[INFO-H-413] Heuristic Optimization - Assignment 1

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1 Implementation

In this section, we describe a short description of the implementation. First, we implemented the iterative improvement algorithm for the permutation flow-shop scheduling problem (PFSP) with the weighted sum of completion times objective. We have to implement 2 different starting solutions : Random ordering of jobs (Uniform Random Picking) and simplified RZ heuristic, 2 pivoting rules : first improvement and best improvement, 3 neighbours rules : Transpose, Exchange, Insert. All combinations of these parameters give 12 different algorithms. We used C++ to implement them. Here, we use the "shuffle" method of C++ to generate uniform random picking.

2 Exercise 1.1 - Iterative Improvement

We made sure that each run of all the algorithms on 1 single instance were based on an initial solution generated using a single seed. (one seed per instance)

We first show the results of all algorithms applied on all instances. Namely, we have 30 instances of 50 jobs and 30 instances of 100 jobs, for a total of 60 instances. We have 12 algorithms, so 12x30 results for jobs 50 and 12x30 results for jobs 100.

We use the following abbreviations to describe the algorithms :

- R = Random
- S = Simplified RZ heuristic
- B = Best improvement
- F = First improvement
- T = Transpose
- E = Exchange
- I = Insert

Significance level used to assess the statistical t-test p-values : $\alpha = 0.01$

2.1 Results of all algorithms on all instances

2.1.1 Weighted sums of completion times

instance	RFT	RFE	RFI	RBT	RBE	RBI	SFT	SFE	SFI	SBT	SBE	SBI
100_20_01	760175	606748	599981	758115	613502	613118	616002	616002	607386	616002	616002	611917
100_20_02	807796	632417	632201	808243	647625	639547	649396	638287	631066	649396	638887	635048
100_20_03	824746	601943	604715	824586	617024	599835	613349	613349	603315	613349	613349	608764
100_20_04	833988	679005	673557	830117	702428	678868	683331	686236	675472	687245	680249	679033
100_20_05	823818	668386	662205	823081	681227	672331	696543	675712	658600	696543	667732	668382
100_20_06	815092	656312	648220	820555	672714	653149	675913	665682	653510	675913	660114	652945
100_20_07	790456	581617	571164	787757	595857	586341	587808	582314	573545	587808	587808	578904
100_20_08	692749	544046	538630	709840	551113	546395	547398	540051	542886	547398	541600	542420
100_20_09	761772	582686	575695	768737	596261	581377	594279	594102	581720	594279	594142	587862
100_20_10	790544	611489	607591	786677	613948	610843	618765	613450	607378	619013	613979	613812
100_20_11	767500	613211	612406	767871	626261	631174	620124	618902	614085	619327	618171	615524
100_20_12	740839	592612	592345	754565	607480	608012	599970	599759	595915	599970	599759	595915
100_20_13	733968	565677	554808	733817	579297	568502	581879	565246	558749	581879	567575	562339
100_20_14	812707	586565	582897	806116	603658	588572	592690	587634	581814	594779	587443	583664
100_20_15	807499	605466	608181	807475	615786	611543	616486	616297	604492	616486	615934	605772
100_20_16	695915	559775	557271	703438	566209	566849	565937	564236	560015	565937	561110	560613
100_20_17	880921	657383	655428	882015	674776	661391	666391	658853	654820	666391	665680	659428
100_20_18	809320	640992	639874	805916	663175	644101	648133	645792	640985	648133	643953	645595
100_20_19	634075	470585	469366	644201	481649	475827	480111	474614	478244	480111	474591	472976
100_20_20	744433	576760	573660	755587	584059	586378	584512	583618	578731	584512	583651	574626
100_20_21	800981	632280	627053	808680	670526	633288	644590	644476	641953	644590	644476	623387
100_20_22	790132	655767	650998	799143	666170	662359	671837	661875	652400	672033	663867	663761
100_20_23	752420	620452	610443	760551	629257	621806	619180	613594	611858	619632	619559	613923
100_20_24	759960	563764	561554	765759	568271	567690	572680	570600	560656	570962	570600	563387
100_20_25	752317	597529	594801	766888	608263	601605	612401	604263	607119	612401	607425	603865
100_20_26	823887	607466	609381	819258	625383	620656	622002	616423	607166	621958	609695	608333
100_20_27	765669	573196	572419	758886	592222	571975	581282	579455	573626	581310	578075	575392
100_20_28	766184	567238	561749	759417	572712	564446	565683	565197	556349	565683	565197	557705
100_20_29	745130	555581	547690	755280	561835	555682	572564	565640	553844	572564	561471	558629
100_20_30	653919	516297	511657	657496	523457	514763	528858	525470	514125	528858	523260	513992

