Problem Set #3

MACSS 40000
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Problem 1

1.
   If the initial guess for savings is $np.ones(\, S-1\,)$, then the variable that won' t bind the constraint is b_2 which is the saving when agent is young, and c_1, which is the saving when period 1. This is saying that the agent saved too much when they are young. Actually, when $c_1 = 1$, that means the agent is not able to consume anything, which is counter intuitive.

2.
   If the initial guess for savings is as the saving distribution in b), then saving distribution and the positiveness of consumption are all violated. This happens particularly because there's negative saving after agent retires. Then at that period, consumption goes to negative. On one hand, saving in the previous period turns to borrowing, which means the agent has to pay off the debt; on the other hand, there's saving required for the next period.

3.
   If the initial guess for savings is what it shows in c), then every variable satisfies the constraint. We will be using this value as our initial guess for the following problem.

4.
   The rule for picking initial value is that you should be careful when agent is young and when agent is old.
   Before agent retires, saving per period b_i should in general be less than 1. When agent is young, the value should be smaller than that when the agent is middle aged. Negative saving can appear when agent is young.
   After agent retires, you should make saving per period less than 0.2. And you normally don't want there to be negative savings since that could sink your consumption when agent is old.

Problem 2

1.
   The solution for $\beta = 0.4412$ is shown as following

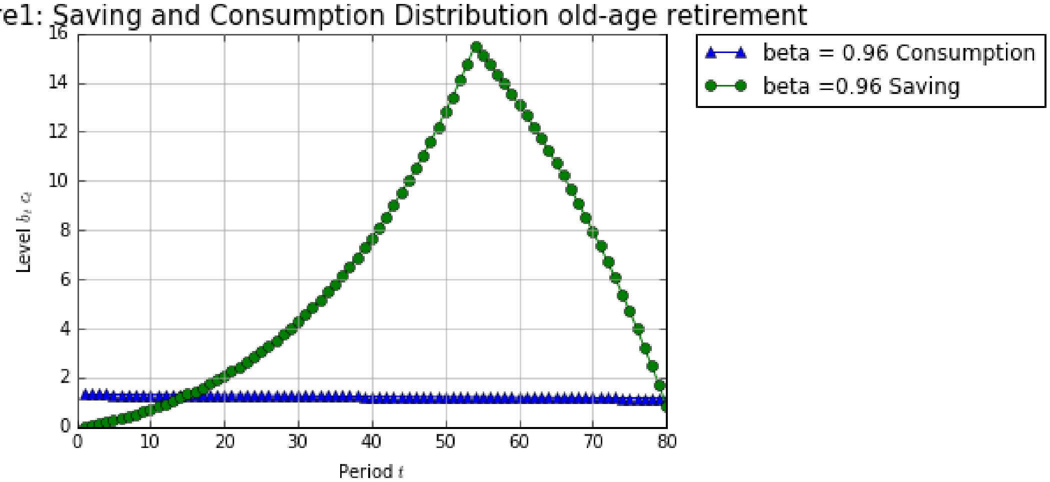
RCerr_ss : 1.27897692437e-13 K_ss : 501.941512378
r_ss : 0.0364593309089
c_ss : [1.3195392 1.31733671 1.31513791 1.31294277 1.31075129 1.30856348
1.30637931 1.30419879 1.30202191 1.29984866 1.29767905 1.29551305
1.29335067 1.29119189 1.28903672 1.28688515 1.28473717 1.28259277
1.28045196 1.27831471 1.27618104 1.27405092 1.27192437 1.26980136
1.26768189 1.26556596 1.26345357 1.26134469 1.25923934 1.25713751
1.25503918 1.25294435 1.25085302 1.24876519 1.24668083 1.24459996
1.24252256 1.24044862 1.23837815 1.23631113 1.23424757 1.23218744
1.23013076 1.22807751 1.22602769 1.22398128 1.22193829 1.21989872
1.2178626 1.21582978 1.2138004 1.2117744 1.20975179 1.20773256
1.20571669 1.20370419 1.20169505 1.19968924 1.19768682 1.19568773
1.19369197 1.19169954 1.18971043 1.18772465 1.18574218 1.18376302
1.18178716 1.1798146 1.17784534 1.17587936 1.17391666 1.17195724
1.17000109 1.1680482 1.16609858 1.1641522 1.16220908 1.1602692
1.15833256 1.15639915]
b_ss : [0.06051915 0.12544728 0.19494146 0.26916448 0.3482851
0.43247822 0.52192513 0.61681373 0.71733879 0.82370217
0.93611311 1.05478847 1.17995304 1.3118398 1.45069023
1.59675463 1.75029242 1.91157249 2.08087354 2.25848444
2.44470458 2.6398443 2.84422525 3.0581808 3.28205649
3.51621046 3.76101393 4.01685164 4.28412238 4.56323946
4.85463129 5.1587419 5.47603151 5.80697712 6.15207315
6.51183201 6.88678485 7.27748215 7.68449449 8.10841324
8.54985135 9.00944411 9.48785001 9.98575152 10.50385601
11.04289664 11.60363332 12.18685366 12.79337394 13.42404037
14.07972986 14.76135134 15.46984689 15.10214627 14.7230554
14.33215563 13.92901305 13.51317797 13.08418425 12.64154879
12.1847709 11.71333162 11.2266931 10.72429784 10.20556805
9.66990488 9.11668765 8.54527305 7.95499432 7.3451604
6.71505504 6.06393589 5.39103351 4.69555046 3.97666018
3.23350602 2.46520007 1.67082209 0.84941826]
w_ss : 1.38005835397
ss_time : 0.061210313305991804
EulErr_ss : [8.76900108e-10 -2.02909356e-11 -6.80737133e-11 -4.77679563e-
11
-3.96938593e-11 -3.15396598e-11 1.78845827e-11 -2.86931590e-12
1.01905151e-10 -1.17553911e-10 1.70652381e-12 1.44645407e-12
-1.38655754e-12 -1.95637950e-12 8.94162522e-12 -3.51190022e-10
3.57360586e-10 -1.51995083e-12 4.24565938e-12 3.17174065e-12
3.70597997e-12 4.12558876e-12 3.67988973e-12 1.70918835e-12
-8.94673224e-12 2.54768429e-11 4.69058126e-12 4.59793315e-12
5.71520609e-12 4.74642547e-12 5.30986366e-12 -2.58570942e-12
-4.01976230e-11 7.93711763e-11 9.49829104e-12 7.26663174e-12
8.84869955e-12 -5.52637935e-11 -2.17115881e-10 3.84103194e-10

