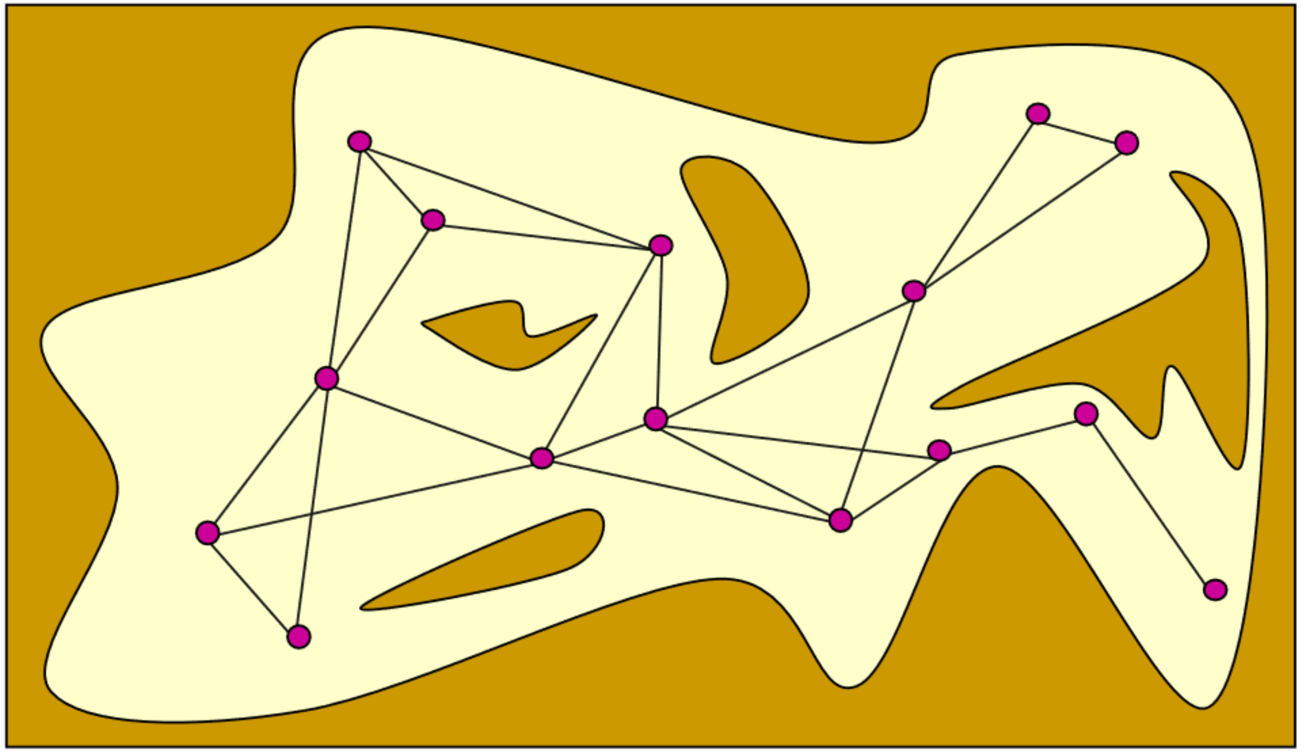


Each node is connected to its neighbors (e.g. within a radius)

PRM

In the offline PRM construction phase we maintain a matrix D[i, j] which contains the total distance of the shortest path from node i to node j.

