Problem Set #3
MACS 40000, Dr. Evans
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1. **Fitting elliptical disutility of labor to CFE function**

   (a)]In the CFE case, we have

   $$v'_{cfe}(l) = (1-l)^{\frac{1}{\theta}}.$$

   In the elliptical case,

   $$v'_{elp}(l) = b\left[1-(1-l)^{\mu}\right]^{\frac{1-\mu}{\mu}}(1-l)^{\mu-1}$$

   (b) My estimates were $b = 0.501$ and $\mu = 1.554$. The plot of the two marginal utilities can be found in figure 1.

2. **Checking feasibility in the steady-state**

   (a) The consumption constraint is violated. Specifically, $c_1 < 0$.

   (b) Similarly to the case above, the consumption constraint is violated once more. This time $c_2 < 0$.

   (c) In this case, none of the constraints are violated.

   (d) Once again, none of the constraints are violated.

   (e) As a thumb rule, high values for the labor supply and low (but positive) values for savings will not violate any of the constraints.

3. **Solve for the steady-state equilibrium**

   (a) The steady state values are

```
{'C_ss': 4.3347304222925924,
 'EulErr_ss': array([ 3.37152511, -2.56403131, -2.19710288, ...,
   2.62509299, 2.67743804,  3.31542748]),
 'K_ss': 1.9763389650259522,
 'L_ss': 8.2417330233076775,
 'RCerr_ss': 5.5511151231257827e-16,
 'Y_ss': 4.999925738767649,
 'b_ss': array([ 0.04176371,  0.10345494,  0.16777769,  0.22660614,
        0.28004111, 0.32058685,  0.33238941,  0.29556384,  0.20815528]),
 'c_ss': array([ 0.35215818,  0.34907219,  0.37643968,  0.40719693,
        0.43346902, 0.46243928,  0.48020632,  0.4981081 , 0.49906487,
        0.47657586]),
 'n_ss': array([ 0.9989684 ,  0.98354507,  0.97375055,  0.94828273,
```

```
        0.91934412, 0.88574648,  0.80147411,  0.70712497,  0.63253415,
        0.39096245]),
 'r_ss': 0.54888291497069996,
 'ss_time': 0.015906726519233416,
 'w_ss': 0.3943286831796281 6}
```

(b) The distribution of consumption and savings is in figure 2; the distribution of labor supply is in figure 3.

(c) When α drops to 0.25, wages increase, since the marginal product of labor has increased. The increase in wages increases the labor supply, increasing L. It also makes consumers richer, so consumption increases and so does aggregate production. Savings drop as a higher fraction of consumer's wealth comes from labor. Despite the decrease in α, making capital less valuable for production, the increase in L and in total production end up increasing the marginal product of capital, thus increasing the interest rate as well.

The new steady state values are as below:

```
{'C_ss': 5.4922493741343672,
 'K_ss': 1.6744890292626833,
 'L_ss': 9.2954952934901325,
 'Y_ss': 6.0558481694147988,
 'b_ss': array([ 0.02596725,  0.09787919,  0.17396547,  0.24743017,
        0.31016607, 0.29233795,  0.28944248,  0.18709221,  0.05020824]),
 'c_ss': array([ 0.43384328,  0.43078356,  0.46165551,  0.5045682 ,
        0.53112749, 0.59709924,  0.63608323,  0.69120299, 0.64256411,
        0.56332176]),
 'n_ss': array([ 0.94105545,  0.9986618 ,  0.98685773,  0.98093893,
        0.92800383, 0.82526723,  0.95632237,  0.86894888,  0.81761307,
        0.99182601]),
 'r_ss': 0.56755417949303399,
 'ss_time': 0.010697268452076969,
 'w_ss': 0.48861152457813578}
```

4. **Solve for the non-steady-state equilibrium time path**

   (a) The plots are in figures 4 (capital), 5 (interest rate) and 6 (wages).