-6.75152156e-11 7.32947036e-12 1.02754472e-11 1.86145543e-11
-2.21194729e-10 5.73069014e-09 -5.96163496e-09 -6.88890057e-08
6.94014998e-08 -1.91361593e-10 4.76239603e-10 -4.43101111e-11
-2.55401256e-11 2.42814657e-11 6.58447741e-11 4.14592094e-09
2.33814349e-08 -2.84071584e-08 -2.76465084e-09 -7.76559883e-09
5.01497721e-09 5.67585290e-09 8.38305758e-10 -9.09416542e-10
1.02996278e-10 1.25889021e-09 1.11144283e-09 -1.48010493e-10
-1.27870659e-09 -1.18992749e-09 2.74971157e-11 8.30636449e-10
4.51612081e-10 -5.10402831e-12 -3.61411567e-10 -8.37553360e-11
5.05032016e-10 1.32456601e-10 -4.04850486e-10]
Y_ss : 123.992935187
C_ss : 98.8958595684

2.

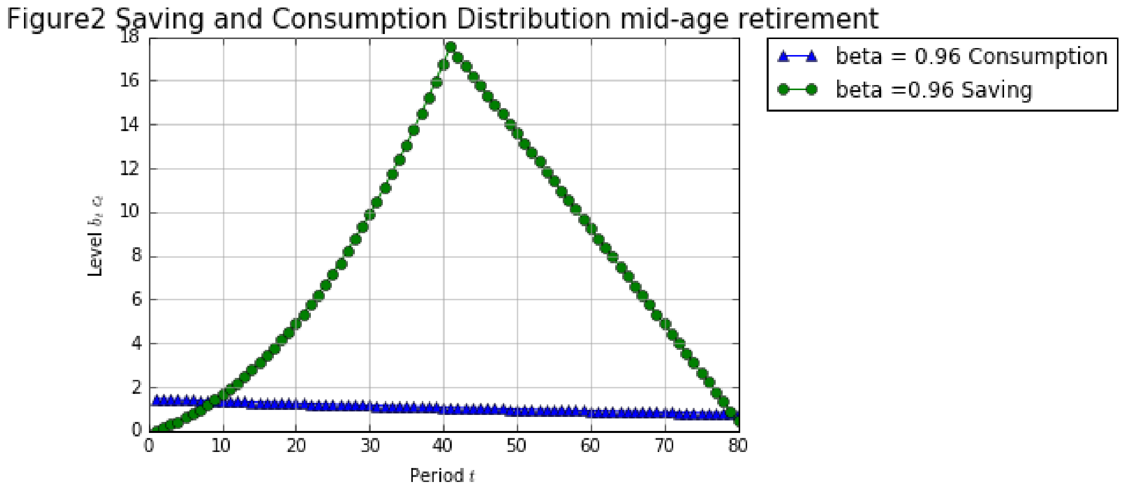
Now we include the graph for the steady state consumption and saving $\{c_i\}_{i=1}^S$, $\{b_i\}_{i=2}^S$.

