

# 1 Appendix: Out-of-Sample MSE

Table 1: Descriptive statistics of the out-of-sample MSE of each model. “MLP” refers to the multi-layer perceptron; “RW” is the random walk; “Ridge CV”, “LASSO CV”, and “Enet CV” are the Ridge, LASSO, and Elastic Net regressions with parameters chosen via cross-validation; “BRidge” is the Bayesian Ridge; “BLASSO” is the Bayesian LASSO; and “MA” is the moving average model suggested by **Atkeson2001**.

| Model    | MSE    | 1    | 2    | 3    | 6    | 12   | 3M   | 6M   | 12M  |
|----------|--------|------|------|------|------|------|------|------|------|
| LSTM     | Min.   | 0.14 | 0.30 | 0.40 | 0.37 | 0.39 | 0.24 | 0.28 | 0.30 |
|          | Q1     | 0.24 | 0.46 | 0.50 | 0.50 | 0.51 | 0.37 | 0.47 | 0.41 |
|          | Median | 0.32 | 0.51 | 0.55 | 0.55 | 0.57 | 0.42 | 0.55 | 0.49 |
|          | Q3     | 0.40 | 0.60 | 0.60 | 0.64 | 0.62 | 0.49 | 0.68 | 0.57 |
|          | Max.   | 0.76 | 1.12 | 0.96 | 1.48 | 1.15 | 0.73 | 1.21 | 1.15 |
| ConvLSTM | Min.   | 0.09 | 0.31 | 0.39 | 0.39 | 0.43 | 0.22 | 0.27 | 0.18 |
|          | Q1     | 0.15 | 0.40 | 0.44 | 0.45 | 0.46 | 0.30 | 0.39 | 0.29 |
|          | Median | 0.17 | 0.42 | 0.45 | 0.46 | 0.46 | 0.33 | 0.43 | 0.34 |
|          | Q3     | 0.19 | 0.44 | 0.46 | 0.46 | 0.47 | 0.37 | 0.50 | 0.40 |
|          | Max.   | 0.25 | 0.47 | 0.47 | 0.47 | 0.48 | 0.59 | 0.86 | 0.62 |
| MLP      | Min.   | 0.75 | 0.86 | 0.88 | 0.89 | 0.93 | 0.88 | 0.90 | 0.99 |
|          | Q1     | 0.92 | 1.10 | 1.10 | 1.10 | 1.09 | 1.05 | 1.09 | 1.11 |
|          | Median | 0.99 | 1.16 | 1.16 | 1.16 | 1.18 | 1.11 | 1.15 | 1.17 |
|          | Q3     | 1.07 | 1.22 | 1.21 | 1.25 | 1.25 | 1.17 | 1.24 | 1.24 |
|          | Max.   | 1.31 | 1.36 | 1.34 | 1.48 | 1.48 | 1.32 | 1.42 | 1.43 |
| RW       | Min.   | 1.53 | 1.52 | 1.60 | 1.51 | 1.38 | 1.46 | 1.46 | 1.48 |
|          | Q1     | 1.89 | 1.84 | 1.87 | 1.88 | 1.81 | 1.90 | 1.94 | 1.91 |
|          | Median | 2.04 | 1.99 | 1.99 | 2.05 | 1.97 | 2.01 | 2.06 | 2.07 |
|          | Q3     | 2.15 | 2.11 | 2.16 | 2.19 | 2.18 | 2.14 | 2.19 | 2.24 |
|          | Max.   | 2.54 | 2.55 | 2.61 | 2.54 | 2.66 | 2.43 | 2.53 | 2.62 |
| Ridge CV | Min.   | 0.66 | 0.88 | 0.83 | 0.80 | 0.82 | 0.87 | 0.95 | 0.94 |
|          | Q1     | 0.80 | 0.99 | 0.99 | 0.99 | 0.97 | 0.97 | 0.99 | 0.99 |
|          | Median | 0.85 | 1.01 | 1.02 | 1.03 | 1.03 | 0.99 | 1.00 | 1.00 |

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| Model    | MSE    | 1    | 2    | 3    | 6    | 12   | 3M   | 6M   | 12M  |
|----------|--------|------|------|------|------|------|------|------|------|
| BRidge   | Q3     | 0.89 | 1.02 | 1.03 | 1.04 | 1.07 | 1.01 | 1.02 | 1.01 |
|          | Max.   | 1.30 | 1.07 | 1.07 | 1.11 | 1.18 | 1.19 | 1.20 | 1.15 |
|          | Min.   | 0.61 | 0.94 | 0.85 | 0.87 | 0.85 | 0.83 | 0.88 | 0.94 |
|          | Q1     | 0.78 | 1.03 | 1.03 | 1.03 | 1.02 | 0.97 | 1.00 | 1.02 |
|          | Median | 0.85 | 1.07 | 1.08 | 1.08 | 1.10 | 1.01 | 1.04 | 1.05 |
| Max.     | Q3     | 0.91 | 1.11 | 1.11 | 1.14 | 1.15 | 1.07 | 1.10 | 1.11 |
|          | 1.20   | 1.26 | 1.23 | 1.32 | 1.36 | 1.21 | 1.26 | 1.26 |      |
| LASSO CV | Min.   | 0.68 | 0.84 | 0.84 | 0.86 | 0.81 | 0.88 | 0.95 | 0.95 |
|          | Q1     | 0.78 | 1.00 | 1.00 | 0.99 | 0.98 | 0.96 | 1.00 | 1.00 |
|          | Median | 0.81 | 1.01 | 1.01 | 1.02 | 1.03 | 1.00 | 1.00 | 1.00 |
|          | Q3     | 0.87 | 1.01 | 1.02 | 1.04 | 1.07 | 1.00 | 1.00 | 1.00 |
|          | Max.   | 1.08 | 1.09 | 1.12 | 1.15 | 1.22 | 1.07 | 1.08 | 1.05 |
| LASSO    | Min.   | 0.68 | 0.84 | 0.83 | 0.79 | 0.81 | 0.84 | 0.94 | 0.98 |
|          | Q1     | 0.78 | 1.00 | 0.99 | 0.99 | 0.97 | 0.96 | 0.99 | 0.99 |
|          | Median | 0.83 | 1.01 | 1.01 | 1.02 | 1.02 | 0.98 | 1.00 | 1.00 |
|          | Q3     | 0.90 | 1.01 | 1.02 | 1.04 | 1.07 | 1.00 | 1.00 | 1.00 |
|          | Max.   | 1.03 | 1.03 | 1.03 | 1.06 | 1.13 | 1.12 | 1.05 | 1.04 |
| Enet CV  | Min.   | 0.49 | 0.84 | 0.83 | 0.78 | 0.80 | 0.83 | 0.91 | 0.91 |
|          | Q1     | 0.71 | 0.98 | 0.98 | 0.98 | 0.95 | 0.93 | 0.96 | 0.97 |
|          | Median | 0.76 | 1.00 | 1.00 | 1.01 | 1.01 | 0.96 | 0.98 | 0.99 |
|          | Q3     | 0.83 | 1.01 | 1.01 | 1.03 | 1.06 | 0.98 | 1.00 | 1.00 |
|          | Max.   | 0.97 | 1.01 | 1.02 | 1.05 | 1.13 | 1.00 | 1.00 | 1.00 |
| SVR      | Min.   | 0.83 | 0.86 | 0.83 | 0.81 | 0.81 | 0.90 | 0.89 | 0.94 |
|          | Q1     | 0.90 | 1.01 | 1.01 | 1.00 | 1.00 | 0.97 | 0.99 | 1.00 |

