Coastal Vulnerability Formalization

Core Definition

"Coastal vulnerability refers to the expected amount of damage that a coastal ecosystem will face as the outcome from its exposure and lack of resilience to coastal hazards"

Formal Mathematical Structure

$$CV(ce) = \mathbb{E}_{h \sim P(h|e_{ch})}[D(ce, h, lor_{ch}(ce))]$$

Where:

- CV(ce) = Coastal vulnerability of coastal ecosystem ce
- $\mathbb{E}_{h \sim P(h|e_{ch})}[\cdot]$ = Expected value over hazard scenarios h distributed according to exposure characteristics
- D(·) = Damage function mapping (ecosystem, hazard realization, resilience) → damage
- ce = Coastal ecosystem state vector
- h = Specific hazard realization (random variable)
- $P(h|e_{ch})$ = Probability distribution of hazard scenarios conditional on exposure characteristics
- $lor_{ch}(ce)$ = Lack of resilience function dependent on ecosystem state

Term-by-Term Formalization

0. Coastal Ecosystem (ce)

- **Definition**: The specific coastal ecosystem under assessment, characterized by its geographical, biological, physical, and functional attributes.
- **Mathematical Representation**: A tuple or set of features.
$$ce = (G, B, P, F)$$
Where:
- G: Geographic attributes (e.g., location coordinates, spatial extent/area A_{ce}, boundaries, connectivity to other ecosystems).
- B: Biotic attributes (e.g., species composition, biodiversity metrics, population densities, habitat types like mangrove, coral reef, saltmarsh, seagrass bed).
- P: Physical/Abiotic attributes (e.g., geomorphology, sediment type, bathymetry, water quality parameters, structural complexity).
- F: Functional attributes (e.g., ecosystem services provided like coastal protection, carbon sequestration rate, nursery function, recreational value).
- **Domain**:
- $G \in \text{SpatialObjects}$
- $B \in \text{BioticFeatureSpace}$
- $P \in \text{PhysicalFeatureSpace}$
- $F \in \text{FunctionalFeatureSpace}$
- **Notes**:
- The specific attributes chosen for B, P, F will depend on the assessment's scope and the type of damage being quantified.
- ce properties directly inform `lor_ch` (e.g., $A(ce), R(ce), S(ce)$) and Ψ(ce) in the damage function.

1. Coastal Vulnerability

- **Definition**: The primary metric representing the anticipated impact on a coastal ecosystem
- **Mathematical Representation**:
$$\text{CV}(ce) = \mathbb{E}_{h \sim P(h|e_{ch})}[D(ce, h, \text{lor}_{ch}(ce))]$$
- **Domain**: $\mathbb{R}_{>0}$ (non-negative real numbers)
- **Constraints**:
- $\text{CV}(ce) \geq 0$ (non-negativity)
- $\text{CV}(ce) \leq \phi(ce)$ where $\phi(ce)$ is the maximum possible damage (e.g., total ecosystem area, biomass, or functional capacity)

2. **Expected Amount of Damage**

- **Definition**: Statistical expectation of damage over all possible hazard scenarios
- **Mathematical Representation**:

$$\mathbb{E}_{h \sim P(h|e_{ch})}[D] = \int_{\mathcal{H}} p(h|e_{ch}) \cdot D(ce, h, lor_{ch}(ce)) dh$$

  Where:
  - $\mathcal{H}$: Space of all hazard scenarios h
  - $p(h|e_{ch})$: Conditional probability density function of hazard h given exposure characteristics
  - $D(ce, h, lor_{ch}(ce))$: Damage function for specific hazard realization h
  - **Properties**:
  - Probabilistic integration over uncertainty in hazards
  - Future-oriented (models stochastic future events)
  - Dimensionally consistent (units depend on damage metric chosen)
  - **Discrete Event Alternative**:

$$CV(ce) = \sum_{i \in T} \lambda_i \cdot \mathbb{E}[D(ce, h_i, lor_{ch}(ce))]$$

  Where λ_i is the annual frequency of hazard type i and T is the set of discrete hazard types.

