# Problem Set #1

Weijia

1.

2. a) $\max_{c_{1,t},c_{2,t+1}} (1-\beta)\,ln\,(c_{1,t}) + \beta ln\,(c_{2,t+1}) \quad s.t. \quad p_t c_{1,t} + p_{t+1} c_{2,t+1} = p_{t+1} e_2$

$$\Rightarrow \quad L = (1-\beta)\,ln\,(c_{1,t}) + \beta ln\,(c_{2,t+1}) - \lambda\,(p_t c_{1,t} + p_{t+1} c_{2,t+1} - p_t e_1 - p_{t+1} e_2)$$

$$\frac{\partial L}{\partial c_{1,t}} = 0 \;\Rightarrow\; \frac{1-\beta}{c_{1,t}} - \lambda p_t = 0$$

$$\frac{1-\beta}{c_{1,t}} = \lambda p_t$$

$$c_{1,t} = \frac{1-\beta}{\lambda p_t}$$

$$\frac{\partial L}{\partial c_{2,t+1}} = 0 \;\Rightarrow\; \frac{\beta}{c_{2,t+1}} = \lambda p_{t+1}$$

$$c_{2,t+1} = \frac{\beta}{\lambda p_{t+1}}$$

now subject to budget constraint

$$\Rightarrow \; p_t c_{1,t} + p_{t+1} c_{2,t+1} = p_t e_1 + p_{t+1} e_2$$

$$p_t \left(\frac{1-\beta}{\lambda p_t}\right) + p_{t+1}\left(\frac{\beta}{\lambda p_{t+1}}\right) = p_t e_1 + p_{t+1} e_2$$

$$\lambda = \frac{1}{p_t e_1 + p_{t+1} e_2}$$

$$\Rightarrow \; c_{1,t} = \frac{(1-\beta)\,(p_t e_1 + p_{t+1} e_2)}{p_t}$$

$$c_{2,t+1} = \frac{\beta\,(p_t e_1 + p_{t+1 e_2})}{p_{t+1}}$$

b) Now we are maximizing $\max_{c_{2,1}} \beta ln\left(c_{2,t+1}\right)$ s.t $\quad p_1 c_{2,1} = p_1 e_2$

$$\Rightarrow \; L = \beta ln\left(c_{2,1}\right) - \lambda\left(p_1 c_{2,1} - p_1 e_2\right)$$

$$\Rightarrow \; \frac{\partial L}{\partial c_{2,1}} = \frac{\beta}{c_{2,1}} - \lambda p_1 = 0$$

$$\frac{\beta}{c_{2,1}} = \lambda p_1$$
$$c_{2,1} = \frac{\beta}{\lambda p_1}$$

$$p_1 c_{2,1} - p_1 e_2 = 0$$
$$p_1\left(\frac{\beta}{\lambda p_1}\right) - p_1 e_2 = 0$$
$$\frac{\beta}{\lambda} = p_1 e_2$$
$$\lambda = \frac{\beta}{p_1 e_2}$$

$$\Rightarrow c_{2,1} = \frac{\beta}{p_1}\frac{p_1 e_2}{\beta} = e_2$$

c) from b):

$$c_{2,1} = e_2$$

it follows:

$$c_{1,1} + c_{2,1} = e_1 + e_2$$
$$\Rightarrow c_{1,1} = e_1$$

then

$$p_1 c_{1,1} + p_2 c_{2,2} = p_1 e_1 + p_2 e_2$$
$$p_1 e_1 + p_2 c_{2,2} = p_1 e_1 + p_2 e_2$$
$$c_{2,2} = e_2$$

by induction, $\{c_{1,t}, c_{2,t}\}_{t=1}^{\infty} = \{e_1, e_2\} \Rightarrow p_t \quad$ *undetermined*

□