Problem Set #1

MACS 40000, Dr. Evans
Alexandre Sollaci

1. **Git and GitHub.com**

2. **2-period-lived OG economy**

   (a) Since we're only interested in households born in periods $t \geq 1$, we can ignore the initial old for now. To find the optimal consumption for these households, we can solve the planner's problem:

   $$\max_{\{c_{1,t},c_{2,t+1}\}_{t=1}^{\infty}} \quad \sum_{t=1}^{\infty}(1-\beta)\ln(c_{1,t}) + \beta\ln(c_{2,t+1})$$

   $$\text{s.t.} \qquad c_{1,t} + c_{2,t} = e_1 + e_2 \quad \forall t \geq 1.$$

   The FOC's are

   $$[c_{1,t}] : \quad \frac{1-\beta}{c_{1,t}} = \lambda$$

   $$[c_{2,t}] : \quad \frac{\beta}{c_{2,t}} = \lambda$$

   These imply

   $$\frac{1-\beta}{\beta}\frac{c_{2,t}}{c_{1,t}} = 1.$$

   Plugging this into the market clearing constraint, we can find

   $$c_{1,t} = \beta(e_1 + e_2) \quad \forall t \geq 1$$

   and

   $$c_{2,t} = (1-\beta)(e_1 + e_2) \quad \forall t \geq 2.$$

   (b) The optimization problem for the initial old has the straightforward solution $c_{2,1} = e_2$ for all $p_1 > 0$.

   (c) As mentioned in part (b), the initial old have $c_{2,1} = e_2$. Plugging this into the market clearing condition

   $$c_{1,1} + c_{2,1} = e_1 + e_2,$$

   we get $c_{1,1} = e_1$.
   The budget constraint for young household in $t = 1$ now requires

   $$p_1 c_{1,1} + p_2 c_{2,2} = p_1 e_1 + p_2 e_2.$$

   Since $c_{1,1} = e_1$, we must have that $c_{2,2} = e_2$.

Repeating the same logic, we it is easy to see that $c_{1,t} = e_1$ and $c_{2,t} = e_2$ for all $t \geq 1$. To find equilibrium prices, we can use the marginal rates of substitution found by solving the household's problem in (1):

$$\begin{aligned} \max_{c_{1,t},c_{2,t}} \quad & (1-\beta)\ln(c_{1,t}) + \beta \ln(c_{2,t+1}) \\ \text{s.t.} \quad & p_t c_{1,t} + p_{t+1} c_{2,t+1} = p_t e_1 + p_{t+1} e_2 \end{aligned}$$

Taking FOC's, we get

$$[c_{1,t}]: \quad \frac{1-\beta}{c_{1,t}} = \lambda p_t$$

and

$$[c_{2,t+1}]: \quad \frac{\beta}{c_{2,t+1}} = \lambda p_{t+1}.$$

This implies

$$\frac{1-\beta}{\beta}\frac{c_{2,t+1}}{c_{1,t}} = \frac{p_t}{p_{t+1}}.$$

Plugging in the equilibrium values for consumption, we find

$$\frac{p_t}{p_{t+1}} = \frac{1-\beta}{\beta}\frac{e_2}{e_1}.$$

Since $p_1 = 1$, we find

$$p_2 = \frac{\beta}{1-\beta}\frac{e_1}{e_2}.$$

Then

$$\frac{p_2}{p_3} = \frac{1-\beta}{\beta}\frac{e_2}{e_1}$$

implies

$$p_3 = \left(\frac{\beta}{1-\beta}\frac{e_1}{e_2}\right)^2.$$

Following this procedure, we find

$$p_t = \left(\frac{\beta}{1-\beta}\frac{e_1}{e_2}\right)^{t-1}.$$

This equilibrium does no coincide with the optimal solution found in part (a). The reason for this is that the initial generation (born in $t = 1$) do not have access to a technology that allows them to transfer consumption over time, given that the initial old will consume their endowment no matter what. The lack of of this technology for the first generation spills over to all subsequent generations, which might render the competitive equilibrium socially inefficient (in particular, this is true if $\beta \neq \frac{e_1}{e_1+e_2}$).