Cointegration-based spread trading applied to the Foreign Exchange market

Thys Lynsey Van Overloop Hans Van Drom Karen
Vanstreels Dieter Trippaers Veerle Vermeir Jellen

July 23, 2015

1 Introduction

The exchange rate for a certain currency pair A/B can be interpreted as a price, expressing the number of units of the counter currency B that must be paid to obtain one unit of the base currency A. FOREX traders believing that certain exchange rates revert towards a particular equilibrium might try to make a profit by employing a mean reversion strategy on those exchange rates. In its simplest form, a mean reversion strategy could be summarized as "buy a currency trading at a price sufficiently lower than its equilibrium exchange rate and sell it once mean reversion has occurred".

This succinct description fails however to convey the more complex issues facing a trader, starting with the difficulties in determining whether an exchange rate time series actually behaves like a mean reverting process. Furthermore, even if the underlying stochastic process implies reversion to some equilibrium exchange rate price, this does not exclude the possibility of sustained and potentially large deviations from this equilibrium. The longer the deviation from the mean persists the higher the risk for substantial equity drawdowns. Ideally, the trader should first obtain high confidence that the time series under consideration is indeed mean reverting and that the half life of mean reversion is sufficiently low, before allocating capital.

From a fundamental viewpoint it can be hypothesized that most currency exchange rates will be non-stationary and thus non mean reverting. However, even if this is the case then it might still be possible that the value of some portfolio consisting of a linear combination of several currency pairs is stationary. A trader might be able to benefit from this by utilizing a mean reversion strategy on such a portfolio by effectively buying and selling the correct amount of the individual exchange rates. Such a strategy is called a "spread trading" strategy.

In this paper we start with a small overview on the concept of stationarity and the relevant statistical methods that can be applied to test individual time series for mean reversion. The term "half life of mean reversion" is also introduced. We continue with an explanation on cointegration and statistical methods that can be used to create stationary processes out of multiple non-stationary time series. Next, we apply the theory and methods in practice: We show that individual USD-based currency exchange rates generally do not behave in a mean reverting fashion, as expected. However, we manage to combine multiple cointegrating exchange rates into mean reverting portfolios. Finally, we apply a mean reverting "spread trading" strategy to one of those portfolios.

2 Univariate time series analysis

A time series process is called weakly stationary if its mean and variance are time-invariant and if the correlation between two observations depends only on the lag. When we observe the raw time series plots of various currency pairs, we notice that typically they do not seem to fulfill the conditions for stationarity. Most often these time series behave like random walks with drifts. Explosiveness or trend-stationarity can generally not apply to the price series that we take under consideration. Hence, to test the exchange rates for stationarity we need to exclude the possibility that the time series process contains a unit root.

2.1 Augmented Dickey - Fuller (ADF) unit root test

The Augmented Dickey-Fuller test is used for determining whether a single time series process has a unit root, and thus is non-stationary. It is an extension of the basic Dickey-Fuller test as it includes higher-order autoregressive terms in the test equation. Assuming that the series has a drift but no linear trend, the model equation is:

$$\Delta y_t = a_0 + \gamma y_{t-1} + \sum_{i=1}^{n} \beta_i \Delta y_{t-i} + \epsilon_t$$

with the innovation sequence ϵ_t i.i.d. and $\mathrm{E}[\epsilon_t] = 0$.

Under the null hypothesis, γ equals 0: the process has a unit root, meaning that innovations have a permanent effect and the series is non-stationary. If the null hypothesis can be rejected in a one-sided test in favor of the alternative hypothesis γ <0, the series is stationary and innovations have only a transitory effect. To determine whether the null hypothesis can be rejected, we use a test statistic $\tau_\mu = \frac{\hat{\gamma}}{SE(\hat{\gamma})}$ which has a specific distribution determined by Dickey and Fuller.

2.2 Phillips - Perron (PP) unit root test

Assume that the underlying time process is autoregressive with drift, i.e. of the following form:

$$y_t = a_0 + \gamma y_{t-1} + u_t$$

where the innovations u_t are allowed to be heteroskedastic and serially correlated, but still satisfy $\mathrm{E}[u_t] = 0$ $\forall$t.

The null hypothesis is that γ equals 1, meaning that the series has a unit root. The alternative hypothesis is for γ <1, in which case the series is stationary. The PP test statistic is a non-parametric adjustment of the t-test statistic that was used in the ADF test. The adjusted test statistic compensates for the PP innovation process not being white noise as the ADF innovation process is and its critical values have been determined through simulation.

