Problem Set #3

MACS 40000, Dr. Evans

Alexandre Sollaci

1. **Checking Feasibility in the steady state**

    (a)   b_cnstr = array([
                True, True, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False], dtype=bool),

                c_cnstr = array([
                True, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False], dtype=bool),

                K_cnstr = False

    $c\_1 <= 0$ constraint is violated.

    (b)   b_cnstr = array([
                False, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False, False,
                False, False, False, False, False, False, False, False, False,
                False, False, False, True, True, False, False, False, False,
                False, False, True, True, False, False, False, False, False,
                False, True, True, False, False, False, False], dtype=bool),

                c_cnstr = array([
                False, False, False, False, False, False, False, False, False,

```
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False, False, False,
            False, False, False,  True, False, False, False, False, False,
            False, False,  True, False, False, False, False, False, False,
            False,  True, False, False, False, False, False, False]), dtype=bool),

            K_cnstr = False
```

Consumption is violated at time 58 ,66 and 74

(c)
```
            b_cnstr = array([
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False]), dtype=bool),
    c_cnstr = array([
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False, False, False,
            False, False, False, False, False, False, False, False]), dtype=bool),
    K_cnstr = False
```

No constraints is violated.

(d) A feasible initial savings should be close to zero. A good initial guess for $b_{s,2}$ can be $b_{s,1}$

2. **Solve for the steady state equilibrium.**

(a) My outputs for get_SS function are:

```
{'K_ss': 11.399999999999997,

  'RCerr_ss': -2.7977620220553945e-14,

 'EulErr_ss': array([
```

```
        105.85829576,  -26.77805249,   -8.18270577,    3.52562556,
          2.1742705 ,    3.92404865,    2.3389081 ,    6.51633869,
        119.21987288,  -26.77805249,   -8.18270577,    3.52562556,
          2.1742705 ,    3.92404865,    2.3389081 ,    6.51633869,
        119.21987288,  -26.77805249,   -8.18270577,    3.52562556,
          2.1742705 ,    3.92404865,    2.3389081 ,    6.51633869,
        119.21987288,  -26.77805249,   -8.18270577,    3.52562556,
          2.1742705 ,    3.92404865,    2.3389081 ,    6.51633869,
        119.21987288,  -26.77805249,   -8.18270577,    3.52562556,
          2.1742705 ,    3.92404865,    2.3389081 ,    6.51633869,
        119.21987288,  -26.77805249,   -8.18270577,    3.52562556,
          2.1742705 ,    3.92404865,    2.3389081 ,    6.51633869,
        119.21987288,  -26.77805249,   -8.18270577,    3.52562556,
         45.13843785,   28.17704857,    9.26166599,  365.68284489,
        181.13258295,  181.13258295,  181.13258295,  181.13258295,
        181.13258295,  181.13258295,  181.13258295,  181.13258295,
        181.13258295,  181.13258295,  181.13258295,  181.13258295,
        181.13258295,  181.13258295,  181.13258295,  181.13258295,
        181.13258295,  181.13258295,  181.13258295,  181.13258295,
        181.13258295,  181.13258295,  -108.85258134]),

'b_ss': array([  0.06051915,    0.12544728,    0.19494146,    0.26916448,
          0.3482851 ,    0.43247822,    0.52192513,    0.61681373,
          0.71733879,    0.82370217,    0.93611311,    1.05478847,
          1.17995304,    1.3118398 ,    1.45069023,    1.59675463,
          1.75029242,    1.91157249,    2.08087354,    2.25848443,
          2.44470458,    2.6398443 ,    2.84422524,    3.05818079,
          3.28205648,    3.51621046,    3.76101392,    4.01685163,
          4.28412237,    4.56323945,    4.85463128,    5.15874189,
          5.4760315 ,    5.80697711,    6.15207313,    6.511832  ,
          6.88678483,    7.27748213,    7.68449447,    8.10841322,
          8.54985132,    9.00944409,    9.48784999,    9.9857515 ,
         10.50385598,   11.04289661,   11.60363329,   12.18685362,
         12.79337396,   13.42404039,   14.07972987,   14.76135135,
         15.46984691,   15.10214629,   14.72305542,   14.33215564,
         13.92901307,   13.51317797,   13.08418424,   12.64154879,
         12.18477091,   11.71333163,   11.2266931 ,   10.72429784,
         10.20556806,    9.66990489,    9.11668766,    8.54527306,
          7.95499433,    7.34516041,    6.71505505,    6.06393589,
          5.39103352,    4.69555046,    3.97666018,    3.23350602,
          2.46520007,    1.67082209,    0.84941826]),

  'ss_time': 0.1033099999999969,

  'c_ss': array([ 0.37693197,  0.2473104 ,  0.36314776,  0.52936354,
```

3

```
       0.62747143,  0.61822828,  0.65936354,  0.57314776,  0.2473104 ,
       0.36314776,  0.52936354,  0.56822828,  0.62747143,  0.61822828,
       0.65936354,  0.57314776,  0.2473104 ,  0.36314776,  0.52936354,
       0.56822828,  0.62747143,  0.61822828,  0.65936354,  0.57314776,
       0.2473104 ,  0.36314776,  0.52936354,  0.56822828,  0.62747143,
       0.61822828,  0.65936354,  0.57314776,  0.2473104 ,  0.36314776,
       0.52936354,  0.56822828,  0.62747143,  0.61822828,  0.65936354,
       0.57314776,  0.2473104 ,  0.36314776,  0.52936354,  0.56822828,
       0.62747143,  0.61822828,  0.65936354,  0.57314776,  0.2473104 ,
       0.36314776,  0.52936354,  0.56822828,  0.33392585,  0.3246827 ,
       0.36581796,  0.16960218,  0.16960218,  0.16960218,  0.16960218,
       0.16960218,  0.16960218,  0.16960218,  0.16960218,  0.16960218,
       0.16960218,  0.16960218,  0.16960218,  0.16960218,  0.16960218,
       0.16960218,  0.16960218,  0.16960218,  0.16960218,  0.16960218,
       0.16960218,  0.16960218,  0.16960218,  0.16960218,  0.26960218]),

'w_ss': 0.36693197470166844,


'r_ss': 0.96215783574927194,


'Y_ss': 32.967426650119137 }
```