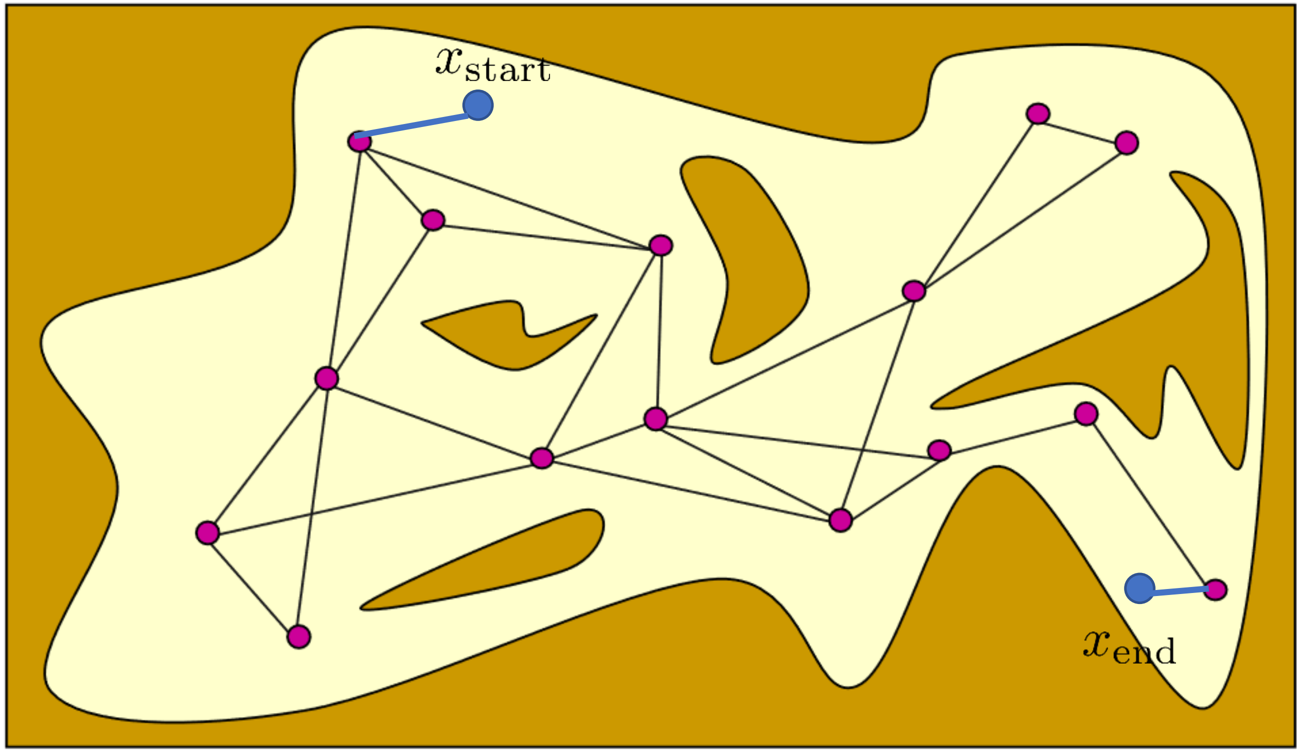
We can do this with an all pairs shortest paths algorithm and then incrementally update D as new nodes are added.



PRM

In the online PRM query phase we are given two endpoints (not vertices of the graph) and we want to find the shortest path between them, by making use of the matrix D[i, j] that was precomputed in the offline phase.

We can incorporate the endpoints in the graph and add 1 row and 1 column to D



PRM

$V \leftarrow \{x_{\mathrm{init}}\} \cup \{\texttt{SampleFree}_i\}_{i=1,\ldots,n}$; $E \leftarrow \emptyset$;

foreach $v \in V$ **do**

  $U \leftarrow \texttt{Near}(G = (V, E), v, r) \setminus \{v\}$;

  foreach $u \in U$ **do**

    if $\texttt{CollisionFree}(v, u)$ **then** $E \leftarrow E \cup \{(v, u), (u, v)\}$

return $G = (V, E)$;

To perform a query (A->B) we need to connect A and B to the PRM.
We can do this by nearest neighbor search (kd-trees, hashing etc.)

