

## COMP417 Introduction to Robotics and Intelligent Systems

Lecture 4: Sensors and Actuators

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## Quick recap from last week

• What is a non-holonomic constraint?

#### Noise

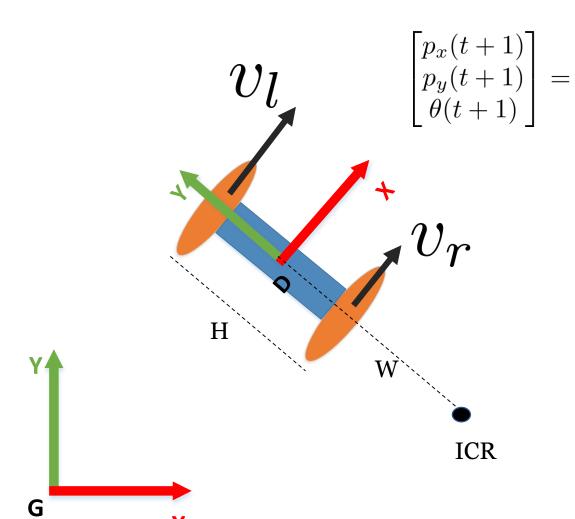
Anything that we do not bother modelling with our model

• Example 1: "assume frictionless surface"

• Example 2: Taylor series expansion (only first few terms are dominant)

• With models, can be thought of as approximation error.

### What type of vehicle is this?



$$\begin{bmatrix} p_x(t+1) \\ p_y(t+1) \\ \theta(t+1) \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x(t) - ICR_x \\ p_y(t) - ICR_y \\ \theta(t) \end{bmatrix} + \begin{bmatrix} ICR_x \\ ICR_y \\ \omega\delta t \end{bmatrix}$$

$$ICR = [p_x - W\sin\theta, p_y + W\cos\theta]$$

#### Special cases:

- moving straight  $v_l = v_r$
- in-place rotation  $v_l = -v_r$
- rotation about the left wheel  $v_l = 0$

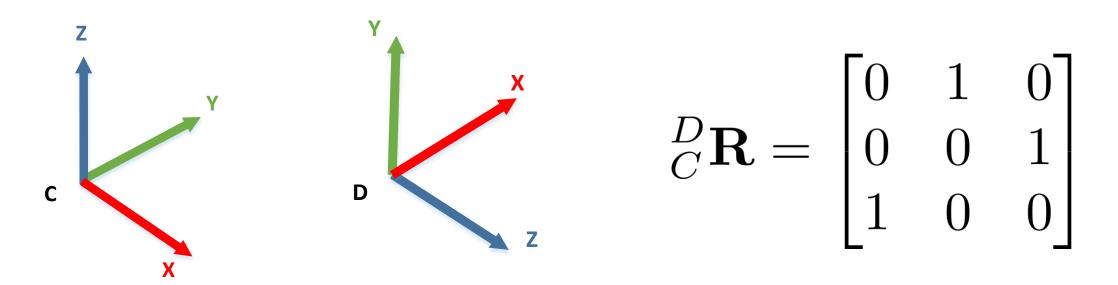
#### The Dubins car is non-holonomic

- Dubins car is constrained to move straight towards the direction it is currently heading. It cannot move sideways. It needs to "parallel park" to move laterally.
- In a small time interval dt the vehicle is going to move by  $\delta p_x$  and  $\delta p_y$  in the global frame of reference. Then from the dynamical system:

$$\begin{array}{lcl} \delta p_x \sin(\theta) & = & v_x \cos(\theta) \sin(\theta) dt \\ \delta p_y \cos(\theta) & = & v_x \sin(\theta) \cos(\theta) dt \end{array} \longrightarrow \delta p_x \sin(\theta) = \delta p_y \cos(\theta) \longrightarrow v_x \sin(\theta) = v_y \cos(\theta)$$

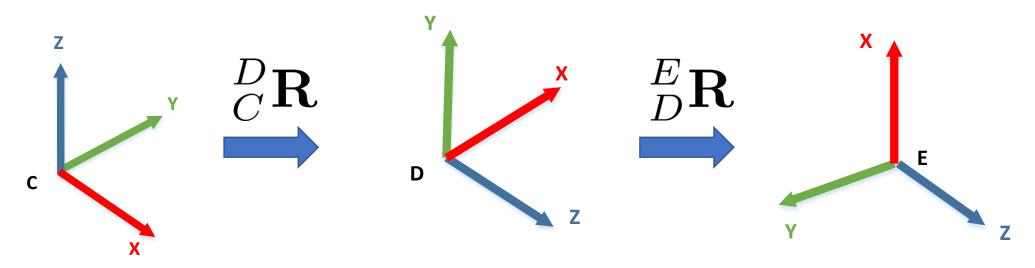
Car is constrained to move along the line of current heading, i.e. non-holonomic

## Example: finding a rotation matrix that rotates one vector to another



This matrix transforms the x-axis of frame C to the z-axis of frame D. Same for y and z axes.

