$$C_{ijkl} \xleftarrow{} \text{81 components}$$

$$C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

Symmetries: $C_{ijkl} = C_{jikl}$ (54 components)

and $C_{ijkl} = C_{ijlk}$ (36 components)

and $C_{ijkl} = C_{klij}$ (only 21 components!)

like a 6x6 symmetric matrix

Only non-zero:

1. $$C_{1111} = C_{2222} = C_{3333} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

$$= \frac{\mu(3\lambda+2\mu)}{\lambda+\mu} \cdot \frac{1}{\frac{2(\lambda+\mu)}{2(\lambda+\mu)} + \frac{\lambda}{2(\lambda+\mu)}} \cdot \frac{1}{1-2\frac{\lambda}{2(\lambda+\mu)}}$$

$$= \frac{\cancel{\mu}\,\cancel{(3\lambda+2\mu)}}{\cancel{\lambda+\mu}} \cdot \frac{\lambda+2\mu}{\cancel{2(\lambda+\mu)}} \cdot \frac{\cancel{2(\lambda+\mu)}}{\cancel{3\lambda+2\mu}} \cdot \frac{\cancel{2}\,\cancel{(\lambda+\mu)}}{\cancel{2}\cancel{\mu}}$$

$$= \boxed{\lambda + 2\mu}$$

2. $C_{1122} = C_{1133} = C_{2211} = C_{3311} = C_{2233} = C_{3322} =$

$$= \frac{E\nu}{(1+\nu)(1-2\nu)} = \frac{\cancel{\mu}(3\lambda+\cancel{2\mu})}{\cancel{\lambda+\mu}} \cdot \frac{\lambda}{2(\cancel{\lambda+\mu})} \cdot \frac{\cancel{2(\lambda+\mu)}}{(\cancel{3\lambda+2\mu})} \cdot \frac{\cancel{2(\lambda+\mu)}}{\cancel{2\mu}}$$

$$= \boxed{\lambda}$$

3. $C_{2323} = C_{3131} = C_{1212} = \boxed{\mu}$

$= C_{2121} = C_{1313} = C_{3232}$

$= C_{3223}$ etc!

Dirichle - Penalty Parameter:

$$Z_{jl} = \beta \frac{d}{h_1} n_i C_{ijkl} n_k$$

$$\Rightarrow \quad Z_{11} = \beta \frac{d}{h_1} n_i C_{i1k1} n_k$$

$$= \beta \frac{d}{h_1} \left[n_i C_{i111} n_1 + n_i C_{i121} n_2 + n_i C_{i131} n_3 \right]$$

$$= \beta \frac{d}{h_1} \Big[n_1 C_{1111} n_1 + n_2 \cancel{C_{2111}}^{0} n_1 + n_3 \cancel{C_{3111}}^{0} n_1$$
$$+ n_1 \cancel{C_{1121}}^{0} n_2 + n_2 C_{2121} n_2 + n_3 \cancel{C_{3121}}^{0} n_2$$
$$+ n_1 \cancel{C_{1131}}^{0} n_3 + n_2 \cancel{C_{2131}}^{0} n_3 + n_3 C_{3131} n_3 \Big]$$

$$\boxed{Z_{11} = \beta \frac{d}{h_1}\left[(\lambda+2\mu)n_1^2 + \mu n_2^2 + \mu n_3^2\right]}$$

$$Z_{12} =$$

$$\beta \frac{d}{h_1}\Big[n_1 C_{1112} n_1 + n_2 C_{2112} n_1 + n_3 C_{3112} n_1$$

$$+ n_1 C_{1122} n_2 + n_2 C_{2122} n_2 + n_3 C_{3122} n_2$$

$$+ n_1 C_{1132} n_3 + n_2 C_{2132} n_3 + n_3 C_{3132} n_3 \Big]$$

$$=$$

$$\beta \frac{d}{h_1}\Big[n_1 \cancelto{0}{C_{1112}} n_1 + n_2 C_{2112} n_1 + n_3 \cancelto{0}{C_{3112}} n_1$$

$$+ n_1 C_{1122} n_2 + n_2 \cancelto{0}{C_{2122}} n_2 + n_3 \cancelto{0}{C_{3122}} n_2$$

