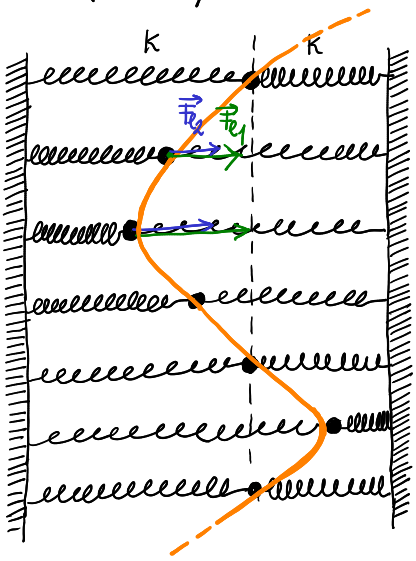
COMPUNEREA OSCILAȚIILOR

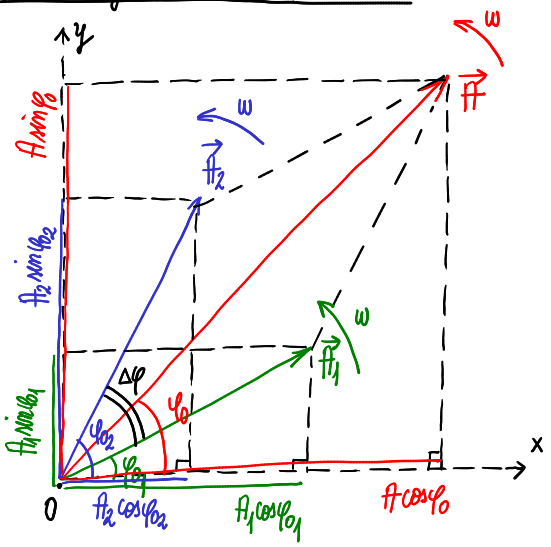
COMPUNEREA OSCILAȚIILOR PARALELE DE FRECVENȚE EGALE ($\omega_1=\omega_2=\omega$)

$$x_1(t)=A_1\sin(\omega t+\varphi_{01})$$
$$x_2(t)=A_2\sin(\omega t+\varphi_{02})$$
$$\oplus \quad x(t)=A\sin(\omega t+\varphi_0)$$

$$\vec{F}_{e_1}=-k\cdot\vec{x}_1$$
$$\vec{F}_{e_2}=-k\cdot\vec{x}_2$$
$$\oplus \quad \vec{F}=\vec{F}_{e_1}+\vec{F}_{e_2}=-k\cdot(\vec{x}_1+\vec{x}_2)=-k\cdot\vec{x}$$
$$\Rightarrow x(t)=x_1(t)+x_2(t)$$



Reprezentarea fazorială a $x_1(t)$, $x_2(t)$, $x(t)$



A $\quad \vec{A}=\vec{A}_1+\vec{A}_2$

$$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\Delta\varphi}\text{, unde } \Delta\varphi=\varphi_{02}-\varphi_{01}$$

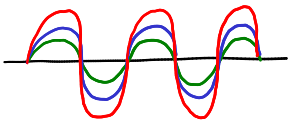
Cazuri particulare:

- **OSCILAȚII ÎN FAZĂ**

$\Delta\varphi=0, 2\pi, 4\pi \dots 2k\pi$

$\cos\Delta\varphi=1$

$$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos(2k\pi)}=A_1+A_2=A_{max}$$

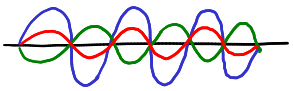


- **OSCILAȚII ÎN OPOZIȚIE DE FAZĂ**

$\Delta\varphi=\pi, 3\pi, 5\pi \dots (2k+1)\pi$

$\cos\Delta\varphi=-1$

$$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos[(2k+1)\pi]}=|A_1-A_2|=A_{min}$$



- **OSCILAȚII ÎN CUADRATURĂ**

$\Delta\varphi=\frac{\pi}{2}$

$\cos\Delta\varphi=0$

$$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\left(\frac{\pi}{2}\right)}=\sqrt{A_1^2+A_2^2}$$

