Characteristics of Stationary Time Series

1. The fluctuations of the time series occur around a constant and long-term mean.
2. They have a constant variance that does not depend on time.
3. The autocorrelation function p_k decreases rapidly as k increases.
4. A shock has a temporary effect.

Dickey-Fuller Test (Monte-Carlo Simulation)

Assume the time series model $Y_t = a_1 * Y_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim N(0, \sigma^2)$

- If $|a_1| < 1$, then the process is a stationary first-order autoregressive process (AR(1)).
- If $|a_1| = 1$, then the process is a random walk (non-stationary)

Steps for Monte Carlo Simulation Dickey-Fuller Test:

1. Create data from a random walk timeseries
2. Estimate an autoregressive first order model AR(1)
3. Compute the quantity $Q = \frac{\hat{a}_1 - 1}{s.e}$, where $\hat{a}_1$ is the estimate of a_1 for the AR(1) model.
4. Repeat steps 1,2,3 thousands of times and save the quantity Q.
5. Construct empirical critical values
6. Compare the quantity Q from the actual (observed) data to the simulated critical values.

H_0 : Non-Stationary $(a_1 = 1)$ vs H_1: $Stationary$ $(a_1 < 1)$

$$Q = \frac{\hat{a}_1 - 1}{s.e}$$

If $Q \leq c$, where c is the critical value at a given significance level (from monte carlo simulation) then reject the null hypothesis, else fail to reject the null hypothesis.