3. **Exposure to Coastal Hazards**

- **Definition**: Characterization of hazard probability distributions affecting the ecosystem
- **Mathematical Representation**:

$$e_{ch} = \{(\lambda_i, F_i)\}_{i \in T}$$

  Where:
  - λ_i: Annual frequency of hazard type i (events/year)
  - $F_i(h)$: Cumulative distribution function of intensity for hazard type i
  - T: Set of relevant hazard types (e.g., storms, sea-level rise)
  - **Hazard Realization Sampling**:

$$h \sim P(h|e_{ch}) = \sum_{i \in T} \frac{\lambda_i}{\sum_j \lambda_j} \cdot f_i(h)$$

  Where $f_i(h)$ is the probability density function derived from $F_i(h)$
- **Domain**:
  - $\lambda_i > 0$ (positive real), F_i is a valid CDF
- **Constraints**:
  - $\sum_i \lambda_i < \infty$ (finite total hazard rate)
- **Ecosystem Feedback**: $e_{ch} = F_{EH}(ce)$ where ecosystem state modifies exposure through protective services

4. **Lack of Resilience**

- **Definition**: Inability to absorb/recover from hazards, quantified via *resilience factors*
- **Mathematical Representation**:

$$lor_{ch}(ce) = 1 - (w_1 \cdot A(ce) + w_2 \cdot R(ce) + w_3 \cdot S(ce))$$

  Where:
  - $A(ce) \in [0, 1]$: Adaptive capacity (e.g., species diversity)
  - $R(ce) \in [0, 1]$: Recovery potential (e.g., sediment replenishment rate)
  - $S(ce) \in [0, 1]$: Structural integrity (e.g., mangrove density)
  - w_j: Weights where $\sum w_j = 1$
  - **Domain**: $\text{lor}_{ch} \in [0, 1]$
  (0 = fully resilient, 1 = no resilience)
  - **Critical Constraint**:
  Damage D must be a monotonically non-decreasing function of $lor_{ch}(ce)$ (higher lack of resilience → higher damage)
  - **Weight Determination**: Use multi-criteria decision analysis (MCDA) or expert elicitation with consistency checks
  - **Sensitivity Analysis**: Mandatory sensitivity analysis on all weight parameters (w_j) to assess robustness of lor_{ch} calculations

5. **Damage Function**

- **Definition**: Mechanism translating hazard impact into quantifiable ecosystem damage
- **Mathematical Representation** (One Possible Functional Form):
$$D(ce, h, lor_{ch}(ce)) = f(h) \cdot g(lor_{ch}(ce)) \cdot \Psi(ce)$$

  Where:
  - $f(h)$: Hazard intensity multiplier (e.g., $\text{storm surge height}^2$)
  - $g(lor_{ch}(ce))$: Resilience scaling term (e.g., $e^{k \cdot lor_{ch}(ce)}$ with $k > 0$)
  - $\Psi(ce)$: Ecosystem's maximum damage potential (e.g., total area, biomass, or service value)
  - **Alternative Forms**: Additive models ($D = f(h) + g(lor_{ch}) + \Psi(ce)$) or more complex interaction terms may be appropriate depending on the specific ecosystem-hazard system
  - **Domain**: $\mathbb{R}_{\geq 0}$ (non-negative)
  - **Enforced Constraints**:
  - $\frac{\partial D}{\partial h} > 0$, $\frac{\partial D}{\partial lor_{ch}} > 0$ (damage monotonicity)
  - $D(ce, h, lor_{ch}(ce)) \leq \phi(ce)$ ∀ h (upper physical bound enforced via saturation)
  - $\lim_{h \to \infty} D(ce, h, lor_{ch}(ce)) = \phi(ce)$ (damage approaches but never exceeds maximum)

6. **Spatio-Temporal Dynamics**

- **Definition**: Accounting for changes in coastal vulnerability over a defined time horizon and across spatial extents.
- **Mathematical Representation**:
  - **Time Dependence**: Key variables become functions of time $t \in [0, T_H]$, where T_H is the time horizon.
    - Ecosystem: $ce(t) = (G(t), B(t), P(t), F(t))$
    - Exposure: $e_{ch}(t) = \{\lambda_i(t), I_i(t)\}_{i \in T}$
    - Lack of Resilience: $\text{lor}_{ch}(t) = 1 - (w_1 \cdot A(ce(t)) + w_2 \cdot R(ce(t)) + w_3 \cdot S(ce(t)))$
    - **Per-Event Expected Damage** at time t: $D_{event}(t) = \mathbb{E}_{h \sim P(h|e_{ch}(t))}[D(ce(t), h, lor_{ch}(ce(t)))]$
    - **Annual Expected Damage** at time t:
      $D_{annual}(t) = \sum_{i \in T} \lambda_i(t) \cdot \mathbb{E}_{h \sim F_i(t)}[D(ce(t), h, lor_{ch}(ce(t)))]$
      Where $\lambda_i(t)$ is the annual frequency of hazard type i at time t
  - **Vulnerability over Time Horizon** (Discrete Time Framework):
    - **Average Annual Vulnerability**:
$$CV_{T_H,avg}(ce) = \frac{1}{T_H} \sum_{t=1}^{T_H} D_{annual}(t)$$