Figure 1: II - WCT - 100 jobs

50_20_01	2531238	1824493	1826057	2536346	1879060	1872799	1861380	1841685	1826045	1861380	1845234	1841547
50_20_02	2367512	1846854	1847197	2379098	1884818	1866512	1872704	1848930	1835871	1872704	1863155	1846248
50_20_03	2293914	1716690	1708918	2283530	1752581	1738197	1754072	1717060	1723543	1754127	1721012	1710347
50_20_04	2689705	1983321	1983814	2685223	2077061	2002707	2020637	2001547	1988731	2020637	2011889	1988201
50_20_05	2230023	1567494	1567423	2252661	1618411	1594455	1620783	1605878	1575143	1620783	1598541	1573492
50_20_06	2392840	1694205	1684230	2380311	1763033	1732625	1733575	1717737	1696702	1733575	1715744	1706546
50_20_07	2456359	1802277	1800650	2501436	1868175	1841075	1858524	1841406	1819283	1858520	1844222	1815190
50_20_08	2382063	1787750	1807846	2392547	1845078	1827869	1832647	1803533	1782956	1832773	1801845	1806087
50_20_09	2518320	1872993	1876126	2544532	1925847	1897013	1942239	1894310	1878784	1943186	1911077	1896771
50_20_10	2354470	1684796	1685929	2413134	1748418	1742725	1736909	1726517	1696334	1736909	1723451	1703455
50_20_11	2608688	1843142	1847680	2636518	1889387	1884003	1911880	1882893	1864799	1911880	1896240	1882134
50_20_12	2489980	1742058	1736716	2513073	1784708	1761413	1771565	1768318	1722874	1772760	1756633	1722556
50_20_13	2403179	1751212	1755628	2413907	1787655	1770408	1781784	1763725	1754316	1781784	1763863	1752756
50_20_14	2459301	1736644	1742642	2478993	1790802	1759828	1786624	1770530	1739054	1786624	1768605	1765016
50_20_15	2127351	1471858	1476714	2115288	1506868	1549640	1513447	1500577	1481735	1513447	1503622	1501537
50_20_16	2624249	1875114	1878491	2640288	1911843	1910536	1931321	1886855	1887347	1932581	1914988	1888359
50_20_17	2464270	1796873	1797633	2435952	1869607	1833419	1857456	1840702	1828708	1856652	1813377	1812047
50_20_18	2580228	1846607	1837925	2585575	1938130	1884663	1917307	1873291	1854862	1917358	1896268	1869216
50_20_19	2411578	1706325	1711494	2418559	1769605	1752178	1760403	1759250	1729931	1760403	1759250	1721923
50_20_20	2593963	1828696	1847452	2613042	1880741	1857611	1874273	1855337	1843492	1874273	1853848	1852163
50_20_21	2397236	1734899	1736928	2392598	1819527	1765184	1794948	1786989	1749757	1795109	1787796	1734645
50_20_22	2394266	1780875	1769969	2411668	1835897	1808994	1820415	1793708	1768638	1820415	1807884	1782273
50_20_23	2530289	1808681	1838914	2556660	1862678	1861803	1870555	1839433	1818445	1870555	1854905	1833881
50_20_24	2421200	1731612	1730751	2450080	1787732	1752359	1778055	1722336	1738987	1778055	1742005	1734255
50_20_25	2622090	1738011	1747502	2624627	1822404	1783345	1789791	1770067	1757994	1789791	1783862	1760246
50_20_26	2171056	1595819	1595107	2182613	1626170	1621831	1647057	1620038	1596557	1647057	1632164	1597788
50_20_27	2494768	1902856	1913886	2511507	1940712	1942851	1960250	1928315	1901675	1960250	1922398	1920572
50_20_28	2486337	1770111	1765607	2479117	1805498	1810286	1822294	1783342	1769285	1822294	1792323	1771378
50_20_29	2727907	2029579	2028125	2733820	2058749	2070146	2086646	2059206	2031565	2088428	2077695	2063390
50_20_30	2433937	1798751	1807821	2476849	1866937	1829568	1840390	1828517	1800993	1840390	1832161	1818074

