Problem Set #3

(a)

MACS 40000, Dr. Evans Alexandre Sollaci

1. Checking Feasibility in the steady state

b_cnstr = array([

```
False, False, False, False, False, False, False, False, False,
         False, False, False, False, False, False, False, False,
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         False, False, False, False, False, False, False, False,
         False, False, False, False, False, False, False, False,
         False, False, False, False, False, False, False, False,
         False, False, False, False, False, False], dtype=bool),
         c_cnstr = array([
         True, False, False, False, False, False, False, False,
         False, False, False, False, False, False, False, False, False,
         False, False, False, False, False, False, False], dtype=bool),
         K_{cnstr} = False
   c_{-1} \le 0 constraint is violated.
(b)
         b_cnstr = array([
         False, False, False, False, False, False, False, False, False,
         False, False, False, False, False, False, False, False, False,
         False, False, False, False, False, False, False, False,
         False, False, False, False, False, False, False, False, False,
         False, False, False, False, False, False, False, False,
         False, False, False, False, False, False, False, False,
         False, False, False, True, False, False, False, False,
         False, False,
                       True, True, False, False, False, False,
         False, True, True, False, False, False, False], dtype=bool),
         c_cnstr = array([
```

True, False, False, False, False, False, False, False,

False, False, False, False, False, False, False, False, False,

```
False, Fa
```

K_cnstr = False

Consumption is violated at time 58,66 and 74

```
(c)
         b_cnstr = array([
         False, False, False, False, False, False, False, False, False,
         False, False, False, False, False, False, False, False,
         False, False, False, False, False, False, False, False,
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         False, False, False, False, False, False, False], dtype=bool),
   c_cnstr = array([
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         False, False, False, False, False, False, False, False,
         False, False, False, False, False, False, False, False,
         False, False, False, False, False, False, False], dtype=bool),
```

K_cnstr = False

No constraints is violated.

(d) A feasible initial savings should be close to zero. A good initial guess for b_s,2 can be b_s,1

2. Solve for the steady state equilibrium.

(a) My outputs for get_SS function are:

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   'RCerr_ss': -2.7977620220553945e-14,
   'EulErr_ss': array([
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                          3.92404865,
                                          2.3389081 ,
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                                         -8.18270577,
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                                         -8.18270577,
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-8.18270577,

3.52562556,

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'w_ss': 0.36693197470166844,

'r_ss': 0.96215783574927194,

'Y_ss': 32.967426650119137 }

(b) (The numbers are incorrect) Intuitively, consumers leave work early so total endowment decreased, thus decreasing total consumption. Interest rate and wages are not dependent on labor supply n_s,t, thus shall not be effected. Savings increases at younger ages and shrink after retirement. Consumers tend to save more to keep their buying power. Aggregate capital supply increases.

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{'K_ss': 11.399999999999997,
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