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YIELDING OF STEEL SHEETS CONTAINING SLITS

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Summary

Yielding at the end of a slit in a sheet is investigated, and a relation is obtained between extent of plastic yielding and external load applied. To verify this relation, panels containing internal and edge slits were loaded in tension and lengths of plastic zones were measured.

1. Introduction

Notches, such as holes and slits, generally reduce the strength of sheet material. If no inelastic deformation is permissible, stresses must nowhere exceed the initial yield stress, though this may be subject to a size effect. If large-scale yielding is to be avoided, the applied load divided by minimum area of section should not reach the plastic flow stress. It is the purpose of the present work to trace the spread of plasticity from a centre of stress concentration as loads are increased.

Relaxation methods can often be applied to a particular problem for determining the field of plastic strain produced by given loads (Allen and Southwell 1950). However, in the present work, a prototype problem is sought which will allow a direct calculation of extent of yielding as a function of external load. When a very thin sheet containing a straight cut is loaded in a direction perpendicular to the cut, it may be expected that yielding will be confined to a very narrow band lying along the line of the cut.

2. Analysis of the Problem

An ideal elastic–plastic material is considered which flows after yielding at a constant uniaxial tensile stress Y. Uniform tensile stress T is applied to the edges of an infinite sheet in a direction perpendicular to an internal cut of total length $2l$. Yielding must occur over some length s measured from the end of the cut, as shown in Fig. 1 (a). It is suggested that the sheet may be considered to deform elastically under the action of the external stress together with a tensile stress Y distributed over part of the surface of a hypothetical cut of length $2a$ as shown in Fig. 1 (b). Since internally applied forces are in static equilibrium, the distribution of stress in the sheet must be independent of elastic constants (Coker and Filon 1957, p. 518).

The problem of a straight cut loaded over part of its edge has been examined by Muskhelishvili (1953, p. 340). His stress functions were found to assume a

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simple form when account was taken of the symmetry of the present configuration. It is convenient to introduce variables α and β defined by $x = a \cosh \alpha$, $l = a \cos \beta$ (see Fig. 1). The stress σ_y acting at points on $y = 0$ was determined in the form of a series in ascending powers of α having a leading term $\sigma_y = -2Y\beta/\pi\alpha$. The

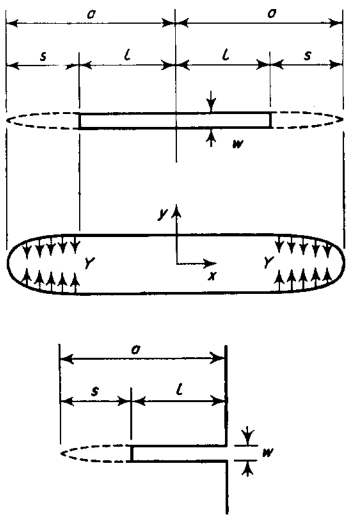

FIG. 1. Geometry of slit: (a) internal slit, (b) internal stresses acting on slit, (c) edge slit.

analogous expression for stress due to the external loading is $\sigma_y = T/\alpha$. When these stresses are superimposed, it is necessary that the stress at the point $\alpha = 0$ (i.e. $x = a$) should not be infinite, so the coefficient of $1/\alpha$ must vanish, viz. $T - 2Y\beta/\pi = 0$. This readily leads to the relation

$$\frac{s}{a} = 2 \sin^2 \left(\frac{\pi}{4} \frac{T}{Y}\right). \tag{1}$$

When T/Y approaches unity, this relation gives $s/a \simeq 1 - (\pi/2)(1 - T/Y)$. The last expression may be alternatively derived from stress functions given by WESTERGAARD (1939). By Saint-Venant's principle, stresses at $x = a$ should not depend on the exact distribution of stress over the segment $-l < x < l$ if l/a is small. Hence the partial loading required may be simulated by an inward tensile stress Y acting over the whole segment $-a < x < a$ together with central concentrated forces of magnitude $2lY$ acting outwards.

When T/Y is very small, equation (1) gives $s/l = 1{\cdot}23\,(T/Y)^2$. This relation gives the scale of plastic deformation if yielding actually occurs, but it need not imply that yielding must occur for any stress, however small.

3. TEST PANELS

The above analysis refers to an infinite sheet in a state of plane stress having a geometry defined by the length of slit. In practice, a sheet must have a definite thickness and a limited width. It was assumed that the sheet would be effectively of infinite size if the total length of slit never exceeded one fifth of the sheet width. This point has been discussed by FROST and DUGDALE (1958). If the sheet is to yield when the tensile stress at any point reaches the uniaxial yield stress, it must be possible for shearing to occur without constraint on planes at 45° to the plane of the sheet. With two exceptions indicated later, slits were made of width w equal to the sheet thickness (0·050 in.).

