PAPER • OPEN ACCESS

An Improved Grey-Wolf Optimization Algorithm Based on Circle Map

To cite this article: Yongliang Wang *et al* 2020 *J. Phys.: Conf. Ser.* **1682** 012020

View the article online for updates and enhancements.

An Improved Grey-Wolf Optimization Algorithm Based on Circle Map

Yongliang Wang[1,2,3], Ting Wang[1,2*], Shuxian Dong[1,2,3] and Chen Yao[1,2]

[1] State Key Laboratory of Robotics, Shenyang Institute of Automation, Chinese Academy of Sciences, Shenyang 110016, China

[2] Institutes for Robotics and Intelligent Manufacturing, Chinese Academy of Sciences, Shenyang 110169, China

[3] University of Chinese Academy of Sciences, Beijing 100049, China

[*]Corresponding author's e-mail: wangting@sia.cn

Abstract. To overcome the shortcomings of grey wolf optimization algorithm (GWO) such as being easy to fall into the local optimum, and the slow convergence rate in the later stage, an adaptive weighted grey-wolf optimization algorithm based on the Circle map is proposed. Firstly, in this algorithm, Circle chaotic map, which enhances the diversity of the initialization population, is introduced into the initialization of population, therefore, the search space can be searched more thoroughly; Secondly, trigonometric function and the beta distribution are introduced in the convergence factor 'a' and the population position update formula, which improve the convergence speed in the later period of the algorithm; Finally, the simulation experiments on the four common test functions on CEC2017 show that under the same experimental conditions, the improved grey wolf optimization algorithm improves the solution accuracy and convergence speed significantly, and its performance is obviously better than other smart optimization algorithms and other improved GWO algorithms.

1. Introduction

Grey wolf optimization (GWO) algorithm is a population-based metaheuristic algorithm[1], which is written by Mirjalili et al in 2014. This algorithm is inspired by the behavior of grey wolves during round-up and hunting. Compared with the experimental [2] of function optimization test, GWO can achieve better results than Genetic Algorithms (GA), particle swarm optimization (PSO) and differential evolution (DE) partial functions. Currently, GWO algorithm has been successfully applied to resource allocation scheduling system [3-4], flow prediction [5], perceptron training [6], displacement prediction [7-8], oil utilization improvement [9], PID controller optimization [10-11], feature selection [12], leather segmentation [13] and other fields.

However, the basic GWO algorithm has the disadvantages of easy to fall into local optimization and slow convergence in the later stage of search. Aiming at the shortcomings of the GWO, many improved methods are put forward, and the test experiments are carried out on the corresponding function set. [14] literature is inspired by particle swarm optimization algorithm to propose a stochastic dynamic adjustment strategy for control parameters. however, the improved algorithm also has some limitations. poor performance on Rosenbrock functions. a new position update formula is proposed in the literature [15], which makes the algorithm have the ability to jump out of the local optimum. although the improved algorithm has obvious improvement in search accuracy, stability and

convergence speed, the convergence rate is still slow in the later stage. Literature [16] introduces three operators of genetic algorithms into GWO, A genetic-grey wolf hybrid algorithm is proposed. Global convergence has been improved, The mutation operation for elite individuals effectively prevents the algorithm from falling into local optimal values, The experiment results show, The algorithm is effective in solving some functions, But as the uncertainty of mutation operations, It has the opposite effect on other functions. A grey wolf optimization algorithm based on anti-chaos sequence is proposed in the literature [17], By introducing Logistic chaotic map search method to improve this algorithm. However, There is no clear explanation in judging when to use chaos search. Literature [18] integrates the optimal position information of the individual in the PSO into the position update formula, And the tent chaos map is introduced for population initialization, But in the middle and high dimensions, The optimization ability of the algorithm is weakened. So far, Both the improved GWO algorithms are difficult to achieve the optimal both in improving the convergence speed and avoiding falling into the local optimum.

In basic GWO algorithm, the convergence factor is linearly reduced from 2 to 0, but in the actual optimization problem, as the complexity of the search process, the convergence factor of linear variation leads to the weak search ability of the algorithm. In addition, the first three levels of wolf weight in the position update equation are equal. Otherwise, in nature area, the higher rank of wolf plays a more important role in the hunting process of grey wolf. In view of the above shortcomings, a nonlinear variation mode of convergence factor and the position updating equation of adaptive weight are designed in this paper. At the same time, the beta distribution is introduced into the search ability of the coordinated algorithm in the position update equation. During population initialization, in order to ensure the convergence speed of the algorithm, Circle chaotic map is introduced to enhance the global convergence speed of the algorithm. The simulation results show that the performance of the algorithm is significantly improved.

