Problem Set 1

Dalya Elmalt

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1 Question 2

1.1 The 2-period Household Problem

The household maximizes this life time utility function

$\max_{c_{1,t},c_{2,t+1}} (1-\beta)ln(c_{1,t}) + \beta ln(c_{2,t+1}) \;\; s.t. \;\; p_t c_{1,t} + p_{t+1} c_{2,t+1} = p_t e_1 + p_{t+1} e_2$

thus he is solving

$\max_{c_{1,t},c_{2,t+1}} (1-\beta)ln(c_{1,t}) + \beta ln(c_{2,t+1}) + \lambda \left(p_t e_1 + p_{t+1} e_2 - p_t c_{1,t} - p_{t+1} c_{2,t+1}\right)$

FOC:

$$[c_{1,t}] : \frac{(1-\beta)}{c_{1,t}} = \lambda p_t \tag{1}$$

$$[c_{2,t+1}] : \frac{\beta}{c_{2,t+1}} = \lambda p_{t+1} \tag{2}$$

$$[\lambda] : p_t e_1 + p_{t+1} e_2 = p_t c_{1,t} + p_{t+1} c_{2,t+1} \tag{3}$$

By dividing (1) by (2) we get

$$\frac{c_{2,t+1}}{c_{1,t}} = \frac{p_t}{p_{t+1}} \; \frac{\beta}{(1-\beta)} \tag{4}$$

By substituting $c_{2,t+1}$ from (4) in (3), we get that

$p_t e_1 + p_{t+1} e_2 = p_t c_{1,t} + p_{t+1}(c_{1,t} \frac{p_t}{p_{t+1}} \; \frac{\beta}{(1-\beta)})$, thus

$$c^*_{1,t} = (1-\beta)(e_1 + \frac{p_{t+1}}{p_t} e_2) \tag{5}$$

and by subtituitng $c^*_{1,t}$ in (4), we get that

$$c^*_{2,t+1} = \beta(\frac{p_t}{p_{t+1}} e_1 + e_2) \tag{6}$$

$\{c^*_{1,t}, c^*_{2,t+1}\}$ is the partial comptitive equilibrium solution to the 2-period household problem.

1.2 Initial Old Agent Optimal Consumption

The initial old agent wants to

$$\max_{c_{2,1}} \quad \beta ln(c_{2,1}) \quad s.t \quad p_1 c_{2,1} \; = \; p_1 e_2 \tag{7}$$

so he chooses to consume his entire endowment at period 1 (his last period), thus the solution is $c^*_{2,1} = e_2$.

1.3 Competitive Equilibrium of Consumption and Prices

To solve for equilibrium, we enforce the resource constratint that
$c_{1,t} + c_{2,t} = e_1 + e_2, \qquad \forall t$

We know the initial old agent will set $c_{2,1} = e_2$, thus the best that the young agent at period 1 will be able to do is to set $c_{1,1} = e_1$, since he is bounded by the resource constraint.
Thus in period 2, the old agent $c_{2,2}$ will want to maximize his utility by setting $c_{2,2} = e_2$, leaving the new young agent with only e_1 to consume, thus the competitive equilibrium consumptions $\{c_{1,t}, c_{2,t}\}_{t=1}^{\infty} = \{e_1, e_2\} \ \forall t$
As for prices, we know that $p_1 = 1$, but since every agent ends up consuming his endowment in every period, no trade occurs, thus prices are indeterminante.
This equilibrium is different from my answer in 1, since ideally agents want to be able to transfer consumption across periods, but the initial old agent's decision disrupted that, and we end up with this world where gains from trade are lost and people can only consume their endowments in each period.