Problem Set #4

MACS 40000, Dr. Evans
Due Tuesday, Nov. 8 at 10:30am

1. **Estimating distribution of bequest recipients (4 points).** Go to the 2013 Survey of Consumer Finances (SCF) page and download the 2013 main data file (`p13i6.dta`) and summary data file (`rscfp2013.dta`). I am recommending that you download the Stata format versions, but you could also download the ASCII versions. You can read the Stata versions into python as a pandas DataFrame using the following commands.

```
import pandas as pd
df_main = pd.read_stata('p13i6.dta')
df_summ = pd.read_stata('rscfp2013.dta')
```

SCF respondents can report up to three separate bequests over their lifetimes up to the date of the survey. These bequest amounts are located in variables `X5804`, `X5809`, and `X5814` with corresponding dates (year) of those bequests in variables `X5805`, `X5810`, and `X5815`. Age of respondent at the time of the survey is in variable `X8022`. Two variables in the summary file are also important. `Net Worth` gives the net worth in 2013 dollars of the respondent at the time of the survey, and `wgt` gives a population weight that allows the user of the data to reweight the respondent to make the survey results representative of the population.

   (a) Using the 2013 SCF main data (`p13i6.dta`) and summary data (`rscfp2013.dta`), caluculate the distribution of *total bequests* (`bq_tot`) by age ζ_s for ages 21 to 100. Plot ζ_s as a histogram. Make sure that you use the `wgt` variable to calculate the percentages. Note that your definition of *total bequests* will not include bequests in the data received by individuals younger than 21. And because the age variable is at the time of the survey, the recipient age must be adjusted by the year of the bequest in order to calculate the age distribution. Further, note that the bequest amounts must be inflation adjusted (use CPI adjustment factors of 2013=1.0000, 2012=0.9854, and 2011=0.9652). Lastly, only include bequests from the years 2011 to 2013.

   (b) Now assume that you care about both the distribution of bequests by age and by lifetime income. Suppose that the `net worth` variable in the summary data file (`rscfp2013.dta`) is a good proxy for lifetime income. Create four categories of net worth that represent the four quartiles (0-25%, 25-50%, 50-75%, and 75-100%). Caluculate the distribution of *total bequests* (`bq_tot`) by net worth group j and by age s $\zeta_{j,s}$ for net worth groups $j = \{1, 2, 3, 4\}$ and for ages $s = \{21, 22...100\}$. Plot $\zeta_{j,s}$ as a 3D histogram. Maks sure that you use the `wgt` variable to calculate the percentages.

2. **Solve for the steady-state equilibrium (3 points).** Use the calibration from Section 10.7 and the steady-state equilibrium Definition 10.1. Use your estimated ζ_s from part (a) of Exercise 1. Write a function named `get_SS()` that has the following form,

```
ss_output = get_SS(params, bvec_guess, SS_graphs)
```

where the inputs are a tuple of the parameters for the model `params = (beta, sigma, chi_b, zeta_s, A, alpha, delta, SS_tol, EulDiff)`, an initial guess of the steady-state savings `bvec_guess`, and a Boolean `SS_graphs` that generates a figure of the steady-state distribution of consumption and savings if it is set to `True`.

The output object `ss_output` is a Python dictionary with the steady-state solution values for the following endogenous objects.

```
ss_output = {
  'b_ss': b_ss, 'c_ss': c_ss, 'w_ss': w_ss, 'r_ss': r_ss,
  'K_ss': K_ss, 'Y_ss': Y_ss, 'C_ss': C_ss, 'b_err_ss': b_err_ss,
  'RCerr_ss': RCerr_ss, 'ss_time': ss_time}
```

Let `ss_time` be the number of seconds it takes to run your steady-state program. You can time your program by importing the time library. And let the object `b_err_ss` be the steady-state vector of Euler errors for the savings decisions given in ratio form $\frac{\beta(1+\bar{r})u'(\bar{c}_{s+1})}{u'(\bar{c}_s)} - 1$ or difference form $\beta(1+\bar{r})u(\bar{c}_{s+1}) - u'(\bar{c}_s)$. The object `RCerr_ss` is a resource constraint error which should be close to zero. It is given by $\bar{Y} - \bar{C} - \delta\bar{K}$.

(a) Solve numerically for the steady-state equilibrium values of $\{\bar{c}_s\}_{s=1}^{S}$, $\{\bar{b}_s\}_{s=2}^{S+1}$, $\bar{w}$, $\bar{r}$, $\bar{K}$, $\bar{Y}$, $\bar{C}$, the S Euler errors and the resource constraint error. List those values. Time your function. How long did it take to compute the steady-state?

(b) Generate a figure that shows the steady-state distribution of consumption and savings by age $\{\bar{c}_s\}_{s=1}^{S}$ and $\{\bar{b}_s\}_{s=2}^{S+1}$.

3. **Solve for the non-steady-state equilibrium time path (3 points).** Use time path iteration (TPI) to solve for the non-steady state equilibrium transition path of the economy. Let the initial state of the economy be given by the following distribution of savings,

$$\{b_{s,1}\}_{s=2}^{S+1} = \{x(s)\bar{b}_s\}_{s=2}^{S+1} \quad \text{where} \quad x(s) = \frac{(1.5 - 0.87)}{79}(s-2) + 0.87$$

where the function of age $x(s)$ is simply a linear function of age s that equals 0.87 for $s = 2$ and equals 1.5 for $s = S+1 = 81$. This gives an initial distribution where there is more inequality than in the steady state. The young have less than their steady-state values and the old have more than their steady-state values. You'll have to choose a guess for T and a time path updating parameter $\xi \in (0,1)$, but I can assure you that $T < 300$. Use an L^2 norm for your distance measure (sum of squared percent deviations), and use a convergence parameter of $\varepsilon = 10^{-9}$. Use a linear or quadratic initial guess for the time path of the aggregate capital stock from the initial state K_1^1 to the steady state K_T^1 at time T.

(a) Plot the equilibrium time paths of the aggregate capital stock $\{K_t\}_{t=1}^{T+5}$, wage $\{w_t\}_{t=1}^{T+5}$, and interest rate $\{r_t\}_{t=1}^{T+5}$.

(b) Also plot the equilibrium time path for savings $\{b_{25,t}\}$ of every person age $s = 25$ in every period.