### Compound rotations



$$_{C}^{E}\mathbf{R}=_{D}^{E}\mathbf{R}_{C}^{D}\mathbf{R}$$

### Converting axis-angle to quaternion

 Given angle theta and axis v the equivalent quaternion representation is

$$\mathbf{q} = \left[\sin(\theta/2)v_1, \sin(\theta/2)v_2, \sin(\theta/2)v_3, \cos(\theta/2)\right]$$

$$\mathbf{q} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} + w$$

• Just like in the case of rotation matrices we denote the source and target frames of the rotation quaternion:  ${}^W_B{f q}$ 

### Today: Sensors and Actuators

• Our last introductory/background material lecture before we start talking about algorithms — the main topic of the course.

#### • Sensors:

- Characteristics and types
- Measurement noise
- Required bandwidth

#### • Actuators:

- Types of motors
- Pulse-Width Modulation

### Types of sensors

- General classification:
  - contact vs. non-contact
  - active vs. passive
  - sampling rate: fast vs. slow
  - local vs. non-local
- General examples:
  - vision
  - laser
  - radar
  - sonar
  - compass, gyroscope, accelerometer
  - touch (tactile)
  - infrared

#### Sensors

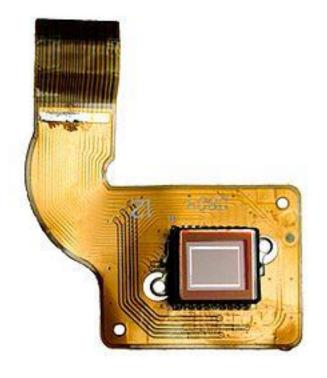
• Devices that can sense and measure physical properties of the environment.

- Key phenomenon is **transduction** (conversion of energy from one form to another). E.g.:
  - Imaging sensors: light to pixel voltages
  - Depth sensors: mechanical pressure to voltage
- Measurements are **noisy**, and difficult to interpret

#### Sensors: general characteristics

- Sensitivity: (change of output) ÷ (change of input)
- Linearity: constancy of (output ÷ input)
- Measurement range: [min, max] or {min, max}
- Response time: time required for input change to cause output change
- Accuracy: difference between measurement and actual
- Repeatability/Drift: difference between repeated measures
- Resolution: smallest observable increment
- Bandwidth: required rate of data transfer
- SNR: signal-to-noise ratio

## Sensors: vision



CCD image sensor

#### **CCD** (charge-coupled device) imaging sensors:

- Capacitor array accumulates electric charge proportional to light intensity.
- Each capacitor's charge is transferred to its neighbor.
- Last capacitor's charge gets amplified and output as voltage.
- (+) High-quality, low-noise images
- (-) Higher power consumption
- (-) Slow readout
- (-) Specialized fabrication

voltage → analog-to-digital converter → pixel value in {0, 255}

#### CMOS (complementary metal-oxide semi-conductor) imaging sensors:

- One amplifier per pixel
- (+) Low power
- (+) Fast readout
- (+) Easier to fabricate
- (-) Poor low-light sensitivity
- (-) Higher noise

#### CCD vs CMOS

#### **CCD Image Sensor CMOS Image Sensor** Photo Sensor (b) Pixel (a) Pixel (a) Amplifier (y) Photo Sensor (b) Light (Light-sensitive Region) (Light-sensitive Region) Light ∡ Signal Charge (Electrons) Vertical CCD (c) Pixel-select Switch (e) Pixel Row (j) Column Signal Wire (f) (Micro Wire) Column-select Switch (g) Column Row Signal Wire (i) Circuit (h) (Micro Wire) Output O Amplifier (x) Horizontal CCD (d)

### Global vs. Rolling Shutter

Shutter = mechanism that allows light to hit the imaging sensor

Shutter "speed" = Exposure time = time duration in which the sensor is exposed to light







Rolling shutter

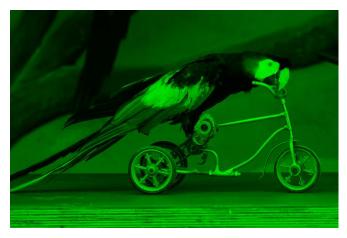
### Reading RGB images from a camera



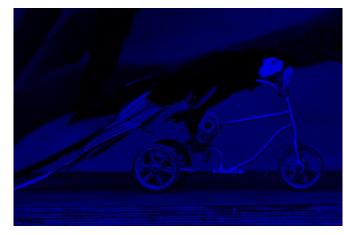
Each pixel contains an intensity value from 0...255



600 x 1000 pixels



600 x 1000 pixels



600 x 1000 pixels

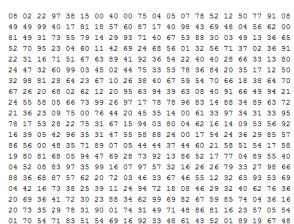
#### Reading images from a camera



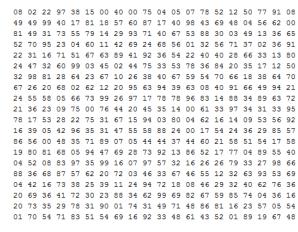
A matrix of  $600 \times 1000 \times 3 =$   $\sim 1.8 \text{ million numbers}$ 