   (b) See figures 7 (labor supply) and 8 (savings)

Figures

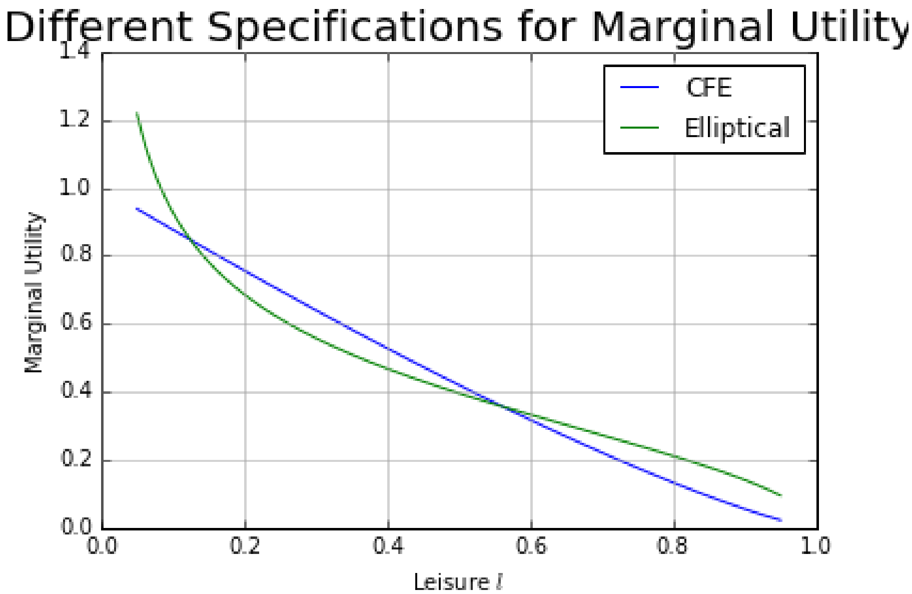

Figure 1: Marginal utility of leisure in CFE and Elliptical specifications.

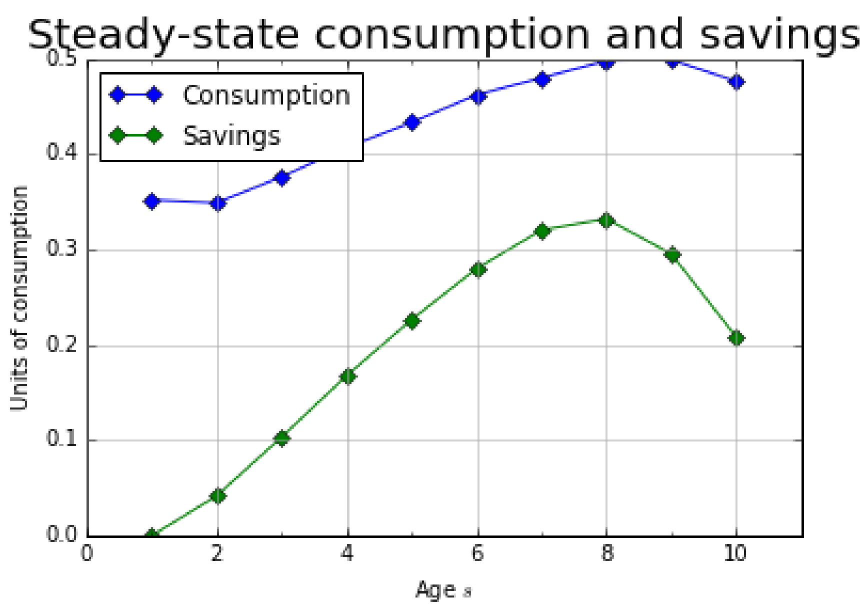

Figure 2: Distribution of consumption and savings in steady state.