2. Basic grey wolf optimization algorithm

In the basic GWO algorithm, We set the population size of the wolf is N and the search space is d dimension, the position of the ith wolf in the space is the global optimal solution.

According to [1], the position of the grey wolf surrounding the prey is updated as follows:

$$\mathbf{D} = \left| \mathbf{C} \cdot \mathbf{X}_p(t) - \mathbf{X}(t) \right|, \tag{1}$$

$$\mathbf{X}(t+1) = \mathbf{X}_p(t) - \mathbf{A} \cdot \mathbf{D}, \tag{2}$$

where the t is the current number of iterations, $\mathbf{X}_p = (x_1, x_2, \cdots, x_d)$ is the prey position, $\mathbf{A} \cdot \mathbf{D}$ is the bounding step, and the vector A and C are defined as

$$\mathbf{A} = 2(\mathbf{r}_1 - \mathbf{E}) \cdot \mathbf{a}, \tag{3}$$

$$\mathbf{C} = 2\mathbf{r}_2, \tag{4}$$

where r_1 and r_2 are the 1 row d column random vectors between interval [0,1], and each element is 1 row d column vector of 1 in the E. a is the convergence factor vector, decreasing linearly from 2 to 0 as the number of iterations increases, the equal of a is as follows:

$$\mathbf{a} = 2(1 - t / t_{\max}) \cdot \mathbf{E}^T, \tag{5}$$

According to the (1)-(5) formula, the predation position of other grey wolf individuals guided by the first three levels of wolves in the predation process is updated as (8)

$$\mathbf{D}_\alpha = \left| \mathbf{C}_1 \cdot \mathbf{X}_\alpha - \mathbf{X} \right|, \mathbf{D}_\beta = \left| \mathbf{C}_2 \cdot \mathbf{X}_\beta - \mathbf{X} \right|,$$
$$\mathbf{D}_\delta = \left| \mathbf{C}_3 \cdot \mathbf{X}_\delta - \mathbf{X} \right|, \tag{6}$$

$$\mathbf{X}_1 = \mathbf{X}_\alpha - \mathbf{A}_1 \cdot \mathbf{D}_\alpha, \mathbf{X}_2 = \mathbf{X}_\beta - \mathbf{A}_2 \cdot \mathbf{D}_\beta,$$
$$\mathbf{X}_3 = \mathbf{X}_\delta - \mathbf{A}_3 \cdot \mathbf{D}_\delta, \tag{7}$$

$$\mathbf{X}(t+1) = \frac{\mathbf{X}_1 + \mathbf{X}_2 + \mathbf{X}_3}{3}. \tag{8}$$

3. Improved grey wolf optimization algorithm

This section is an introduction of an improved grey wolf optimization algorithm based on Circle map. we modify three parts of this algorithm, the details are as follows.

3.1. Population initialization based on the Circle map

Chaos theory is widely used in group intelligence algorithms to enhance the diversity of initialized groups to improve the optimization performance of their algorithms because of its randomness and non-repetition. Compared with random search, chaos theory can exploit the search space thoroughly. To sum up, in order to make the initial population individuals use the information of solution space as much as possible, this paper introduces the Circle map in chaos theory into the population initialization of the improved GWO algorithm. What's more, in this chaotic map, only when $a = 0.5, b = 2.2$, this map is into chaos. The mathematical model of Circle map is [19]:

$$x_{k+1} = x_k + a - \mathrm{mod}(\frac{b}{2\pi} * \sin(2\pi x_k), 1) \tag{9}$$

$$\mathbf{x}_{i,j} = \mathbf{x}_{\min,j} + \mathbf{x}_{k,j} \cdot (\mathbf{x}_{\max,j} - \mathbf{x}_{\min,j}), \tag{10}$$

Concrete steps to generate the initial population using Circle chaotic map are shown in algorithm 1.