Each pixel contains an intensity value from 0...255

600 x 1000 pixels



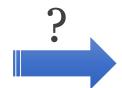
600 x 1000 pixels



600 x 1000 pixels

### Computer/robot vision

```
08 02 22 97 38 15 00 40 00 75 04 05 07 78 52 12 50 77 91 08
        40 17 81 18 57 60 87 17 40 98 43 69 48 04 56 62 00
81 49 31 73 55 79 14 29 93 71 40 67 53 88 30 03 49 13
52 70 95 23 04 60 11 42 69 24 68 56 01 32 56 71 37 02 36 91
22 31 16 71 51 67 63 89 41 92 36 54 22 40 40 28 66 33 13 80
24 47 32 60 99 03 45 02 44 75 33 53 78 36 84 20 35 17 12 50
32 98 81 28 64 23 67 10 26 38 40 67 59 54 70 66 18 38 64 70
67 26 20 68 02 62 12 20 95 63 94 39 63 08 40 91 66 49 94 21
24 55 58 05 66 73 99 26 97 17 78 78 96 83 14 88 34 89 63 72
21 36 23 09 75 00 76 44 20 45 35 14 00 61 33 97 34 31 33 95
78 17 53 28 22 75 31 67 15 94 03 80 04 62 16 14 09 53 56 92
86 56 00 48 35 71 89 07 05 44 44 37 44 60 21 58 51 54 17 58
19 80 81 68 05 94 47 69 28 73 92 13 86 52 17 77 04 89 55 40
04 52 08 83 97 35 99 16 07 97 57 32 16 26 26 79 33 27 98 66
88 36 68 87 57 62 20 72 03 46 33 67 46 55 12 32 63 93 53 69
04 42 16 73 38 25 39 11 24 94 72 18 08 46 29 32 40 62 76 36
20 69 36 41 72 30 23 88 34 62 99 69 82 67 59 85 74 04 36 16
20 73 35 29 78 31 90 01 74 31 49 71 48 86 81 16 23 57 05 54
01 70 54 71 83 51 54 69 16 92 33 48 61 43 52 01 89 19 67 48
```



- 1. I'm seeing a parrot
- 2. I'm seeing a toy bicycle
- 3. The parrot is riding the bicycle
- 4. The bicycle is on top of a desk
- 5. Is this physically plausible?
- 6. Where is the parrot in 3D w.r.t. the camera?
- 7. Where will the parrot go next?
- 8. What is the speed of the parrot?