Figure 2: II - WCT - 50 jobs

2.1.2 Relative Percentage Deviations

instance	RFT	RFE	RFI	RBT	RBE	RBI	SFT	SFE	SFI	SBT	SBE	SBI
100_20_01	27.704700	1.929913	0.793099	27.358633	3.064543	3.000034	3.484528	3.484528	2.037093	3.484528	3.484528	2.798273
100_20_02	29.799371	1.618885	1.584177	29.871196	4.062557	2.764557	4.347127	2.562096	1.401802	4.347127	2.658506	2.041643
100_20_03	39.140102	1.551763	2.019418	39.113109	4.096028	1.196130	3.476031	3.476031	1.783229	3.476031	3.476031	2.702511
100_20_04	25.106770	1.857727	1.040471	24.526080	5.371418	1.837176	2.506672	2.942452	1.327741	3.093812	2.044340	1.861928
100_20_05	26.014611	2.239089	1.293618	25.901877	4.203302	2.842533	6.546100	3.359704	0.742182	6.546100	2.139051	2.238477
100_20_06	26.705985	2.023647	0.765746	27.555208	4.573337	1.531959	5.070621	3.480213	1.588076	5.070621	2.614668	1.500247
100_20_07	39.810922	2.872784	1.023922	39.333540	5.391466	3.708335	3.967809	2.996065	1.445059	3.967809	3.967809	2.392925
100_20_08	30.192239	2.245643	1.227784	33.404248	3.573784	2.687104	2.875604	1.494840	2.027638	2.875604	1.785953	1.940060
100_20_09	32.991618	1.726441	0.505938	34.207581	4.096390	1.497913	3.750369	3.719468	1.557794	3.750369	3.726451	2.630077
100_20_10	31.585176	1.781669	1.132851	30.941518	2.190967	1.674143	2.992751	2.108075	1.097398	3.034030	2.196127	2.168330
100_20_11	27.012339	1.479301	1.346083	27.073735	3.638924	4.451969	2.623322	2.421095	1.623938	2.491428	2.300123	1.862075
100_20_12	26.812781	1.440091	1.394388	29.162323	3.985115	4.076180	2.699594	2.663476	2.005481	2.699594	2.663476	2.005481
100_20_13	32.978467	2.487929	0.518711	32.951109	4.955566	2.999755	5.423366	2.409842	1.232732	5.423366	2.831804	1.883159
100_20_14	41.845070	2.375583	1.735392	40.694715	5.358894	2.725873	3.444605	2.562160	1.546372	3.809206	2.528824	1.869260
100_20_15	35.630232	1.696094	2.152114	35.626201	3.429476	2.716807	3.547050	3.515305	1.532498	3.547050	3.454334	1.747491
100_20_16	26.244687	1.547775	1.093529	27.609421	2.714955	2.831056	2.665612	2.357037	1.591313	2.665612	1.789955	1.699795
100_20_17	36.147139	1.599138	1.296991	36.316217	4.287242	2.218578	2.991333	1.826328	1.203024	2.991333	2.881447	1.915195
100_20_18	28.425576	1.714732	1.537324	27.885418	5.234804	2.208078	2.847889	2.476412	1.713621	2.847889	2.184593	2.445151
100_20_19	36.211408	1.090636	0.828772	38.386666	3.467395	2.216718	3.137003	1.956143	2.735936	3.137003	1.951202	1.604269
100_20_20	31.702109	2.038072	1.489633	33.675430	3.329382	3.739650	3.409525	3.251362	2.386774	3.409525	3.257200	1.660534
100_20_21	29.628503	2.326409	1.480486	30.874487	8.516034	2.489541	4.318625	4.300176	3.891860	4.318625	4.300176	0.887191
100_20_22	22.945978	2.038539	1.296474	24.348106	3.657265	3.064267	4.539061	2.988955	1.514628	4.569559	3.298914	3.282420
100_20_23	24.875526	2.973432	1.312287	26.224989	4.434756	3.198149	2.762325	1.835243	1.547127	2.837341	2.825225	1.889846
100_20_24	37.583437	2.064041	1.663942	38.633290	2.879990	2.774806	3.678197	3.301633	1.501368	3.367170	3.301633	1.995789
100_20_25	27.957690	1.630603	1.166612	30.435995	3.456293	2.323869	4.160104	2.775954	3.261716	4.160104	3.313762	2.708260
100_20_26	37.246792	1.194411	1.513420	36.475672	4.179105	3.391660	3.615883	2.686508	1.144435	3.608553	1.565728	1.338840
100_20_27	35.801219	1.663663	1.525852	34.598167	5.038168	1.447103	3.097819	2.773777	1.739929	3.102785	2.529017	2.053152
100_20_28	39.354576	3.170010	2.171663	38.123785	4.165628	2.662197	2.887184	2.798790	1.189504	2.887184	2.798790	1.436135
100_20_29	36.674957	1.907063	0.459661	38.536714	3.054198	1.925588	5.022158	3.752128	1.588455	5.022158	2.987432	2.466140
100_20_30	28.418334	1.391764	0.480548	29.120795	2.797863	1.090513	3.858526	3.193182	0.965221	3.858526	2.759176	0.939102

Figure 3: II - RPD - 100 jobs

50_20_01	41.243450	1.806976	1.894248	41.528478	4.851823	4.502458	3.865276	2.766292	1.893578	3.865276	2.964327	2.758592
50_20_02	30.754090	1.998940	2.017883	31.393967	4.095634	3.084621	3.426596	2.113594	1.392365	3.426596	2.899219	1.965471
50_20_03	36.552254	2.191228	1.728576	35.934114	4.327750	3.471498	4.416506	2.213253	2.599174	4.419780	2.448508	1.813641
50_20_04	38.396330	2.049983	2.075349	38.165713	6.873289	3.047472	3.970043	2.987785	2.328349	3.970043	3.519923	2.301078
50_20_05	44.892306	1.845506	1.840893	46.363176	5.153760	3.597256	5.307877	4.339447	2.342488	5.307877	3.862737	2.235217
50_20_06	44.100980	2.027967	1.427255	43.346462	6.172909	4.341686	4.398897	3.445105	2.178341	4.398897	3.325083	2.771163
50_20_07	37.720708	1.048285	0.957064	40.248040	4.742989	3.223573	4.201886	3.242131	2.001761	4.201662	3.400016	1.772278
50_20_08	35.771088	1.896869	3.042286	36.368647	5.164409	4.183543	4.455876	2.796457	1.623624	4.463057	2.700246	2.942028
50_20_09	35.965187	1.123703	1.292855	37.380384	3.977313	2.420553	4.862324	2.274617	1.436361	4.913453	3.179875	2.407487
50_20_10	41.829561	1.489455	1.557705	45.363388	5.321944	4.979007	4.628660	4.002663	2.184486	4.628660	3.817972	2.613444
50_20_11	43.626493	1.477840	1.727688	45.158729	4.023950	3.727523	5.262347	3.666410	2.670209	5.262347	4.401255	3.624621
50_20_12	45.475047	1.778315	1.466213	46.824237	4.270107	2.909115	3.502238	3.312534	0.657506	3.572055	2.629848	0.638927
50_20_13	40.304584	2.240853	2.498672	40.930916	4.368501	3.361571	4.025735	2.971398	2.422073	4.025735	2.979455	2.330996
50_20_14	43.873461	1.596747	1.947641	45.025478	4.765086	2.953052	4.520666	3.579138	1.737737	4.520666	3.466522	3.256560
50_20_15	46.800930	1.567688	1.902784	45.968505	3.983604	6.935148	4.437597	3.549485	2.249265	4.437597	3.759609	3.615731
50_20_16	42.057088	1.504558	1.687363	42.925319	3.492790	3.422038	4.547182	2.140127	2.166760	4.615389	3.663036	2.221543
50_20_17	38.625151	1.081371	1.124125	37.032149	5.172953	3.137232	4.489410	3.546930	2.872219	4.444182	2.009788	1.934970
50_20_18	41.459093	1.238864	0.762880	41.752238	6.256545	3.325256	5.114939	2.701794	1.691438	5.117735	3.961491	2.478386
50_20_19	43.913804	1.826987	2.135453	44.330403	5.603297	4.563319	5.054156	4.985349	3.235703	5.054156	4.985349	2.757816
50_20_20	43.967488	1.494419	2.535396	45.026391	4.382968	3.099230	4.023988	2.973021	2.315612	4.023988	2.890380	2.796861
50_20_21	40.261421	1.508320	1.627036	39.990053	6.459874	3.280285	5.021766	4.556087	2.377656	5.031186	4.603305	1.493459
50_20_22	37.538258	2.302102	1.675609	38.537914	5.462833	3.917394	4.573472	3.039292	1.599150	4.573472	3.853631	2.382410
50_20_23	42.423112	1.805753	3.507486	43.907464	4.845097	4.795846	5.288472	3.536699	2.355342	5.288472	4.407576	3.224192
50_20_24	43.045356	2.304252	2.253384	44.751597	5.619842	3.529992	5.048121	1.756223	2.739970	5.048121	2.918274	2.460401
50_20_25	52.998600	1.412709	1.966507	53.146633	6.337029	4.057941	4.434065	3.283172	2.578714	4.434065	4.088108	2.710118
50_20_26	38.644120	1.909357	1.863888	39.382152	3.847578	3.570489	5.181427	3.455988	1.956486	5.181427	4.230357	2.035098
50_20_27	33.971732	2.185419	2.777742	34.870634	4.218327	4.333194	5.267539	3.552595	2.121998	5.267539	3.234846	3.136787
50_20_28	43.504060	2.165602	1.905644	43.087343	4.208035	4.484385	5.177451	2.929256	2.117927	5.177451	3.447613	2.238729
50_20_29	36.927313	1.874733	1.801750	37.224116	3.338922	3.910994	4.739212	3.361861	1.974421	4.828659	4.289916	3.571877
50_20_30	37.475614	1.598520	2.110819	39.899404	5.449857	3.339151	3.950408	3.279788	1.725155	3.950408	3.485611	2.689938