Algorithm 1:

Step1： Initialize the population initial value $x(i,j)$ randomly and Set the population scale N , the dimension d and the maximum chaotic iterative step number k ;

Step2： Iterate by formula (9) and j , k increased by 1 to produce $x_{k,j}$ sequence. With formula (10), the $x_{i,j}$ sequence is generated by i self-increased 1. Finally, the $x_{i,j}$ sequence is initialized population matrix;

Step3： If the iteration reaches the maximum number of times, jump to the step4, or return to the step2;

Step4： Terminate the run and save the sequence.

3.2. Adaptive adjustment strategies

The convergence factor in the GWO algorithm affects the iteration of the whole algorithm and finally solve the global optimal solution, and consider that the first three levels of wolves have different abilities in the process of finding prey. In the basic grey wolf optimization algorithm, the optimal solution, the suboptimal solution and the third optimal solution are regarded as equally important. Therefore, beta distribution [20] and trigonometric functions are introduced to disturb the a of convergence factor and weight parameters ω . The nonlinear variation of the above parameters affects the convergence speed and search ability of the algorithm.

Beta distribution is a density function that satisfies binomial distribution and Bernoulli distribution, is distributed between [0,1]. Beta function formula and probability distribution function are:

$$B(b_1, b_2) = \int_0^1 t^{b_1-1}(1-t)^{b_2-1} dt, b_1 > 0, b_2 > 0, \tag{11}$$

$$f(x) = \frac{x^{b_1-1}(1-x)^{b_2-1}}{B(b_1, b_2)}, 0 < x < 1, \tag{12}$$

As a result, this paper proposes an adjustment strategy for convergence factor a

$$a = 2 - 2\tanh(\frac{1}{\xi} \cdot (\frac{t}{M_{iter}}) \cdot \pi) + 0.2B(b_1, b_2) \tag{13}$$

where $\xi = 2\text{-}2\tanh(\frac{t}{M_{iter}} \cdot \pi)$ .A weighted position updating method is proposed to increase the convergence rate of the GWO algorithm, as follows:

$$X(t+1) = \omega_1 X_1 + \omega_2 X_2 + \omega_3 X_3, \tag{14}$$

$$\omega_1 = \omega_{1\min} + (\omega_{1\max} - \omega_{1\min})\cos(2\pi \frac{t}{M_{iter}}) \tag{15}$$
$$+ \sigma B(b_1, b_2),$$

$$\omega_{1\min} = 0.6, \omega_{1\max} = 0.8, \tag{16}$$

$$\omega_3 = \omega_{3\min} + (\omega_{3\max} - \omega_{3\min})\sin(2\pi \frac{t}{M_{iter}}), \tag{17}$$
$$- \sigma B(b_1, b_2)$$

$$\omega_{3\min} = 0.08, \omega_{3\max} = 0.1, \tag{18}$$

$$\omega_2 = 1 - \omega_1 - \omega_3. \tag{19}$$

σ is the adjustment factor of inertial weight, which controls the offset degree of inertial weight $\omega_{1,2,3}$ after fusion with beta distribution, which improves the convergence speed and global search ability of the algorithm. In this paper, $\sigma = 0.1$.

3.3. CGWO algorithm description

The CGWO of the improved algorithm (An improved grey wolf optimization algorithm based on Circle map) is implemented as follows:

Algorithm 2:

Step1：Initialization population size is N, maximum chaotic iteration step k, correlation coefficient value μ of Circle map, dimension d, maximum iteration number M_{iter}, initial position X of grey wolf population, initialization inertia weight adjustment coefficient σ, inertial weight and other parameters ω;

Step2：Use algorithm 1 to initialize the $\{x_i^d, i=1^d, 2^d, \cdots, N^d\}$ of grey wolf population;

Step3: Calculate the fitness value $\{f(x_i^d), i=1^d, 2^d, \cdots, N^d\}$ of each individual in the group, and record the first three optimal individual α, β, δ, and their corresponding positions are x_α, x_β, x_δ respectively;

Step4：i, j increase by 1. Calculate the distance control parameter a according to formula (13), According to formula (3), (4) Calculating parameter A, C value, According to formula (6), (7), (14) Updating individual locations;

Step5：Recalculate the fitness value $\{f(x_i), i=1,2,\cdots,N\}$ of the individual in the group according to the updated individual position, and update the corresponding position x_α, x_β, x_δ, iteration number of the first three optimal individuals by 1;

Step6：Determine whether the termination condition is satisfied, if satisfied, return the global optimal fitness value, otherwise return the step4 for cycle iteration.