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| Model   | MSE    | 1    | 2    | 3    | 6    | 12   | 3M   | 6M   | 12M  |
|---------|--------|------|------|------|------|------|------|------|------|
| RF      | Median | 0.92 | 1.03 | 1.03 | 1.05 | 1.06 | 0.99 | 1.02 | 1.03 |
|         | Q3     | 0.94 | 1.06 | 1.05 | 1.08 | 1.10 | 1.02 | 1.04 | 1.06 |
|         | Max.   | 0.99 | 1.14 | 1.14 | 1.16 | 1.35 | 1.08 | 1.17 | 1.20 |
|         | Min.   | 0.75 | 0.92 | 0.89 | 0.87 | 0.84 | 0.87 | 0.94 | 0.93 |
|         | Q1     | 0.87 | 1.05 | 1.04 | 1.05 | 1.03 | 1.01 | 1.02 | 1.02 |
| BART    | Median | 0.93 | 1.09 | 1.09 | 1.10 | 1.10 | 1.05 | 1.05 | 1.07 |
|         | Q3     | 1.00 | 1.13 | 1.13 | 1.15 | 1.17 | 1.08 | 1.10 | 1.10 |
|         | Max.   | 1.21 | 1.36 | 1.27 | 1.30 | 1.47 | 1.15 | 1.24 | 1.22 |
|         | Min.   | 0.73 | 0.91 | 0.85 | 0.86 | 0.83 | 0.88 | 0.92 | 0.92 |
|         | Q1     | 0.86 | 1.01 | 1.02 | 1.02 | 1.02 | 0.96 | 0.98 | 1.00 |
| Bagging | Median | 0.91 | 1.05 | 1.06 | 1.07 | 1.08 | 1.00 | 1.01 | 1.02 |
|         | Q3     | 0.96 | 1.08 | 1.10 | 1.11 | 1.13 | 1.04 | 1.04 | 1.04 |
|         | Max.   | 1.23 | 1.16 | 1.27 | 1.27 | 1.35 | 1.12 | 1.14 | 1.13 |
|         | Min.   | 0.80 | 0.97 | 0.89 | 0.87 | 0.94 | 0.90 | 0.91 | 0.95 |
|         | Q1     | 0.94 | 1.12 | 1.08 | 1.08 | 1.10 | 1.05 | 1.05 | 1.05 |
| kNN     | Median | 0.99 | 1.16 | 1.14 | 1.15 | 1.17 | 1.10 | 1.09 | 1.12 |
|         | Q3     | 1.06 | 1.21 | 1.21 | 1.21 | 1.23 | 1.15 | 1.14 | 1.17 |
|         | Max.   | 1.31 | 1.42 | 1.41 | 1.32 | 1.54 | 1.28 | 1.40 | 1.27 |
|         | Min.   | 0.95 | 0.86 | 0.90 | 0.85 | 0.88 | 0.92 | 0.92 | 0.88 |
|         | Q1     | 0.98 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.01 | 1.00 |
| Huber   | Median | 1.00 | 1.03 | 1.04 | 1.05 | 1.05 | 1.02 | 1.03 | 1.03 |
|         | Q3     | 1.02 | 1.07 | 1.07 | 1.07 | 1.11 | 1.05 | 1.05 | 1.06 |
|         | Max.   | 1.07 | 1.14 | 1.14 | 1.17 | 1.26 | 1.12 | 1.11 | 1.14 |
|         | Min.   | 0.67 | 1.68 | 1.51 | 1.54 | 1.54 | 1.42 | 1.38 | 1.55 |

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| <b>Model</b> | <b>MSE</b> | <b>1</b> | <b>2</b> | <b>3</b> | <b>6</b> | <b>12</b> | <b>3M</b> | <b>6M</b> | <b>12M</b> |
|--------------|------------|----------|----------|----------|----------|-----------|-----------|-----------|------------|
| Theil-Sen    | Q1         | 0.93     | 1.99     | 2.03     | 1.98     | 2.01      | 1.67      | 1.88      | 1.92       |
|              | Median     | 1.04     | 2.18     | 2.25     | 2.32     | 2.20      | 1.85      | 2.04      | 2.14       |
|              | Q3         | 1.19     | 2.42     | 2.43     | 2.55     | 2.52      | 2.04      | 2.25      | 2.44       |
|              | Max.       | 1.49     | 3.22     | 3.29     | 3.28     | 3.19      | 2.45      | 2.93      | 2.97       |
|              | Min.       | 0.67     | 1.77     | 1.51     | 1.66     | 1.59      | 1.46      | 1.45      | 1.62       |
| Median       | Q1         | 0.94     | 2.08     | 2.10     | 2.14     | 2.11      | 1.71      | 1.93      | 1.97       |
|              | Median     | 1.04     | 2.31     | 2.33     | 2.44     | 2.33      | 1.90      | 2.09      | 2.19       |
|              | Q3         | 1.20     | 2.55     | 2.58     | 2.68     | 2.60      | 2.11      | 2.30      | 2.52       |
|              | Max.       | 1.53     | 3.44     | 3.49     | 3.49     | 3.36      | 2.57      | 2.97      | 3.02       |
|              | Min.       | 0.74     | 0.87     | 0.88     | 0.87     | 0.86      | 0.94      | 0.93      | 0.90       |
| Factors      | Q1         | 0.93     | 1.01     | 1.01     | 1.02     | 1.02      | 1.01      | 1.01      | 1.01       |
|              | Median     | 1.01     | 1.04     | 1.05     | 1.07     | 1.08      | 1.06      | 1.05      | 1.03       |
|              | Q3         | 1.08     | 1.08     | 1.08     | 1.11     | 1.13      | 1.10      | 1.11      | 1.08       |
|              | Max.       | 1.36     | 1.28     | 1.24     | 3.69     | 1.29      | 1.27      | 1.42      | 2.89       |
|              | Min.       | 0.24     | 0.37     | 0.38     | 0.35     | 0.39      | 0.50      | 0.45      | 0.39       |
| GARCH        | Q1         | 0.49     | 0.72     | 0.83     | 0.79     | 0.82      | 0.67      | 0.60      | 0.64       |
|              | Median     | 0.64     | 0.93     | 1.05     | 1.11     | 1.09      | 0.72      | 0.70      | 0.70       |
|              | Q3         | 0.89     | 1.34     | 1.52     | 1.52     | 1.46      | 0.82      | 0.80      | 0.83       |
|              | Max.       | 2.19     | 3.09     | 3.31     | 3.67     | 3.93      | 1.20      | 1.30      | 1.38       |
|              | Min.       | 0.38     | 0.83     | 0.99     | 0.84     | 1.10      | 0.53      | 0.49      | 0.36       |
| VECM         | Q1         | 0.63     | 1.28     | 1.45     | 1.27     | 1.42      | 0.78      | 0.79      | 0.77       |
|              | Median     | 0.70     | 1.44     | 1.63     | 1.48     | 1.57      | 0.91      | 0.96      | 1.13       |
|              | Q3         | 0.77     | 1.65     | 1.94     | 1.70     | 1.82      | 1.05      | 1.15      | 1.38       |
|              | Max.       | 1.06     | 2.23     | 2.95     | 2.43     | 3.31      | 1.51      | 1.59      | 2.08       |
|              | Min.       | 0.38     | 0.83     | 0.99     | 0.84     | 1.10      | 0.53      | 0.49      | 0.36       |

