IEOR 4404 Simulation **Fall 2024**

Homework 2

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Problem 1 Let $X \sim Bin(n, p)$ be a binomial random variable with parameters n and p. Recall that the p.m.f of X is given by

$$p(k) = \mathbb{P}(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, 1, 2, \cdots, n.$$

(a) Verify that

$$p(k+1) = \frac{n-k}{k+1} \cdot \frac{p}{1-p} \cdot p(k), \quad k = 0, 1, \cdots, n-1.$$

(b) Write an algorithm (or "pseudo code") for the inverse transform method for generating X that utilizes the relation in part (a), in a way that you do not have to compute all the p.m.f. values, and also you only have to compute the c.d.f. values on the fly if needed.

(c) Implement your answer in part (b) on a computer to generate 100 copies of $Bin(10, 2/3)$ and plot their distribution.

Problem 2 In this problem we will generate a negative binomial random variable with parameters r, p in three different ways. This random variable, called $NB(r, p)$, is the number of independent Bernoulli trials needed to get r successes, where each Bernoulli trial has success probability p. It has a p.m.f. given by

$$p(k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}, k = r, r+1, \ldots$$

(a) Verify the relation

$$p(k+1) = \frac{k(1-p)}{k+1-r} p(k).$$

(b) Use the relation in part (b) to give an algorithm for generating $NB(r, p)$.

(c) Write down the relationship between $NB(r, p)$ and $Geom(p)$. Use it to obtain an algorithm to generate $NB(r, p)$.

(d) Using the interpretation that $NB(r, p)$ counts the number of independent $Ber(p)$ trials required to accumulate r successes, obtain yet another approach for generating $NB(r, p)$.

(e) Implement your algorithms in parts (a), (b) and (c) on a computer for generating $NB(2, 1/3)$. For each algorithm, generate 100 copies of X and plot their distribution. Are the plots similar?

Problem 3 Write an algorithm to generate the random variable X where

$$\mathbb{P}(X = j) = \left(\frac{1}{2}\right)^{j+1} + \left(\frac{1}{2}\right) \frac{2^{j-1}}{3^j}, j = 1, 2, \ldots$$

Implement your algorithm on a computer to generate 100 copies of X, and plot their distribution.

Problem 4 A fair die is to be continually rolled until all possible outcomes $1, 2, \ldots, 6$ have occurred at least once, and we are interested in the total number of rolls in this experiment. Implement an algorithm on a computer to generate 100 copies of the total number of rolls, and plot the distributions of these 100 copies.