Conclusions/Inference/Deduction/Estimation

Structured numbers

### Computer/robot vision

```
08 02 22 97 38 15 00 40 00 75 04 05 07 78 52 12 50 77 91 08
52 70 95 23 04 60 11 42 69 24 68 56 01 32 56 71 37 02 36 91
22 31 16 71 51 67 63 89 41 92 36 54 22 40 40 28 66 33 13 80
24 47 32 60 99 03 45 02 44 75 33 53 78 36 84 20 35 17 12 50
32 98 81 28 64 23 67 10 26 38 40 67 59 54 70 66 18 38 64 70
67 26 20 68 02 62 12 20 95 63 94 39 63 08 40 91 66 49 94 21
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78 17 53 28 22 75 31 67 15 94 03 80 04 62 16 14 09 53
86 56 00 48 35 71 89 07 05 44 44 37 44 60 21 58 51 54 17 58
19 80 81 68 05 94 47 69 28 73 92 13 86 52 17 77 04 89 55 40
04 52 08 83 97 35 99 16 07 97 57 32 16 26 26 79 33 27 98 66
88 36 68 87 57 62 20 72 03 46 33 67 46 55 12 32 63 93 53 69
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```

Structured numbers

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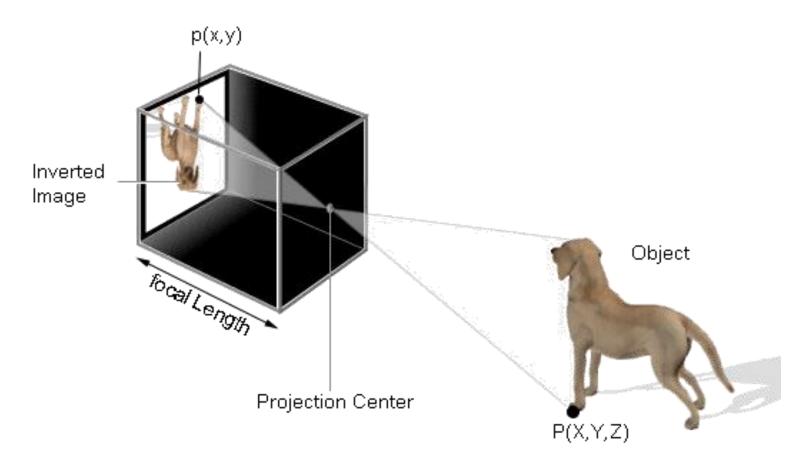
Conclusions/Inference/Deduction/Estimation

#### Camera lenses

- Lens determines:
  - image distortion
  - focus
  - sharpness or blur
- Lens characteristics:
  - focal length
  - aperture
  - depth-of-field

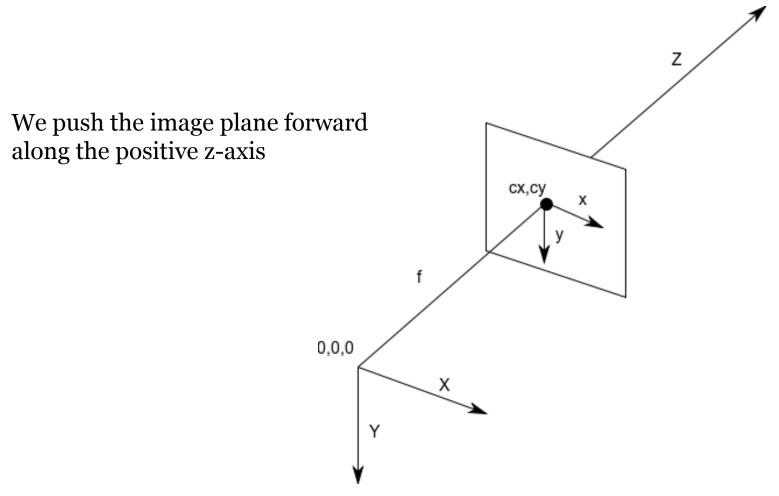


#### Pinhole Camera Model

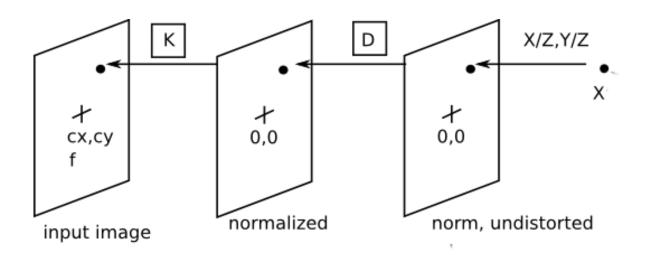


We know **approximately** how a 3D point (X,Y,Z) projects to pixel (x,y) We call this the **pinhole projection model** 

## Pinhole Camera: Avoiding to think about image inversion

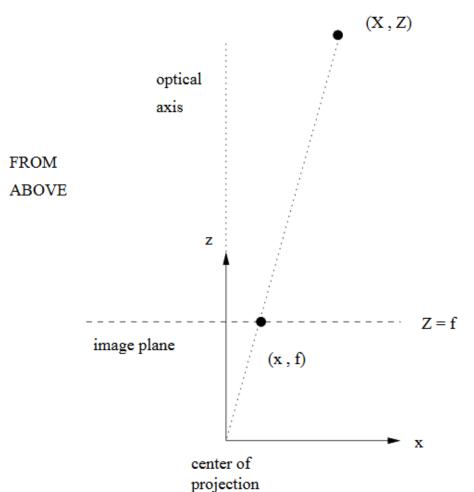


## Pinhole Camera: From 3D points to pixels



- (1)  $[x,y] = \pi(X,Y,Z)$  perspective projection
- (2)  $[x^*, y^*] = D(x,y)$  lens distortion
- (3)  $[u,v,1] = K[x^*,y^*,1]$ ' pixel coordinates

# (1) Perspective projection $[x,y] = \pi(X,Y,Z)$



By similar triangles: x/f = X/Z

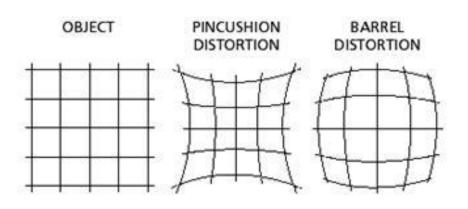
So, x = f \* X/Z and similarly y = f \* Y/Z

Problem: we just lost depth (Z) information by doing this projection, i.e. depth is now uncertain.

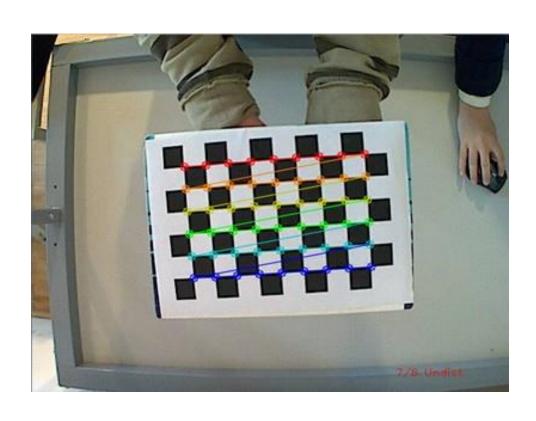