2.3 Kwiatkowski - Phillips - Schmidt - Shin (KPSS) stationarity test

A common problem with the previous two tests is that it is often hard to reject the null hypothesis of a unit root. The KPSS test circumvents this problem by making the absence of a unit root the null hypothesis, thus reversing the burden of proof. Suppose the time series process is modeled by:

$$y_t = r_t + \epsilon_t \; and \; r_t = r_{t-1} + u_t$$

with the innovations ϵ_t assumed to come from a stationary process and the u_t are i.i.d. with a mean equal to zero and a variance σ_u^2. An equivalent expression for this model is:

$$y_t = r_0 + k \sum_{i=1}^{t} \xi_i + \epsilon_t$$

with the ξ_t assumed to be i.i.d. with an expected value of zero and variance equal to one.

The null hypothesis of the time series being stationary around a level r_0 can then be simply expressed as $\sigma_u^2 = 0$ or k = 0. The computation of the test statistic will not be further discussed here.

2.4 Half life of mean reversion

To perform further analysis on the mean reversion properties of the time series we can consider the γ coefficient in the ADF equation to be a measure of how long it takes for the time series to mean revert. The smaller the γ, the less time is needed for mean reversion to occur. The equation used in the ADF test can be transformed to a differential form while ignoring the drift and the lagged differences. We obtain the Ornstein-Uhlenbeck formula for mean-reverting process:

$$dy(t) = (\gamma y(t-1) + \mu)dt + d\varepsilon$$

where $d\varepsilon$ is some Gaussian noise. γ is determined from the discrete linear regression of $\Delta y(t)$ against y(t-1) and carried over to the differential form. Writing the equation in this form gives us the expected value of y(t):

$$E[y(t)] = y_0 exp(\gamma t) - \mu/\gamma(1 - exp(\gamma t))$$

Because γ is negative for a mean-reverting process, the expected value of the price decays exponentially to the value $-\mu/\gamma$. The half-life of decay is equal to

-log(2)/γ. Hence, If λ is very close to zero, a mean-reverting strategy will not be very profitable because the half-life of mean reversion will be very long.

3 Multivariate time series analysis and cointegration

If there exist two or more time series for which some linear combination has a lower order of integration than the time series individually then it can be said that the time series are cointegrated. As an example we consider 2 non-stationary financial time series x_t and y_t that are both $I(1)$ and we add them to the following regression model:

$$y_t = \beta + \gamma x_t + u_t$$

If we estimate the model and rearrange the result then we obtain the following:

$$\hat{y_t} - \hat{\gamma} x_t = \hat{\beta} + \hat{u_t} = \hat{z_t}$$

If $\hat{z_t}$ is a stationary ($I(0)$) process then it can be said that x_t and y_t are cointegrated with cointegrating coefficient γ and cointegrating vector [1 - $\hat{\gamma}$]

3.1 Error correction model (ECM)

When modeling $I(1)$ variables the usual response is to take the differences of the individual variables and then use these differences in the subsequent modeling process. The model that one may consider estimating is:

$$\Delta y_t = \beta_1 \Delta x_t + u_t$$

However, when the relationship between the variables is important, such a procedure is inadvisable because this model has nothing to say about whether x and y have a long term equilibrium relationship. Fortunately, there is a class of models that can overcome this problem by using combinations of first differences and lagged levels of cointegrated variables. Consider the following equation:

$$\Delta y_t = \beta_1 \Delta x_t + \beta_2 (y_{t-1} - \gamma x_{t-1}) + u_t$$

with $\beta_2(y_{t-1} - \gamma x_{t-1})$ as the lagged error correction term and cointegrating coefficient γ.

This model is known as an error correction model and the interpretation is as follows: y changes between $t-1$ and t as a result of changes in the values of the explanatory variable(s) x between $t-1$ and t, and also to correct for any disequilibrium that existed during the previous period. γ defines the long term relationship between x and y, β_1 the short term relationship. β_2 measures the proportion of last period's equilibrium error that is corrected and can be seen as the speed of adjustment back to equilibrium.

3.2 Engle-Granger test

The Engle-Granger two-step method is a method for testing for cointegration and is conducted as follows: In the first stage we verify that the individual time series are $I(1)$ by using one of the univariate testing methods described above. Next, we estimate the cointegrating regression by using OLS and we obtain the cointegrating vector. The residuals $\hat{u}_t$ can now be tested for stationarity. If the residuals are stationary then we can conclude that the variables are cointegrated.

In the second stage, Engle and granger suggest a model containing only first differences when the residuals are non-stationary while an error correction model should be used if the variables are indeed cointegrated:

$$\Delta y_t = \beta_1 \Delta x_t + \beta_2(\hat{u}_{t-1}) + v_t$$

The interpretation of this error correction model was explained above. Inferences on the results of this equation can be made since all variables in the regression are stationary. It should also be noted that we can not perform any hypothesis tests about the cointegrating relationship that was estimated in the first stage.

3.3 Johansen test

With the Johansen test several $I(1)$ time series can be tested for cointegration. The test is more generally applicable than the Engle-Granger test because it allows for more than one cointegrating relationship while hypothesis tests can also be performed on the cointegrating vectors. In this section we first give a brief overview on so called Vector Autoregressive Models and then we continue with an explanation on the actual testing procedure.