Figure 4: II - RPD - 50 jobs

2.1.3 Average relative percentage deviations

instance	RFT	RFE	RFI	RBT	RBE	RBI	SFT	SFE	SFI	SBT	SBE	SBI
50	31.751610	1.922562	1.261697	32.298874	4.106828	2.576408	3.658226	2.848966	1.697465	3.680001	2.787209	1.998792
100	40.803956	1.745111	1.903740	41.528801	4.892967	3.783494	4.573138	3.211950	2.118196	4.581665	3.514129	2.505994

Figure 5: II - average relative percentage deviations

2.1.4 Average computation times

instance	RFT	RFE	RFI	RBT	RBE	RBI	SFT	SFE	SFI	SBT	SBE	SBI
50	0.001588	0.192114	0.338045	0.001836	0.097348	0.110222	0.000134	0.014352	0.033766	0.000183	0.021346	0.032333
100	0.018534	6.025141	9.871000	0.018694	1.954387	2.259440	0.001121	0.397353	0.771604	0.001637	0.425360	0.633853

Figure 6: II - average computation times

2.2 Statistical tests

	A	B	C
1	RFT , RFE	1.45195335513809e-12	1.67132856965264e-11
2	RFT , RFI	1.2107011922515e-12	1.67132856965264e-11
3	RFT , RBT	0.000384024248959285	0.000198207408241016
4	RFT , RBE	1.49747380004171e-12	1.67132856965264e-11
5	RFT , RBI	1.23005899314382e-12	1.67132856965264e-11
6	RFT , SFT	1.38781847821521e-12	1.67132856965264e-11
7	RFT , SFE	1.38550600341999e-12	1.67132856965264e-11
8	RFT , SFI	1.25359345859921e-12	1.67132856965264e-11
9	RFT , SBT	1.38973070019891e-12	1.67132856965264e-11
10	RFT , SBE	1.27033991484837e-12	1.67132856965264e-11
11	RFT , SBI	1.26515922757485e-12	1.67132856965264e-11
12	RFE , RFI	0.304915079404254	0.463875951465839
13	RFE , RBT	1.53445219735901e-12	1.67132856965264e-11
14	RFE , RBE	3.35581498109123e-11	1.67132856965264e-11
15	RFE , RBI	1.28818025529477e-09	3.31827283251242e-10
16	RFE , SFT	1.88780531730147e-11	1.94489567709484e-11
17	RFE , SFE	1.29702662832809e-08	4.40493702946702e-10
18	RFE , SFI	0.00734284800962505	0.0285186607242488
19	RFE , SBT	1.83956416091295e-11	1.94489567709484e-11
20	RFE , SBE	2.91629166786632e-09	4.1205427984959e-11
21	RFE , SBI	1.97699690198988e-05	1.5774912575696e-05
22	RFI , RBT	1.27769463251488e-12	1.67132856965264e-11
23	RFI , RBE	1.57881909411787e-11	1.67132856965264e-11
24	RFI , RBI	2.99980678376051e-10	2.62991876740124e-11
25	RFI , SFT	5.2495047689435e-12	1.67132856965264e-11
26	RFI , SFE	9.2996293323004e-09	4.0088752836935e-10
27	RFI , SFI	0.0685044365868979	0.00652597472959805
28	RFI , SBT	5.21834792762024e-12	1.67132856965264e-11
29	RFI , SBE	1.86204378691196e-09	7.82242510995495e-11
30	RFI , SBI	9.24409512149523e-06	3.17698675029324e-07
31	RBT , RBE	1.59110396315169e-12	1.67132856965264e-11
32	RBT , RBI	1.31322337085303e-12	1.67132856965264e-11
33	RBT , SFT	1.46946606712068e-12	1.67132856965264e-11

