Estimating the probability of event occurrence

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Abstract

English abstract goes here.

Sammanfattning

Träutensilierna i ett tryckeri äro ingalunda en oviktig faktor, för trevnadens, ordningens och ekonomiens upprätthållande, och dock är det icke sällan som sorgliga erfarenheter göras på grund af det oförstånd med hvilket kaster, formbräden och regaler tillverkas och försäljas Kaster som äro dåligt hopkomna och af otillräckligt.

Contents

Introduction

In complex systems, errors can occur intermittently and in a non-deterministic way, which makes it harder to diagnose real errors among spurious ones. In manufacturing for instance, intermittent errors could be due to physical properties, either internal, like bad contacts, or external, e.g. extreme temperatures. In any case, these errors are often hard to troubleshoot and require close attention. By analogy with *flaky tests* in computer science, we will also refer to these as *flaky errors*.

Non-deterministic errors are often considered as unreliable and therefore discarded, which creates an important risk of ignoring a real error. On the other hand, troubleshooting each occurrence of a flaky error is very time-consuming and is not always an option. Therefore, it is critical to detect when flaky errors occur at an unexpected rate and to pinpoint when the rate of failure is likely to have evolved. This enables engineers to understand which elements have an impact on the error. In computer science, the usual workaround for flaky tests is to re-run tests that failed a certain number of times until they pass. In other fields, this is not always possible as errors are triggered in a production environment.

In this thesis, we intend to estimate the underlying probability of occurrence of an error. We assume its distribution to be piecewise stationary. This corresponds to the fact that the probability of occurrence changes when a part breaks, wears out, or is repaired. Given this assumption, estimating the underlying probability of occurrence is equivalent to finding its change points.

Chapter 1

Pilot study

1.1 Objective

The desired outcome of the degree project is to detect change points in the probability of occurrence of events. This has two main interests:

- Alert when events start occurring more frequently.
- Troubleshoot errors by pinpointing when the probability of occurrence changed.

For instance, in the case of intermittent errors due to bad contact, the detection of change in frequency could enable users to understand that high vibrations triggered the bad contact, and that some specific maintenance work fixed it.

According to [1], two binomial distributions $Bin(n, p)$ and $Bin(n, p+\varepsilon)$ (where n is the number of observations, and $p \gg \varepsilon$ the probability of occurrence of the event) can be distinguished after

$$n \sim \frac{K}{\varepsilon^2}$$

observations, for some K independent of ε. So a change in probability of 0.1 should be detected within the order of $n \sim 100$ observations.

1.2 Research question

How precisely and how quickly can change points in frequency be detected? How big a change in the frequency of occurrence

are we able to detect, and how many observations after the change are required to guarantee that the frequency has changed with high confidence?

Examination method First, we will generate sequences of probabilities of occurrence in a *realistic* way (See 1.4), and derive events based on those probabilities. Then, we will try to estimate the underlying probabilities based on the events. Once this is set up, define a metric to measure the error in the estimation of the underlying sequence of probabilities, and compare different candidate models to find the one that performs best.

Depending on the complexity of the data, we will either use the mathematical definition or approximate it using Monte Carlo methods to evaluate the performance of models.

As we will generate the data, it is important to clearly define the assumptions. Otherwise, the results will be biased by the way we generated the data and will not correctly evaluate the models. We will generate probabilities of occurrences that either change abruptly, for instance when a part breaks, or that slowly increase, to model the natural deterioration of a part (as explained in 1.4).

Expected scientific results The hypothesis being tested is that we are able to detect *significant* changes in probability of occurrence, *shortly* after the change. We want to define as few parameters as possible, such as a confidence threshold for the null hypothesis, or the transition probability for the Hidden Markov Model.

Given these, we want to measure:

- the minimal increase or decrease in the frequency of occurrence that we are able to detect
- the minimal number of events required to detect a change

To measure the accuracy of our estimations, we can use the $\mathcal{L}_2$ score to measure the error with the true underlying distribution. We will also measure the error in the detection of a change point, to determine whether we are able to pinpoint the factor that triggered the change.

1.3 News value

In complex systems, it may be too time-consuming to monitor all the errors that occur. in that case, unusual errors are carefully troubleshooted, but intermittent and non-deterministic errors can end up being ignored. In that case, being able to flag when an error occurs more frequently than usual can be vital. Indicating precisely when the frequency increased significantly is also very useful to quickly understand the reason for this increase in frequency and troubleshoot the error.

1.4 Generating events

To generate our data, we will simulate part failures that lead to an increased probability of triggered events. Interestingly, the lifetime of organisms, devices, structures, materials in both biological and engineering sciences have very similar behaviors. For example, business mortality [2], failures in the air-conditioning equipment of aircrafts or in semiconductors [3] and integrated circuit modules [4] all have similar behaviors. These can be modeled with a mixture of exponential or Weibull-Lomax distributions. In particular, the Weibull distribution [5] is the most widely used to model the lifetime of parts [6], as it has a limited number of parameters which can easily be interpreted, and captures both the *infant mortality* of defective parts and the exponential distribution of events that occur independently at a constant average rate, for normal parts. The two parameters are used to reflect these two elements, the defects in the material and the average rate of failure.

1.5 Detection of probability change-points

The detection of a change point can be formulated as a null hypothesis that determines whether all events were drawn from the same binomial distribution [7]. To generalize this to multiple change points, we could use a sliding window. However, this is often unstable because the change point detection is performed on small regions, and therefore isn't always statistically significant [8], [9]. Therefore, we will also explore binary segmentation or dynamic programming to generalize our algorithm to sequences of events with multiple change points [10], while still running in linear time [11].

Another lead could be to use Hidden Markov Models [12], which seems well-adapted to our use case, as we are trying to guess the hidden probability state that was used to generate the events. Chis and Harrison [13] suggest a solution to adapt the model to online problems by estimating its updated parameters. This can be used to avoid recomputing the hidden state with new values.

Chapter 2

Problem definition

2.1 Data generation

2.1.1 Definitions

As we will generate our data, it is important to clearly specify how the data is generated and what assumptions are made.

We consider a complex system composed of many different physical parts, and equipped with a logger that triggers an error when an unusual behavior is detected. This logging systems stores a sequence of observations t, for which events either occurred or did not. These observations could be of various time scales, either periodic (e.g. 1 per second) or based on iterations (e.g. one per cycle in manufacturing). We consider a single event e, independently of the other ones that were recorded by the logging system.

Definitions

- We call *observation* t each iteration where a value is stored by the logger.
- We call *event* or *error* e a specific kind of event that was recorded by a logging system. For each observation t, we note $e(t) = 1$ if the event occurred and $e(t) = 0$ otherwise. We note $p(t) = \mathbb{P}[e(t) = 1]$ the probability that e occurs at instant t.
- We call *part* a physical part of our system; it can be anything from a valve to a sensor. The failure or wear of such a part could increase

the probability of occurrence of e, and maintenance or replacement of the part could decrease it.

- We call change point any of these changes affecting a part which has an impact on e.

Assumptions We assume that events occur with a fixed probability p, independent of previous events, between two consecutive change points. This can still be generalized to any case by considering many change points.

2.1.2 Lifetime of a part

As mentioned in 1.4

2.2 Model comparison

Chapter 3

Hidden Markov Models

3.1 Parameters and output

3.2 Tuning the Hidden Markov Model

3.3 Results and Discussion

Chapter 4

Null hypothesis

4.1 Detection of single change points

4.2 Generalization to multiple change points

4.3 Results and Discussion

Chapter 5

Conclusion

5.1 Discussion

5.2 Future Work

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