4. Experimental analysis

In this section, we do some experiments to proof the priority of our algorithm.

4.1. Experimental Test Function

To verify the applicability and efficiency of this algorithm, Choose the test functions commonly used in the CEC2017, with the basic GWO algorithm [1], PSO algorithm, EGWO algorithm [14], EEGWO algorithm[2], IGWO algorithm[15]. Among them, EGWO the population initialization method, convergence factor and position update formula are all changed, EEGWO changed the position update formula and convergence factor according to PSO inspiration, IGWO only change the position update formula. In the test function, f1、f2 as a unimodal function, f3 is a multi - peak function, f4, f5 a

hybrid function, f6 is a compound function, The domain of all functions is [-100, 100]. Function definitions are shown in Table 1:

Table1. CEC2017 Test Functions

Num	The name of function	Scope	Solution
f1	Bent Cigar Function	[-100,100]	100
f2	Sum of Different Power Function	[-100,100]	200
f3	Rosenbrock's Function	[-100,100]	400
f4	Hybrid Function 2	[-100,100]	1200
f5	Hybrid Function 9	[-100,100]	1900
f6	Composition Function 8	[-100,100]	2800

4.2. Parameter settings

To ensure the fairness of the comparison results, the parameters of all algorithms are as follows: the population size N is 90. In order to increase the difficulty of optimization, the dimension d is set to 100. Maximum number of iterations M_{iter} is set to 5000. According to the literature [20], the parameters of beta distribution in CGWO algorithm are set to $b_1 = 1, b_2 = 2$.

4.3. Experimental results and analysis

In order to avoid the influence of randomness and contingency on the experimental results, In this experiment, comparison algorithms in middle high - dimensional 100, In order to verify the ability of CGWO algorithm in dealing with various complex problems. Especially in the case of 100 dimensions, the results are helpful to judge the stability and robustness of the algorithm. Count the fitness mean and variance of each function, The experimental results are shown in Table 2 and Figure 1. Table 2 shows the CGWO algorithm with other basic intelligent optimization algorithms and the improved grey wolf optimization algorithm in dimension 100, In the table, The bold part represents the optimal solution on the same function. Figure 1 is the convergence curve of each algorithm optimization process, The convergence of the algorithm in each function is represented. The tests were performed on Intel Core i7-4710MQ、8G memory ,2.50 GHz computers, programming is implemented on the Matlab R2018b.

4.3.1. Accuracy of solution

From the experimental data in Table 2, when the dimension is 100, the experimental data in Table 2 shows that the solution accuracy of the improved algorithm is smaller than that of other algorithms, which shows that the CGWO algorithm has better stability. CGWO algorithm is stable and robust when other algorithms are disturbed by dimension disaster.

Table2. A comparison of the results of CGWO algorithm and other intelligent optimization algorithms in 100D for 6 functions

Func	PSO		GWO		EGWO		EEGWO		IGWO		CGWO	
	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
f1	1.20E+11	5.91E+20	3.24E+10	7.53E+19	4.77E+10	4.31E+20	1.39E+11	4.74E+20	1.48E+10	3.59E+19	6.36E+09	6.22E+18
f2	4.44E+148	4.15E+298	5.14E+131	7.93E+264	3.30E+140	1.66E+282	9.11E+149	1.60E+301	1.78E+119	9.46E+239	3.52E+114	3.46E+230
f3	2.59E+04	7.92E+07	3.10E+03	1.87E+06	8.64E+03	2.04E+07	2.08E+04	4.73E+07	1.86E+03	1.39E+05	1.45E+03	6.28E+04
f4	5.37E+10	3.45E+20	5.45E+09	7.91E+18	1.52E+10	1.62E+20	6.52E+10	6.52E+20	2.27E+09	2.45E+18	1.08E+09	6.11E+17
f5	1.23E+09	1.16E+18	6.15E+07	5.69E+15	2.78E+08	1.70E+17	3.97E+09	1.68E+19	6.40E+07	5.88E+15	1.20E+07	1.04E+14
f6	1.74E+04	8.55E+06	6.82E+03	7.20E+05	9.36E+03	7.43E+06	1.36E+04	3.18E+07	5.32E+03	5.45E+05	4.41E+03	6.73E+04