φ_0 $\quad A\cos\varphi_0=A_1\cos\varphi_{01}+A_2\cos\varphi_{02}$

$$A\sin\varphi_0=A_1\sin\varphi_{01}+A_2\sin\varphi_{02}$$

$$\div \quad \operatorname{tg}\varphi_0=\frac{A_1\sin\varphi_{01}+A_2\sin\varphi_{02}}{A_1\cos\varphi_{01}+A_2\cos\varphi_{02}}$$

$$\varphi_0=\operatorname{arctg}\frac{A_1\sin\varphi_{01}+A_2\sin\varphi_{02}}{A_1\cos\varphi_{01}+A_2\cos\varphi_{02}}$$

$$x(t)=A\sin\varphi_0$$

$$\boxed{x(t)=\sqrt{A_1^2+A_2^2+2A_1A_2\cos(\varphi_{02}-\varphi_{01})}\cdot\sin\left(\omega t+\operatorname{arctg}\frac{A_1\sin\varphi_{01}+A_2\sin\varphi_{02}}{A_1\cos\varphi_{01}+A_2\cos\varphi_{02}}\right)}$$

CAZ PARTICULAR: Amplitudini egale

(A) $A_1 = A_2 = A$

$$\mathcal{A} = \sqrt{A^2 + A^2 + 2AA\cos\Delta\varphi}$$

$$\mathcal{A} = A\sqrt{2 + 2\cos\Delta\varphi}$$

$$\mathcal{A} = 2A\sqrt{\frac{2+2\cos\Delta\varphi}{4}}$$

$$\mathcal{A} = 2A\sqrt{\frac{1+\cos\Delta\varphi}{2}}, \text{ dar } \sqrt{\frac{1+\cos\alpha}{2}} = \cos\frac{\alpha}{2}$$

$$\mathcal{A} = 2A\cos\frac{\Delta\varphi}{2}$$

$$\Rightarrow \boxed{x(t) = 2A\cos\left(\frac{\varphi_2-\varphi_1}{2}\right)\sin\left(\omega t + \frac{\varphi_1+\varphi_2}{2}\right)}$$

(φ_0)

$$\varphi_0 = \operatorname{arctg}\frac{A\sin\varphi_{01} + A\sin\varphi_{02}}{A\cos\varphi_{01} + A\cos\varphi_{02}}$$

$$\varphi_0 = \operatorname{arctg}\frac{2\cdot\sin\left(\frac{\varphi_{01}+\varphi_{02}}{2}\right)\cdot\cos\left(\frac{\varphi_{01}-\varphi_{02}}{2}\right)}{2\cdot\cos\left(\frac{\varphi_{01}+\varphi_{02}}{2}\right)\cdot\cos\left(\frac{\varphi_{01}-\varphi_{02}}{2}\right)}$$

$$\varphi_0 = \frac{\varphi_{01}+\varphi_{02}}{2}$$

COMPUNEREA OSCILAȚIILOR PARALELE DE FRECVENȚE APROPRIATE ($\Delta\omega = \omega_2 - \omega_1$, $\Delta\omega \ll \omega_1$, $\Delta\omega \ll \omega_2$)

FENOMENUL BĂTĂILOR

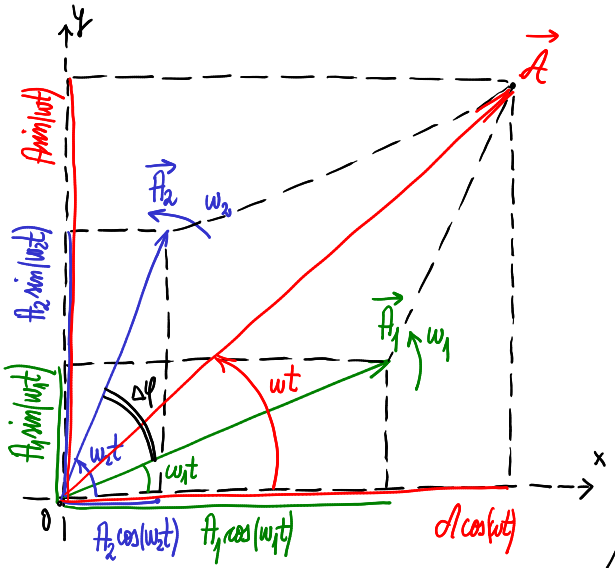
$$x_1(t) = A_1\sin(\omega_1 t)$$
$$x_2(t) = A_2\sin(\omega_2 t)$$
$$\oplus$$
$$x(t) = A\sin(\omega t)$$

(A)

$$\vec{\mathcal{A}} = \vec{A_1} + \vec{A_2}$$

$$\mathcal{A} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\Delta\varphi}, \text{ unde } \Delta\varphi = (\omega_2 - \omega_1)t$$

$$\mathcal{A} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos[(\omega_2-\omega_1)t]}$$



!Obs Fazorul mai rapid va depăși fazorul mai lent! Defazajul $\Delta\varphi$ pariază în timp.