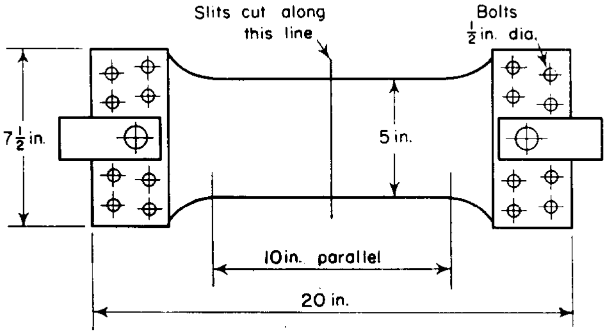

FIG. 2. Test panel.

Load was transmitted to the test panels through bolted joints as shown in Fig. 2. Measurements with extensometers on a panel without slits showed that stresses over the central section did not vary from the average value by more than 2 per cent. Steel produced by the Air Blown Bessemer process was selected, as this steel responds to strain-etching. The chemical analysis was: carbon 0·05 per cent, manganese 0·40 per cent, nitrogen 0·013 per cent. When cutting operations were completed, the panels were annealed at 900 °C, precautions being taken to avoid scaling.

TABLE 1. *Experimental values*

Internal slits			*Edge slits*		
Half-length of slit l (in.)	*Applied stress T* (tons/in^2)	*Plastic zone length s* (in.)	*Length of slit l* (in.)	*Applied stress T* (tons/in^2)	*Plastic zone length s* (in.)
0·50	3·3	0·042	0·50	3·7	0·057
0·40	5·0	0·088	0·50	4·6	0·093
0·25	6·2	0·087	0·40	5·7	0·116
0·25	6·9	0·122	0·40	7·0	0·197
0·25	8·0	0·185	0·25	8·3	0·204
0·25	10·0	0·470	0·25	9·5	0·350
0·10*	11·2	0·394	0·15*	10·8	0·448

*Width of slit w = 0·10 in.

4. EXPERIMENTAL RESULTS

Tensile load was applied to each panel for 2 min. The stress T was calculated from the total width of the test section (5 in.) taking no account of the presence of slits (FROST and DUGDALE 1958). After being loaded, panels were aged at 250 °C for a few hours and were dissected for examination. Portions including the slit and plastically deformed zone were polished on both sides and etched with Fry's reagent (JEVONS 1925). The plastic zone tapered to a fine point, and its length was often uncertain to the extent of 0·003 in. It was measured with a steel rule and magnifying glass. Average values of the four measurements for each panel are shown in Table 1.

A strip was cut from each panel for a tensile test. When results for all panels were averaged, lower yield stresses of 12·0, 12·5 and 13·0 tons/in^2 were obtained for strain rates of 10^{-6}, 10^{-5} and 10^{-4} per second. For plotting Fig. 3, a value of 12·5 tons/in^2 was selected. Strain hardening was found to begin at a strain of 1·5 per cent.

5. Discussion of Results

In Fig. 3 experimental values are shown in relation to the curve calculated from equation (1). It should be noted that an edge slit was considered to have a length equivalent to half the total length of an internal slit, as shown in Fig. 1. It is thought that the experimental points fall sufficiently near the calculated curve to

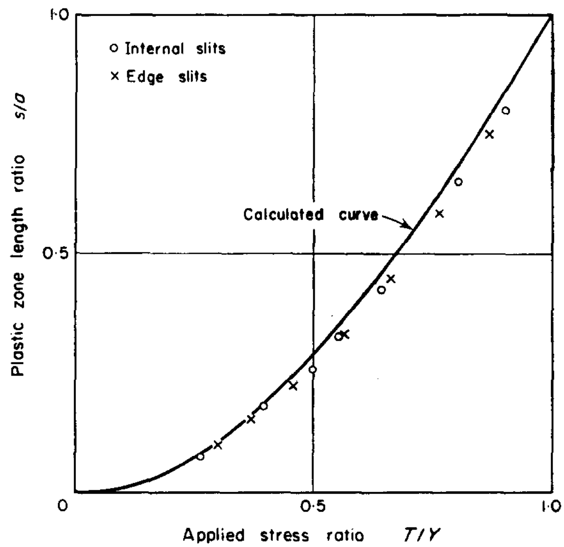

FIG. 3. Test results.

show that the analysis for internal slits is correct and that it applies also to edge slits. It can be seen that all experimental points fall slightly to the right of the curve. However, they may be brought into agreement with the curve by an adjustment of only about 3 per cent in the yield stress Y. As the yield stress corresponds to an arbitrarily chosen strain rate, it may be unwise to attach any significance to this discrepancy.

Acknowledgment

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