(b) (The numbers are incorrect) Intuitively, consumers leave work early so total endowment decreased, thus decreasing total consumption. Interest rate and wages are not dependent on labor supply n_s,t, thus shall not be effected. Savings increases at younger ages and shrink after retirement. Consumers tend to save more to keep their buying power. Aggregate capital supply increases.

```
{'K_ss': 11.399999999999997,
 'RCerr_ss': 3.8160925368973229
  'EulErr_ss': array([
        105.85829576,  -26.77805249,   -8.18270577,    3.52562556,
          2.1742705 ,    3.92404865,    2.3389081 ,    6.51633869,
        119.21987288,  -26.77805249,   -8.18270577,    3.52562556,
          2.1742705 ,    3.92404865,    2.3389081 ,    6.51633869,
        119.21987288,  -26.77805249,   -8.18270577,    3.52562556,
          2.1742705 ,    3.92404865,    2.3389081 ,    6.51633869,
        119.21987288,  -26.77805249,   -8.18270577,    3.52562556,
          2.1742705 ,    3.92404865,    2.3389081 ,    6.51633869,
        119.21987288,  -26.77805249,   -8.18270577,    3.52562556,
          2.1742705 ,    3.92404865,    2.3389081 ,    6.51633869,
        119.21987288,  -26.77805249,   -8.18270577,    3.52562556,
          2.1742705 ,    3.92404865,    2.3389081 ,    6.51633869,
        119.21987288,  -26.77805249,   -8.18270577,    3.52562556,
         45.13843785,   28.17704857,    9.26166599,  365.68284489,
```

```
            181.13258295,  181.13258295,  181.13258295,  181.13258295,
            181.13258295,  181.13258295,  181.13258295,  181.13258295,
            181.13258295,  181.13258295,  181.13258295,  181.13258295,
            181.13258295,  181.13258295,  181.13258295,  181.13258295,
            181.13258295,  181.13258295,  181.13258295,  181.13258295,
            181.13258295,  181.13258295, -108.85258134]),
  'RCerr_ss': 6.7515483345107121,
 'Y_ss': 32.967426650119137,
 'b_ss': array([
            0.06051915,   0.12544728,   0.19494146,   0.26916448,
            0.3482851 ,   0.43247822,   0.52192513,   0.61681373,
            0.71733879,   0.82370217,   0.93611311,   1.05478847,
            1.17995304,   1.3118398 ,   1.45069023,   1.59675463,
            1.75029242,   1.91157249,   2.08087354,   2.25848443,
            2.44470458,   2.6398443 ,   2.84422524,   3.05818079,
            3.28205648,   3.51621046,   3.76101392,   4.01685163,
            4.28412237,   4.56323945,   4.85463128,   5.15874189,
            5.4760315 ,   5.80697711,   6.15207313,   6.511832  ,
            6.88678483,   7.27748213,   7.68449447,   8.10841322,
            8.54985132,   9.00944409,   9.48784999,   9.9857515 ,
           10.50385598,  11.04289661,  11.60363329,  12.18685362,
           12.79337396,  13.42404039,  14.07972987,  14.76135135,
           15.46984691,  15.10214629,  14.72305542,  14.33215564,
           13.92901307,  13.51317797,  13.08418424,  12.64154879,
           12.18477091,  11.71333163,  11.2266931 ,  10.72429784,
           10.20556806,   9.66990489,   9.11668766,   8.54527306,
            7.95499433,   7.34516041,   6.71505505,   6.06393589,
            5.39103352,   4.69555046,   3.97666018,   3.23350602,
            2.46520007,   1.67082209,   0.84941826]),
 'c_ss': array([
          0.37693197,  0.2473104 ,  0.36314776,  0.52936354,  0.56822828,
          0.62747143,  0.61822828,  0.65936354,  0.57314776,  0.2473104 ,
          0.36314776,  0.52936354,  0.56822828,  0.62747143,  0.61822828,
          0.65936354,  0.57314776,  0.2473104 ,  0.36314776,  0.52936354,
          0.56822828,  0.62747143,  0.61822828,  0.65936354,  0.57314776,
          0.2473104 ,  0.36314776,  0.52936354,  0.56822828,  0.62747143,
          0.61822828,  0.65936354,  0.57314776,  0.2473104 ,  0.36314776,
          0.52936354,  0.56822828,  0.62747143,  0.61822828,  0.65936354,
          0.27960218, -0.04623518,  0.06960218,  0.23581796,  0.2746827 ,
          0.33392585,  0.3246827 ,  0.36581796,  0.27960218, -0.04623518,
          0.06960218,  0.23581796,  0.2746827 ,  0.33392585,  0.3246827 ,
          0.36581796,  0.16960218,  0.16960218,  0.16960218,  0.16960218,
          0.16960218,  0.16960218,  0.16960218,  0.16960218,  0.16960218,
          0.16960218,  0.16960218,  0.16960218,  0.16960218,  0.16960218,
          0.16960218,  0.16960218,  0.16960218,  0.16960218,  0.16960218,
```

```
        0.16960218,  0.16960218,  0.16960218,  0.16960218,  0.26960218]),
'r_ss': 0.9621578357492719,
'ss_time': 0.07494100000000259,
  'w_ss':  0.36693197470166844}
```