Problem Set #6

MACS 40000, Dr. Evans
Due Tuesday, Nov. 22 at 10:30am

1. **Fertility rates function (2 points).** Get current U.S. fertility data (births per 1,000 women) from the National Vital Statistics Reports, volume 64, number 1. Use Table 3 from the appendix for fertility rates for the age categories 10-14, 15-17, 18-19, 20-24, 25-29, 30-34, 35-39, 40-44, and 45-49. Assume that the fertility value in each of those age bins is associated with the midpoint age in those bins. We want fertility rates in the form of births per person (both men and women) of a given age. For example, if the fertility value for women age 20-24 is 80.7 births per 1,000 women, then the fertility rate associated with the midpoint of age 22 is 0.04035 (80.7/2,000). Write a function `get_fert()` that has the following form,

   ```
   fert_rates = get_fert(totpers, graph)
   ```

   where `totpers` is the number of periods an agent lives ($E+S$), `fert_rates` is a vector of fertility rates of length `totpers` for each model period, and `graph` is a Boolean that equals `True` if you want to output a plot of the fertility rates. Inside the `get_fert()` function, use the function `scipy.interpolate.interp1d()` to fit a smoothly interpolated fertility rate function of age for 100 evenly spaced periods from 1 to 100. Then any number of periods greater than 100 or less than 100 evenly spans that period of years. Regardless of how many periods an individual's life is in the model (`totpers`), the number of years to which those total model periods correspond is 100. An agent is born at the beginning of year-1 and dies at the end of year-100. The total model periods (`totpers`) span that period of 100 years.

   (a) Plot the interpolated fertility rate function for `totpers`=100.

   (b) Plot the interpolated fertility rate function for `totpers`=80 as scatter points on top of the line you produced in part (a).

   (c) Plot the interpolated fertility rate function for `totpers`=20 as scatter points on top of the line you produced in part (a).

2. **Mortality rates function (1 points).** Look up the infant mortality value (number of births per 1,000 births that die at birth or before they turn one year old). Transform that number into a mortality rate (percent of total births that die at birth or before they turn one year old). Get current U.S. mortality data by age from the Actuarial Life Table, 2011 from the U.S. Social Security Administration, which is available as `mort_rates2011.csv`. These data give mortality rates by age (percent of age-s individuals that died in that year) by gender. You need to calculate the total fertility rate (both men and women) by age using the population of men and women, respectively, in each of those age groups. Again, assume that agents in your model live for 100 years. Those years start at the beginning of year-1 and finish at the end of year-100. In the data, let the mortality rate from age=0 correspond to the mortality rate of year-1, and let the mortality rate in the data from age-98 correspond to the mortality rate of year-99. Set the mortality rate of year-100 to 1.0, such that everyone dies after 100 years. Write a function `get_mort()` that has the following form,

```
mort_rates, infmort_rate = get_mort(totpers, graph)
```

where `totpers` is the number of periods an agent lives ($E + S$), `mort_rates` is a vector of mortality rates of length `totpers` for each model period (percent of individuals age-s that die each period), and `graph` is a Boolean that equals `True` if you want to output a plot of the mortality rates. Note that when `totpers`< 100, each model period has a duration that is longer than a year. So mortality rates will not just be a simple interpolation along the mortality rate curve when `totpers`=100, as in part (a). You will have to calculate the cumulative mortality rate for the model age bin given the annual mortality rates and total population levels for the given ages from the data.

(a) Plot the mortality rate function with the infant mortality rate at age ($s = 0$) for `totpers`=100.

(b) Plot the mortality rate function with the infant mortality rate at age ($s = 0$) for `totpers`=80 as scatter points in the same plot of the line you produced in part (a).

(c) Plot the mortality rate function with the infant mortality rate at age ($s = 0$) for `totpers`=20 as scatter points in the same plot of the line you produced in part (a).

3. **Immigration rates function (2 points).** We will treat immigration rates as a residual based on two-consecutive periods of population data by age, mortality rates by age, and fertility rates by age. Get population level data by age from the comma-delimited file `pop_data.csv` taken from the U.S. Census Bureau Population Estimates. This file has U.S. population totals by age (ages 0 to 99) for years 2012 and 2013. For the model with $E + S = 100$, treat age-0 in the data as model age 1 and age-99 in the data as model age 99. Let the immigration rates i_s be implicitly defined by the population dynamics equations (9.1) and explicitly defined in Section 9.7.4 as equations (9.42). Write a function `get_imm_resid()` that has the following form,

```
imm_rates = get_imm_resid(totpers, graph)
```

where `totpers` is the number of periods an agent lives $(E + S)$, `imm_rates` is a vector of immigration rates of length `totpers` for each model period (percent of individuals age-s immigrate into the country), and `graph` is a Boolean that equals `True` if you want to output a plot of the mortality rates. Note that negative immigration rates imply the people of age-s are moving out of the country. You will want to call your `get_fert()` and `get_mort()` functions inside of this `get_imm_resid()` function.

   (a) Plot the immigration rates for `totpers`=100.

   (b) Plot the immigration rates for `totpers`=80.

   (c) Plot the immigration rates for `totpers`=20.

4. **Time path of the population distribution (1 points).** Let $E = 20$ and $S = 80$. For the following problems, use the fertility rates f_s, mortality rates ρ_s, and immigration rates i_s fro problems (1), (2), and (3). You must use the stationary version of the population dynamics equations (9.43).