Figure1: Saving and Consumption Distribution old-age retirement



3.

Now we can generate a graph for the new exogenous labor supply.



Now we can see that, with early retirement, the saving goes down earlier for the agents, which is intuitive because early retirement means less labor supply and hence fewer wages after retirement. Thus the agent can keep up with previous savings.

As for the consumption, since the lifetime income is decreasing, we now have the lifetime consumption is also less than when agent has a higher exogenous labor supply (except for the first few periods).

This is a compound effect of wage movement and saving movement.

When the agent has less time to work(that's when the agent retires earlier), their consumption smoothing incentive makes them save more when they have higher exogenous labor supply. That is exactly why mid-age agent has higher saving than old-age agent. That is one of the reasons why the old-age retiring agent has higher consumption for most of the time period.

But the mid-age retiring agent actually has higher consumption when they are young. This is caused by the higher wage, which is direct result of higher MPL caused by higher rate of saving.

But in general, because of the stark difference in exogenous labor supply, it's harder for the agent to smooth their lifetime consumption (that is why we can see that there is more downward sloping trend in the mid-age retiring agent consumption path).

Thus in this economy, when agent retires earlier, their lifetime saving will go down and hence less consumption throughout his/her lifetime except for the first few periods.

Problem 3

1.

The following is the time path for aggregate capital for the whole economy.

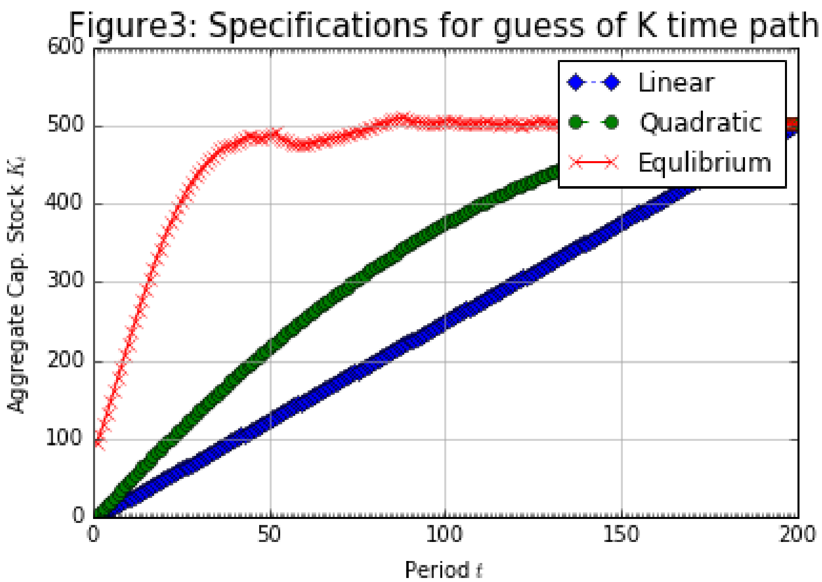


Figure 4 shows the time path for wage w:

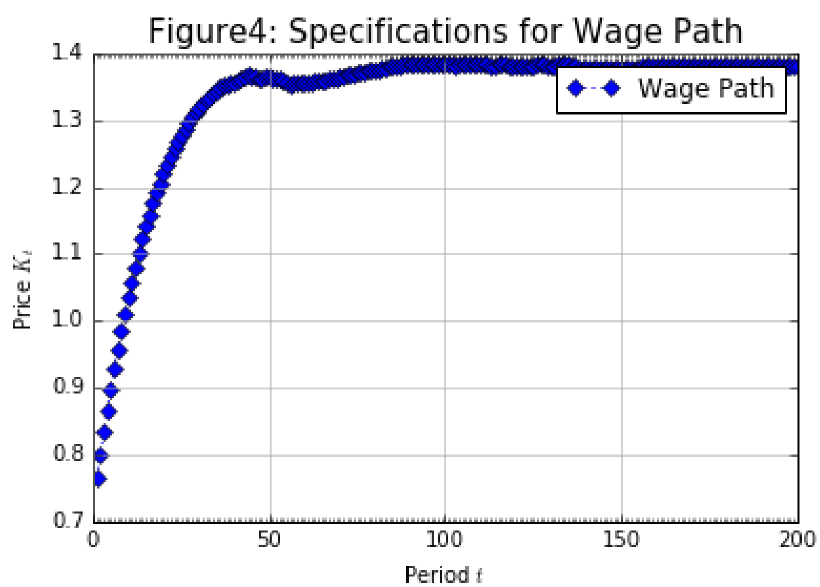
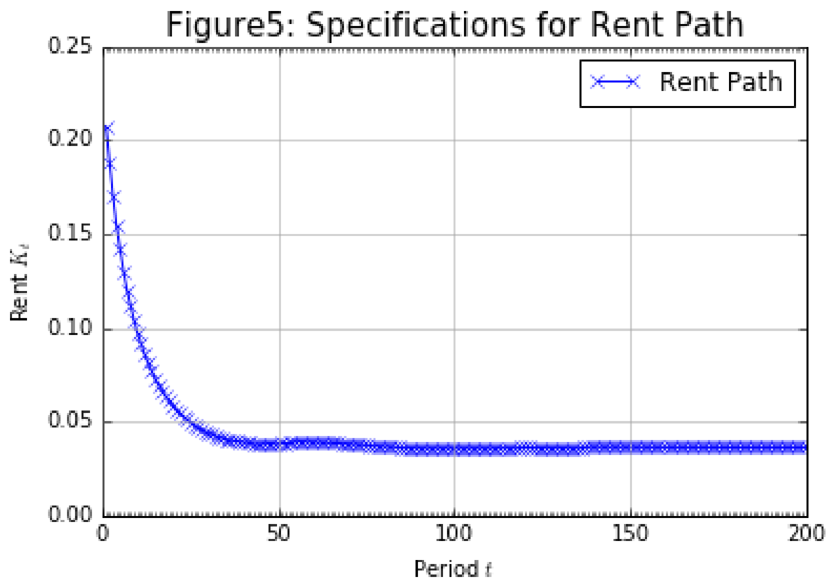
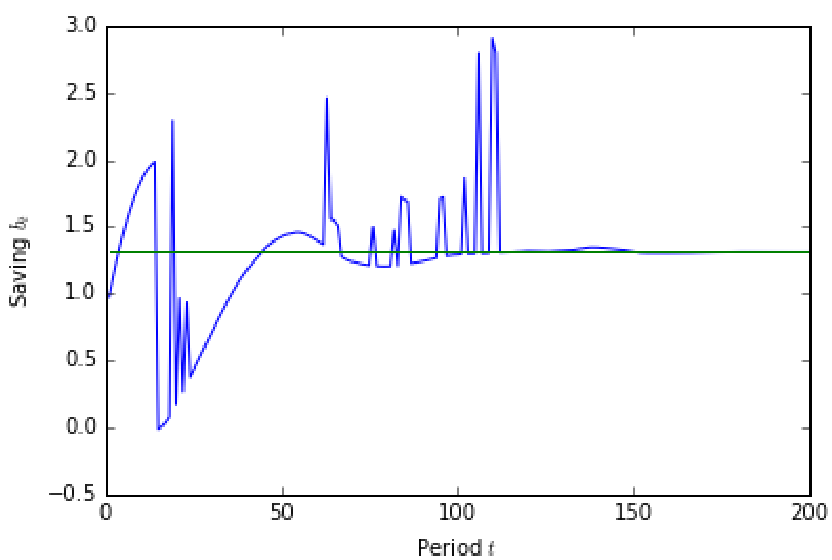


Figure 5 shows the time path for rental rate r:



We can see that wage path demonstrate the same trend as capital and yet rent path demonstrates the exactly opposite trend. But they all converge at approximately 7 periods.

2.



The graph doesn't look particularly nice. The optimization process doesn't converge as expected.
But in the picture it's likely that we will have a $b_1 5$ that is above the steady state.

3.

Under this case, the total capital falls within 0.0001 range of the steady state after approximately 85 periods.