COMP417 Midterm

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A. Each of the following questions is worth 1 point

1. Adding two rotation matrices always results in another rotation matrix. True or False?

2. Encoders are sensors that measure the discretized angle of a motor. True or False?

3. The Kinect sensor measures depth based on the principle of time-of-flight. True or False?

4. Is the transition cost function $g(\mathbf{x}, \mathbf{u}) = \mathbf{x}^T \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \mathbf{x} + \mathbf{u}^T \mathbf{u}$ valid for use in LQR control? Justify your answer.

5. A free-falling accelerometer in a vacuum would measure $9.81 m/s^2$. True or False?

6. A gyroscope measures the angular velocity of a body. True or False?

7. Pulse-Width Modulation enables almost continuous control of LEDs and motors using a discrete voltage source. True or False?

8. The heuristic $h(\mathbf{x}) = ||\mathbf{x} - \mathbf{x}_{\text{goal}}||^2$ is an admissible and monotonic heuristic for A^*. True or False?

9. Dijkstra's shortest path algorithm and A^* always return the exact same path. True or False?

10. If left to perform infinitely many iterations the RRT algorithm will always converge to the shortest path. True or False?

11. The purpose of including the derivative term in the PID is to predict the value of the error in the near future. True or False?

12. The best way to start tuning a PID controller is to set the proportional and the derivative gains to zero, and tweak the integral gain. True or False?

13. You have a point ${}^{B}\mathbf{p}$ expressed in the coordinates of the body frame B. You are also given the rotation matrix ${}^{W}_{B}\mathbf{R}$ from the body frame, B, to the world frame, W. Finally, you are given the translation vector ${}^{W}\mathbf{t}_{WB}$ which denotes the origin of the body frame with respect to the world frame, expressed in coordinates of the world frame. Show how to express the point above in the world frame W.

14. The following is a valid rotation matrix. True or False?

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

15. A potential field that is the weighted sum of a single repulsive term (due to a single obstacle) and a single attractive term (due to a single destination), can never have a local minimum other than at the goal. True or False?

16. Name a data structure that can be used for fast nearest neighbour queries.

17. Quadtrees and Octrees are balanced trees. True or False?

18. Visibility-graph path planning produces shortest paths even in 3D. True or False?

19. In A^* once a node gets popped from the priority queue its cost-to-come never gets updated again, but its heuristic value for the cost-to-go can get updated. True or False?

20. The matrices $\mathbf{A}$ and $\mathbf{B}$ in the linear dynamics model $\mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t + \mathbf{B}\mathbf{u}_t$ need to be positive definite if we want to apply LQR. True or False?

21. Which planning algorithm would you use for a robot arm with 10 joints?
(a) RRT with path smoothing
(b) A^* on a discretized grid of the state space.

22. A^* can only be used for holonomic systems. True or False?

23. Which planning algorithm would you use for a self-driving car?
(a) RRT without path smoothing
(b) A^* on a discretized grid of the state space with path smoothing
(c) Either of the above

24. A^* allows negative costs for the edge weights, but Dijkstra's algorithm does not. True or False?

25. A^* with the heuristic $h(\mathbf{x}) = 0$ is Dijkstra's algorithm. True or False?

B. Bayesian Updates

1. [2pts] Prove Bayes' rule, namely that

$$p(A|B) = \frac{p(B|A)p(A)}{p(B)}$$

2. [2pts] Prove the conditional Bayes rule, namely that

$$p(A|B, C) = \frac{p(B|A, C)p(A|C)}{p(B|C)}$$

3. Consider the problem in which a robot is trying to determine if a door is open or not by taking a series of noisy sensor measurements. Assume the door is static over time. Let m be the binary random variable that denotes whether the door is open. The robot's initial belief, before any measurements are obtained, is $p(m = \text{open}) = 0.5$. The robot's sensor takes binary measurements z of whether the door is open, according to the following sensor model:

$p(z = \text{open} \mid m = \text{open}) = 0.8$
$p(z = \text{open} \mid m = \text{closed}) = 0.4$

which does not depend on time. Suppose that the first measurement is $z_1 = \text{closed}$ and the second measurement is $z_2 = \text{closed}$.

(a) [3pts] What is the robot's belief about whether the door is open given the first measurement?

(b) [3pts] What is the robot's belief about whether the door is open given the first two measurements? You may assume that measurements are independent given the state of the door.