PRM

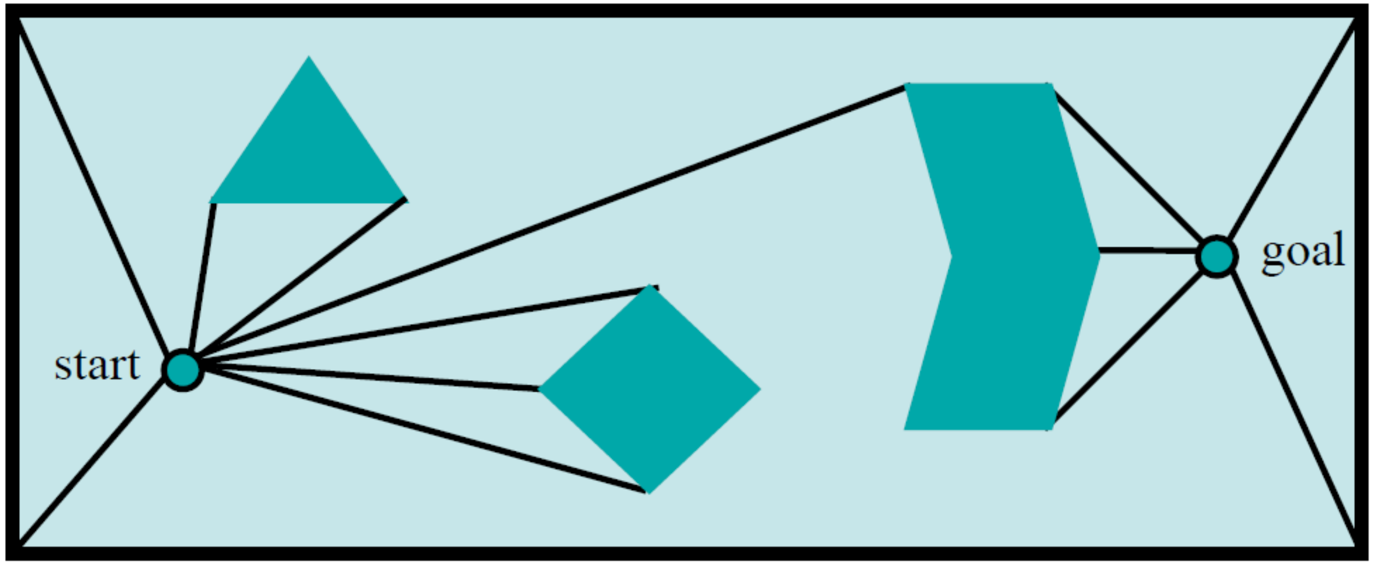
$V \leftarrow \{x_{\text{init}}\} \cup \{\texttt{SampleFree}_i\}_{i=1,\ldots,n}$; $E \leftarrow \emptyset$;
foreach $v \in V$ **do**
    $U \leftarrow \texttt{Near}(G = (V, E), v, r) \setminus \{v\}$;
    foreach $u \in U$ **do**
        if $\texttt{CollisionFree}(v, u)$ **then** $E \leftarrow E \cup \{(v, u), (u, v)\}$
return $G = (V, E)$;

Range search can be done efficiently using a kd-tree

To perform a query (A->B) we need to connect A and B to the PRM.
We can do this by nearest neighbor search (kd-trees, hashing etc.)