Figure 7: II - all paired p-values part 1. B = t-test, C = wilcoxon

34	RBT , SFE	1.46427045600748e-12	1.67132856965264e-11
35	RBT , SFI	1.33508594119917e-12	1.67132856965264e-11
36	RBT , SBT	1.47122056614724e-12	1.67132856965264e-11
37	RBT , SBE	1.33940265265547e-12	1.67132856965264e-11
38	RBT , SBI	1.33623498278424e-12	1.67132856965264e-11
39	RBE , RBI	4.81007358863804e-06	2.78679427712337e-08
40	RBE , SFT	0.0847775809066388	0.0410678008563904
41	RBE , SFE	9.07466123273456e-09	1.21280005249813e-10
42	RBE , SFI	1.48368252168166e-11	1.67132856965264e-11
43	RBE , SBT	0.0990009548467116	0.0403442902198315
44	RBE , SBE	9.84691598997035e-08	1.60419554601932e-08
45	RBE , SBI	4.07659117625809e-10	4.3301918258828e-11
46	RBI , SFT	2.6790843296592e-06	7.54639191071673e-08
47	RBI , SFE	0.00402128199671634	0.0279891444856437
48	RBI , SFI	2.41443886868117e-09	2.87797884564981e-10
49	RBI , SBT	2.41675359082082e-06	7.24395754196259e-08
50	RBI , SBE	0.170769485331541	0.83956956888465
51	RBI , SBI	1.64777326996168e-08	1.13693223956167e-08
52	SFT , SFE	1.03650370990994e-08	6.71899558839375e-11
53	SFT , SFI	5.47121228440714e-12	1.67132856965264e-11
54	SFT , SBT	0.0495447754691748	0.0313220795037409
55	SFT , SBE	3.93401218743567e-09	6.53489941484003e-11
56	SFT , SBI	1.83771996544497e-11	1.67132856965264e-11
57	SFE , SFI	1.01490468882716e-08	1.34205251325615e-09
58	SFE , SBT	1.0014705030523e-08	3.59910232594763e-11
59	SFE , SBE	0.0161769937383313	0.0504108745574905
60	SFE , SBI	5.27333386335289e-05	2.06253965106104e-07
61	SFI , SBT	5.70353696291477e-12	1.67132856965264e-11
62	SFI , SBE	1.96993463278142e-09	3.47915858286267e-10
63	SFI , SBI	0.00280264105201087	0.000793433779461824
64	SBT , SBE	3.08857636888741e-09	5.28272486561804e-11
65	SBT , SBI	1.75714917902182e-11	1.67132856965264e-11
66	SBE , SBI	1.40800372006313e-07	2.26770415822908e-10

Figure 8: II - all paired p-values part 2. B = t-test, C = wilcoxon

Hereabove are the pvalues of the Student t-test and wilcoxon-test for each combination of algorithm (66 in total). The set of values used for the tests are the relative percentage deviations of the weighted sums of completion times of the algorithms over all the instances. Among all of them, only those pvalues under are above α (the significance

level, here we choose $\alpha = 0.01$), which means that we cannot reject the null hypothesis (the null hypothesis H_0 for the t-test states that there is no statistical difference between the two sets of values", which means that those pairs of algorithms are not different in terms of RPDs and if we run the algorithms enough times, the difference will probably disappear.

	A	B	C
1	RFE-RFI	0.304915100	0.463876000
2	RFI-SFI	0.068504437	0.006525975
3	RBE-SFT	0.084777580	0.041067800
4	RBE-SBT	0.099000950	0.040344290
5	RBI-SBE	0.170769500	0.839569600
6	SFT-SBT	0.049544780	0.031322080
7	SFE-SBE	0.016176990	0.050410870

Figure 9: II - significant p-values among all p-values. B = t-test, C = wilcoxon

But in order to answer the questions hereunder, we will use only 24 different combinations, which are relevant to the questions asked. Here is the table containing the pvalues we will consider :

	A	B	C
1	RFI-SFI	0.0685044365868979	0.00652597472959805
2	RFE-SFE	1.29702662832809e-08	4.40493702946702e-10
3	RFT-SFT	1.38781847821521e-12	1.67132856965264e-11
4	RBI-SBI	1.64777326996168e-08	1.13693223956167e-08
5	RBE-SBE	9.84691598997035e-08	1.60419554601932e-08
6	RBT-SBT	1.47122056614724e-12	1.67132856965264e-11
7	RFI-RBI	2.99980678376051e-10	2.62991876740124e-11
8	RFE-RBE	3.35581498109123e-11	1.67132856965264e-11
9	RFT-RBT	0.000384024248959285	0.000198207408241016
10	SFI-SBI	0.00280264105201087	0.000793433779461824
11	SFE-SBE	0.0161769937383313	0.0504108745574905
12	SFT-SBT	0.0495447754691748	0.0313220795037409
13	RFE-RFI	0.304915079404254	0.463875951465839
14	RBE-RBI	4.81007358863804e-06	2.78679427712337e-08
15	SFE-SFI	1.01490468882716e-08	1.34205251325615e-09
16	SBE-SBI	1.40800372006313e-07	2.26770415822908e-10
17	RFT-RFI	1.2107011922515e-12	1.67132856965264e-11
18	RBT-RBI	1.31322337085303e-12	1.67132856965264e-11
19	SFT-SFI	5.47121228440714e-12	1.67132856965264e-11
20	SBT-SBI	1.75714917902182e-11	1.67132856965264e-11
21	RFT-RFE	1.45195335513809e-12	1.67132856965264e-11
22	RBT-RBE	1.59110396315169e-12	1.67132856965264e-11
23	SFT-SFE	1.03650370990994e-08	6.71899558839375e-11
24	SBT-SBE	3.08857636888741e-09	5.28272486561804e-11