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| <b>Model</b> | <b>MSE</b> | <b>1</b> | <b>2</b> | <b>3</b> | <b>6</b> | <b>12</b> | <b>3M</b> | <b>6M</b> | <b>12M</b> |
|--------------|------------|----------|----------|----------|----------|-----------|-----------|-----------|------------|
| SETAR        | Min.       | 0.55     | 0.71     | 0.79     | 0.75     | 0.65      | 1.04      | 0.98      | 0.67       |
|              | Q1         | 0.80     | 0.99     | 1.11     | 1.04     | 1.04      | 1.27      | 1.33      | 1.53       |
|              | Median     | 0.93     | 1.16     | 1.27     | 1.20     | 1.17      | 1.42      | 1.55      | 1.81       |
|              | Q3         | 1.06     | 1.33     | 1.48     | 1.36     | 1.41      | 1.59      | 1.74      | 2.01       |
|              | Max.       | 1.37     | 1.75     | 2.10     | 1.83     | 2.01      | 1.96      | 2.23      | 2.66       |
| MA           | Min.       | 0.65     | 0.78     | 0.76     | 0.71     | 0.71      | 0.78      | 0.72      | 0.67       |
|              | Q1         | 0.84     | 1.01     | 0.99     | 0.92     | 0.92      | 1.01      | 0.93      | 0.88       |
|              | Median     | 0.93     | 1.13     | 1.12     | 1.06     | 1.07      | 1.12      | 1.06      | 1.02       |
|              | Q3         | 1.03     | 1.27     | 1.27     | 1.22     | 1.24      | 1.24      | 1.20      | 1.19       |
|              | Max.       | 1.33     | 1.64     | 1.65     | 1.58     | 1.62      | 1.60      | 1.56      | 1.55       |
| SARIMA       | Min.       | 0.54     | 0.64     | 0.75     | 0.87     | 0.85      | 0.49      | 0.47      | 0.39       |
|              | Q1         | 0.70     | 0.93     | 1.02     | 1.09     | 1.10      | 0.67      | 0.60      | 0.61       |
|              | Median     | 0.77     | 1.02     | 1.13     | 1.24     | 1.22      | 0.75      | 0.69      | 0.68       |
|              | Q3         | 0.82     | 1.14     | 1.28     | 1.35     | 1.34      | 0.81      | 0.78      | 0.77       |
|              | Max.       | 1.05     | 1.54     | 1.61     | 1.82     | 1.82      | 1.06      | 1.05      | 1.22       |
| ARFIMA       | Min.       | 0.51     | 0.62     | 0.88     | 0.82     | 0.82      | 0.45      | 0.46      | 0.44       |
|              | Q1         | 0.69     | 0.83     | 1.02     | 1.08     | 1.08      | 0.62      | 0.63      | 0.62       |
|              | Median     | 0.75     | 0.95     | 1.14     | 1.19     | 1.21      | 0.70      | 0.71      | 0.71       |
|              | Q3         | 0.82     | 1.11     | 1.38     | 1.34     | 1.35      | 0.79      | 0.81      | 0.81       |
|              | Max.       | 1.12     | 1.43     | 1.76     | 1.92     | 1.83      | 1.09      | 1.00      | 1.03       |
| GradBoost    | Min.       | 0.75     | 0.92     | 0.92     | 0.92     | 0.90      | 0.91      | 0.90      | 0.97       |
|              | Q1         | 0.84     | 1.07     | 1.09     | 1.09     | 1.07      | 1.03      | 1.03      | 1.05       |
|              | Median     | 0.92     | 1.13     | 1.16     | 1.14     | 1.15      | 1.07      | 1.08      | 1.10       |
|              | Q3         | 0.98     | 1.19     | 1.20     | 1.20     | 1.23      | 1.12      | 1.13      | 1.15       |

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| Model      | MSE    | 1    | 2    | 3    | 6    | 12   | 3M   | 6M   | 12M  |
|------------|--------|------|------|------|------|------|------|------|------|
| AdaBoost   | Max.   | 1.13 | 1.39 | 1.35 | 1.37 | 1.45 | 1.27 | 1.27 | 1.37 |
|            | Min.   | 0.80 | 0.91 | 0.86 | 0.84 | 0.80 | 0.90 | 0.96 | 0.94 |
|            | Q1     | 0.88 | 1.02 | 1.03 | 1.03 | 1.02 | 0.99 | 1.02 | 1.02 |
|            | Median | 0.93 | 1.06 | 1.06 | 1.07 | 1.10 | 1.04 | 1.06 | 1.06 |
|            | Q3     | 0.97 | 1.09 | 1.10 | 1.10 | 1.15 | 1.08 | 1.09 | 1.11 |
| Bayes Reg. | Max.   | 1.10 | 1.22 | 1.22 | 1.24 | 1.38 | 1.19 | 1.30 | 1.26 |
|            | Min.   | 0.76 | 0.96 | 1.01 | 1.00 | 0.98 | 0.97 | 0.99 | 1.00 |
|            | Q1     | 0.99 | 1.22 | 1.22 | 1.23 | 1.24 | 1.19 | 1.20 | 1.23 |
|            | Median | 1.07 | 1.33 | 1.32 | 1.33 | 1.34 | 1.30 | 1.31 | 1.33 |
|            | Q3     | 1.20 | 1.43 | 1.42 | 1.42 | 1.50 | 1.40 | 1.42 | 1.46 |
|            | Max.   | 1.56 | 1.74 | 1.95 | 2.08 | 2.10 | 1.90 | 1.71 | 1.75 |

Table 2: Average MSE reduction delivered by each model with respect to random walk. Negative values mean that the model increased the MSE. The winning model is highlighted in bold font.

| Model           | 1            | 2            | 3            | 6            | 12           | 3M           | 6M           | 12M          |
|-----------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| LSTM            | 83.6%        | 72.8%        | 72.4%        | 71.6%        | 70.9%        | 78.4%        | 71.4%        | 75.2%        |
| <b>ConvLSTM</b> | <b>91.5%</b> | <b>79.0%</b> | <b>77.9%</b> | <b>77.9%</b> | <b>77.0%</b> | <b>83.4%</b> | <b>77.8%</b> | <b>82.8%</b> |
| MLP             | 50.5%        | 41.5%        | 42.5%        | 42.3%        | 40.8%        | 44.8%        | 43.7%        | 43.4%        |
| RW              | 0.0%         | 0.0%         | 0.0%         | 0.0%         | 0.0%         | 0.0%         | 0.0%         | 0.0%         |
| Ridge CV        | 57.5%        | 49.3%        | 49.9%        | 49.9%        | 49.0%        | 50.6%        | 51.3%        | 51.7%        |
| BRidge          | 57.7%        | 46.0%        | 46.9%        | 46.6%        | 45.7%        | 49.8%        | 49.1%        | 48.9%        |
| LASSO CV        | 58.8%        | 49.3%        | 50.3%        | 50.4%        | 49.1%        | 51.2%        | 51.4%        | 51.8%        |
| BLASSO          | 58.5%        | 49.5%        | 50.3%        | 50.5%        | 49.3%        | 51.6%        | 51.7%        | 51.9%        |

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| <b>Model</b> | <b>1</b> | <b>2</b> | <b>3</b> | <b>6</b> | <b>12</b> | <b>3M</b> | <b>6M</b> | <b>12M</b> |
|--------------|----------|----------|----------|----------|-----------|-----------|-----------|------------|
| Enet CV      | 62.0%    | 50.1%    | 50.8%    | 50.9%    | 50.0%     | 53.0%     | 52.7%     | 52.7%      |
| SVR          | 54.5%    | 48.1%    | 48.9%    | 49.1%    | 47.5%     | 50.7%     | 50.8%     | 50.4%      |
| RF           | 53.3%    | 44.8%    | 46.1%    | 45.9%    | 45.0%     | 48.4%     | 48.6%     | 48.8%      |
| BART         | 55.0%    | 47.1%    | 47.4%    | 47.2%    | 46.2%     | 50.4%     | 50.9%     | 51.1%      |
| Bagging      | 50.4%    | 41.4%    | 43.2%    | 42.5%    | 41.5%     | 45.6%     | 46.9%     | 46.5%      |
| kNN          | 50.3%    | 47.9%    | 48.6%    | 48.4%    | 47.4%     | 49.3%     | 50.2%     | 50.4%      |
| Huber        | 47.4%    | -12.5%   | -11.7%   | -11.9%   | -13.4%    | 7.5%      | -0.3%     | -4.9%      |
| Theil-Sen    | 47.0%    | -18.4%   | -17.5%   | -17.7%   | -19.0%    | 4.8%      | -3.0%     | -7.6%      |
| Factors      | 49.6%    | 46.9%    | 48.2%    | 47.0%    | 46.2%     | 47.3%     | 48.5%     | 48.7%      |
| GARCH        | 65.0%    | 46.0%    | 41.0%    | 40.1%    | 39.3%     | 63.0%     | 65.5%     | 64.8%      |
| VECM         | 65.0%    | 25.7%    | 15.1%    | 17.2%    | 17.4%     | 54.4%     | 52.7%     | 47.3%      |
| SETAR        | 53.6%    | 41.1%    | 35.3%    | 40.0%    | 38.6%     | 28.6%     | 24.2%     | 15.1%      |
| MA           | 54.0%    | 43.1%    | 44.2%    | 46.8%    | 46.4%     | 44.4%     | 48.8%     | 51.2%      |
| SARIMA       | 62.0%    | 47.8%    | 43.0%    | 40.7%    | 38.2%     | 63.0%     | 66.0%     | 66.6%      |
| ARFIMA       | 62.7%    | 51.1%    | 40.5%    | 39.3%    | 38.6%     | 64.7%     | 65.3%     | 65.2%      |
| GradBoost    | 54.6%    | 42.8%    | 43.3%    | 42.9%    | 42.2%     | 46.8%     | 47.5%     | 47.0%      |
| AdaBoost     | 54.0%    | 46.6%    | 47.2%    | 47.2%    | 45.6%     | 48.4%     | 48.7%     | 48.6%      |
| Bayes Reg.   | 45.5%    | 32.9%    | 34.3%    | 33.9%    | 31.4%     | 35.2%     | 35.9%     | 35.2%      |

