variable	value	objective function	interpretation
η^H	0.06	1.0699	New technology benefit
η^M	0.03	1.0699	New combination benefit
τ	200	1.0699	Shape parameter for idea distribution
ξ	1	1.0699	$1/\xi$ is the fraction of viable combinations
λ	1.5	1.0699	scale parameter of the cost distribution
κ	0.2	1.0699	shape parameter of the cost distribution

Table 1: Current values that minimize the objective function

Column #	Moment	Model	Data
1	Fraction of refinements in 1880	55.179%	55%
2	Fraction of new combinations in 1880	39.99%	30%
3	Fraction of new technologies in 1880	4.82%	10%
4	Fraction of refinements in 1930	29.054%	35%
5	Fraction of new combinations in 1930	68.09%	60%
6	Fraction of new technologies in 1930	2.84%	3%
7	Peak of the refinement share	85.705%	60%
8	Year of the peak in refinement share	1845	1870

Table 2: Moments

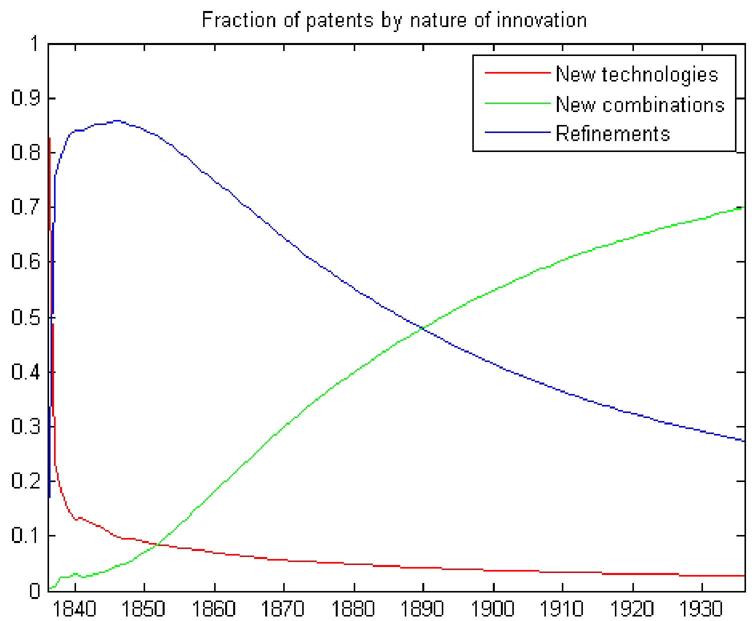

Figure 1: Fraction of patents by type

Rate of growth of the economy

There is a final good, produced with technology

$$Y_t = \frac{L_t^\gamma}{1-\gamma} \sum_{j \in \mathcal{J}} q_{jt}^\gamma k_{jt}^{1-\gamma}.$$

Labor is inelastic, so $L_t \equiv 1$ and the profit maximization of firms implies $q_{jt} = k_{jt}$. This, in turn, gives

$$Y_t = \frac{1}{1-\gamma} \sum_{j \in \mathcal{J}} q_{jt}.$$

Finally, we also have that the quality fo the product of a firm evolves according to

$$q_{j,t+1} = q_{jt} + s_j \bar{q}_t$$

where s_j is the quality of the patent that was purchased and $\bar{q}_t$ is the average quality in year t.

Define the growth rate g_t as

$$g_t = \frac{Y_{t+1} - Y_t}{Y_t} = \frac{\frac{1}{1-\gamma} \sum_{j \in \mathcal{J}} q_{j,t+1} - \frac{1}{1-\gamma} \sum_{j \in \mathcal{J}} q_{jt}}{\frac{1}{1-\gamma} \sum_{j \in \mathcal{J}} q_{jt}}$$

Plugging in $q_{j,t+1}$, we have

$$g_t = \frac{\sum_{j \in \mathcal{J}} s_j \bar{q}_t}{\sum_{j \in \mathcal{J}} q_{jt}}$$

and finally, using the definition of $\bar{q}_t$, we get

$$g_t = \frac{\frac{1}{\mathcal{J}} \sum_{j \in \mathcal{J}} q_{jt} \sum_{j \in \mathcal{J}} s_j}{\sum_{j \in \mathcal{J}} q_{jt}}$$

$$g_t = \frac{1}{\mathcal{J}} \sum_{j \in \mathcal{J}} s_j$$