Problem Set #1

MACS 40000, Dr. Evans
Dan Ehrlich

1. Git and Github.com. Done online.
2. 2 period lived OG economy.
   (a) To solve for the household's (born in period $t \geq 1$) optimal consumption, maximize the household utility function subject to the lifetime budget constraint:
   $$\max_{c_{1,t},c_{2,t+1}} (1-\beta)ln(c_{1,t}) + \beta ln(c_{2,t+1})$$
   $$s.t. p_t c_{1,t} + p_{t+1}c_{2,t+1} = p_t e_1 + p_{t+1}e_2$$
   The FOCs are
   $$[c_{1,t}] : \frac{1-\beta}{c_{1,t}} = \lambda p_t$$
   $$[c_{2,t+1}] : \frac{\beta}{c_{2,t+1}} = \lambda p_{t+1}$$
   $$[\lambda] : p_t c_{1,t} + p_{t+1}c_{2,t+1} = p_t e_1 + p_{t+1}e_2$$
   The Euler equation is
   $$\frac{c_{2,t+1}}{c_{1,t}} = \frac{p_t}{p_{t+1}}\frac{\beta}{1-\beta}$$
   $$c_{2,t+1} = c_{1,t}\frac{p_t}{p_{t+1}}\frac{\beta}{1-\beta}$$
   Substitute into household lifetime budget constraint
   $$p_t c_{1,t} + c_{1,t}p_t\frac{\beta}{1-\beta} = p_t e_1 + p_{t+1}e_2$$
   $$c_{1,t} = (1-\beta)[e_1 + \frac{p_{t+1}}{p_t}e_2]$$
   $$c_{2,t+1} = \beta[\frac{p_t}{p_{t+1}}e_1 + e_2]$$
   (b) Solving for the initial old person's optimal consumption is mathematically trivial. The initial old do not wish to trade with the households born in period $t = 1$ because they will die at the end of this period. And since the intial old want to maximize consumption in period $t = 1$, we can therefore look solely at the budget constraint $p_1 c_{2,1} = p_1 e_2$. Hence, $c_{2,1} = e_2$.

(c) To solve for the competitive equilibrium, use the solution $c_{2,1} = e_2$ from part (b) and substitute into the market clearing condition $c_{1,t} + c_{2,t} = e_1 + e_2$. We see that $c_{1,1} = e_1$. Plugging that into the lifetime budget constraint we get $c_{2,2} = e_2$. Solving the problem forward we get $\{c_{1,t}, c_{2,t}\}_{t=1}^{\infty} = \{e_1, e_2\}$. Consumption is independent of price so for any price we will see the same consumption.

The equilibrium does not equal the solution from part (a) because we have introduced the initial old who do not wish to trade with those born in period $t = 1$. It is possible, however, to find a price for part (a) so that consumption in part (a) would equal consumption in part (c). From part (a), the Euler equation is

$$\frac{c_{2,t+1}}{c_{1,t}} = \frac{p_t}{p_{t+1}} \frac{\beta}{1-\beta}$$

$$\frac{p_t}{p_{t+1}} = \frac{c_{2,t+1}}{c_{1,t}} \frac{1-\beta}{\beta}$$

Plugging in $c_{2,t} = e_2$ and $c_{1,t} = e_1$,

$$\frac{p_t}{p_{t+1}} = \frac{e_2}{e_1} \frac{1-\beta}{\beta}$$

and therefore

$$p_t = (\frac{e_1}{e_2} \frac{\beta}{1-\beta})^{t-1}$$