## 2 Appendix: Out-of-Sample MAE

Table 3: Descriptive statistics of the out-of-sample MAE of each model. “MLP” refers to the multi-layer perceptron; “RW” is the random walk; “Ridge CV”, “LASSO CV”, and “Enet CV” are the Ridge, LASSO, and Elastic Net regressions with parameters chosen via cross-validation; “BRidge” is the Bayesian Ridge; “BLASSO” is the Bayesian LASSO; and “MA” is the moving average model suggested by **Atkeson2001**.

| Model    | Statistic | 1    | 2    | 3    | 6    | 12   | 3M   | 6M   | 12M  |
|----------|-----------|------|------|------|------|------|------|------|------|
| LSTM     | Min.      | 0.29 | 0.42 | 0.47 | 0.49 | 0.49 | 0.40 | 0.43 | 0.45 |
|          | Q1        | 0.40 | 0.55 | 0.57 | 0.57 | 0.56 | 0.49 | 0.55 | 0.51 |
|          | Median    | 0.45 | 0.58 | 0.60 | 0.59 | 0.61 | 0.53 | 0.59 | 0.57 |
|          | Q3        | 0.51 | 0.63 | 0.63 | 0.64 | 0.64 | 0.58 | 0.66 | 0.63 |
|          | Max.      | 0.72 | 0.84 | 0.79 | 1.03 | 0.91 | 0.70 | 0.88 | 0.86 |
| ConvLSTM | Min.      | 0.25 | 0.45 | 0.48 | 0.50 | 0.50 | 0.37 | 0.40 | 0.33 |
|          | Q1        | 0.30 | 0.52 | 0.53 | 0.54 | 0.54 | 0.44 | 0.50 | 0.43 |
|          | Median    | 0.32 | 0.53 | 0.54 | 0.54 | 0.54 | 0.46 | 0.53 | 0.46 |
|          | Q3        | 0.35 | 0.54 | 0.55 | 0.55 | 0.55 | 0.49 | 0.58 | 0.50 |
|          | Max.      | 0.41 | 0.56 | 0.56 | 0.55 | 0.56 | 0.58 | 0.75 | 0.60 |
| MLP      | Min.      | 0.63 | 0.71 | 0.69 | 0.68 | 0.67 | 0.72 | 0.74 | 0.78 |
|          | Q1        | 0.68 | 0.76 | 0.76 | 0.76 | 0.77 | 0.79 | 0.82 | 0.86 |
|          | Median    | 0.71 | 0.79 | 0.79 | 0.80 | 0.80 | 0.82 | 0.85 | 0.88 |
|          | Q3        | 0.74 | 0.82 | 0.82 | 0.84 | 0.83 | 0.85 | 0.88 | 0.91 |
|          | Max.      | 0.84 | 0.88 | 0.87 | 0.95 | 0.91 | 0.91 | 0.96 | 0.98 |
| RW       | Min.      | 0.91 | 0.88 | 0.92 | 0.93 | 0.91 | 0.91 | 0.93 | 0.96 |
|          | Q1        | 1.02 | 1.01 | 1.01 | 1.02 | 1.00 | 1.06 | 1.08 | 1.09 |
|          | Median    | 1.07 | 1.05 | 1.06 | 1.07 | 1.05 | 1.10 | 1.11 | 1.13 |
|          | Q3        | 1.10 | 1.09 | 1.11 | 1.10 | 1.11 | 1.13 | 1.15 | 1.16 |
|          | Max.      | 1.20 | 1.18 | 1.20 | 1.22 | 1.28 | 1.24 | 1.30 | 1.25 |
| Ridge CV | Min.      | 0.56 | 0.64 | 0.64 | 0.63 | 0.63 | 0.67 | 0.71 | 0.74 |
|          | Q1        | 0.62 | 0.69 | 0.69 | 0.68 | 0.69 | 0.75 | 0.77 | 0.80 |
|          | Median    | 0.64 | 0.72 | 0.71 | 0.71 | 0.71 | 0.77 | 0.79 | 0.82 |
|          | Q3        | 0.67 | 0.73 | 0.73 | 0.74 | 0.74 | 0.79 | 0.82 | 0.84 |



Table 3: Descriptive statistics of the out-of-sample MAE of each model. “MLP” refers to the multi-layer perceptron; “RW” is the random walk; “Ridge CV”, “LASSO CV”, and “Enet CV” are the Ridge, LASSO, and Elastic Net regressions with parameters chosen via cross-validation; “BRidge” is the Bayesian Ridge; “BLASSO” is the Bayesian LASSO; and “MA” is the moving average model suggested by **Atkeson2001**.

| Model    | Statistic | 1    | 2    | 3    | 6    | 12   | 3M   | 6M   | 12M  |
|----------|-----------|------|------|------|------|------|------|------|------|
| BRidge   | Max.      | 0.82 | 0.78 | 0.78 | 0.78 | 0.79 | 0.84 | 0.87 | 0.91 |
|          | Min.      | 0.55 | 0.67 | 0.65 | 0.66 | 0.66 | 0.69 | 0.73 | 0.74 |
|          | Q1        | 0.63 | 0.72 | 0.72 | 0.72 | 0.72 | 0.75 | 0.79 | 0.81 |
|          | Median    | 0.65 | 0.75 | 0.74 | 0.76 | 0.75 | 0.78 | 0.82 | 0.84 |
|          | Q3        | 0.68 | 0.78 | 0.77 | 0.78 | 0.78 | 0.80 | 0.84 | 0.86 |
| LASSO CV | Max.      | 0.80 | 0.86 | 0.83 | 0.84 | 0.85 | 0.86 | 0.93 | 0.94 |
|          | Min.      | 0.58 | 0.64 | 0.65 | 0.63 | 0.64 | 0.68 | 0.70 | 0.72 |
|          | Q1        | 0.62 | 0.69 | 0.69 | 0.69 | 0.70 | 0.74 | 0.77 | 0.80 |
|          | Median    | 0.64 | 0.71 | 0.71 | 0.71 | 0.71 | 0.76 | 0.79 | 0.82 |
|          | Q3        | 0.66 | 0.73 | 0.73 | 0.74 | 0.74 | 0.78 | 0.82 | 0.84 |
| BLASSO   | Max.      | 0.76 | 0.78 | 0.77 | 0.79 | 0.81 | 0.84 | 0.88 | 0.89 |
|          | Min.      | 0.53 | 0.64 | 0.65 | 0.63 | 0.64 | 0.67 | 0.70 | 0.72 |
|          | Q1        | 0.63 | 0.69 | 0.69 | 0.68 | 0.69 | 0.74 | 0.77 | 0.80 |
|          | Median    | 0.66 | 0.71 | 0.71 | 0.71 | 0.71 | 0.76 | 0.79 | 0.82 |
|          | Q3        | 0.69 | 0.73 | 0.73 | 0.73 | 0.74 | 0.78 | 0.81 | 0.84 |
| Enet CV  | Max.      | 0.75 | 0.77 | 0.77 | 0.78 | 0.79 | 0.83 | 0.86 | 0.89 |
|          | Min.      | 0.55 | 0.64 | 0.64 | 0.64 | 0.64 | 0.66 | 0.70 | 0.72 |
|          | Q1        | 0.60 | 0.68 | 0.68 | 0.68 | 0.68 | 0.73 | 0.76 | 0.79 |
|          | Median    | 0.63 | 0.71 | 0.71 | 0.71 | 0.71 | 0.75 | 0.78 | 0.81 |
|          | Q3        | 0.65 | 0.73 | 0.72 | 0.73 | 0.73 | 0.77 | 0.81 | 0.83 |
| SVR      | Max.      | 0.74 | 0.77 | 0.77 | 0.78 | 0.79 | 0.81 | 0.86 | 0.87 |
|          | Min.      | 0.59 | 0.64 | 0.66 | 0.65 | 0.65 | 0.68 | 0.73 | 0.74 |
|          | Q1        | 0.65 | 0.70 | 0.70 | 0.70 | 0.70 | 0.74 | 0.78 | 0.80 |
|          | Median    | 0.67 | 0.73 | 0.73 | 0.73 | 0.73 | 0.77 | 0.80 | 0.83 |