3.3.1 Vector autoregressive models (VARs)

A VAR is a systems regression model, which means that it is multivariate. A bivariate VAR, with only two variables y_{1t} and y_{2t}, is the simplest case. Each of the variables current values depend on different combinations of the previous k values of both variables, corrected with white noise error terms ($E(u_{it}) = 0, (i = 1, 2), E(u_{1t}u_{2t}) = 0$).

$$y_{1t} = \beta_{10} + \beta_{11}y_{1t-1} + ... + \beta_{1k}y_{1t-k} + \alpha_{11}y_{2t-1} + ... + \alpha_{1k}y_{2t-k} + u_{1t}$$

$$y_{2t} = \beta_{20} + \beta_{21}y_{2t-1} + ... + \beta_{2k}y_{2t-k} + \alpha_{21}y_{1t-1} + ... + \alpha_{2k}y_{1t-k} + u_{2t}$$

Note that the variables can depend on more than just its own lags, which allows VARs to be more flexible than univariate AR models (the latter can be seen as a restricted case of VAR models). The model can be expanded to g variables $y_{1t}, y_{2t}, y_{3t}, ..., y_{gt}$ each with their own equation.

VAR models can be estimated by using OLS on the individual equations. Optimal model lag length can be obtained by using likelihood ratio tests or by minimizing a multivariate version of an information criterion.

3.3.2 The testing procedure

Suppose that we want to test a set of g variables that are $I(1)$ for cointegration. A VAR with k lags containing these variables could be set up:

$$\underset{g\times 1}{y_t} = \underset{g\times g}{\beta_1}\,\underset{g\times 1}{y_{t-1}} + \underset{g\times g}{\beta_2}\,\underset{g\times 1}{y_{t-2}} + ... + \underset{g\times 1}{u_t}$$

This VAR can be turned into a vector error correction model (VECM):

$$\Delta y_t = \Pi y_{t-k} + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + ... + \Gamma_{k-1} \Delta y_{t-(k-1)} + u_t$$

where $\Pi = (\sum_{i=1}^{k} \beta_i) - I_g$ and $\Gamma_i = (\sum_{j=1}^{i} \beta_j) - I_g$.

Notice the comparability between this set of equations and the testing equation of an ADF test. The Johansen test will examine the Π matrix by looking at its rank: If the variables are not cointegrated then the rank of the matrix will not be significantly different from 0. The rank is equal to the number of its eigenvalues (characteristic roots) that are different from zero and can be tested by utilizing one of the following test statistics:

$$\lambda_{trace}(r) = -T \sum_{i=r+1}^{g} ln(1 - \hat{\lambda}_i)$$

and

$$\lambda_{max}(r, r+1) = -T ln(1 - \hat{\lambda}_{r+1})$$

where T is the sample size, r the number of cointegrating vectors under the null hypothesis and $\hat{\lambda}_i$ the estimated value for the ith ordered eigenvalue from the Π matrix. The test statistics assume that the eigenvalues were put in ascending order $(\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_g)$

Each eigenvalue is associated with a different cointegrating vector, which is equal to its corresponding eigenvector. An eigenvalue significantly different from zero indicates a significant cointegrating vector. The trace test tests the null hypothesis of less than or equal to r cointegrating vectors against the alternative of more than r cointegrating vectors. The maximum eigenvalue test tests the null hypothesis of r cointegrating vectors against the alternative of $r+1$ cointegrating vectors. The distribution of the test statistics is non-standard, Johansen provides the critical values. The testing is conducted in a sequence under the null, $r = 0, 1, ..., g-1$ so that r is continually increased until the null is no longer rejected.

4 Trading a mean reversion strategy

In this section we perform data analysis on daily FOREX data that was gathered from the dukascopy Swiss bank trading platform. We utilize daily currency pair data spanning from January 2008 up to June 2014. We first investigate if we can perform a mean reversion trading strategy on the 7 most liquid USD based currency pair exchange rates. Next, we proceed with cointegration analysis on the time series data and we perform a spread trading strategy on one of the cointegrating portfolios. Results are reported.

4.1 Data modeling

We analyze the 7 most important USD based currency pairs: EUR/USD, GBP/USD, USD/JPY, AUD/USD, USD/CHF, NZD/USD and USD/CAD. We model the time stamps, daily bid open prices and the relevant transaction costs. The volume indicator is also modeled and can be used to detect missing data entry's and to realign the portfolio timestamps where needed.

It is important to note that for "inverse currency pairs" the time series are inverted and transaction costs are recalculated so that the USD currency becomes the quoted pair. The cost of buying or selling one unit of currency pair X is y USD with y being the exchange rate of X/USD.