      - **Discounted Total Expected Damage**:
$$CV_{T_H,total}(ce) = \sum_{t=1}^{T_H} \frac{1}{(1+r)^t} D_{annual}(t)$$
      Where r is the discount rate for temporal preferences
  - **Spatial Explicitness**: Variables become functions of spatial coordinates $\mathbf{s} = (x, y)$ (or a more general spatial vector) within the ecosystem's spatial domain $\mathcal{S}_{ce}$.
    - $ce(\mathbf{s})$, $e_{ch}(\mathbf{s})$, $\text{lor}_{ch}(\mathbf{s})$
    - Spatially Explicit Vulnerability:
$$CV(ce, \mathbf{s}) = \mathbb{E}[D(ce(\mathbf{s}), e_{ch}(\mathbf{s}), \text{lor}_{ch}(\mathbf{s}))]$$

      This results in a vulnerability map.
      - **Combined Spatio-Temporal Vulnerability**:
      - $ce(\mathbf{s}, t)$, $e_{ch}(\mathbf{s}, t)$, $\text{lor}_{ch}(\mathbf{s}, t)$
      - Instantaneous Spatially Explicit Vulnerability:
$$CV(ce, \mathbf{s}, t) = \mathbb{E}[D(ce(\mathbf{s}, t), e_{ch}(\mathbf{s}, t), \text{lor}_{ch}(\mathbf{s}, t))]$$
- **Domain**:
  - $t \in [0, T_H]$
  - $\mathbf{s} \in \mathcal{S}_{ce} \subset \mathbb{R}^2$ (or $\mathbb{R}^3$)
- **Stability Analysis for Feedback Systems**:
  - System converges if $\|F_{EH}(ce_{t+1}) - F_{EH}(ce_t)\| < \epsilon$ for ecosystem feedback
  - Lyapunov stability condition: $\frac{d}{dt}V(ce(t)) < 0$ where V is a suitable energy function
- **Notes**:
  - Discrete time framework better represents annual assessment cycles
  - Discounting prevents unbounded damage accumulation over infinite horizons
  - Spatially explicit models require gridded or vector data for inputs

7. **Interdependencies and Feedbacks**

- **Definition**: Interactions where ecosystem state influences hazard exposure, or where damage/recovery dynamics are complex, or where ecosystems influence each other's vulnerability.
- **Mathematical Representation**:
  - **Ecosystem-Hazard Feedback**: Exposure e_{ch} becomes a function of the ecosystem state ce.
$$e_{ch} = \mathcal{F}_{EH}(ce)$$