   (a) Use the instruction in Section 9.7.5 to solve for and plot the steady-state stationary population distribution $\bar{\omega}_s$.

   (b) Create an $80 \times T + S - 2$ matrix that shows the time path of the stationarized population distribution $\hat{\omega}_{s,t}$ for the economically relevant population $E + 1 \leq s \leq E + S$. Let the first column be $\hat{\omega}_{s,1}$. Calculate $\hat{\omega}_{s,1}$ from the 2013 population by age data from the file `pop_data.csv`. And every column for $t > T$ should be the steady-state stationary population distribution $\bar{\omega}_s$.

   (c) Make a single plot that shows the stationary population distribution at time $t = 1, 10, 30$, and T.

   (d) Plot the population growth rate of the economically relevant population $\tilde{g}_{n,t}$ for $1 \leq t \leq T + S - 2$.

5. **Solve for the steady-state equilibrium (2 points).** Use the calibration from Section 9.7 and the steady-state equilibrium Definition 9.1. Write a function named `get_SS()` that has the following form,

```
ss_output = get_SS(params, bvec_guess, SS_graphs)
```

where the inputs are a tuple of the parameters and objects for the model are,

```
params = (beta, sigma, nvec, L, A, alpha, delta, g_y, mort_rates,
    imm_rates, omega_SS, g_n_SS, SS_tol)
```

an initial guess of the steady-state savings `bvec_guess`, and a Boolean `SS_graphs` that generates a figure of the steady-state distribution of consumption and savings if it is set to `True`.

The output object `ss_output` is a Python dictionary with the steady-state solution values for the following endogenous objects.

```
ss_output = {
  'b_ss': b_ss, 'c_ss': c_ss, 'w_ss': w_ss, 'r_ss': r_ss,
  'K_ss': K_ss, 'Y_ss': Y_ss, 'C_ss': C_ss,
  'EulErr_ss': EulErr_ss, 'RCerr_ss': RCerr_ss,
  'ss_time': ss_time}
```

Let `ss_time` be the number of seconds it takes to run your steady-state program. You can time your program by importing the time library. And let the object `EulErr_ss` be a length-$(S-1)$ vector of the Euler errors from the resulting steady-state solution given in ratio form $\frac{e^{-\sigma g_y}\beta(1+\bar{r})(1-\rho_s)u'(\bar{c}_{s+1})}{u'(\bar{c}_s)} - 1$ or difference form $e^{-\sigma g_y}\beta(1+\bar{r})(1-\rho_s)u'(\bar{c}_{s+1}) - u'(\bar{c}_s)$. The object `RCerr_ss` is a resource constraint error which should be close to zero. It is given by,

$$\bar{Y} - \bar{C} - \Big[(1+\bar{g}_n)e^{g_y} - 1 + \delta\Big]\bar{K} + e^{g_y}\sum_{s=E+2}^{E+S} i_s\bar{\omega}_s\hat{b}_{s+1}$$

(a) Solve numerically for the steady-state equilibrium values of $\{\bar{c}_s\}_{s=E+1}^{E+S}$, $\{\bar{b}_s\}_{s=E+2}^{E+S}$, $\bar{w}$, $\bar{r}$, $\bar{K}$, $\bar{Y}$, $\bar{C}$, the $S-1$ Euler errors and the resource constraint error. List those values. Time your function. How long did it take to compute the steady-state?

(b) Generate a figure that shows the steady-state distribution of consumption and savings by age $\{\bar{c}_s\}_{s=E+1}^{E+S}$ and $\{\bar{b}_s\}_{s=E+2}^{E+S}$.

6. **Solve for the non-steady-state equilibrium time path (2 points).** Use time path iteration (TPI) to solve for the non-steady state equilibrium transition path of the economy. Let the initial state of the economy be given by the following distribution of savings,

$$\{b_{s,1}\}_{s=E+2}^{E+S} = \{x(s)\bar{b}_s\}_{s=E+2}^{E+S} \quad \text{where} \quad x(s) = \frac{(1.5 - 0.87)}{S + E - 2}(s-2) + 0.87$$

where the function of age $x(s)$ is simply a linear function of age s that equals 0.87 for $s = E+2$ and equals 1.5 for $s = E+S$. This gives an initial distribution where there is more inequality than in the steady state. The young have less than their steady-state values and the old have more than their steady-state values. You'll have to choose a guess for T and a time path updating parameter $\xi \in (0,1)$, but I can assure you that $T < 320$. Use an L^2 norm for your distance measure (sum of squared percent deviations), and use a convergence parameter of $\varepsilon = 10^{-9}$. Use a linear or parabolic initial guess for the time path of the aggregate capital stock from the initial state $\hat{K}_1^1$ to the steady state $\hat{K}_T^1$ at time T.

   (a) Plot the equilibrium time paths of the stationary aggregate capital stock $\{\hat{K}_t\}_{t=1}^{T+5}$, wage $\{\hat{w}_t\}_{t=1}^{T+5}$, and interest rate $\{r_t\}_{t=1}^{T+5}$.

   (b) How many periods did it take for the economy to get within 0.00001 of the steady-state aggregate capital stock $\bar{K}$? What is the period after which the aggregate capital stock never is again farther than 0.00001 away from the steady-state?