Problem Set #1

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1. Problem 2 a)
   For the normal agents problem, we have the following setup:

$$\max_{c_{1,t},c_{2,t+1}} \quad (1-\beta)ln(c_{1,t}) + \beta ln(c_{2,t+1})$$
$$s.t. \quad p_t c_{1,t} + p_{t+1} c_{2,t} = P_t e_1 + p_{t+1} e_2$$

Build up the Lagrangian and solve for the first order conditions:

$$L(c_{1,t}, c_{2,t+1}, \lambda) = \quad (1-\beta)ln(c_{1,t}) + \beta ln(c_{2,t+1}) + \lambda(p_t c_{1,t} + p_{t+1} c_{2,t} = P_t e_1 + p_{t+1} e_2)$$

$$[c_{1,t}] \qquad \frac{1-\beta}{c_{1,t}} = \lambda p_t \tag{0.1}$$
$$[c_{2,t+1}] \qquad \frac{\beta}{c_{2,t+1}} = \lambda p_{t+1} \tag{0.2}$$
$$[\lambda] \qquad p_t c_{1,t} + p_{t+1} c_{2,t} = P_t e_1 + p_{t+1} e_2 \tag{0.3}$$

Now derive the eulers equation for this problem using the first two first order conditions:

$$(1-\beta)P_{t+1} c_{2,t+1} = \beta p_t c_{1,t}$$
$$c_{2,t+1} = \beta \frac{p_t}{p_t + 1} e_1 + \beta e_2$$

Substitute this into the budget constraint

$$p_t e_1 + p_{t+1} e_2 - \frac{1}{1-\beta} p_t c_{1,t} = 0$$

Then we can solve for $c_{1,t}$ *and* and $c_{2,t+1}$

$$c_{1,t} = (1-\beta)e_1 + (1-\beta)\frac{p_{t+1}}{p_t} e_2$$
$$c_{2,t+1} = \beta \frac{p_t}{p_t + 1} e_1 + \beta e_2$$

2. Problem 2 b)
   Now we have the maximization problem for the initial agent

$$\max_{c_{2,1}} \quad \beta ln(c_{2,1})$$
$$s.t. \quad p_1 c_{2,1} = p_1 e_2$$

   This is an equality constraint, then we can pin down the solution:

$$c_{2,1} = e_2$$

3. Problem 2 c)
   In order to solve the competitive equilibrium, we have to solve the problem sequentially.

   Thus we have to deal with the initial agent first. We solved this in part b)

   Then the initial agent's consumption $c_{2,1}$ is exactly e_2 .

   This means that there would be no trade between the new agent in the first period, meaning the new agent only has the option of consuming its initial endowment.

   Then we have

$$c_{1,1} = e_1$$

   . In other word, we have the following maximization problem instead of part a):

$$\max_{c_{1,t}, c_{2,t+1}} (1-\beta)ln(c_{1,t}) + \beta ln(c_{2,t+1})$$
$$s.t. \quad c_{1,1} = e_1$$
$$p_t c_{1,t} + p_{t+1} c_{2,t} = P_t e_1 + p_{t+1} e_2$$

   The budget constraint pins down the solutions:

$$c_{1,1} = e_1$$
$$c_{2,2} = e_2$$

   By induction, we can have that:

$$\{c_{1,t}, \ , c_{2,t}\}_{t=1}^{\infty} = \{e_1, e_2\}_{t=1}^{\infty}$$

   At this time, we can't determine the prices for the competitive equilibrium, since price doesn't matter anymore, or in other word, it's no longer the mechanism guiding the agents' behavior.

Comparing the result we have from c) to those in a), we can notice that the results are not necessarily equal to each other. But there does exist one set of prices where the results coincide with each other. That is:

$$P_t = (\frac{\beta e_1}{(1-\beta)e_2})^{t-1} \qquad \forall t \in \{1, 2, 3, ...\}$$

Solving this is easy, we just substitute e_1 and e_2 into a) and by induction we can get the set of price for the following periods. But since the prices in a) is a result of the equilibrium but the price in c) is something that we impose in our answer, there should be fundamental differences.