Towards an Optimal Multi-query Framework based on Model-order Reduction for Non-linear Dynamics

Seminar

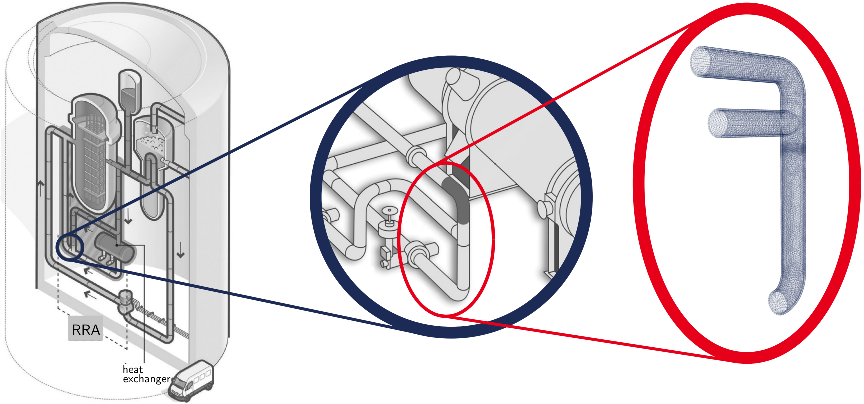
A. Daby-Seesaram

Supervised by D. Néron, A. Fau and P.-É. Charbonnel

29st of January 2024

Industrial context

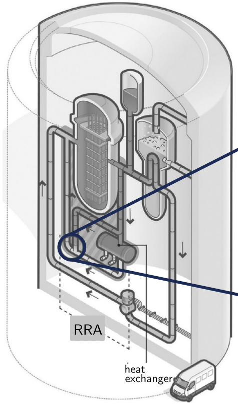
Structural elements



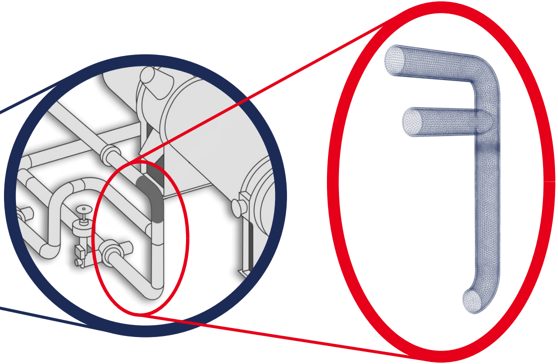
Source: IRSN

Industrial context

Structural elements



Source: IRSN

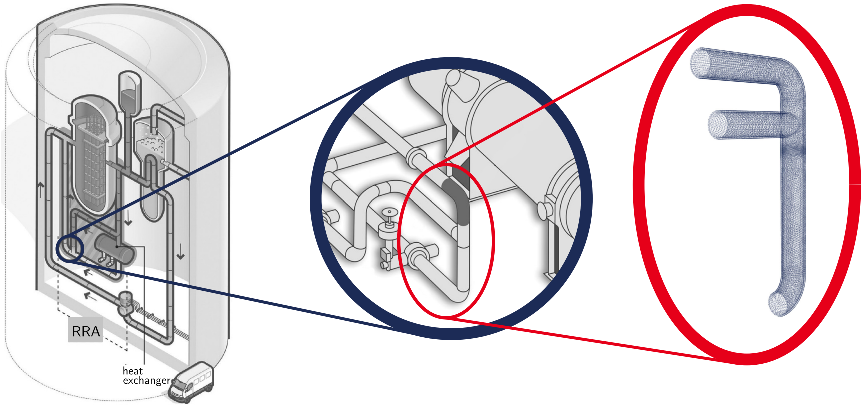


Nominal operation

- Piping component - Cooling system
  - Thermal conditions: $20 - 180\,°\mathrm{C}$
  - Ageing (thermal fatigue)

Industrial context

Structural elements



Source: IRSN

Nominal operation

- Piping component - Cooling system
  - Thermal conditions: $20 - 180\,^{\circ}\mathrm{C}$
  - Ageing (thermal fatigue)

