IEOR 4404 Simulation	**Fall 2024**

Homework 4

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Problem 1 Let G be a cumulative distribution function and suppose that for constants $a < b$, we wish to generate a random variable X with cumulative distribution function

$$F(x) = \frac{G(x) - G(a)}{G(b) - G(a)}, \quad a \leq x \leq b.$$

(a) If random variable Y has cumulative distribution function G, then given what information is F the conditional distribution of Y? Prove your claim.

(b) Show that an acceptance-rejection method to generate X in this case is to generate a random variable Y having cumulative distribution function G and then accept it if it lies between a and b.

Problem 2 Derive an acceptance-rejection method to generate (X, Y) with joint density function

$$f(x, y) = \frac{1}{K} \exp(-x^2 - y^2 + x \sin(xy)),$$

where K is a normalizing constant. Implement your method in a computer to generate 100 pairs of (X, Y), and plot the distributions of X and Y respectively.

Problem 3 Derive a procedure to sample uniformly from the region

$$A = \left\{(x, y) : x \in [-1, 1], |y| \leq |x|^{-1/2}\right\}.$$

Implement your procedure in a computer to generate 100 pairs of (X, Y), and plot their positions on a two-dimensional plane to check that they indeed appear uniformly distributed over the region A.

Problem 4 Consider the discrete random variable X where

$$\mathbb{P}(X = j) = \left(\frac{1}{2}\right)^{j+1} + \left(\frac{1}{2}\right) \frac{2^{j-1}}{3^j}, j = 1, 2, \ldots$$

which appears in HW2 Problem 3. Instead of using the inverse transform method as there, now present a composition method to generate X. Then implement your procedure in a computer to generate 100 copies of X, and plot their distribution.

Problem 4 Present a method to generate a random variable X having cumulative distribution function

$$F(x) = \int_0^\infty x^y e^{-y} dy, 0 \leq x \leq 1.$$

Implement your procedure in a computer to generate 100 copies of X, and plot their distribution.