$$+ n_1 \cancelto{0}{C_{1132}} n_3 + n_2 \cancelto{0}{C_{2132}} n_3 + n_3 \cancelto{0}{C_{3132}} n_3 \Big]$$

$$= \boxed{\beta \frac{d}{h_1} (\mu+\lambda) n_1 n_2}$$

$$Z_{13} = \beta \frac{d}{h_1}\Big[n_1 \cancelto{0}{C_{1113}} n_1 + n_2 \cancelto{0}{C_{2113}} n_1 + n_3 C_{3113} n_1$$

$$+ n_1 \cancelto{0}{C_{1123}} n_2 + n_2 \cancelto{0}{C_{2123}} n_2 + n_3 \cancelto{0}{C_{3123}} n_2$$

$$+ n_1 C_{1133} n_3 + n_2 \cancelto{0}{C_{2133}} n_3 + n_3 \cancelto{0}{C_{3133}} n_3 \Big]$$

$$= \beta \frac{d}{h_1} \left[\mu n_1 n_3 + \lambda n_1 n_3 \right]$$

$$= \beta \frac{d}{h_1} (\mu + \lambda) n_1 n_3$$

$$Z_{21} = \beta \frac{d}{h_1} \Big[n_1 C_{1211} n_1 + n_2 C_{2211} n_1 + n_3 C_{3211} n_1$$

$$+ n_1 C_{1221} n_2 + n_2 C_{2221} n_2 + n_3 C_{3221} n_2$$

$$+ n_1 C_{1231} n_3 + n_2 C_{2231} n_3 + n_3 C_{3231} n_3 \Big]$$

$$\beta \frac{d}{h_1} \Big[n_1 \cancelto{0}{C_{1211}} n_1 + n_2 C_{2211} n_1 + n_3 \cancelto{0}{C_{3211}} n_1$$

$$+ n_1 C_{1221} n_2 + n_2 \cancelto{0}{C_{2221}} n_2 + n_3 \cancelto{0}{C_{3221}} n_2$$

$$+ n_1 \cancelto{0}{C_{1231}} n_3 + n_2 \cancelto{0}{C_{2231}} n_3 + n_3 \cancelto{0}{C_{3231}} n_3 \Big]$$

$$= \beta \frac{d}{h_1} (\mu + \lambda) n_1 n_2 = Z_{12}$$

(Z is probably
Symmetric!)

$$Z_{22} =$$

$$\beta \frac{d}{h_1}\Big[n_1 C_{1212} n_1 + \cancelto{0}{n_2 C_{2212} n_1} + \cancelto{0}{n_3 C_{3212} n_1}$$

$$+ \cancelto{0}{n_1 C_{1222} n_2} + n_2 C_{2222} n_2 + \cancelto{0}{n_3 C_{3222} n_2}$$

$$+ \cancelto{0}{n_1 C_{1232} n_3} + \cancelto{0}{n_2 C_{2232} n_3} + n_3 C_{3232} n_3 \Big]$$

$$= \beta \frac{d}{h_1}\left[\mu n_1^2 + (\lambda + 2\mu) n_2^2 + \mu n_3^2 \right]$$

$$Z_{23} =$$

$$\beta \frac{d}{h_1}\Big[\cancelto{0}{n_1 C_{1213} n_1} + \cancelto{0}{n_2 C_{2213} n_1} + \cancelto{0}{n_3 C_{3213} n_1}$$

$$+ \cancelto{0}{n_1 C_{1223} n_2} + \cancelto{0}{n_2 C_{2223} n_2} + n_3 C_{3223} n_2$$

$$+ \cancelto{0}{n_1 C_{1233} n_3} + n_2 C_{2233} n_3 + \cancelto{0}{n_3 C_{3233} n_3} \Big]$$

$$= \beta \frac{d}{h_1} (\mu + \lambda) n_2 n_3$$

$Z_{31} = Z_{13}$ (could check?)

$Z_{32} = Z_{23}$ (could check)

$Z_{33} =$

$$= \frac{\beta d}{h_1}\left[\mu n_1^2 + \mu n_2^2 + (\lambda + 2\mu) n_3^2\right]$$

For our problems (i.e. no coordinate transform at the moment, all the Z_{je}'s are constant!

So you can just add the

to T_{je} (ie. a scalar plus a matrix).