Extreme conditions

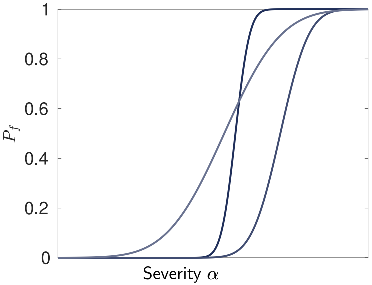
- Seismic hazard
  - Assessing the **risk of failure**

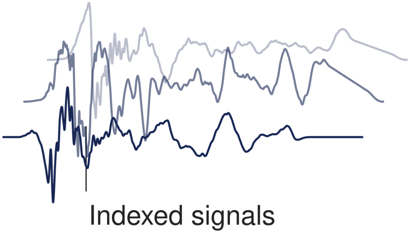
Industrial context

Variability of seismic loading

Account for the variability

- Wide range of plausible earthquake signals (~ 1000)
- Sort the signals by severity (α)
- Compute a failure probability (P_f) for each severity level



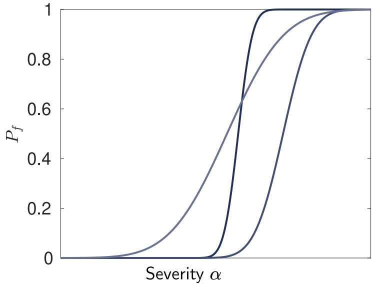


Industrial context

Variability of seismic loading

Account for the variability

- Wide range of plausible earthquake signals (~ 1000)
- Sort the signals by severity (α)
- Compute a failure probability (P_f) for each severity level

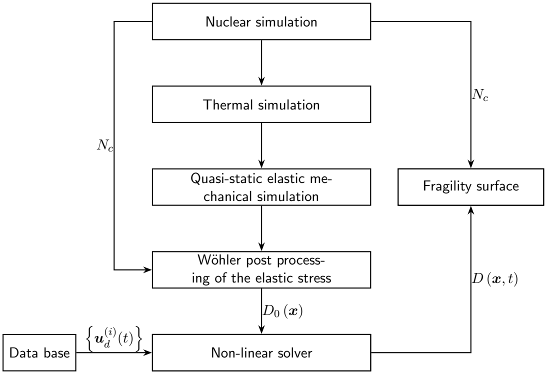


In practice

- Parametrised (lognormal) [Kennedy, et al., 1980]
  - Fewer computations
  - Bias in the probability prediction [Mandal, et al., 2016]
- Simplified mechanical model or high computational cost
- Fine structural description combined with accounting for the ageing of the strucure
  - Workpackage II of NARSIS Project

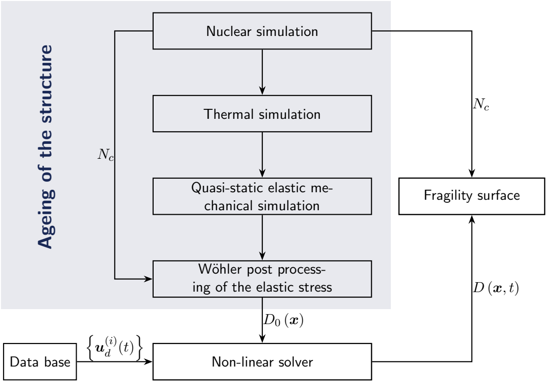
Industrial context

Multi-query requirement [NARSIS WP2]



