Floor Plan

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1 Explanation

It is well known that
$$\sum_{i=1}^{n} i = \frac{n^2+n}{2}$$
But from it we can derive
$$\sum_{i=1}^{n} i = \frac{n^2+n}{2} \iff 2\cdot\sum_{i=1}^{n} i = n^2+n \iff 2\cdot\sum_{i=1}^{n} i - n = n^2$$
$$\iff 2\cdot\sum_{i=1}^{n} i - \sum_{i=1}^{n} 1 = n^2 \iff \sum_{i=1}^{n} 2i-1 = n^2$$
Hence the sum of the n first odds numbers is n^2.
We need to find what numbers can be written has the difference of two squares.
Let $m, k \in \mathbb{N}$ such that $m > k$.
$$m^2 - k^2 = \sum_{i=1}^{m} 2i-1 - \sum_{i=1}^{k} 2i-1 = \sum_{i=k+1}^{m} 2i-1$$
The problem can be simplify has, find two odds numbers, such that the sum of the consecutive odds numbers between the two is n.
Trivially, every odd number can be written as a sum of odds numbers.
The last piece of the puzzle is to notice that the sum of two consecutive odds numbers is always a multiple of 4.
Hence if a number is congruent to 2 mod 4, it can't be written has difference of squares.
To find m and k use the second equality and go backward.