Problem Set #2

MACSS 40000
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Problem 1

1. If the initial guess for savings is [1.0, 1.2], then the variable that won' t bind the constraint is b_2 which is the saving when agent is young, and c_1, which is the saving when agent is young. This is saying that the saving for agent when they are middle aged is too high, such that the consumption when they are young turns to negative.

2. If the initial guess for savings is [0.1, -0.06], then every variable satisfies the constraint.

3. If the initial guess for savings is [0.1, 0.1], then every variable satisfies the constraint. We will be using this value as our initial guess for the following problem.

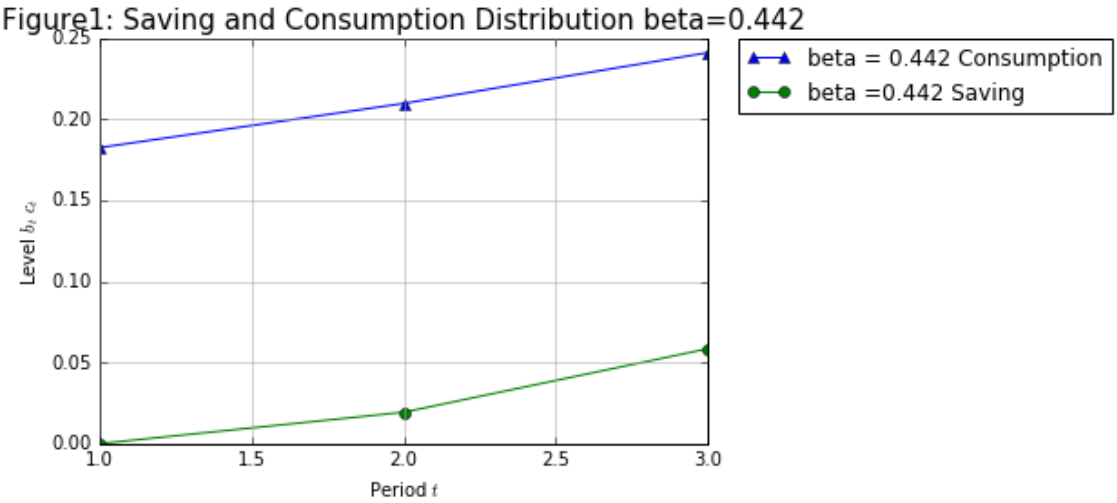
Problem 2

1. The solution for $\beta = 0.4412$ is shown as following

$$
\begin{aligned}
Steady\ State\ Saving &: [0.01931253,\ 0.0584111] \\
Steady\ State\ Consumption &: [0.18241213,\ 0.20961468,\ 0.24087387] \\
Steady\ State\ Total\ Consumption &: 0.632900672939 \\
Steady\ State\ Total\ Capital &: 0.077723625753 \\
Steady\ State\ Wage &: 0.20172465739 \\
Steady\ State\ Rental\ Rate &: 2.43306233913 \\
Resource\ Constraint\ Error &: 6.93889390391e-17 \\
EulerlError &: [-4.57504257e-10,\ 8.86430485e-10] \\
Steady\ State\ Production &: 0.68276037886 \\
Time &: 0.0024846342275850475
\end{aligned}
$$

2.

Now we have the following graph plotted.