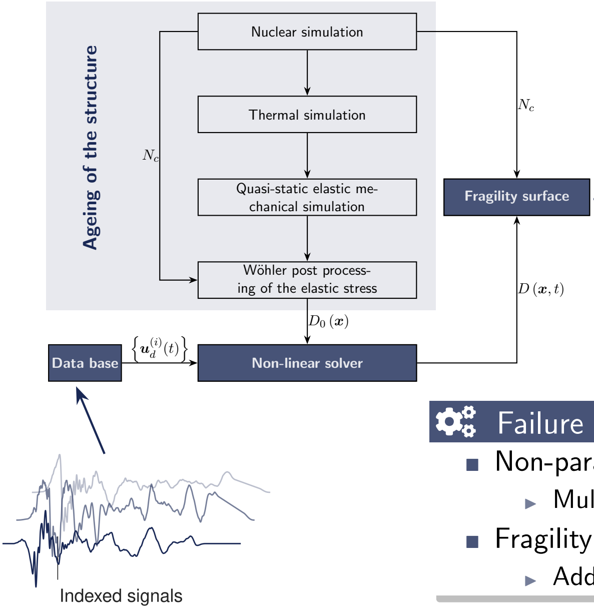
Industrial context

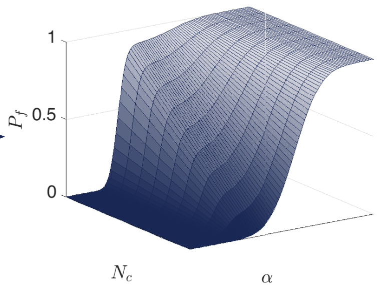
Multi-query requirement [NARSIS WP2]



Industrial context

Multi-query requirement [NARSIS WP2]

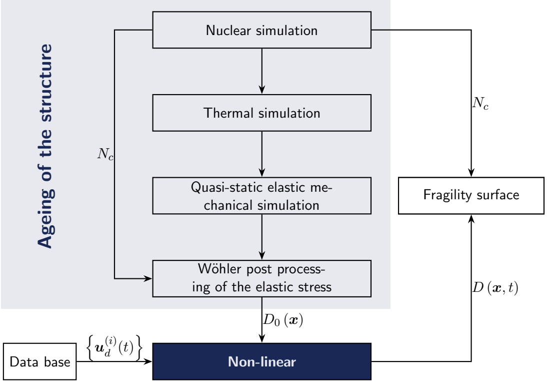




Failure probability under seismic loading

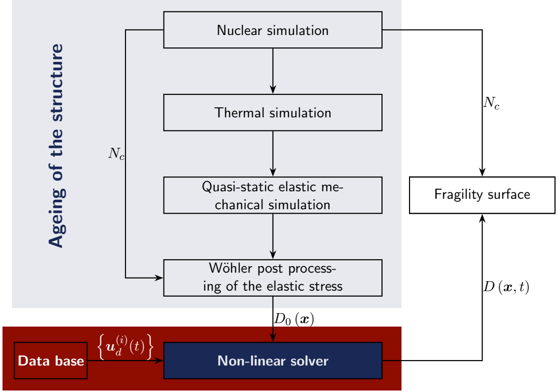
- Non-parametrised signals $[0\,\mathrm{Hz}, 30\,\mathrm{Hz}]$
  - Multiple queries to the solver
- Fragility surface
  - Additional dimension N_c

How to efficiently compute fragility surfaces?



I - Efficient solver

How to efficiently compute fragility surfaces?

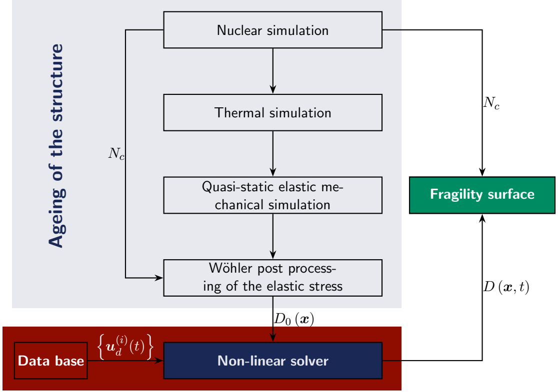


I - Efficient solver

II - Multi-query framework

Outline

How to efficiently compute fragility surfaces?