http://www.cim.mcgill.ca/%7Elanger/558.html

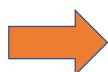
# (2) Lens distortion $[x^*, y^*] = D(x,y)$

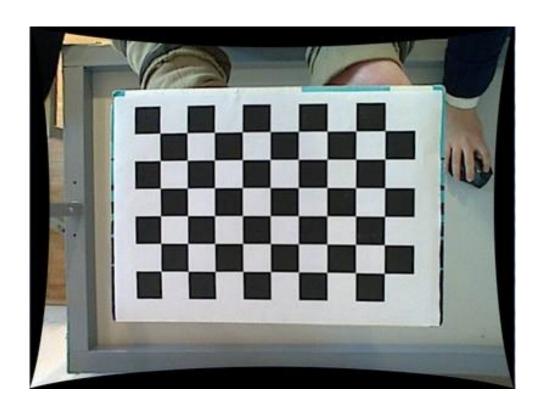




## (2) Estimating parameters of lens distortion: $[x^*, y^*] = D(x,y)$







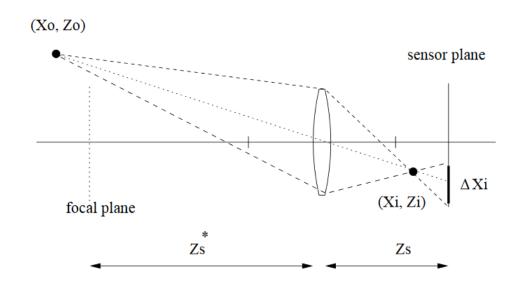
$$x^* = x \frac{1 + k_1 r^2 + k_2 r^4 + k_3 r^6}{1 + k_4 r^2 + k_5 r^4 + k_6 r^6}$$
 where  $r = x^2 + y^2$ 

## (3) From metric to pixel coordinates $[u,v,1] = K[x^*,v^*,1]$

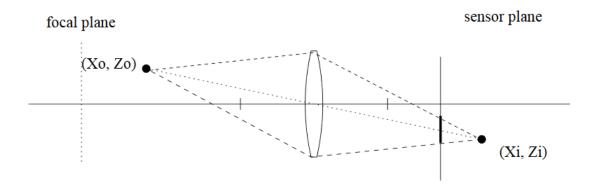
- Let  $f_x$  and  $f_y$  describe the number of pixels per mm in the sensor
- Typically (0,0) is the pixel coordinate of the top left pixel, not of the principal point.
- Let  $c_x$  and  $c_y$  describe the pixel coordinates of the principal point.

The unknown parameters

#### Non-pinhole cameras: thin lens model

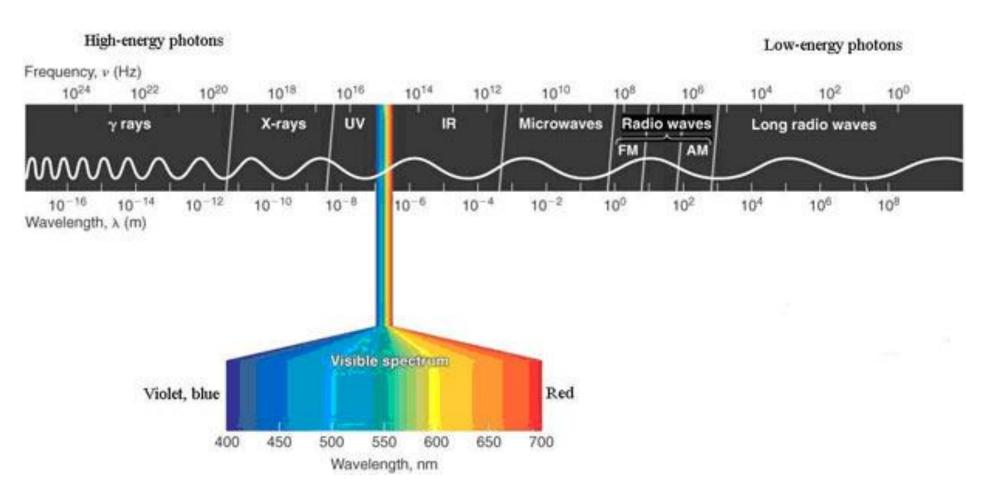


Unlike the pinhole camera, this is able to model blur.

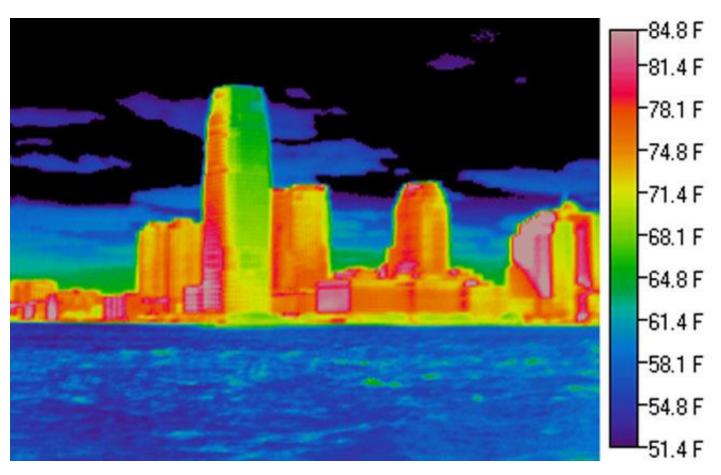


http://www.cim.mcgill.ca/%7Elanger/558.html

## Beyond the visible spectrum: infrared cameras



# Beyond the visible spectrum: infrared cameras



Drawback:
Doesn't work underwater

# Beyond the visible spectrum: infrared cameras





#### Main ideas:

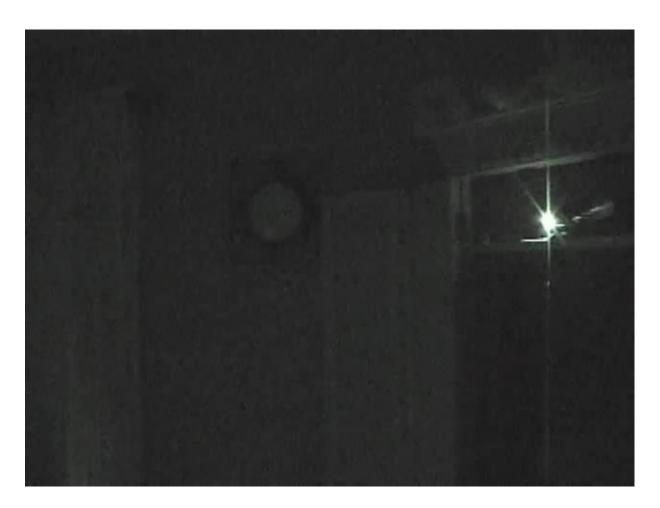
- Active sensing
- Projector emits infrared light in the scene
- Infrared sensor reads the infrared light
- Deformation of the expected pattern allows computation of the depth

#### Drawbacks:

- Does not work outdoors, sunlight saturates its measurements
- Maximum range is [0.5, 8] meters

#### Advantages:

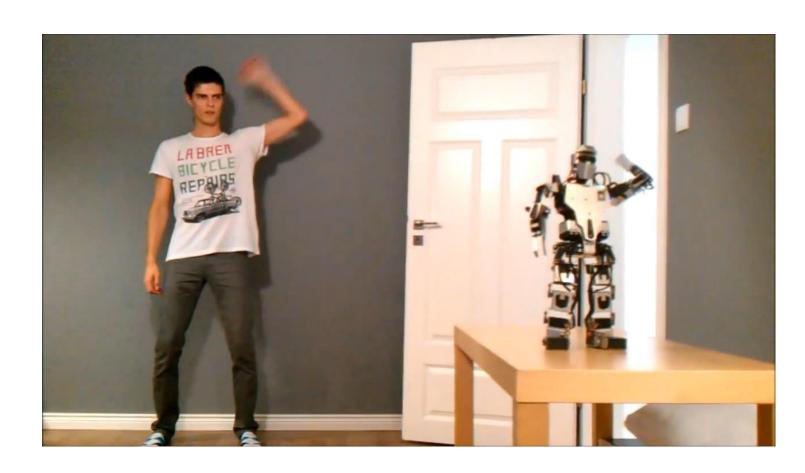
- Real-time depth estimation at 30Hz
- Cheap



Enabled a wave of research, applications, and video games, based on real-time skeleton tracking



Despite their drawbacks RGBD sensors have been extensively used in robotics.

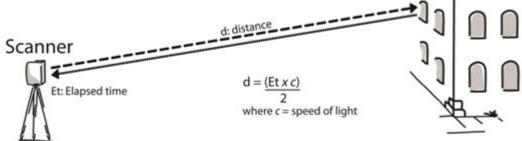


### 3D LIDAR (Light detection and ranging)

Produces a pointcloud of 3D points and intensities

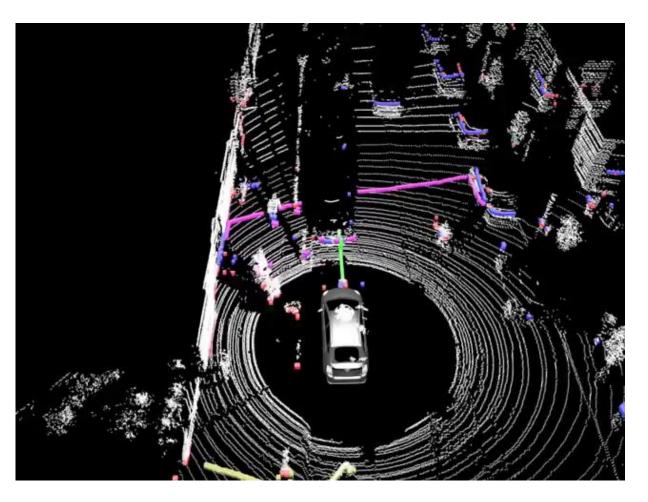