    Where $\mathcal{F}_{EH}$ is a function or model translating ecosystem attributes (e.g., reef height, mangrove density) into modified hazard characteristics (e.g., reduced wave energy).
    The core CV equation becomes: $CV(ce) = \mathbb{E}_{h\sim P(h|\mathcal{F}_{EH}(ce))}[D(ce, h, lor_{ch}(ce))]$.
    - **Damage Accumulation & Recovery Dynamics**: The state of damage D_{state} evolves over time with explicit feedback to ecosystem state.
    - Let $D_{state}(t)$ be the accumulated damage at time t.
    - For a hazard event h_k at time t_k, instantaneous damage $D_{event,k} = D(ce(t_k), h_k, \mathrm{lor}_{ch}(t_k))$.
    - Recovery function $Rec(ce, D_{state}, \Delta t)$ describes damage reduction over Δt.
    - Evolution of damage (discrete time):
$$D_{state}(t + \Delta t) = D_{state}(t) + \sum_{k \text{ s.t. } t_k \in [t, t+\Delta t)} D_{event,k} - Rec(ce(t), D_{state}(t), \Delta t)$$
      - **Critical Feedback**: Ecosystem state and resilience depend on damage state:
        - $ce(t) = \mathcal{F}_{damage}(ce_0, D_{state}(t))$ where ce_0 is the undamaged baseline state
        - $\mathrm{lor}_{ch}(t) = \mathrm{lor}_{ch}(ce(t), D_{state}(t))$ (damaged ecosystems typically less resilient)
      - **Implementation Note**: These feedbacks often require numerical simulation rather than analytical solutions.
  - **Inter-ecosystem Linkages**: Vulnerability of ecosystem ce_j depends on the state of ecosystem ce_i.
    - Let $D_{state,i}$ be the damage state of ce_i.
    - Exposure for ce_j: $e_{ch,j} = \mathcal{F}_{link,E}(ce_i, D_{state,i}, \ldots)$ (e.g., damaged reef ce_i increases wave exposure for seagrass ce_j).
    - Lack of resilience for ce_j: $\mathrm{lor}_{ch,j} = \mathcal{F}_{link,R}(ce_i, D_{state,i}, \ldots)$ (e.g., damaged mangroves ce_i reduce sediment supply for marsh ce_j, affecting its recovery).
    - Then $CV(ce_j|ce_i, D_{state,i}) = \mathbb{E}[D(ce_j, e_{ch,j}(\cdot), \mathrm{lor}_{ch,j}(\cdot))]$.
    - This can lead to a system of coupled CV equations for a network of ecosystems $\mathbf{CE} = \{ce_1, \ldots, ce_N\}$.
- **Notes**:
  - These feedbacks can significantly increase model complexity, often requiring numerical simulation.
  - $\mathcal{F}_{EH}$, Rec, $\mathcal{F}_{link,E}$, $\mathcal{F}_{link,R}$ are complex functions/models specific to the ecosystems and hazards.

8. **Uncertainty Quantification**

- **Definition**: Systematic assessment of uncertainty in $CV(ce)$ arising from uncertainties in input parameters, model structure, and hazard probabilities.
- **Mathematical Representation**:
  - Let $\Theta = \{\theta_1, \theta_2, \ldots, \theta_M\}$ be the set of all uncertain parameters and model choices (e.g., parameters in $p(h)$, weights w_j in lor_{ch}, parameters in $f(h)$ and $g(\mathrm{lor}_{ch})$, choice of functional forms).
  - Each θ_k has an associated probability distribution $P(\theta_k)$ or a set of alternative models.
  - Coastal Vulnerability becomes a random variable conditional on Θ: $CV(ce|\Theta)$.
  - The goal is to find the probability distribution of $CV(ce)$, denoted $P(CV(ce))$.
  - **Methods**:
    - **Monte Carlo Simulation**:
      1. Sample N sets of parameters $\Theta^{(s)}$ from their respective distributions $P(\Theta_k)$.
      2. For each sample $s = 1, \ldots, N$, calculate $CV^{(s)}(ce) = \mathbb{E}[D(ce, e_{ch}(\Theta^{(s)}), \mathrm{lor}_{ch}(\Theta^{(s)}))|\Theta^{(s)}]$.
      3. The ensemble $\{CV^{(s)}(ce)\}$ approximates $P(CV(ce))$.
    - **Sensitivity Analysis**: Evaluate $\frac{\partial CV}{\partial \theta_k}$ to identify key sources of uncertainty.
- **Output**:
  - Not just a point estimate for $CV(ce)$, but also:
    - Mean: $\mathbb{E}_\Theta[CV(ce|\Theta)]$
    - Variance: $Var_\Theta[CV(ce|\Theta)]$
    - Confidence Intervals / Credible Intervals (e.g., 5th-95th percentiles).
    - Probability Density Function (PDF) or Cumulative Distribution Function (CDF) of $CV(ce)$.
- **Notes**:
  - Crucial for decision-making, as it provides a measure of confidence in the CV estimate.

