Governing equations

- t is the time,
- m_i and m_j, the masses of two particles of indices i and j.,
- $\vec{d_{ij}}$ the difference between the two position vectors of the particles,
- G the gravitational constant.

The force applied to the particle i is:

$$\vec{F}(t) = G.\frac{m_i.m_j}{||\vec{d_{ij}}||^2}.\frac{\vec{d_{ij}}}{||\vec{d_{ij}}||}, \tag{1}$$

Now, if there are n particles. The total force applied on a particle i from all the surrounding particles is:

$$\vec{F}(t) = \sum_{j \neq i}^{n} \vec{f_{ij}}(t) = G.m_i.\sum_{j \neq i}^{n} \frac{m_j.\vec{d_{ij}}}{||\vec{d_{ij}}||^3}, \tag{2}$$

According to Newton's third law the acceleration $\vec{a_i}(t)$ can be computed as:

$$\vec{a_i}(t) = \frac{\vec{F_i}(t)}{m_i} = G.\sum_{j \neq i}^{n} \frac{m_j.\vec{d_{ij}}}{||\vec{d_{ij}}||^3}. \tag{3}$$

And this is integrated to compute new velocities (v_i) and positions (p_i) after a time step dt.

$$\vec{v_i}(t + dt) = \vec{v_i}(t) + \vec{a_i}(t).dt. \tag{4}$$

$$p_i(t + dt) = p_i(t) + \vec{v_i}(t).dt + \frac{\vec{a_i}(t).dt^2}{2}. \tag{5}$$