- (x,y,z) in the laser's frame of reference
- Intensity is related to the material of the object that reflects the light

Works based on time-of-flight for each beam to return back to the scanner



Not very robust to adverse weather conditions: rain, snow, smoke, fog etc.

Used in most self-driving cars today for obstacle detection. Range < 100m.



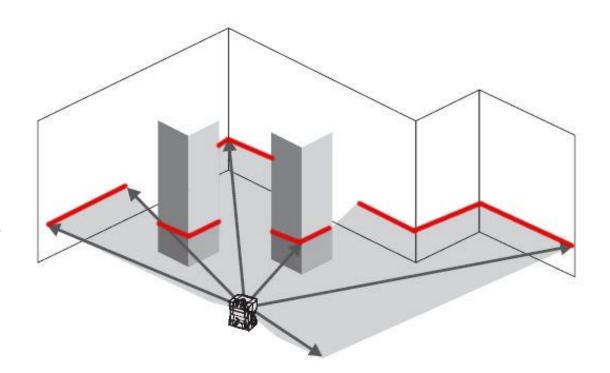
Usually around 1million points in a single pointcloud

# 2D LIDAR (Light detection and ranging)

Produces a scan of 2D points and intensities

- (x,y) in the laser's frame of reference
- Intensity is related to the material of the object that reflects the light

Certain surfaces are problematic for LIDAR: e.g. glass

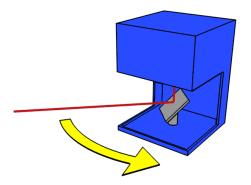


# 2D LIDAR (Light detection and ranging)

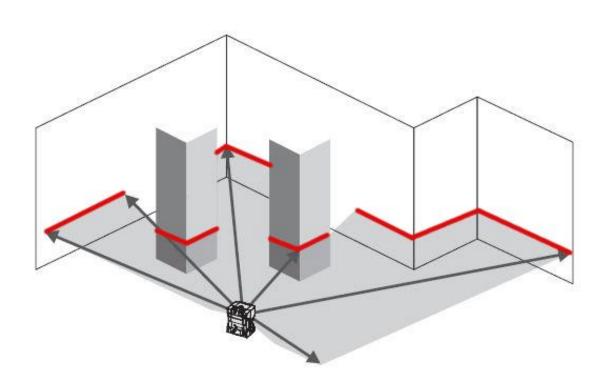
Produces a scan of 2D points and intensities

- (x,y) in the laser's frame of reference
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Certain surfaces are problematic for LIDAR: e.g. glass



Lots of moving parts: motors quickly rotate the laser beam and once complete (angle bound reached) a scan is returned. I.e. points are not strictly speaking time-synchronized, even though we usually treat them as such.



Usually around 1024 points in a single scan.

#### **Inertial Sensors**

- Gyroscopes, Accelerometers, Magnetometers
- Inertial Measurement Unit (IMU)
- Perhaps the most important sensor for 3D navigation, along with the GPS

• Without IMUs, plane autopilots would be much harder, if not impossible, to build

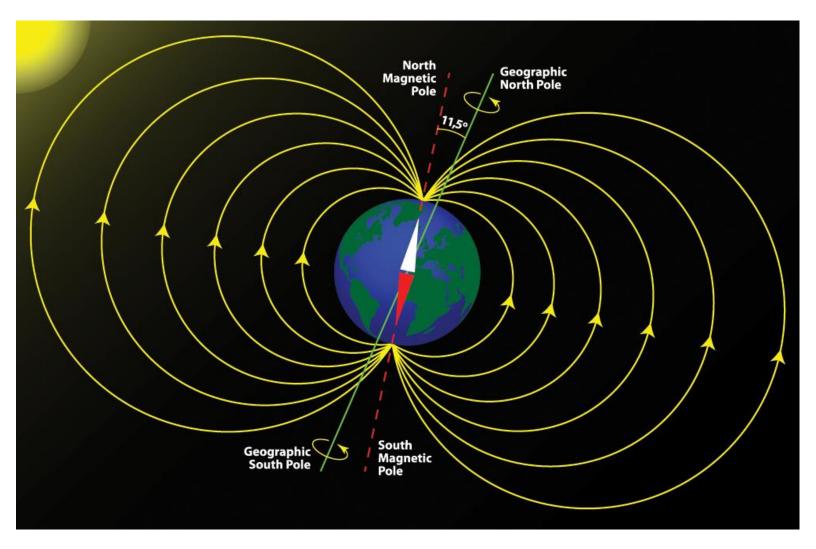
## Magnetometers

#### Drawbacks:

- Needs careful calibration
- Needs to be placed away from moving metal parts, motors

#### Advantages:

Can be used as a compass for absolute heading



## Gyroscopes

- Measure angular velocity in the body frame
- Often affected by noise and bias

$$\omega_{\text{measured}}(t) = \omega_{\text{true}}(t) + b_g(t) + n_g(t)$$