Figure 10: II - relevant p-values comparisons. B = t-test, C = wilcoxon

2.2.1 Question 1 : Which initial solution is preferable ?

To answer this question, we will look at pairs of algorithm 1 to 6 (included) of table 10. The initial rules changes while the rest is fixed. We can see that the only significant p-value is 0.06 of RFI-SFI, which means that they are not different. If we look at table 5, the average RPD of RFI lower than the avg RPD of SFI, so it seems that RFI

is better than SFI, which means that Random is better than SRZ. If we look at table 3 and table 4, most of the values of RFI are better than SFI.

And for the others (that are not significant), and according to table 5 :

- RFE is better than SFE
- RFT is worse than SFT
- RBI is worse than SBI
- RBE is worse than SBE
- RBT is worse than SBT

Those comparisons cannot be confirmed since those pairs of algorithms are statistically different.

2.2.2 Question 2 : Which pivoting rule generates better quality solutions and which is faster ?

To answer this question, we will look at pairs of algorithm 7 to 12 (included) of table 10 aswell as table 5 and table 6. The pivoting rule changes while the rest is fixed. First, pairs which are not statistically different (given their p-values) :

- SFE is worse than SBE for 50 jobs, but better for 100 jobs. And SFE is faster than SBE
- SFT is better than SBT and SFT is faster than SBT

It seems that first improvement is more preferable than best improvement even though we would have expected the best improvement to perform better. Also, solutions with first improvement are faster than solutions with best improvement.

And the pairs which are statistically different :

- RFI is better than RBI and RFI is slower than RBI
- RFE is better than RBE and RFE is slower than RFE
- RFT is better than RBT and RFT is faster than RBT
- SFI is better than SBI and SFI is slower than SBI

Those comparisons cannot be confirmed since those pairs of algorithms are statistically different but all algorithm with first improvement generate better solutions than the one using best improvement. Here, it seems that solutions with first improvement are slower than solutions with best improvement.

2.2.3 Question 3 : Which neighbourhood generates better quality solution and what computation time is required to reach local optima ?

To answer this question, we will look at pairs of algorithm 13 to 24 (included) of table 10. The neighbourhood rule changes, while the rest is fixed.

The only significant p-value belongs to the pair RFE-RFI (0.3), which means that we can compare exchange neighbourhood with insert neighbourhood. RFE is worse than RFI for 50 jobs but better for 100 jobs. RFE is faster than RFI.

Then we have the pairs of algorithms that are statistically different :

- RBE is worse than RBI but faster
- SFE is worse than SFI but faster
- SBE is worse than SBI but faster
- RFT is worse than RFI but faster
- RBT is worse than RBI but faster
- SFT is worse than RFI but faster

- SBT is worse than SBI but faster
- RFT is worse than RFE but faster
- RBT is worse than RBE but faster
- SFT is worse than SFE but faster
- SBT is worse than SBE but faster

Given those results, we can conclude that Transpose is dominated by all other neighbourhood types, Exchange is dominated by Insert in most cases, so the most preferable neighbourhood seems to be Insert. The fastest algorithms to reach local optima are the ones using Transpose neighbourhood

For 50 jobs :

- RFT : 0.0015 sec
- RBT : 0.0018 sec
- SFT : 0.00013 sec
- SBT : 0.00018 sec

3 Exercise 1.2 - Variable Neighbourhood Descent

3.1 Results of all algorithms on all instances

3.1.1 Weighted sums of completion times

instance	RTEI	RTIE	STEI	STIE
100_20_01	606031	608628	610123	607386
100_20_02	634016	645013	637781	635382
100_20_03	601122	604403	598892	599235
100_20_04	675853	678999	676514	674947
100_20_05	659260	672537	664556	662741
100_20_06	654517	656181	653805	655640
100_20_07	575436	580048	576714	579663
100_20_08	540882	541565	538976	542886
100_20_09	583210	581449	588535	588535
100_20_10	611569	609884	610236	606447
100_20_11	613282	614418	611084	614216
100_20_12	591031	593510	596247	595930
100_20_13	557403	562717	562107	565484
100_20_14	580082	578902	589191	584816
100_20_15	605879	603310	601533	608389
100_20_16	559107	557401	556564	556730
100_20_17	653609	667435	660292	661444
100_20_18	634825	636605	641005	645996
100_20_19	470154	470227	477609	478244
100_20_20	574822	569670	577395	582009
100_20_21	628869	627410	630918	637490
100_20_22	647800	658949	662505	659162
100_20_23	613288	618767	615619	615272
100_20_24	560317	562653	558823	557742
100_20_25	602995	599686	600494	597630
100_20_26	610559	611845	608184	615420
100_20_27	574716	567404	575786	573409
100_20_28	556361	558113	558687	558245
100_20_29	552338	559831	551969	553187
100_20_30	522421	513173	516630	512457

