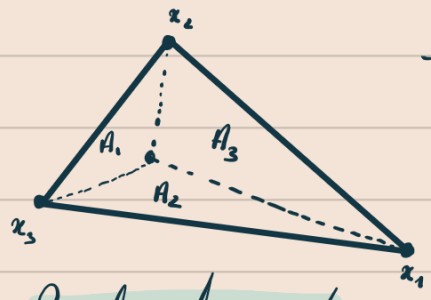
Inverse mapping for T3 element:



$$a_1 = \frac{A_1}{A}\,;\; a_2 = \frac{A_2}{A}\,;\; a_3 = \frac{A_3}{A}$$

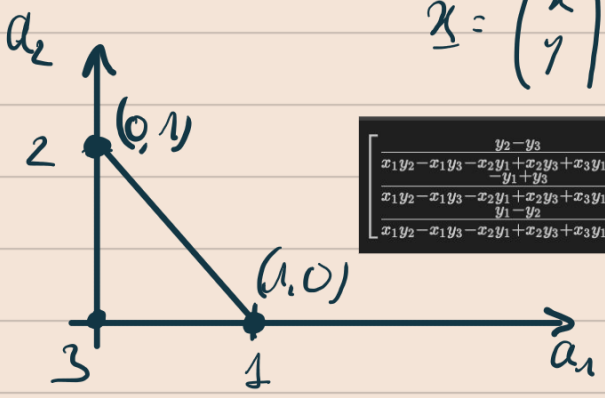
$$a_1 + a_2 + a_3 = 1.$$

Real element

coordonnées d'aire: $\underline{x} = a_1\,\underline{x}^{(1)} + a_2\,\underline{x}^{(2)} + a_3\,\underline{x}^{(3)}$

$$\Rightarrow \underline{x} = a_1\,\underline{x}^{(1)} + a_2\,\underline{x}^{(2)} + \underbrace{(1-a_1-a_2)}_{a_3}\,\underline{x}^{(3)}$$

$$\underline{x} = \begin{pmatrix} x \\ y \end{pmatrix}$$



Reference element

$$\begin{bmatrix} \frac{y_2-y_3}{x_1y_2-x_1y_3-x_2y_1+x_2y_3+x_3y_1-x_3y_2} & \frac{-x_2+x_3}{x_1y_2-x_1y_3-x_2y_1+x_2y_3+x_3y_1-x_3y_2} & \frac{x_2y_3-x_3y_2}{x_1y_2-x_1y_3-x_2y_1+x_2y_3+x_3y_1-x_3y_2} \\ \frac{-y_1+y_3}{x_1y_2-x_1y_3-x_2y_1+x_2y_3+x_3y_1-x_3y_2} & \frac{x_1-x_3}{x_1y_2-x_1y_3-x_2y_1+x_2y_3+x_3y_1-x_3y_2} & \frac{-x_1y_3+x_3y_1}{x_1y_2-x_1y_3-x_2y_1+x_2y_3+x_3y_1-x_3y_2} \\ \frac{y_1-y_2}{x_1y_2-x_1y_3-x_2y_1+x_2y_3+x_3y_1-x_3y_2} & \frac{-x_1+x_2}{x_1y_2-x_1y_3-x_2y_1+x_2y_3+x_3y_1-x_3y_2} & \frac{x_1y_2-x_2y_1}{x_1y_2-x_1y_3-x_2y_1+x_2y_3+x_3y_1-x_3y_2} \end{bmatrix}$$

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \underbrace{\begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{pmatrix}}_{\mathcal{A}} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \Rightarrow \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \mathcal{A}^{-1} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Mapping & inverse mapping

$$a_1 = \xi \qquad N_1 = \xi \quad (= a_1)$$
$$a_2 = \eta \qquad N_2 = \eta \quad (= a_2)$$
$$N_3 = 1 - \xi - \eta \quad (= a_3)$$

For T6, shape function also defined by a_1, a_2, a_3

$\Rightarrow$ inverse mapping available!