CIS 607 Project 1

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Exercise 3

(a)

Did it.

(b)

For $a > 0$, the spatial discretization is given by

$$U_j^{n+1} = U_j^n - \frac{ak}{2h}\left(3U_j^n - 4U_{j-1}^n + U_{j-2}^n\right) + \frac{a^2k^2}{2h^2}\left(U_j^n - 2U_{j-1}^n + U_{j-2}^n\right)$$

let $v = ak/h$, the stability condition is given by $0 \leq v \leq 2$

(c)

For advection function, there is asymmetry in the equations because the equation models translation at speed a. If $a > 0$, the solution moves rightward. If $a < 0$, the solution moves leftward. For this reason, we would like to use the upwind method which can capture this asymmetry from the equation.

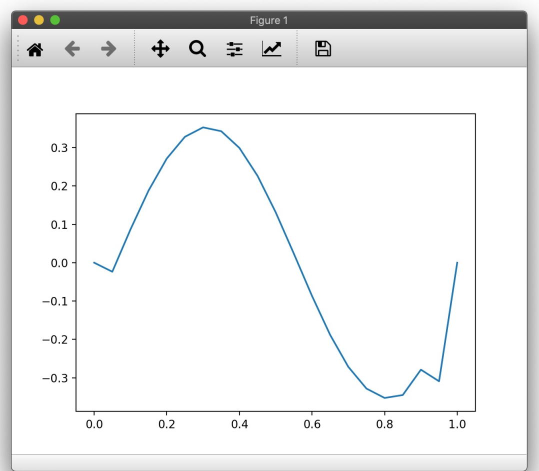
(d)

I modified the "FE_MOL_BS.jl" file for this exercise. The core code that I modified is given by

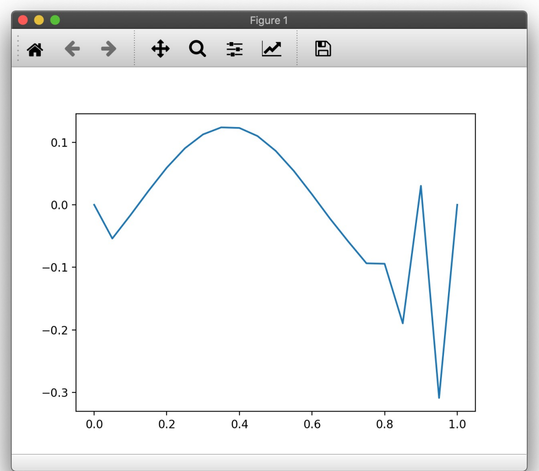
```
A = zeros(N-1,N-1)
A1 = zeros(N-1,N-1)
for j in 1:N-1
    A1[j,j] = 1
end

A2 = zeros(N-1,N-1)
for j in 1:N-1
    A2[j,j] = 3
end
for j in 1:N-2
    A2[j,j+1] = 1
end

for j in 2:N-1
```

```
    A2[j,j-1] = -4
end

A3 = zeros(N-1,N-1)

for j in 1:N-1
    A3[j,j] = 1
end
for j in 1:N-2
    A3[j,j] = 1
end
for j in 2:N-1
    A3[j,j-1] = -2
end

A = A1 + (-a*k/(2h))*A2 + (a^2*k^2/(2*h^2))*A3
```

Given our parameter, $a = 0.5, h = 0.05, \lambda = k/h$, the theoretical stable result for λ given by (b) should be $0 \leq \lambda \leq 4$. My tests show that for $\lambda = 0.5, 2$, the numerical results are good. At time $t = 1, t = 2$, the graph plotted is just a sine wave translated along x direction without distortion. But for $\lambda = 4$, the numerical result is not good. Starting from $t = 1$, the graph has been distorted. See Figure 1 below.



(a) t=1



(b) t=2

Figure 1: $\lambda = 4$

The stability result is obtained given infinitesimal grid width. We are using a rather coarse grid size, this could be the reason why we don't have stable numerical result at $\lambda = 4$. I tested $h = 0.01$, and with this grid size, we can obtain stable numerical result at $\lambda = 4$