Figure 11: VND - WCT - 100 jobs

50_20_01	1822023	1851102	1837548	1824454
50_20_02	1844104	1853484	1852709	1843526
50_20_03	1702645	1718361	1703842	1694390
50_20_04	1998278	1988412	1991113	1989447
50_20_05	1580322	1570671	1579380	1577091
50_20_06	1704531	1685397	1698716	1703923
50_20_07	1800642	1836945	1826225	1818179
50_20_08	1787677	1812508	1790537	1801238
50_20_09	1890309	1879969	1888192	1867388
50_20_10	1690411	1705812	1695634	1704404
50_20_11	1857451	1869377	1853048	1859442
50_20_12	1744919	1759073	1738649	1731405
50_20_13	1751396	1787732	1748427	1754979
50_20_14	1727076	1743233	1756222	1742622
50_20_15	1479482	1483255	1486949	1487923
50_20_16	1880587	1878340	1895673	1897705
50_20_17	1818852	1835388	1799917	1809006
50_20_18	1851609	1866962	1844328	1845931
50_20_19	1715210	1728009	1720955	1712026
50_20_20	1830844	1855307	1837053	1834752
50_20_21	1750512	1757877	1755418	1756185
50_20_22	1785969	1763689	1787088	1776334
50_20_23	1826188	1829931	1832403	1841500
50_20_24	1720702	1736361	1740272	1737329
50_20_25	1738556	1759198	1745247	1756508
50_20_26	1592338	1585586	1608862	1606368
50_20_27	1891507	1900336	1896130	1911710
50_20_28	1757871	1776238	1768458	1780172
50_20_29	2045526	2050766	2030898	2062291
50_20_30	1804570	1818264	1817086	1802229

Figure 12: VND - WCT - 50 jobs

3.1.2 Relative Percentage Deviations

instance	RTEI	RTIE	STEI	STIE
100_20_01	1.809461	2.245741	2.496892	2.037093
100_20_02	1.875817	3.642852	2.480790	2.095311
100_20_03	1.413255	1.966782	1.037040	1.094906
100_20_04	1.384895	1.856827	1.484052	1.248986
100_20_05	0.843138	2.874043	1.653236	1.375607
100_20_06	1.744614	2.003283	1.633934	1.919185
100_20_07	1.779527	2.595269	2.005572	2.527172
100_20_08	1.651015	1.779375	1.292809	2.027638
100_20_09	1.817922	1.510483	2.747570	2.747570
100_20_10	1.794985	1.514519	1.573109	0.942434
100_20_11	1.491050	1.679045	1.127307	1.645617
100_20_12	1.169465	1.593806	2.062311	2.008049
100_20_13	0.988867	1.951644	1.841125	2.452962
100_20_14	1.244079	1.038129	2.833910	2.070323
100_20_15	1.765463	1.333965	1.035494	2.187051
100_20_16	1.426594	1.117112	0.965273	0.995387
100_20_17	1.015863	3.152684	2.048727	2.226769
100_20_18	0.736132	1.018588	1.716795	2.508783
100_20_19	0.998049	1.013731	2.599526	2.735936
100_20_20	1.695209	0.783738	2.150414	2.966704
100_20_21	1.774383	1.538262	2.105987	3.169581
100_20_22	0.798860	2.533664	3.086985	2.566808
100_20_23	1.784458	2.693781	2.171323	2.113733
100_20_24	1.439995	1.862905	1.169521	0.973816
100_20_25	2.560287	1.997476	2.134904	1.647782
100_20_26	1.709657	1.923884	1.314018	2.519424
100_20_27	1.933255	0.636378	2.123033	1.701442
100_20_28	1.191687	1.510343	1.614743	1.534351
100_20_29	1.312218	2.686616	1.244534	1.467945
100_20_30	2.594411	0.778264	1.457159	0.637654

Figure 13: VND - RPD - 100 jobs

50_20_01	1.669150	3.291762	2.535447	1.804800
50_20_02	1.847061	2.365104	2.322302	1.815139
50_20_03	1.355156	2.290699	1.426411	0.863752
50_20_04	2.819581	2.311935	2.450913	2.365190
50_20_05	2.678986	2.051927	2.617781	2.469056
50_20_06	2.649817	1.497534	2.299627	2.613202
50_20_07	0.956615	2.992016	2.390978	1.939863
50_20_08	1.892708	3.308008	2.055721	2.665648
50_20_09	2.058601	1.500340	1.944303	0.821087
50_20_10	1.827694	2.755426	2.142319	2.670610
50_20_11	2.265650	2.922260	2.023234	2.375268
50_20_12	1.945467	2.772403	1.579147	1.155922
50_20_13	2.251595	4.372997	2.078256	2.460781
50_20_14	1.037002	1.982215	2.742095	1.946471
50_20_15	2.093794	2.354155	2.609065	2.676277
50_20_16	1.800825	1.679189	2.617467	2.727465
50_20_17	2.317779	3.247996	1.252609	1.763902
50_20_18	1.513095	2.354812	1.113919	1.201803
50_20_19	2.357210	3.121005	2.700050	2.167201
50_20_20	1.613635	2.971356	1.958241	1.830533
50_20_21	2.421831	2.852755	2.708879	2.753756
50_20_22	2.594727	1.314855	2.659007	2.041245
50_20_23	2.791174	3.001857	3.141000	3.653045
50_20_24	1.659685	2.584825	2.815888	2.642014
50_20_25	1.444509	2.648967	1.834928	2.492006
50_20_26	1.687059	1.255875	2.742286	2.583018
50_20_27	1.575966	2.050092	1.824226	2.660888
50_20_28	1.459145	2.519234	2.070195	2.746293
50_20_29	2.675193	2.938215	1.940941	3.516712
50_20_30	1.927194	2.700669	2.634133	1.794967