Table 3: Descriptive statistics of the out-of-sample MAE of each model. “MLP” refers to the multi-layer perceptron; “RW” is the random walk; “Ridge CV”, “LASSO CV”, and “Enet CV” are the Ridge, LASSO, and Elastic Net regressions with parameters chosen via cross-validation; “BRidge” is the Bayesian Ridge; “BLASSO” is the Bayesian LASSO; and “MA” is the moving average model suggested by **Atkeson2001**.

| Model   | Statistic | 1    | 2    | 3    | 6    | 12   | 3M   | 6M   | 12M  |
|---------|-----------|------|------|------|------|------|------|------|------|
| RF      | Q3        | 0.69 | 0.75 | 0.74 | 0.75 | 0.75 | 0.79 | 0.82 | 0.85 |
|         | Max.      | 0.76 | 0.81 | 0.79 | 0.81 | 0.83 | 0.82 | 0.88 | 0.90 |
|         | Min.      | 0.55 | 0.66 | 0.67 | 0.65 | 0.67 | 0.69 | 0.73 | 0.76 |
|         | Q1        | 0.67 | 0.74 | 0.72 | 0.73 | 0.73 | 0.76 | 0.79 | 0.81 |
|         | Median    | 0.69 | 0.76 | 0.75 | 0.76 | 0.76 | 0.79 | 0.82 | 0.84 |
| BART    | Q3        | 0.71 | 0.78 | 0.77 | 0.78 | 0.79 | 0.81 | 0.84 | 0.87 |
|         | Max.      | 0.79 | 0.83 | 0.87 | 0.87 | 0.92 | 0.86 | 0.92 | 0.92 |
|         | Min.      | 0.60 | 0.64 | 0.66 | 0.64 | 0.66 | 0.67 | 0.71 | 0.74 |
|         | Q1        | 0.66 | 0.71 | 0.71 | 0.71 | 0.72 | 0.75 | 0.78 | 0.80 |
|         | Median    | 0.68 | 0.74 | 0.74 | 0.74 | 0.75 | 0.77 | 0.80 | 0.83 |
| Bagging | Q3        | 0.71 | 0.76 | 0.76 | 0.77 | 0.78 | 0.79 | 0.82 | 0.85 |
|         | Max.      | 0.80 | 0.81 | 0.83 | 0.83 | 0.88 | 0.85 | 0.88 | 0.91 |
|         | Min.      | 0.65 | 0.70 | 0.67 | 0.69 | 0.69 | 0.72 | 0.71 | 0.73 |
|         | Q1        | 0.70 | 0.77 | 0.74 | 0.75 | 0.76 | 0.79 | 0.81 | 0.83 |
|         | Median    | 0.72 | 0.79 | 0.77 | 0.78 | 0.79 | 0.82 | 0.83 | 0.86 |
| kNN     | Q3        | 0.74 | 0.82 | 0.80 | 0.81 | 0.82 | 0.83 | 0.86 | 0.89 |
|         | Max.      | 0.83 | 0.90 | 0.89 | 0.88 | 0.91 | 0.90 | 0.92 | 0.95 |
|         | Min.      | 0.63 | 0.66 | 0.65 | 0.62 | 0.66 | 0.67 | 0.72 | 0.73 |
|         | Q1        | 0.69 | 0.71 | 0.70 | 0.71 | 0.71 | 0.76 | 0.78 | 0.81 |
|         | Median    | 0.71 | 0.73 | 0.72 | 0.73 | 0.73 | 0.78 | 0.81 | 0.83 |
| Huber   | Q3        | 0.73 | 0.75 | 0.75 | 0.76 | 0.77 | 0.80 | 0.83 | 0.85 |
|         | Max.      | 0.78 | 0.79 | 0.80 | 0.82 | 0.83 | 0.84 | 0.90 | 0.91 |
|         | Min.      | 0.63 | 0.99 | 0.94 | 0.94 | 0.96 | 0.92 | 0.92 | 0.99 |
|         | Q1        | 0.72 | 1.10 | 1.09 | 1.11 | 1.10 | 1.01 | 1.09 | 1.11 |

Table 3: Descriptive statistics of the out-of-sample MAE of each model. “MLP” refers to the multi-layer perceptron; “RW” is the random walk; “Ridge CV”, “LASSO CV”, and “Enet CV” are the Ridge, LASSO, and Elastic Net regressions with parameters chosen via cross-validation; “BRidge” is the Bayesian Ridge; “BLASSO” is the Bayesian LASSO; and “MA” is the moving average model suggested by **Atkeson2001**.

| Model     | Statistic | 1    | 2    | 3    | 6    | 12   | 3M   | 6M   | 12M  |
|-----------|-----------|------|------|------|------|------|------|------|------|
| Theil-Sen | Median    | 0.76 | 1.15 | 1.15 | 1.17 | 1.16 | 1.07 | 1.14 | 1.16 |
|           | Q3        | 0.79 | 1.22 | 1.22 | 1.23 | 1.25 | 1.10 | 1.20 | 1.24 |
|           | Max.      | 0.87 | 1.46 | 1.38 | 1.45 | 1.43 | 1.24 | 1.37 | 1.41 |
|           | Min.      | 0.63 | 1.01 | 0.95 | 0.99 | 1.00 | 0.94 | 0.92 | 1.00 |
|           | Q1        | 0.72 | 1.11 | 1.12 | 1.13 | 1.13 | 1.02 | 1.09 | 1.12 |
| Factors   | Median    | 0.76 | 1.19 | 1.18 | 1.20 | 1.20 | 1.08 | 1.16 | 1.18 |
|           | Q3        | 0.79 | 1.25 | 1.26 | 1.27 | 1.28 | 1.12 | 1.22 | 1.26 |
|           | Max.      | 0.87 | 1.48 | 1.41 | 1.50 | 1.48 | 1.25 | 1.38 | 1.44 |
|           | Min.      | 0.61 | 0.65 | 0.63 | 0.66 | 0.65 | 0.71 | 0.74 | 0.74 |
|           | Q1        | 0.69 | 0.71 | 0.71 | 0.71 | 0.71 | 0.76 | 0.79 | 0.81 |
| GARCH     | Median    | 0.72 | 0.74 | 0.74 | 0.74 | 0.74 | 0.79 | 0.82 | 0.83 |
|           | Q3        | 0.76 | 0.76 | 0.76 | 0.77 | 0.77 | 0.82 | 0.85 | 0.86 |
|           | Max.      | 0.85 | 0.83 | 0.81 | 1.50 | 0.88 | 0.90 | 0.95 | 1.32 |
|           | Min.      | 0.38 | 0.49 | 0.48 | 0.49 | 0.49 | 0.53 | 0.50 | 0.50 |
|           | Q1        | 0.56 | 0.67 | 0.68 | 0.70 | 0.71 | 0.62 | 0.61 | 0.62 |
| VECM      | Median    | 0.63 | 0.74 | 0.79 | 0.81 | 0.81 | 0.67 | 0.65 | 0.67 |
|           | Q3        | 0.72 | 0.89 | 0.93 | 0.93 | 0.95 | 0.71 | 0.70 | 0.73 |
|           | Max.      | 0.99 | 1.20 | 1.24 | 1.33 | 1.39 | 0.86 | 0.86 | 0.94 |
|           | Min.      | 0.48 | 0.71 | 0.80 | 0.69 | 0.81 | 0.59 | 0.54 | 0.49 |
|           | Q1        | 0.61 | 0.89 | 0.95 | 0.90 | 0.94 | 0.70 | 0.69 | 0.72 |
| SETAR     | Median    | 0.64 | 0.93 | 1.02 | 0.96 | 0.99 | 0.75 | 0.78 | 0.85 |
|           | Q3        | 0.68 | 1.02 | 1.12 | 1.03 | 1.06 | 0.81 | 0.86 | 0.94 |
|           | Max.      | 0.77 | 1.16 | 1.36 | 1.24 | 1.46 | 0.97 | 1.04 | 1.20 |
|           | Min.      | 0.51 | 0.62 | 0.64 | 0.65 | 0.60 | 0.78 | 0.79 | 0.65 |