4.3.2. Convergence process

Figure 1(a)~(f) shows the convergence process of six different intelligent optimization algorithms in the case of 100 dimensions. Compared with other intelligent optimization algorithms, it can be seen that the convergencespeed and late convergence effects of CGWO algorithm are more prominent in high dimensional cases. Therefore, in the solution of higher dimensional functions such as 100 dimension, the convergence speed and the ability of CGWO algorithm to jump out of the local minimum are more obvious, and the slow convergence speed in the later stage is obviously improved. CGWO convergence effect of the algorithm is better and the convergence and late convergence speed are more advantageous with the increase of dimension
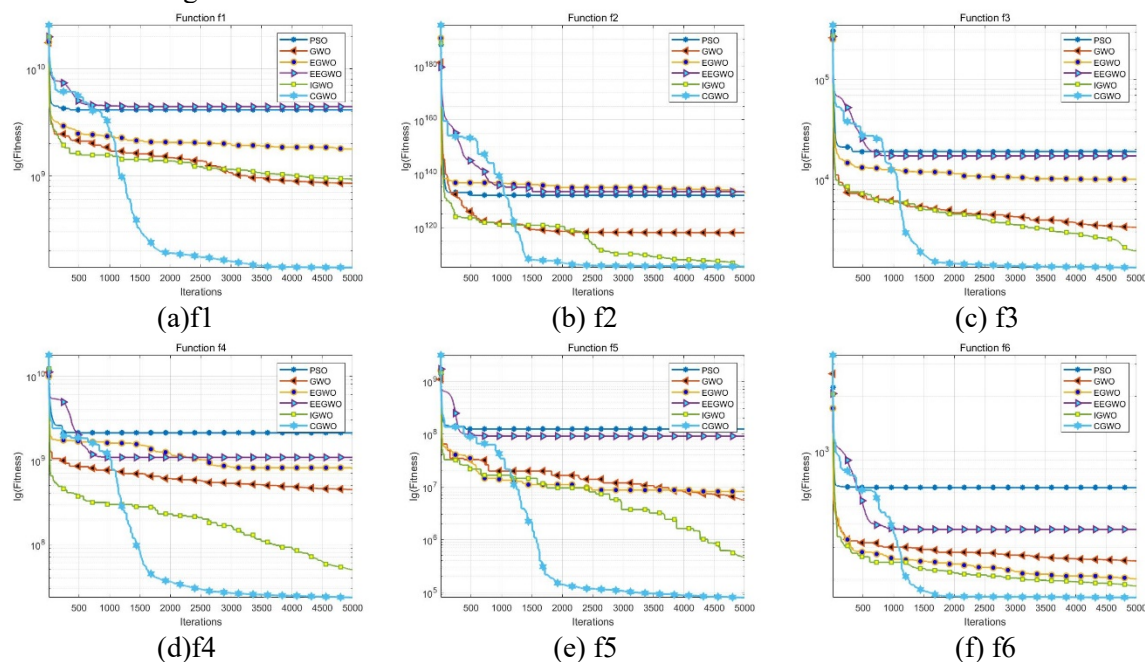

(a)f1	(b) f2	(c) f3
(d)f4	(e) f5	(f) f6

Figure 1. Comparison of Convergence Performance of 6 Intelligent Optimization Algorithms for 6 Functions in 100 D

5. Conclusion

This paper presents a grey wolf optimization algorithm based on Circle map and adaptive weight for the problems of basic GWO that are easy to fall into local optimization, prone to converge early and low precision. Circle map in chaos is introduced in population initialization stage. Compared with random search, the introduction of Circle map can make the algorithm search the search space completely and thoroughly. By modifying the position update equation and convergence factor, the convergence speed of the algorithm is improved, and the search and development ability of the algorithm is balanced. simulation experiments show that the improved CGWO algorithm in this paper has higher accuracy and faster convergence speed in the optimization results of four kinds of standard functions: single peak function, multi-peak function, compound function and mixed function. compared with other basic types of intelligent optimization algorithms and improved GWO algorithms, it has better convergence performance and higher solution accuracy and stability. moreover, the algorithm has better stability and robustness in solving high-dimensional functions, and the solution ability is better than other algorithms. In the next step, the performance of the algorithm will be further improved and applied to practical optimization problems.