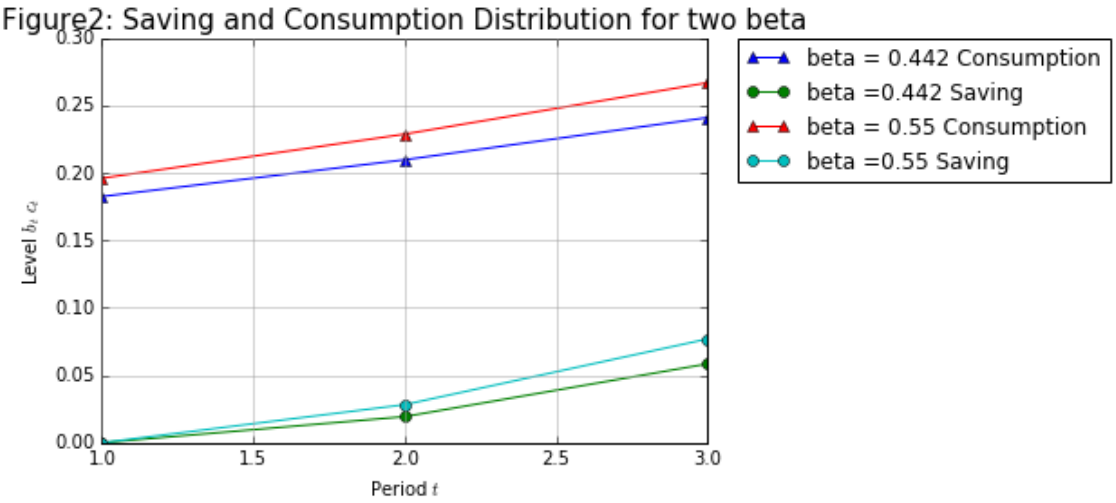


3.

When $\beta = 0.55$, then we can have the following steady state result:

$$
\begin{aligned}
\textit{Steady State Saving} &: [0.02817692,\ 0.07686545] \\
\textit{Steady State Consumption} &: [0.19597528,\ 0.22861594,\ 0.26669307] \\
\textit{Steady State Total Consumption} &: 0.691284290355 \\
\textit{Steady State Total Capital} &: 1.88637650572 \\
\textit{Steady State Wage} &: 0.224152195934 \\
\textit{Steady State Rental Rate} &: 2.43306233913 \\
\textit{Resource Constraint Error} &: 9.71445146547e-17 \\
\textit{EulerlError} &: [-6.87805368e-12,\ -2.52953214e-12] \\
\textit{Steady State Production} &: 0.758668970855 \\
\textit{Time} &: 0.0009397435205755755
\end{aligned}
$$

In convenience, we plot the steady state consumption and capital under these two stages together.



This grpah shows that when *beta* gets higher, consumption and saving for each agent in each stage of their lives get higher.

Notice that when β is increasing, this means that the agent is getting more and more patient. Then they would be willing to save to get a higher utility in the future. This effect causes the total capital $K \uparrow$. Then the rental rate $r \downarrow$. Accordingly, marginal product of labor $MPL \uparrow$ is getting higher, thus the wage for each period is getting higher. This is particularly good for each agent since they are getting paid more each period. Consumption under this circumstance is also higher, even for the young agent and this comes entirely as a result of higher saving leading to higher wages. Normally if we have a higher savings rate, the economy tends to have higher production growth(though there's no growth in this problem). If the older generation save for the new generation, then the welfare of the new generation tends to go up. Higher saving rate leads to an economy with not only higher production but also with higher consumption.

Notice that, by simply making the agent more patient, the production, consumption and saving for the whole economy is getting higher. This can shed lights on the social security system.

Problem 3

1. The following is the time path for aggregate capital for the whole economy.

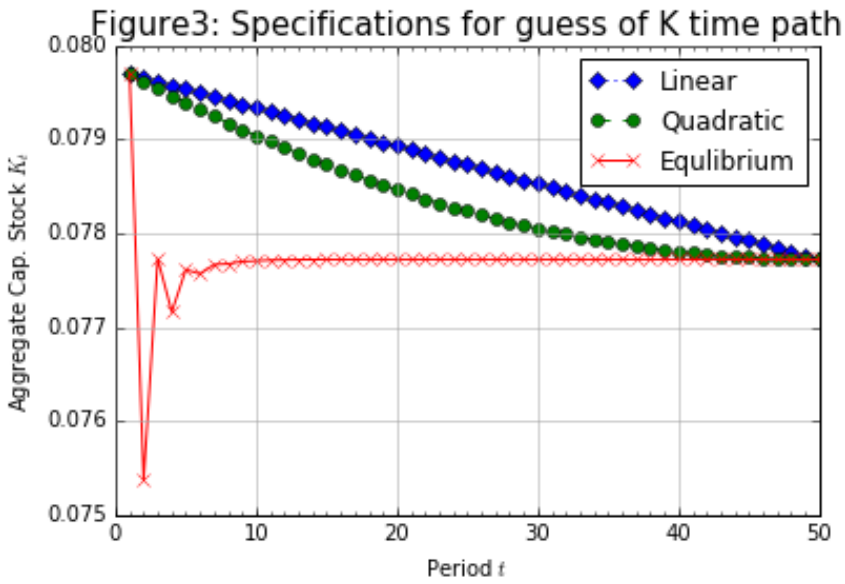


Figure 4 shows the time path for wage w:

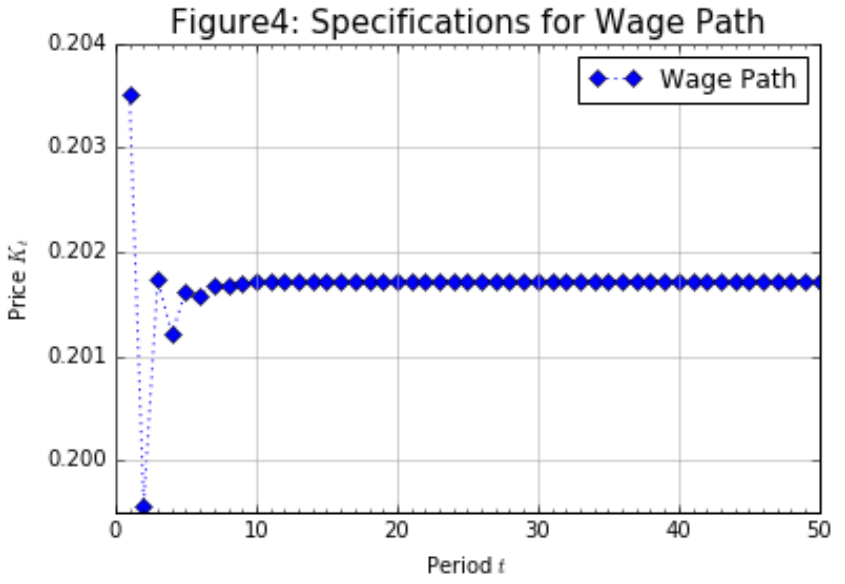
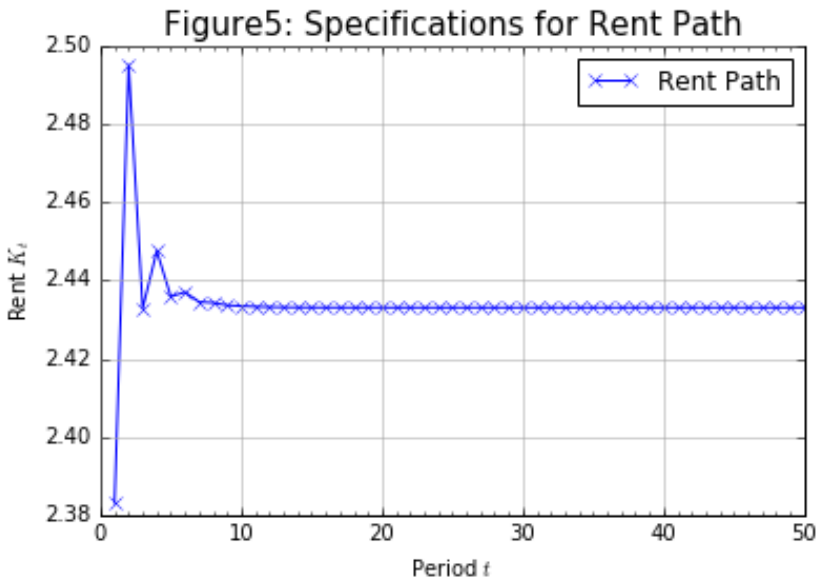


Figure 5 shows the time path for rental rate r:



We can see that wage path demonstrate the same trend as capital and yet rent path demonstrates the exactly opposite trend. But they all converge at approximately 7 periods.

2.

Under this case, the total capital falls within 0.0001 range of the steady state after 7 periods, meaning $T = 7$.