Figure 14: VND - RPD - 50 jobs

3.1.3 Average relative percentage deviations

instance	RTEI	RTIE	STEI	STIE
50	1.524820	1.827773	1.840270	1.938201
100	1.972930	2.533683	2.241046	2.240597

Figure 15: VND - average relative percentage deviations

3.1.4 Average computation times

instance	RTEI	RTIE	STEI	STIE
50	0.117659	0.146266	0.052422	0.024498
100	2.133100	3.434033	0.768017	0.446567

Figure 16: VND - average computation times

For the percentage improvement over the usage of a single neighbourhood, in particular, the exchange and the insert one, we compare the following pairs :

- RTEI - RFE
- RTIE - RFI
- STEI - SFE
- STIE - SFI

since we are using first improvement pivoting rule in VND and we only consider exchange and insert

3.1.5 Percentage improvement

instance	RTEI	RTIE	STEI	STIE
50	26.084472	-30.970807	54.812408	-12.420582
100	-11.547265	-24.862741	43.323710	-5.462904

Figure 17: VND - percentage improvement over single neighbourhood type

3.2 Statistical tests

	A	B	C
1	RTEI , RTIE	0.000438298229634962	0.00034173196463598
2	RTEI , STEI	0.0116801652500986	0.00745193863009051
3	RTEI , STIE	0.0136646833313008	0.0103021684047635
4	RTIE , STEI	0.0611777634130448	0.152191968712822
5	RTIE , STIE	0.0771500247122845	0.287441227006125
6	STEI , STIE	0.880626331772809	0.912849026424428

Figure 18: VND - all p-values. B = t-test, C = wilcoxon

Hereabove are the pvalues of the Student t-test and wilcoxon-test for each combination of algorithm (6 in total). The set of values used for the tests are the relative percentage deviations of the weighted sums of completion times of the algorithms over all the instances. Among all of them, only those pvalues under are above α (the significance level, here we choose $\alpha = 0.01$), which means that we cannot reject the null hypothesis (the null hypothesis H_0 for the t-test states that there is no statistical difference between the two sets of values", which means that those pairs of algorithms are not different in terms of RPDs and if we run the algorithms enough times, the difference will probably disappear.

	A	B	C
1	RTEI-STEI	0.011680165	0.007451939
2	RTEI-STIE	0.013664680	0.010302170
3	RTIE-STEI	0.061177760	0.152191970
4	RTIE-STIE	0.077150020	0.287441230
5	STEI-STIE	0.880626300	0.912849000
6			

Figure 19: VND - significant p-values among all p-values. B = t-test, C = wilcoxon

If we compare the 6 combinations of algorithms by looking at the table 15 and table 18, we have :

First, we can notice that all pairs of algorithms are statistically not different (p-value above α) except the first one : RTEI with RTIE. RTEI is better than RTIE (generate better quality solutions in average) and is also faster (if we look at the average computing times). But they are statistically different.

Statistically not different pairs :

- RTEI is better than STEI and also faster
- RTEI is better than STIE but slower
- RTIE is better than STEI for 50 jobs but worse for 100 jobs. However, STEI is faster than RTIE
- RTIE is better than STIE for 50 jobs but worse for 100 jobs. However, STIE is faster than RTIE
- STEI is better than STIE for 50 jobs but worse for 100 jobs. However, STIE is faster than STEI

Given those results, we can try to answers these questions :

3.2.1 Question 1 : Which initial solution is preferable ?

For this question, we need RTEI-STEI and RTIE-STIE. RTEI is better than STEI and RTEI is better for 50 jobs, so in general, the algorithm with random initial solution generate better quality solutions. However, the algorithms with SRZ are faster.

3.2.2 Question 2 : Which neighbourhood generates better quality solution and what computation time is required to reach local optima ?

For this question, we need to fix the initial solution and change the neighbourhood : RTEI-RTIE, RTEI-STIE, RTIE-STEI, STEI-STIE. According to the comparisons above, we can deduce that solutions generated by "TEI" is generally better than "TIE" in terms of solution quality.

If we look at table 16, STIE seems the most performant algorithm (in terms of computing time). 0.02 and 0.44 seconds in average for 50 and 100 jobs respectively.

3.3 Improvement

"With the results of the experiments, quantify the improvement of the VND over a local search in a single neighbourhood and examine which ordering of the neighbourhoods in the VND is preferable." The former question can be answered with table 17. We show it hereunder again. The latter has already been answered in the previous section.

instance	RTEI	RTIE	STEI	STIE
50	26.084472	-30.970807	54.812408	-12.420582
100	-11.547265	-24.862741	43.323710	-5.462904

Figure 20: VND - percentage improvement over single neighbourhood type

We can see that STEI is fully improving over a single local search using exchange neighbourhood. RTEI is also improving for 50 jobs. However, for 100 jobs, it is not improving. RTIE and STIE are not improving at all. We observe that the VND solutions that use the TEI ordering of neighbourhoods generate an improvement, which confirms the conclusion depicted in the previous section.

4 Code

In order to run the code, simply compile with the makefile (C++) provided then specify the different arguments :

- make
- ./flowshopRUN

```
./flowshopRUN II <init> (R, SRZ) <pivoting> (FI, BI) <neighbour> (T, E, I) <instance_file>
./flowshopRUN VND <init> (R, SRZ) <n1> <n2> <n3> (T E I or T I E) <instance_file>"
./flowshopRUN runAllExperiments"

e.g. ./flowshopRUN II R FI T instances/50_20_01
e.g. ./flowshopRUN VND SRZ T E I instances/100_20_01
e.g. ./flowshopRUN runAllExperiments
```

To run the statistical tests, simply run the R file named ptest.R in results directory.