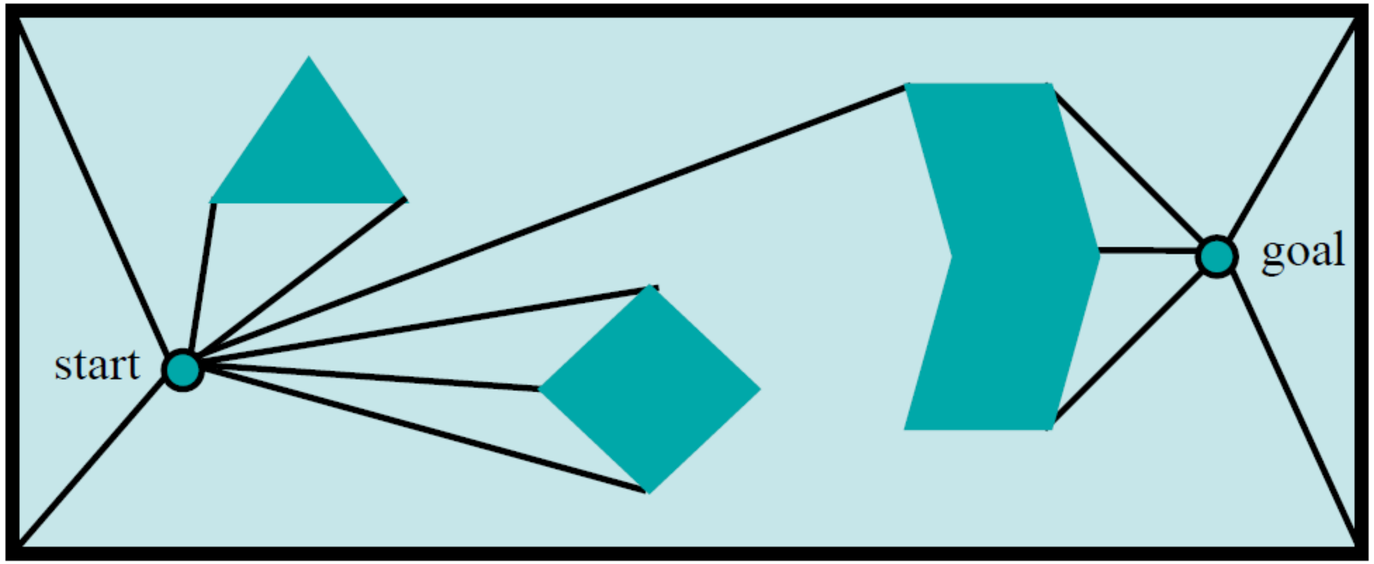
Visibility Graph Path Planning

- First, draw lines of sight from the start and goal to all "visible" vertices and corners of the world.



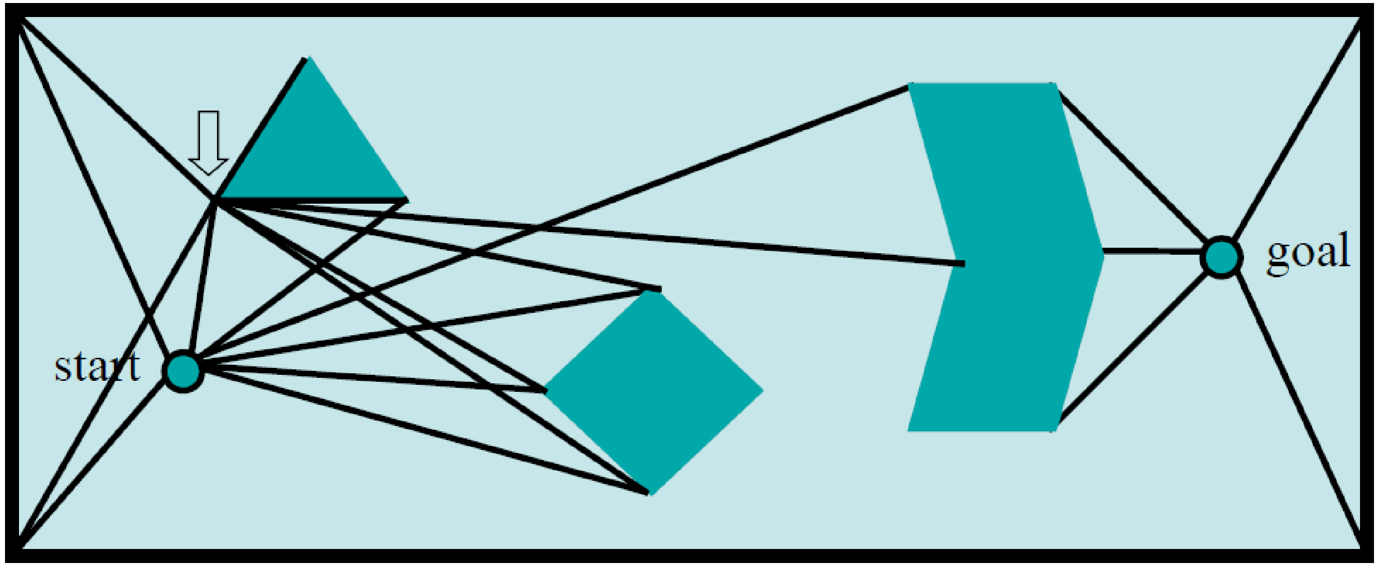
Visibility Graph Path Planning

- First, draw lines of sight from the start and goal to all "visible" vertices and corners of the world.