I - Efficient solver

II - Multi-query framework

III - Application to classification

I - Efficient solver

Requirements:
- Low-frequency dynamics
- Ductile damage
- Low computational cost

Reduced-order model

Low-rank approximation of the solution

Full-order discretised model

- N spatial shape functions
  - Span a finite spatial space of dimension N
- Requires computing N associated temporal functions

Reduced-order model

- $m \ll N$ spatial modes
  - Similar to global shape functions
  - Span a smaller space
- Galerkin projection

Finding the reduced-order basis

- Linear Normal Modes (eigenmodes of the structure) [Hansteen & Bell, 1979]
  - Do not account for the loading and behaviour specifics
- Proper Orthogonal Decomposition [Chatterjee, 2000],[Radermacher & Reese, 2013]
  - Require wise selection of the snapshots and *a priori* costly computations
- Reduced basis method [Maday & Rønquist, 2002] with EIM [Barrault et al., 2004]
  - Rely on prior expensive computations

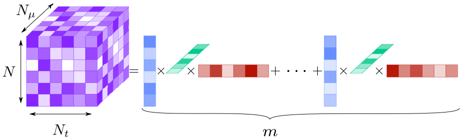
Reduced-order model

Proper Generalised Decomposition (PGD)

Choice of separation of variables

Extracoordinates [Chinesta et al., 2011]

- $$\boldsymbol{u}(\boldsymbol{x}, \omega, \boldsymbol{\mu}) = \sum_{i=1}^{m} \overline{\boldsymbol{u}}^i(\boldsymbol{x}) \; \lambda_\omega^i(\omega) \prod_{j=1}^{n} M_j^i(\boldsymbol{\mu})$$
    - Expression of the full parametrised solution
    - Single computation
    - Problem dependent



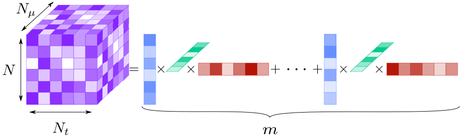
Reduced-order model

Proper Generalised Decomposition (PGD)

Choice of separation of variables

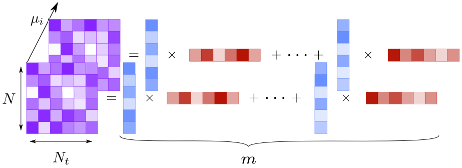
Extracoordinates [Chinesta et al., 2011]

- $\boldsymbol{u}(\boldsymbol{x}, \omega, \boldsymbol{\mu}) = \sum_{i=1}^{m} \overline{\boldsymbol{u}}^i(\boldsymbol{x})\ \lambda_\omega^i(\omega) \prod_{j=1}^{n} M_j^i(\boldsymbol{\mu})$
  - Expression of the full parametrised solution
  - Single computation
  - Problem dependent



Out of the decomposition [Ladevèze, 1985]

- $\boldsymbol{u}_\mu(\boldsymbol{x}, \omega) = \sum_{i=1}^{m} \overline{\boldsymbol{u}}^i(\boldsymbol{x})\ \lambda_\omega^i(\omega)$
  - More versatile [Scanff et al., 2022]
  - Suited to non-parametrised problems

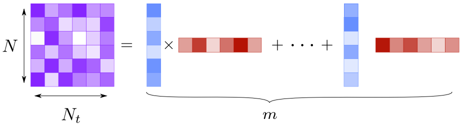


Reduced-order model

Proper Generalised Decomposition (PGD)

PGD

- $\boldsymbol{u}(x,\omega) = \sum_{i=1}^{m} \overline{\boldsymbol{u}}^{i}(x)\ \lambda_{\omega}^{i}(\omega)$
  - Spatial modes $\overline{\boldsymbol{u}}^{i}(x)$
  - Frequency modes $\lambda_{\omega}^{i}(\omega)$
- For a given basis $\{\overline{\boldsymbol{u}}^{i}(\boldsymbol{x})\}_{i\in[\![1,m]\!]}$
  - Update the associated frequency functions
    - Similar to POD
- Add new modes on the fly
  - Generate modes targeted for the current problem
  - Greedy algorithm



Requirements

- Linear equations
  - Linearisation strategy for non-linear problems
- Global computations in space and time
  - Non-incremental scheme