Table 3: Descriptive statistics of the out-of-sample MAE of each model. “MLP” refers to the multi-layer perceptron; “RW” is the random walk; “Ridge CV”, “LASSO CV”, and “Enet CV” are the Ridge, LASSO, and Elastic Net regressions with parameters chosen via cross-validation; “BRidge” is the Bayesian Ridge; “BLASSO” is the Bayesian LASSO; and “MA” is the moving average model suggested by **Atkeson2001**.

| Model     | Statistic | 1    | 2    | 3    | 6    | 12   | 3M   | 6M   | 12M  |
|-----------|-----------|------|------|------|------|------|------|------|------|
| MA        | Q1        | 0.64 | 0.73 | 0.78 | 0.76 | 0.75 | 0.87 | 0.92 | 1.00 |
|           | Median    | 0.68 | 0.78 | 0.85 | 0.81 | 0.81 | 0.93 | 0.98 | 1.07 |
|           | Q3        | 0.72 | 0.84 | 0.91 | 0.87 | 0.89 | 0.99 | 1.06 | 1.14 |
|           | Max       | 0.82 | 0.95 | 1.06 | 0.99 | 0.98 | 1.13 | 1.21 | 1.39 |
|           | Min.      | 0.48 | 0.52 | 0.51 | 0.49 | 0.49 | 0.53 | 0.50 | 0.48 |
| SARIMA    | Q1        | 0.61 | 0.68 | 0.66 | 0.64 | 0.64 | 0.68 | 0.64 | 0.62 |
|           | Median    | 0.68 | 0.76 | 0.75 | 0.73 | 0.74 | 0.75 | 0.73 | 0.72 |
|           | Q3        | 0.76 | 0.85 | 0.85 | 0.84 | 0.86 | 0.83 | 0.83 | 0.84 |
|           | Max.      | 0.97 | 1.10 | 1.10 | 1.09 | 1.12 | 1.07 | 1.07 | 1.09 |
|           | Min.      | 0.56 | 0.64 | 0.68 | 0.74 | 0.72 | 0.56 | 0.57 | 0.50 |
| ARFIMA    | Q1        | 0.67 | 0.77 | 0.81 | 0.83 | 0.84 | 0.65 | 0.62 | 0.62 |
|           | Median    | 0.71 | 0.81 | 0.85 | 0.88 | 0.88 | 0.69 | 0.67 | 0.67 |
|           | Q3        | 0.74 | 0.85 | 0.91 | 0.93 | 0.95 | 0.73 | 0.71 | 0.71 |
|           | Max.      | 0.84 | 1.00 | 1.04 | 1.10 | 1.09 | 0.84 | 0.81 | 0.93 |
|           | Min.      | 0.58 | 0.61 | 0.72 | 0.70 | 0.73 | 0.53 | 0.51 | 0.52 |
| GradBoost | Q1        | 0.65 | 0.73 | 0.80 | 0.82 | 0.83 | 0.63 | 0.64 | 0.63 |
|           | Median    | 0.69 | 0.77 | 0.85 | 0.87 | 0.87 | 0.66 | 0.68 | 0.68 |
|           | Q3        | 0.72 | 0.84 | 0.94 | 0.92 | 0.93 | 0.70 | 0.71 | 0.72 |
|           | Max.      | 0.86 | 0.93 | 1.07 | 1.12 | 1.07 | 0.83 | 0.80 | 0.83 |
|           | Min.      | 0.59 | 0.68 | 0.70 | 0.71 | 0.67 | 0.71 | 0.75 | 0.74 |
|           | Q1        | 0.66 | 0.75 | 0.76 | 0.75 | 0.75 | 0.78 | 0.80 | 0.83 |
|           | Median    | 0.68 | 0.77 | 0.78 | 0.78 | 0.78 | 0.80 | 0.83 | 0.86 |
|           | Q3        | 0.71 | 0.80 | 0.80 | 0.80 | 0.81 | 0.82 | 0.85 | 0.88 |
|           | Max.      | 0.77 | 0.89 | 0.86 | 0.88 | 0.92 | 0.90 | 0.91 | 0.93 |

Table 3: Descriptive statistics of the out-of-sample MAE of each model. “MLP” refers to the multi-layer perceptron; “RW” is the random walk; “Ridge CV”, “LASSO CV”, and “Enet CV” are the Ridge, LASSO, and Elastic Net regressions with parameters chosen via cross-validation; “BRidge” is the Bayesian Ridge; “BLASSO” is the Bayesian LASSO; and “MA” is the moving average model suggested by **Atkeson2001**.

| Model      | Statistic | 1    | 2    | 3    | 6    | 12   | 3M   | 6M   | 12M  |
|------------|-----------|------|------|------|------|------|------|------|------|
| AdaBoost   | Min.      | 0.60 | 0.66 | 0.66 | 0.67 | 0.65 | 0.69 | 0.73 | 0.75 |
|            | Q1        | 0.66 | 0.72 | 0.72 | 0.72 | 0.72 | 0.76 | 0.79 | 0.82 |
|            | Median    | 0.69 | 0.74 | 0.74 | 0.74 | 0.75 | 0.78 | 0.82 | 0.84 |
|            | Q3        | 0.71 | 0.76 | 0.77 | 0.77 | 0.78 | 0.81 | 0.84 | 0.86 |
|            | Max.      | 0.80 | 0.83 | 0.81 | 0.83 | 0.87 | 0.89 | 0.93 | 0.93 |
| Bayes Reg. | Min.      | 0.65 | 0.71 | 0.74 | 0.72 | 0.73 | 0.75 | 0.76 | 0.77 |
|            | Q1        | 0.73 | 0.82 | 0.82 | 0.82 | 0.83 | 0.85 | 0.86 | 0.90 |
|            | Median    | 0.77 | 0.86 | 0.86 | 0.87 | 0.87 | 0.88 | 0.91 | 0.93 |
|            | Q3        | 0.81 | 0.90 | 0.90 | 0.90 | 0.92 | 0.92 | 0.95 | 0.98 |
|            | Max.      | 0.93 | 1.01 | 1.01 | 1.07 | 1.05 | 1.06 | 1.08 | 1.09 |