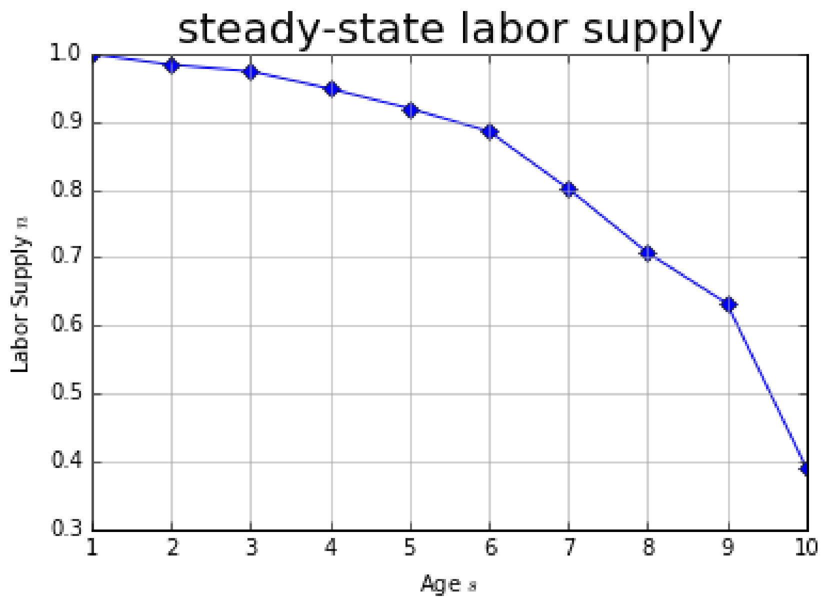

Figure 3: Distribution of labor supply in steady state.

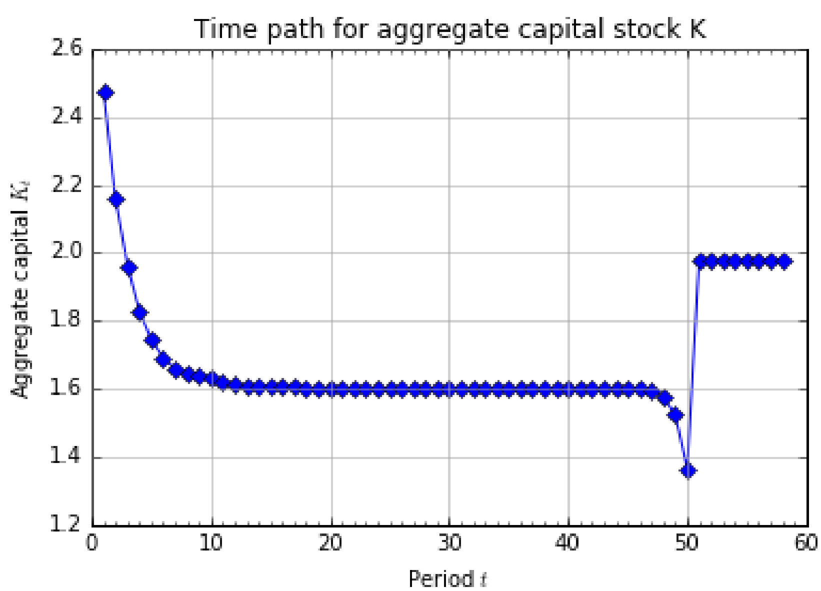

Figure 4: Time path of capital to Steady State.

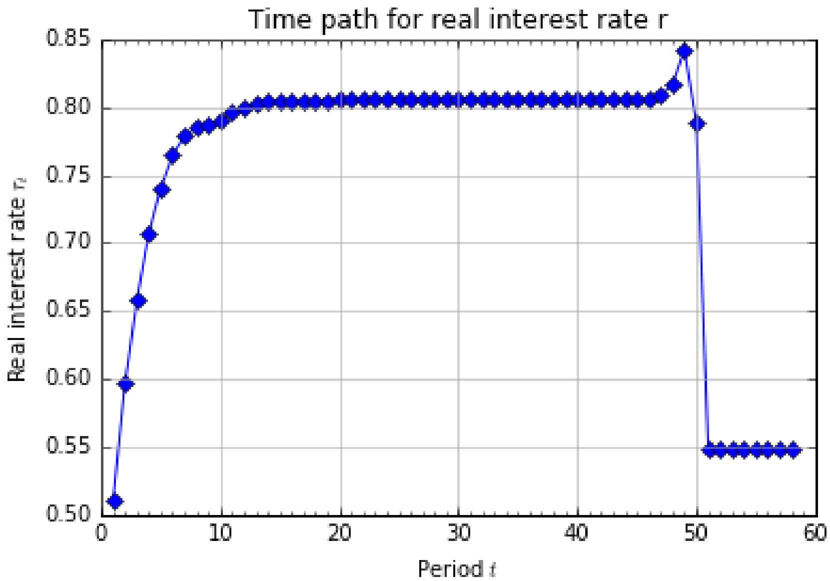

Figure 5: Time path of interest rate to Steady State.

