Deceptive Dice

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1 Explanation

Let $n, k \in \mathbb{N}$ and k=1.
Let X be the random variable of the value of the roll. F_i is the expected value with i throws left. If the value we roll is bigger then F_{i-1} then we stop. If not, then we reroll, since we expected a better score. Therefor F_k is calculated recursively.
The base case is i=1. Let X be the random variable of the value of the roll.

$$F_1 = \mathbb{E}[X] = \sum_{i=1}^{n} i \cdot \mathbb{P}[X = i] = \sum_{i=1}^{n} \frac{i}{n} = \frac{n+1}{2}$$

The recursif case would be

$$F_i = \mathbb{P}[X \geq F_{i-1}] \cdot \mathbb{E}[X | X \geq F_{i-1}] + \mathbb{P}[X < F_{i-1}] \cdot F_{i-1}$$

$$F_i = \frac{n - \lfloor F_{i-1} \rfloor}{n} \cdot \sum_{i=\lfloor F_{i-1} \rfloor}^{n} i \cdot \mathbb{P}[X = i | X \geq F_{i-1}] + \frac{\lfloor F_{i-1} \rfloor}{n} \cdot F_{i-1}$$

$$F_i = \frac{n - \lfloor F_{i-1} \rfloor}{n} \cdot \frac{n \cdot (n+1) - \lfloor F_{i-1} \rfloor \cdot (\lfloor F_{i-1} \rfloor + 1)}{2 \cdot (n - \lfloor F_{i-1} \rfloor)} + \frac{\lfloor F_{i-1} \rfloor}{n} \cdot F_{i-1}$$

$$F_i = \frac{n \cdot (n+1) - \lfloor F_{i-1} \rfloor \cdot (\lfloor F_{i-1} \rfloor + 1)}{2 \cdot n} + \frac{\lfloor F_{i-1} \rfloor}{n} \cdot F_{i-1}$$