Table 4: Average MAE reduction delivered by each model with respect to random walk. Negative values mean that the model increased the MSE. The winning model is highlighted in bold font.

| Model           | 1            | 2            | 3            | 6            | 12           | 3M           | 6M           | 12M          |
|-----------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| LSTM            | 57.9%        | 44.5%        | 43.5%        | 43.5%        | 42.1%        | 51.6%        | 46.9%        | 49.5%        |
| <b>ConvLSTM</b> | <b>69.7%</b> | <b>49.7%</b> | <b>48.9%</b> | <b>49.2%</b> | <b>48.0%</b> | <b>58.2%</b> | <b>52.1%</b> | <b>58.8%</b> |
| MLP             | 33.1%        | 24.5%        | 25.7%        | 25.6%        | 23.8%        | 25.7%        | 23.3%        | 22.0%        |
| RW              | 0.0%         | 0.0%         | 0.0%         | 0.0%         | 0.0%         | 0.0%         | 0.0%         | 0.0%         |
| Ridge CV        | 39.8%        | 31.6%        | 32.9%        | 33.1%        | 31.8%        | 30.2%        | 28.8%        | 27.1%        |
| BRidge          | 39.1%        | 28.7%        | 30.1%        | 29.9%        | 28.4%        | 29.2%        | 26.5%        | 25.3%        |
| LASSO CV        | 39.7%        | 32.6%        | 33.7%        | 33.4%        | 32.3%        | 30.4%        | 28.9%        | 27.3%        |
| BLASSO          | 38.3%        | 32.4%        | 33.7%        | 33.8%        | 32.4%        | 30.6%        | 28.9%        | 27.3%        |
| Enet CV         | 41.2%        | 32.6%        | 33.7%        | 33.7%        | 32.3%        | 31.4%        | 29.6%        | 27.7%        |

Table 4: Average MAE reduction delivered by each model with respect to random walk. Negative values mean that the model increased the MSE. The winning model is highlighted in bold font.

| <b>Model</b> | <b>1</b> | <b>2</b> | <b>3</b> | <b>6</b> | <b>12</b> | <b>3M</b> | <b>6M</b> | <b>12M</b> |
|--------------|----------|----------|----------|----------|-----------|-----------|-----------|------------|
| SVR          | 36.9%    | 30.4%    | 31.6%    | 32.5%    | 30.4%     | 29.7%     | 28.1%     | 26.1%      |
| RF           | 35.7%    | 27.4%    | 29.3%    | 29.7%    | 27.1%     | 28.5%     | 26.4%     | 25.6%      |
| BART         | 36.3%    | 29.2%    | 30.5%    | 30.5%    | 28.8%     | 30.0%     | 28.5%     | 26.6%      |
| Bagging      | 32.5%    | 24.6%    | 27.4%    | 26.8%    | 24.7%     | 25.6%     | 25.7%     | 23.6%      |
| kNN          | 33.2%    | 30.3%    | 32.0%    | 31.6%    | 30.4%     | 29.4%     | 27.6%     | 26.0%      |
| Huber        | 29.2%    | -10.0%   | -7.8%    | -9.0%    | -11.2%    | 2.5%      | -2.0%     | -2.9%      |
| Theil-Sen    | 28.6%    | -14.0%   | -10.6%   | -11.9%   | -14.5%    | 1.4%      | -4.2%     | -4.6%      |
| Factors      | 32.2%    | 29.8%    | 30.9%    | 30.5%    | 29.2%     | 27.6%     | 26.7%     | 26.0%      |
| GARCH        | 40.7%    | 29.1%    | 25.4%    | 23.8%    | 23.0%     | 39.3%     | 41.4%     | 40.6%      |
| VECM         | 39.9%    | 11.2%    | 3.8%     | 5.9%     | 5.4%      | 31.3%     | 29.8%     | 24.6%      |
| SETAR        | 36.3%    | 25.5%    | 20.4%    | 24.6%    | 23.0%     | 15.7%     | 12.3%     | 5.3%       |
| MA           | 35.9%    | 27.9%    | 29.9%    | 31.9%    | 29.6%     | 32.0%     | 34.3%     | 36.5%      |
| SARIMA       | 33.7%    | 22.4%    | 19.9%    | 18.1%    | 15.9%     | 37.2%     | 40.3%     | 40.9%      |
| ARFIMA       | 35.4%    | 27.0%    | 20.1%    | 18.6%    | 17.1%     | 39.5%     | 38.8%     | 39.8%      |
| GradBoost    | 36.3%    | 26.1%    | 26.7%    | 27.2%    | 26.0%     | 27.4%     | 25.6%     | 24.0%      |
| AdaBoost     | 35.3%    | 29.4%    | 30.1%    | 30.0%    | 28.8%     | 28.7%     | 26.6%     | 25.6%      |
| Bayes Reg.   | 27.7%    | 18.0%    | 18.9%    | 19.6%    | 17.3%     | 19.7%     | 18.4%     | 17.3%      |

Table 5: Ranks produced by comparing the models according to their average MAE through the simulations.

The two best models are highlight in bold font.

| <b>Model</b>    | <b>1</b> | <b>2</b> | <b>3</b> | <b>6</b> | <b>12</b> | <b>3M</b> | <b>6M</b> | <b>12M</b> |
|-----------------|----------|----------|----------|----------|-----------|-----------|-----------|------------|
| <b>LSTM</b>     | <b>2</b> | <b>2</b> | <b>2</b> | <b>2</b> | <b>2</b>  | <b>2</b>  | <b>2</b>  | <b>2</b>   |
| <b>ConvLSTM</b> | <b>1</b> | <b>1</b> | <b>1</b> | <b>1</b> | <b>1</b>  | <b>1</b>  | <b>1</b>  | <b>1</b>   |
| MLP             | 20       | 20       | 17       | 17       | 19        | 20        | 21        | 21         |
| RW              | 26       | 24       | 24       | 24       | 24        | 26        | 24        | 24         |

Table 5: Ranks produced by comparing the models according to their average MAE through the simulations.

The two best models are highlight in bold font.

| <b>Model</b> | <b>1</b> | <b>2</b> | <b>3</b> | <b>6</b> | <b>12</b> | <b>3M</b> | <b>6M</b> | <b>12M</b> |
|--------------|----------|----------|----------|----------|-----------|-----------|-----------|------------|
| Ridge CV     | 6        | 6        | 6        | 6        | 6         | 11        | 11        | 10         |
| BRidge       | 8        | 13       | 11       | 13       | 13        | 15        | 17        | 17         |
| LASSO CV     | 7        | 3        | 5        | 5        | 5         | 10        | 9         | 9          |
| BLASSO       | 9        | 5        | 3        | 3        | 4         | 9         | 10        | 8          |
| Enet CV      | 3        | 4        | 4        | 4        | 3         | 7         | 8         | 7          |
| SVR          | 10       | 7        | 8        | 7        | 8         | 13        | 13        | 12         |
| RF           | 15       | 15       | 14       | 14       | 15        | 17        | 18        | 15         |
| BART         | 13       | 11       | 10       | 11       | 10        | 12        | 12        | 11         |
| Bagging      | 21       | 19       | 15       | 16       | 17        | 21        | 19        | 20         |
| kNN          | 19       | 8        | 7        | 9        | 7         | 14        | 14        | 13         |
| Huber        | 23       | 25       | 25       | 25       | 25        | 24        | 25        | 25         |
| Theil-Sen    | 24       | 26       | 26       | 26       | 26        | 25        | 26        | 26         |
| Factors      | 22       | 9        | 9        | 10       | 11        | 18        | 15        | 14         |
| GARCH        | 4        | 12       | 18       | 19       | 12        | 4         | 3         | 4          |
| VECM         | 5        | 23       | 23       | 23       | 23        | 8         | 7         | 18         |
| SETAR        | 11       | 18       | 19       | 18       | 18        | 23        | 23        | 23         |
| MA           | 14       | 14       | 13       | 8        | 9         | 6         | 6         | 6          |
| SARIMA       | 18       | 21       | 21       | 22       | 21        | 5         | 4         | 3          |
| ARFIMA       | 16       | 16       | 20       | 21       | 20        | 3         | 5         | 5          |
| GradBoost    | 12       | 17       | 16       | 15       | 16        | 19        | 20        | 19         |
| AdaBoost     | 17       | 10       | 12       | 12       | 14        | 16        | 16        | 16         |
| Bayes Reg.   | 25       | 22       | 22       | 20       | 22        | 22        | 22        | 22         |

### 3 Appendix: Out-of-Sample MAPE

Since MAPE is closely related to MAE and MSE, for the sake of brevity, we are here providing only the ranks of the models according to their respective MAPE.