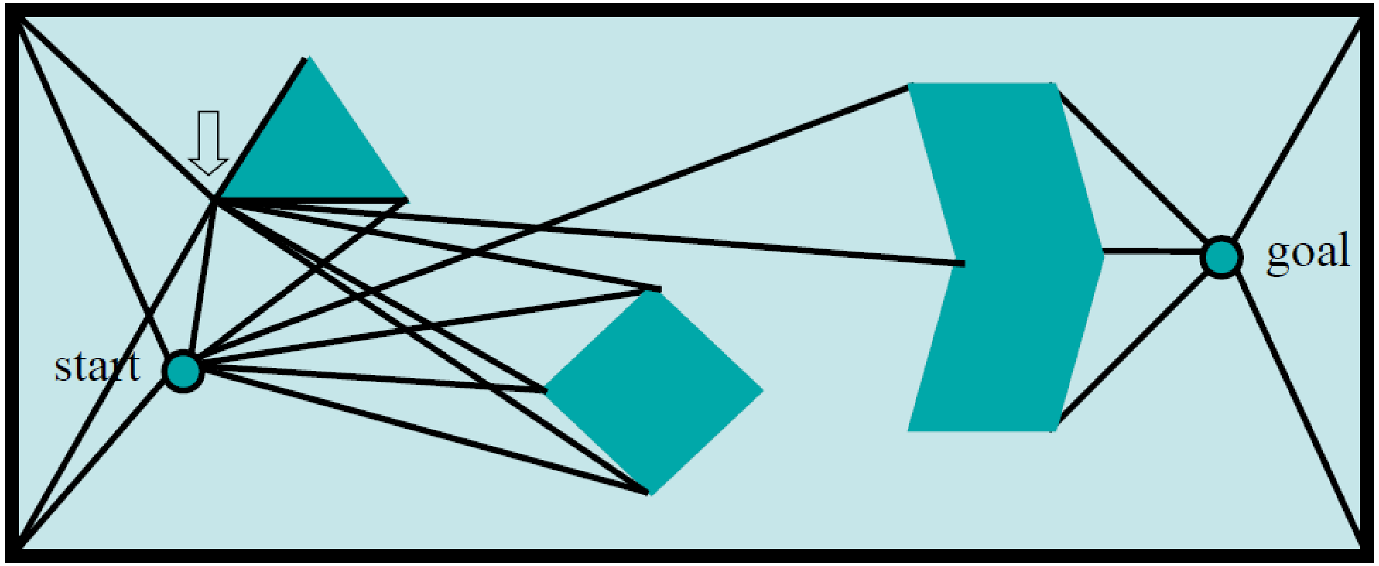
Visibility Graph Path Planning

- Second, draw lines of sight from every vertex of every obstacle like before. Remember lines along edges are also lines of sight.



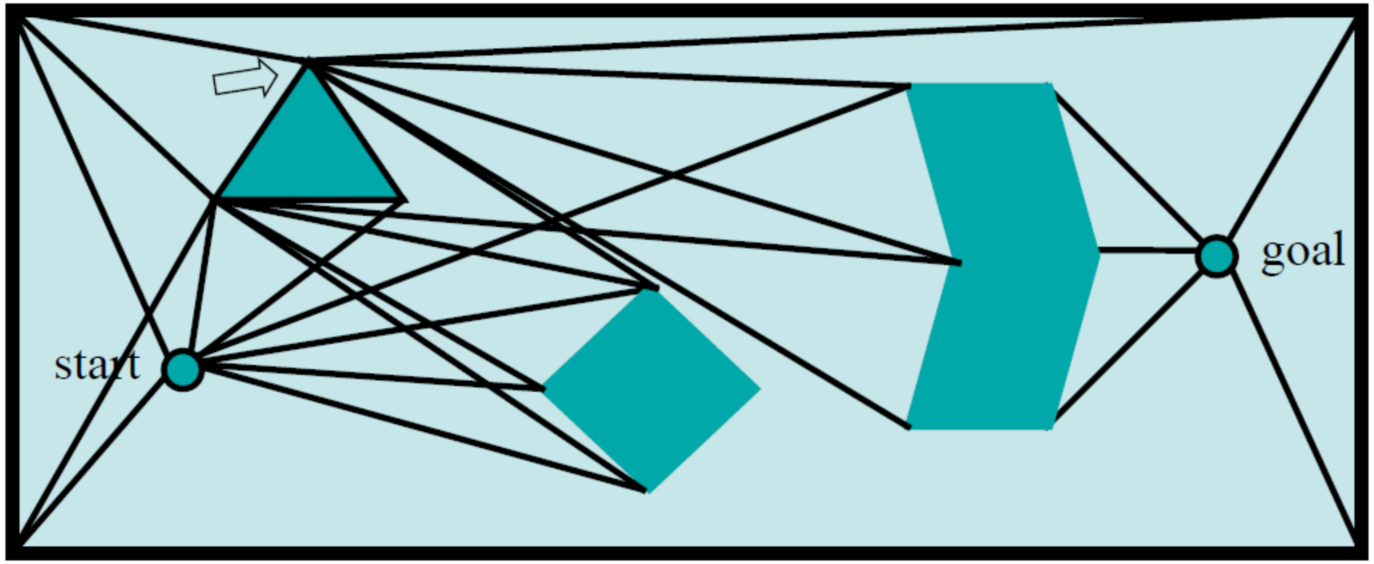
Visibility Graph Path Planning

- Second, draw lines of sight from every vertex of every obstacle like before. Remember lines along edges are also lines of sight.