4.2 Univariate analysis of individual exchange rates

Before we consider trading a mean reversion strategy on one of the 7 currency pairs in our portfolio we should first test if the time series are indeed behaving in a mean reverting manner.

Unfortunately, we can show that all 7 timeseries are non-stationary and integrated of order $I(1)$ with at least 95% confidence. An ADF test is used to first reject the stationarity of the levels of the series. Next, an ADF test is executed on the differenced levels of the series. Results indicate that the differenced series are stationary with high confidence, implying that the original levels are $I(1)$.

While performing these tests we assume that the timeseries contain a drift but no linear trend. The maximum lag length is set to $k = \left\lfloor 12 \times \frac{T}{100}^{(\frac{1}{4})} \right\rfloor$ and the BIC information criterion is used to select the optimal model.

4.3 Cointegration analysis on multiple exchange rates

It will be difficult to perform a mean reverting strategy on the individual exchange rates since they are $I(1)$. However, It is useful to investigate if there exist linear combinations of 2 or more currency pairs that show mean reverting behavior. To investigate this we perform a cointegration analysis on all possible portfolios that contain up to a maximum of 4 individual currency pairs.

To perform this analysis, we utilize the Johansen test for cointegration. We first obtain the optimal lag length for the VECM by subtracting 1 from the optimal VAR lag length solution. We assume a constant but no trend in the cointegrating regression and request the trace teststatistic in the output of the test. If the trace statistic for $r \leq 0$ is rejected with at least 90% confidence then we can assume that there is at least one significant cointegrating vector. In our analysis we detect that 16 out of 91 tested portfolios are cointegrated.

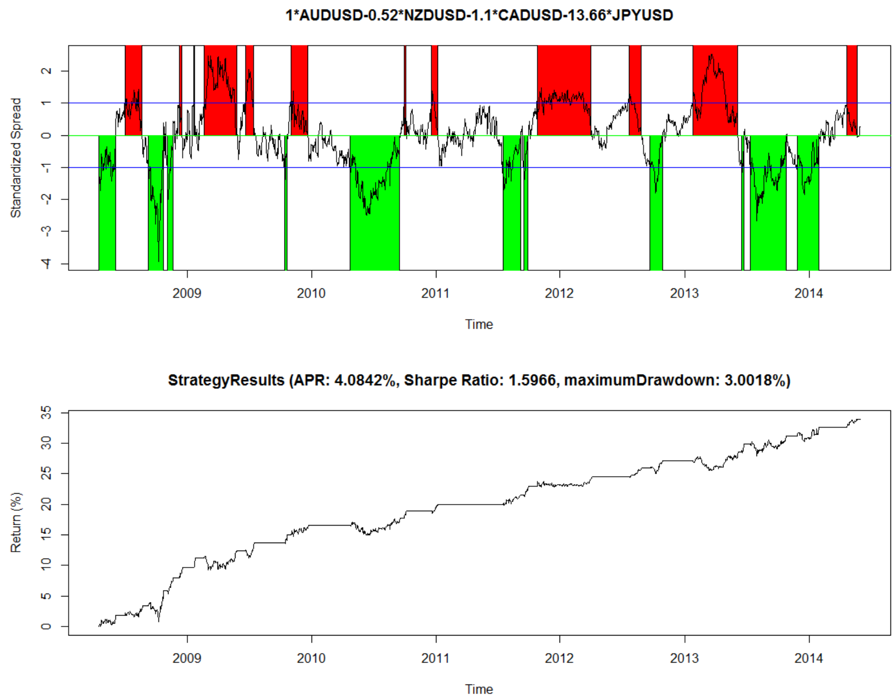

Figure 1: Spread trading strategy applied to a cointegrated portfolio

4.4 The Strategy

We execute our strategy on the cointegrating portfolio with the lowest half life of mean reversion. However, there are multiple portfolios that have the same minimum half life of 30 days. We pick the portfolio that consists of AUD/USD, NZD/USD, CAD/USD and JPY/USD. The cointegrating vector with highest eigenvalue is $[1 - 0.52 - 1.1 - 13.66]$. This eigenvector represents the so called "hedge ratio's": For every unit of AUD/USD that we want to buy we should sell 0.52 units of NZD/USD, sell 1.1 units of CAD/USD and sell 13.66 units of JPY/USD.

Our trading rules are defined as follows. If the value of the stationary portfolio deviates more than 1 standard deviation to the upside then we sell the portfolio. In practice, this means that we buy or sell a certain amount of the individual currency pairs depending on the hedge ratio's. We close out our positions when the portfolio value mean reverts. Vice versa for the long scenario, we buy the portfolio when we deviate 1 standard deviation to the downside and close out upon mean reversion. The strategy is illustrated in the figure above. Green and red backgrounds represent periods where we have a long or short position in the portfolio. Trading results are also illustrated.