Table 6: Ranks produced by comparing the models according to their average MAE through the simulations.

The two best models are highlight in bold font.

| Model           | 1        | 2        | 3        | 6        | 12       | 3M       | 6M       | 12M      |
|-----------------|----------|----------|----------|----------|----------|----------|----------|----------|
| <b>LSTM</b>     | <b>2</b> | <b>2</b> | <b>2</b> | <b>2</b> | <b>2</b> | <b>2</b> | <b>2</b> | <b>2</b> |
| <b>ConvLSTM</b> | <b>1</b> | <b>1</b> | <b>1</b> | <b>1</b> | <b>1</b> | <b>1</b> | <b>1</b> | <b>1</b> |
| MLP             | 21       | 22       | 21       | 22       | 20       | 20       | 21       | 21       |
| RW              | 26       | 25       | 24       | 24       | 24       | 26       | 24       | 24       |
| Ridge CV        | 6        | 7        | 6        | 6        | 6        | 9        | 9        | 10       |
| BRidge          | 7        | 8        | 7        | 9        | 7        | 15       | 17       | 17       |
| LASSO CV        | 8        | 3        | 5        | 5        | 5        | 10       | 11       | 9        |
| BLASSO          | 3        | 5        | 3        | 3        | 4        | 4        | 3        | 8        |
| Enet CV         | 5        | 4        | 4        | 4        | 3        | 7        | 8        | 7        |
| SVR             | 10       | 6        | 8        | 7        | 8        | 13       | 13       | 12       |
| RF              | 15       | 15       | 14       | 14       | 15       | 17       | 18       | 18       |
| BART            | 4        | 11       | 10       | 11       | 10       | 11       | 10       | 11       |
| Bagging         | 20       | 19       | 15       | 16       | 17       | 21       | 19       | 20       |
| kNN             | 19       | 13       | 11       | 13       | 13       | 14       | 14       | 13       |
| Huber           | 23       | 24       | 25       | 25       | 25       | 24       | 25       | 25       |
| Theil-Sen       | 25       | 26       | 26       | 26       | 26       | 25       | 26       | 26       |
| Factors         | 22       | 9        | 9        | 10       | 11       | 18       | 15       | 14       |
| GARCH           | 13       | 12       | 18       | 19       | 12       | 12       | 12       | 4        |
| VECM            | 9        | 23       | 23       | 23       | 23       | 8        | 7        | 15       |
| SETAR           | 11       | 18       | 19       | 18       | 18       | 23       | 23       | 23       |
| MA              | 14       | 14       | 13       | 8        | 9        | 6        | 6        | 6        |
| SARIMA          | 18       | 21       | 17       | 17       | 21       | 5        | 4        | 3        |
| ARFIMA          | 16       | 16       | 20       | 21       | 19       | 3        | 5        | 5        |



Table 6: Ranks produced by comparing the models according to their average MAE through the simulations.  
The two best models are highlight in bold font.

| <b>Model</b> | <b>1</b> | <b>2</b> | <b>3</b> | <b>6</b> | <b>12</b> | <b>3M</b> | <b>6M</b> | <b>12M</b> |
|--------------|----------|----------|----------|----------|-----------|-----------|-----------|------------|
| GradBoost    | 17       | 17       | 16       | 15       | 16        | 19        | 20        | 19         |
| AdaBoost     | 12       | 10       | 12       | 12       | 14        | 16        | 16        | 16         |
| Bayes Reg.   | 24       | 20       | 22       | 20       | 22        | 22        | 22        | 22         |

## 4 Appendix: Out-of-Sample $R^2$

Analogously to the case of MAPE, we are simply providing the ranks of each model according to their respective out-of-sample  $R^2$ .

Table 7: Ranks produced by comparing the models according to their average  $R^2$  through the simulations.  
The two best models are highlight in bold font.

| <b>Model</b>    | <b>1</b> | <b>2</b> | <b>3</b> | <b>6</b> | <b>12</b> | <b>3M</b> | <b>6M</b> | <b>12M</b> |
|-----------------|----------|----------|----------|----------|-----------|-----------|-----------|------------|
| <b>LSTM</b>     | <b>2</b> | <b>2</b> | <b>2</b> | <b>2</b> | <b>2</b>  | <b>2</b>  | <b>2</b>  | <b>2</b>   |
| <b>ConvLSTM</b> | <b>1</b> | <b>1</b> | <b>1</b> | <b>1</b> | <b>1</b>  | <b>1</b>  | <b>1</b>  | <b>1</b>   |
| MLP             | 17       | 17       | 16       | 15       | 15        | 18        | 19        | 19         |
| RW              | 25       | 22       | 22       | 22       | 22        | 22        | 22        | 22         |
| Ridge CV        | 26       | 26       | 26       | 26       | 26        | 26        | 26        | 26         |
| BRidge          | 9        | 6        | 5        | 5        | 5         | 10        | 9         | 8          |
| LASSO CV        | 24       | 25       | 25       | 25       | 25        | 25        | 25        | 25         |
| BLASSO          | 8        | 5        | 4        | 4        | 4         | 8         | 8         | 7          |
| Enet CV         | 7        | 4        | 3        | 3        | 3         | 7         | 6         | 6          |
| SVR             | 12       | 7        | 6        | 6        | 6         | 9         | 11        | 12         |
| RF              | 16       | 14       | 11       | 12       | 12        | 14        | 15        | 13         |
| BART            | 10       | 10       | 9        | 9        | 9         | 11        | 10        | 10         |
| Bagging         | 18       | 18       | 14       | 14       | 14        | 17        | 18        | 18         |
| kNN             | 19       | 8        | 7        | 7        | 7         | 12        | 12        | 11         |

Table 7: Ranks produced by comparing the models according to their average  $R^2$  through the simulations. The two best models are highlight in bold font.

| <b>Model</b> | <b>1</b> | <b>2</b> | <b>3</b> | <b>6</b> | <b>12</b> | <b>3M</b> | <b>6M</b> | <b>12M</b> |
|--------------|----------|----------|----------|----------|-----------|-----------|-----------|------------|
| Huber        | 21       | 23       | 23       | 23       | 23        | 23        | 23        | 23         |
| Theil-Sen    | 22       | 24       | 24       | 24       | 24        | 24        | 24        | 24         |
| Factors      | 20       | 11       | 8        | 10       | 10        | 15        | 16        | 14         |
| GARCH        | 4        | 13       | 17       | 17       | 16        | 5         | 4         | 5          |
| VECM         | 3        | 21       | 21       | 21       | 21        | 6         | 7         | 16         |
| SETAR        | 15       | 19       | 19       | 18       | 17        | 21        | 21        | 21         |
| MA           | 13       | 15       | 12       | 11       | 8         | 19        | 13        | 9          |
| SARIMA       | 6        | 9        | 15       | 16       | 19        | 4         | 3         | 3          |
| ARFIMA       | 5        | 3        | 18       | 19       | 18        | 3         | 5         | 4          |
| GradBoost    | 11       | 16       | 13       | 13       | 13        | 16        | 17        | 17         |
| AdaBoost     | 14       | 12       | 10       | 8        | 11        | 13        | 14        | 15         |
| Bayes Reg.   | 23       | 20       | 20       | 20       | 20        | 20        | 20        | 20         |