References

[1] Mirjalili S, Lewis A. Grey Wolf Optimizer[J]. Advances in Engineering Software, 2014, 69: 46-61.

[2] Long W, Jiao J, Liang X. An exploration-enhanced grey wolf optimizer to solve high-dimensional numerical optimization[J]. Engineering Applications of Artificial Intelligence, 2018, 68: 63-80.

[3] Tikhamarine Y, Souag-Gamane D. A new intelligent method for monthly streamflow prediction: hybrid wavelet support vector regression based on grey wolf optimizer[J]. Arabian Journal of Geosciences, 2019, 12(17).

[4] Yang Y, Yang B, Wang S, et al. An Improved Grey Wolf Optimizer Algorithm for Energy-Aware Service Composition in Cloud Manufacturing[J]. The International Journal of Advanced Manufacturing Technology. 2019, 105(7-8): 3079-91.

[5] Ahmad Z, Amir S, Mahdi J. Identification of influential users in social network using grey wolf optimization algorithm[J]. Expert Systems With Applications, 2020, 142.

[6] Yan Fu, Xu Jianzhong, Li Fengshu. Chaotic grey wolf optimization algorithm trains multi-layer perceptors [J]. Journal of Electronics and Information, 2019, 41(04): 115-122.(in Chinese)

[7] Lian C , Zeng Z , Yao W , et al. Displacement prediction model of landslide based on a modified ensemble empirical mode decomposition and extreme learning machine[J]. Natural Hazards, 2013, 66(2):759-771.

[8] Yu Z , Shi X , Zhou J , et al. Prediction of Blast-Induced Rock Movement During Bench Blasting: Use of Grey Wolf Optimizer and Support Vector Regression[J]. Natural Resources Research, 2019(9).

[9] Zhao B, Ren Y, Gao D. Energy utilization efficiency evaluation model of refining unit Based on Contourlet neural network optimized by improved grey optimization algorithm[J]. Energy, 2019, 185: 1032-1044.

[10] Yang Bo, Shu HC, Zhu DN. Optimal Passive Fractional pid Control of PV Inverter Based on Group Wolf Optimization [J]. Control and Decision-Making, 2019, 11(15): 11-12. (in Chinese)

[11] Hatta N M , Zain A M , Sallehuddin R , et al. Recent studies on optimisation method of Grey Wolf Optimiser (GWO): a review (2014–2017)[J]. Artificial Intelligence Review, 2018.

[12] Abdel-Basset M , El-Shahat D , El-Henawy I , et al. A new fusion of grey wolf optimizer algorithm with a two-phase mutation for feature selection[J]. Expert Systems with Applications, 2019, 139:112824.

[13] Senel F A, Gekne F, Yyuksel A S. A novel hybrid PSO-GWO algorithm for optimization problems[J]. Engineering with Computers, 2018, 35(4): 1359-1373.

[14] Long Wen, Cai Shaohong, Jiao Jianjun. An improved grey wolf optimization algorithm [J]. Journal of Electronic Science, 2019, 47(01): 171-177. (in Chinese)

[15] Xing Yin, Chen Chuang, Liu Lilong. An improved grey wolf algorithm for solving the optimal solution of functions [J]. Computer Simulation, 2018, 35(09): 264-268. (in Chinese)

[16] Gu Qinghua, Li Xuexian, Lu Caiwu, et al. Genetic grey wolf hybrid algorithm for solving high dimensional complex functions[J]. Control and Decision-Making, 2019, 11(23): 1-8. (in Chinese)

[17] Gupta S, Deep K. An opposition-based chaotic Grey Wolf Optimizer for global optimisation tasks[J]. Journal of Experimental & Theoretical Artificial Intelligence, 2018, 31(5): 751-779.

[18] Teng Z J, Lv J L, Guo L W. An improved hybrid grey wolf optimization algorithm[J]. Soft Computing, 2018, 23(15): 6617-6631.

[19] Tavazoei M S, Haeri M. Comparison of different one-dimensional maps as chaotic search pattern in chaos optimization algorithms[J]. Applied Mathematics and Computation, 2007, 187(2): 1076-1085.

[20] Yuan X, Chen C, Jiang M, Yuan Y. Prediction interval of wind power using parameter optimized Beta distribution based LSTM model[J]. Applied Soft Computing, 2019, 82.