9. Normalization and Aggregation for Indices

- **Definition**: Standardizing $CV(ce)$ values for comparison across different ecosystems or damage units, and combining them into composite indices for broader assessment.
- **Mathematical Representation**:
  - **Normalization**: Transforming $CV(ce)$ to a common scale, typically [0, 1].
    - **Method 1 (Min-Max Scaling)** - Use for relative comparisons:
$$CV_{norm}(ce) = \frac{CV(ce) - CV_{min}}{CV_{max} - CV_{min}}$$
Where CV_{min} and CV_{max} are minimum/maximum values across comparison set.
- **Limitation**: Highly sensitive to outliers; single extreme value compresses all other rankings
- **Recommendation**: Remove outliers or use robust percentile-based scaling (e.g., 5th-95th percentiles)
- **Method 2 (Maximum Damage Scaling)** - Use for absolute vulnerability:
$$CV_{norm}(ce) = \frac{CV(ce)}{\phi(ce)}$$
Where $\phi(ce)$ is the maximum possible damage. Represents proportion of ecosystem at risk.
    - **Method 3 (Z-Score Normalization)** - Alternative for outlier robustness:
$$CV_{norm}(ce) = \frac{CV(ce) - \mu_{CV}}{\sigma_{CV}}$$
Where μ_{CV} and σ_{CV} are mean and standard deviation of CV values.
- **Selection Criteria**: Use Method 1 for ecosystem rankings (with outlier handling), Method 2 for absolute risk assessment, Method 3 when outliers are problematic.
- **Aggregation**: Combining CV values from multiple ecosystems ($ce_k, k = 1, \dots, N_{eco}$) into a single index.
- **Regional Vulnerability Index (RVI)**:
$$RVI = \sum_{k=1}^{N_{eco}} W_k \cdot CV_{norm}(ce_k)$$
Where:
      - W_k: Weight assigned to ecosystem ce_k determined via MCDA or expert elicitation
      - $\sum_{k=1}^{N_{eco}} W_k = 1$ for weighted average interpretation
      - **Weight Determination**: Use Analytic Hierarchy Process (AHP) or similar MCDA methods
      - **Mandatory Sensitivity Analysis**: Assess RVI robustness to weight variations ($W_k \pm \delta$)
- **Domain**:
  - $CV_{norm}(ce) \in [0, 1]$ (enforced by normalization)
  - $RVI \in [0, 1]$ (if W_k sum to 1 and $CV_{norm} \in [0, 1]$)
- **Consistency Checks**:
  - Sensitivity analysis on weight choices
  - Rank correlation analysis between different weighting schemes
- **Notes**:
  - Method selection significantly affects rankings - document rationale
  - Provide uncertainty bounds on aggregated indices

10. Adaptation and Management Interventions

- **Definition**: Evaluating the potential of human actions to modify coastal vulnerability by altering exposure, resilience, or ecosystem characteristics.
- **Mathematical Representation**:
  - Let $\mathcal{M}$ be a set of possible management interventions $M_j \in \mathcal{M}$.
  - An intervention M_j can modify:
    - Ecosystem characteristics: $ce \to ce'(M_j)$
    - Exposure: $e_{ch} \to e'_{ch}(M_j, ce')$ (e.g., building a seawall, restoring a reef)
    - Lack of Resilience: $\text{lor}_{ch} \to \text{lor}'_{ch}(M_j, ce')$ (e.g., habitat restoration improving $A(ce')$, $R(ce')$, $S(ce')$).
  - **Vulnerability with Intervention**:
$$CV(ce, M_j) = \mathbb{E}[D(ce'(M_j), e'_{ch}(M_j, ce'(M_j)), \text{lor}'_{ch}(M_j, ce'(M_j)))]$$

- **Effectiveness of Intervention (Vulnerability Reduction)**:

$$\Delta CV(M_j) = CV_{baseline}(ce) - CV(ce, M_j)$$

Where $CV_{baseline}(ce)$ is the vulnerability without intervention.
    - **Cost-Effectiveness Analysis**:
        - Let $C(M_j)$ be the cost of implementing intervention M_j.
        - Cost-Effectiveness Ratio: $CER(M_j) = \frac{C(M_j)}{\Delta CV(M_j)}$ (cost per unit of vulnerability reduced).
        - Benefit-Cost Ratio (if ΔCV can be monetized): $BCR(M_j) = \frac{\text{Monetized}(\Delta CV(M_j))}{C(M_j)}$.
- **Optimization**:
    - Find $M_j^* \in \mathcal{M}$ that maximizes $\Delta CV(M_j)$ subject to a budget constraint $C(M_j) \leq B_{max}$.
    - Or, find M_j^* that minimizes $CER(M_j)$.
- **Notes**:
    - This formalization allows for a quantitative comparison of different adaptation strategies.
    - Modeling the effect of M_j on $ce, e_{ch}, \text{lor}_{ch}$ is a significant research and modeling challenge itself.
    - The framework can also be used to assess maladaptation, where $CV(ce, M_j) > CV_{baseline}(ce)$.