• We integrate it to get 3D orientation (Euler angles, quaternions rotation matrices), but there is drift due to noise and bias

#### Accelerometers

- Measure linear acceleration relative to freefall (measured in g)
- A free-falling accelerometer in a vacuum would measure zero g
- An accelerometer resting on the surface of the earth would measure 1g
- Also affected by bias and noise. Usually modelled as:

$$a_{\text{measured}}(t) = R({}_{G}^{I}q(t))({}^{G}a - {}^{G}g)(t) + b_{a}(t) + n_{a}(t)$$

Where g is the gravity vector, I is the IMU body frame and G is the fixed world frame

Double integration to get position is very noisy. Errors grow quadratically with time.

#### Inertial Measurement Unit

- Combines measurements from accelerometer, gyroscope, and magnetometer to output an estimate of orientation with reduced drift.
- Does not typically provide a position estimate, due to double integration.
- Runs at 100-1000Hz
- Expect yaw drift of 5-10 deg/hour on most modern low-end IMUs

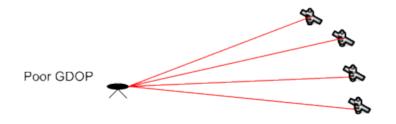
### Global Positioning System: Satellites

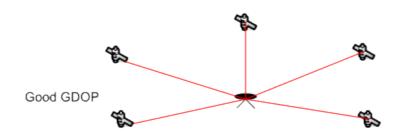
- Each GPS satellite periodically transmits:
  - [Coarse/Acquisition code] A 1023-bit pseudorandom binary sequence (PRN code), which repeats every 1 ms, unique for each satellite (no correlation with other satellites).
  - [Navigation frame] A 1500-bit packet that contains
    - GPS date, time, satellite health
    - Detailed orbital data for the satellite, accurate for the next ~4hrs
    - PRN codes and status of all satellites in the network
    - Takes 12.5mins to transmit
  - [Precision code] A 6.2-terabit code for military use.
- Carrier frequencies are 1575.42 MHz (L1) and 1227.60 MHz (L2)

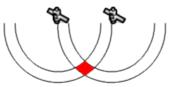
### Global Positioning System: Receivers

- Each (civilian) GPS receiver:
  - Knows the PRN codes for each satellite in advance
  - Correlates received PRN signal with database PRN signal → time shift → noisy distance to satellite
  - If 4 or more satellite PRN codes are received, it does **trilateration** to compute latitude and longitude