Visibility Graph Path Planning

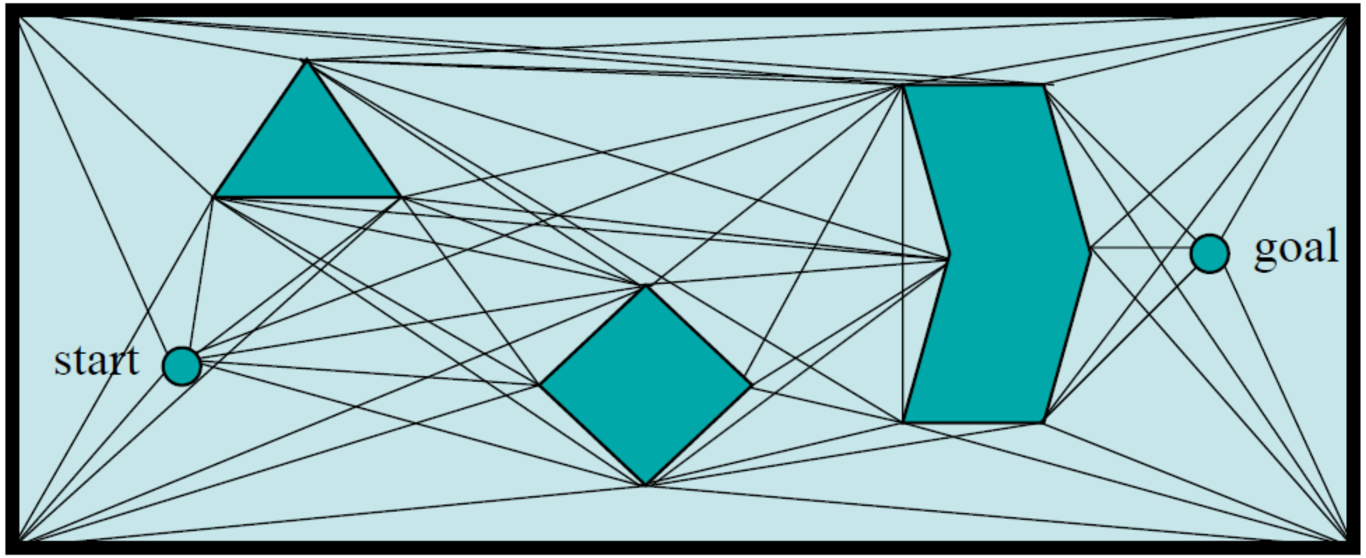
- Second, draw lines of sight from every vertex of every obstacle like before. Remember lines along edges are also lines of sight.



Visibility Graph Path Planning

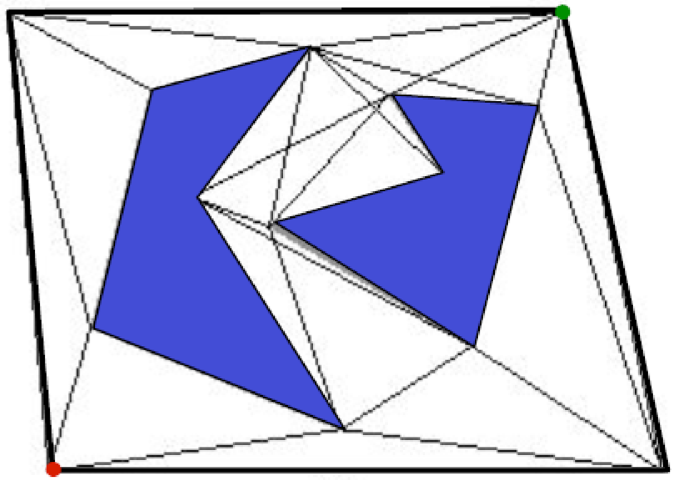
- Repeat until you're done.