Figure 6: Time path of wages to Steady State.

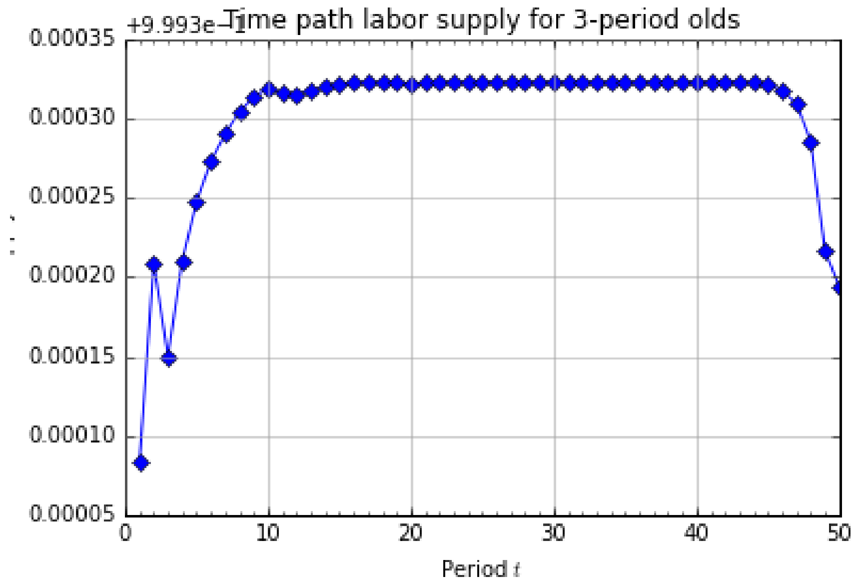

Figure 7: Equilibrium time path labor supply for 3-period olds.

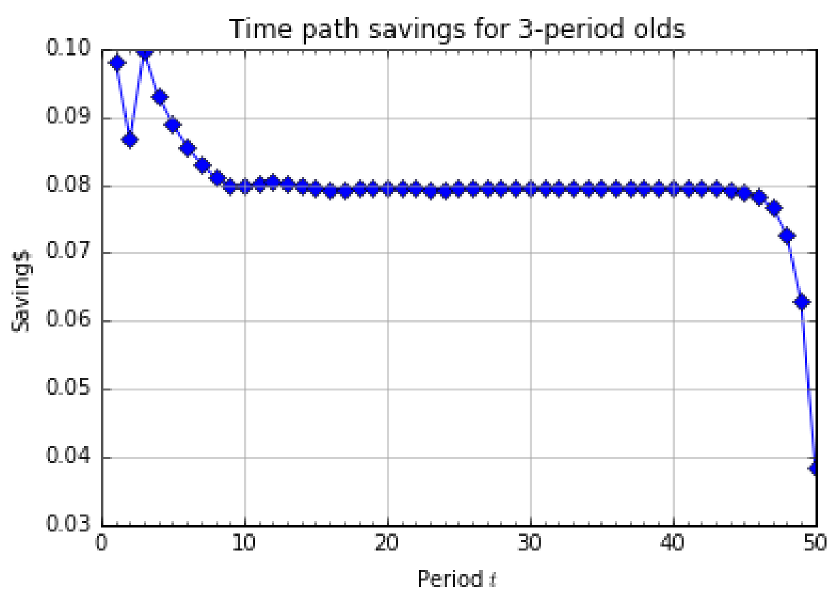

Figure 8: Equilibrium time path savings for 3-period olds.