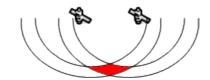
### Global Positioning System: Receivers and Dilution of Precision







Well spaced satellites- low uncertainty of position

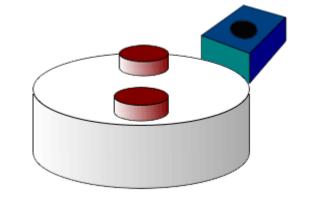


Poorly spaced satellites – high uncertainty of position

Geometry in 2-D (GPS Basics, 2000)

#### Hall Effect Sensor

• Varies its voltage in response to a magnetic field



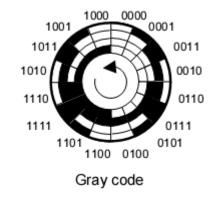
• Used as a proximity switch, to measure a full rotation of a wheel for example

• Used to measure rate of rotation of wheels

# Rotary Encoder

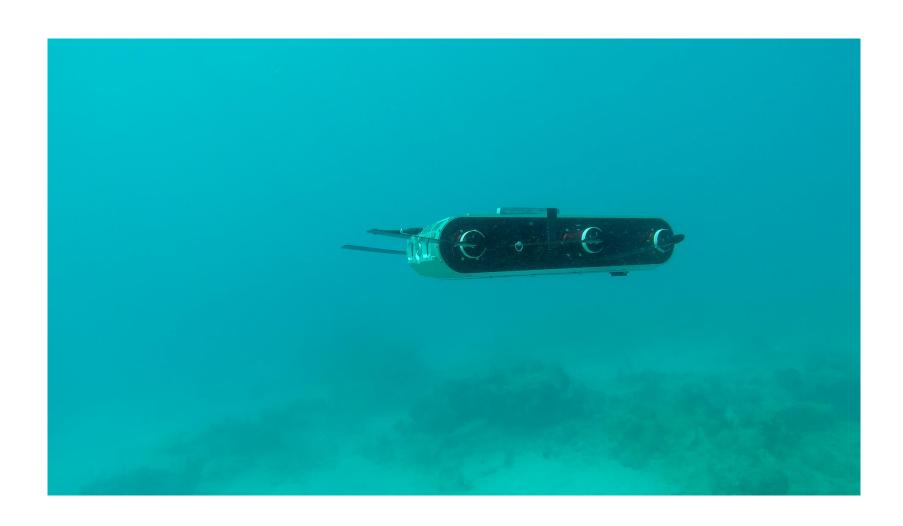
• Contains an analog to digital converter for encoding the angle of a shaft/motor/axle

• Usually outputs the discretized absolute angle of the shaft/motor/axle

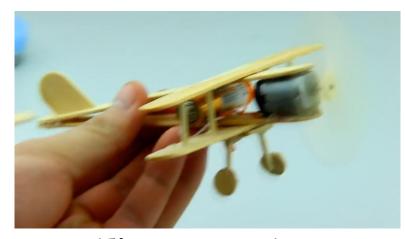


• Useful in order to know where different shafts are relative to each other.

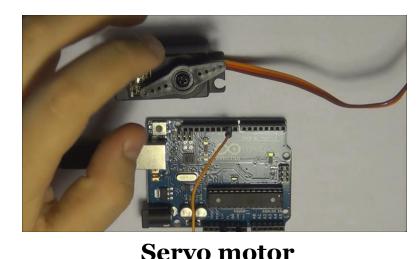
# Example: flippers on the Aqua robot



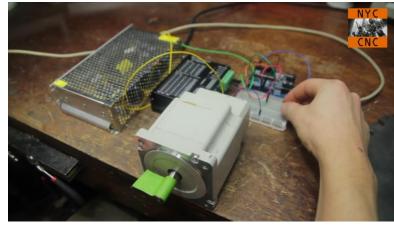
#### Actuators



DC (direct current) motor
They turn continuously at high RPM
(revolutions per minute) when
voltage is applied. Used in
quadrotors and planes, model cars
etc.

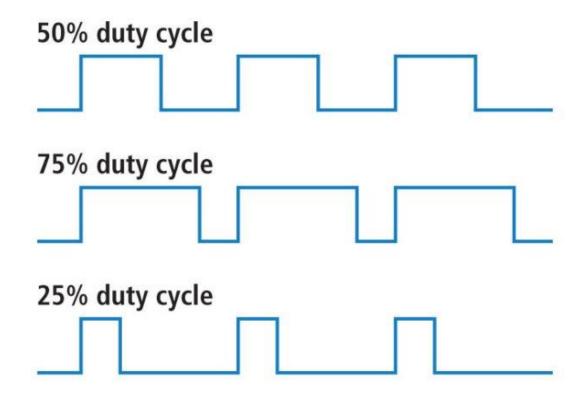


Usually includes: DC motor, gears, control circuit, position feedback
Precise control without free rotation
(e.g. robot arms, boat rudders)
Limited turning range: 180 degrees



Positioning feedback and no positioning errors.
Rotates by a predefined step angle.
Requires external control circuit.
Precise control without free rotation.
Constant holding torque without powering the motor (good for robot arms or weight-carrying systems).

#### Pulse Width Modulation



Used for creating analog/continuous behavior when voltage applied is discrete.

Main idea: turn on and off the motor fast enough so average voltage is the desired target.

Used in dimming LEDs, controlling the speed of DC motors, controlling the position of servo motors.