Visibility graph

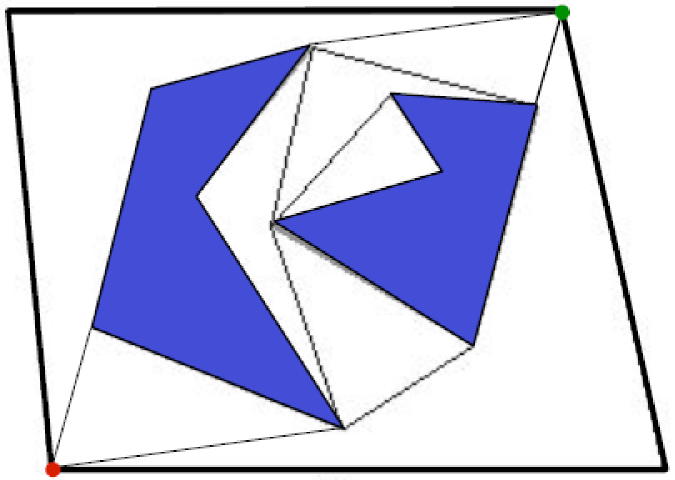


Can use graph search on visibility graph to find shortest path

Visibility Graph Path Planning



Full visibility graph



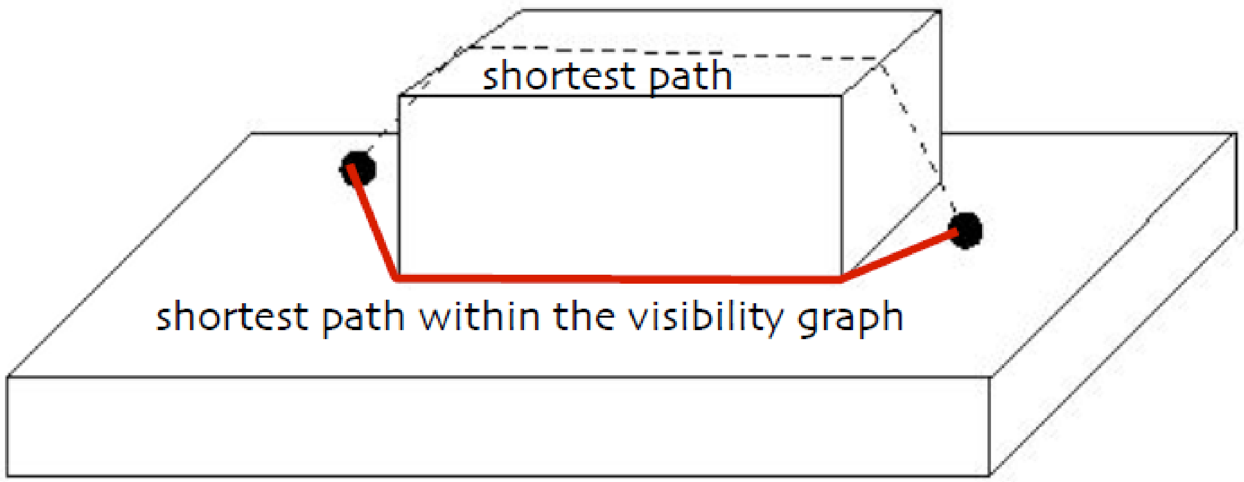
Reduced visibility graph, i.e., not including segments that extend into obstacles on either side.

(but keeping endpoints' roads)

Potential problem: shortest path touches obstacle corners. Need to dilate obstacles.

Visibility Graph Path Planning

Visibility graphs do not preserve their optimality in higher dimensions:



Path smoothing

- Plans obtained from any of these planners are not going to be smooth
- A plan is a sequence of states: $\pi = (\mathbf{x}_{\text{src}}, \mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N, \mathbf{x}_{\text{dest}})$
- We can get a smoother path $\text{smooth}(\pi) = (\mathbf{x}_{\text{src}}, \mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_N, \mathbf{x}_{\text{dest}})$ by minimizing the following cost function

$$f(\mathbf{y}_1, ..., \mathbf{y}_N) = \sum_{t=1}^{N} ||\mathbf{y}_t - \mathbf{x}_t||^2 + \alpha \sum_{t=1}^{N} ||\mathbf{y}_t - \mathbf{y}_{t-1}||^2$$

Stay close to the old path

Penalize squared length

- May need to stop smoothing when smooth path comes close to obstacles.