$\Rightarrow$ { La un moment dat fazorii vor fi în fază ($\Delta\varphi = 0 \Rightarrow \mathcal{A}_{max} = A_1 + A_2$)
La alt moment fazorii vor fi în opoziție de fază ($\Delta\varphi = \pi \Rightarrow \mathcal{A}_{min} = A_1 - A_2$).

$\Rightarrow$ Amplitudinea ba va fi maximă, ba va fi minimă!

$\Rightarrow$ FENOMENUL BĂTĂILOR $\mathcal{A} \in [\mathcal{A}_{min}, \mathcal{A}_{max}]$

(φ_0)

$$A\sin\omega t = A_1\sin(\omega_1 t) + A_2\sin(\omega_2 t)$$
$$A\cos\omega t = A_1\cos(\omega_1 t) + A_2\cos(\omega_2 t)$$

$$\operatorname{tg}\omega t = \frac{A_1\sin(\omega_1 t) + A_2\sin(\omega_2 t)}{A_1\cos(\omega_1 t) + A_2\cos(\omega_2 t)}$$

$$\Rightarrow \omega t = \operatorname{arctg}\frac{A_1\sin(\omega_1 t) + A_2\sin(\omega_2 t)}{A_1\cos(\omega_1 t) + A_2\cos(\omega_2 t)}$$

$x(t) = A\sin(\omega t)$

$$\boxed{x(t) = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos[(\omega_2-\omega_1)t]}\cdot\sin\left(\operatorname{arctg}\frac{A_1\sin\omega_1 t + A_2\sin\omega_2 t}{A_1\cos\omega_1 t + A_2\cos\omega_2 t}\right)}$$

CAZ PARTICULAR: Amplitudini egale

(A) $A_1 = A_2 = A$

$$\mathcal{A} = \sqrt{A^2 + A^2 + 2AA\cos\Delta\varphi}$$

$$\mathcal{A} = A\sqrt{2 + 2\cos\Delta\varphi}$$

$$\mathcal{A} = 2A\sqrt{\frac{2+2\cos\Delta\varphi}{4}}$$

$$\mathcal{A} = 2A\sqrt{\frac{1+\cos\Delta\varphi}{2}}, \text{ dar } \sqrt{\frac{1+\cos\alpha}{2}} = \cos\frac{\alpha}{2}$$

$$\mathcal{A} = 2A\cos\frac{\Delta\varphi}{2}$$

$$\mathcal{A} = 2A\cos\left[\frac{(\omega_2-\omega_1)t}{2}\right]$$

$\Rightarrow$

$$x(t) = \underbrace{2A\cos\left[\frac{(\omega_2-\omega_1)t}{2}\right]}_{A(t)}\sin\left[\frac{(\omega_1+\omega_2)t}{2}\right]$$

$A(t)$ – Amplitudine variabilă în timp!
Amplitudine armonică!

(φ$_0$) $\omega t = \operatorname{arctg}\dfrac{A\sin\omega_1 t + A\sin\omega_2 t}{A\cos\omega_1 t + A\cos\omega_2 t}$

$$\omega t = \operatorname{arctg}\frac{2\sin\left(\frac{\omega_1 t+\omega_2 t}{2}\right)\cos\left(\frac{\omega_1 t-\omega_2 t}{2}\right)}{2\cos\left(\frac{\omega_1 t+\omega_2 t}{2}\right)\cos\left(\frac{\omega_1 t-\omega_2 t}{2}\right)}$$

$$\omega t = \frac{(\omega_1+\omega_2)t}{2}$$

$\Rightarrow$ Amplitudinea ba va fi maximă, ba va fi minimă!

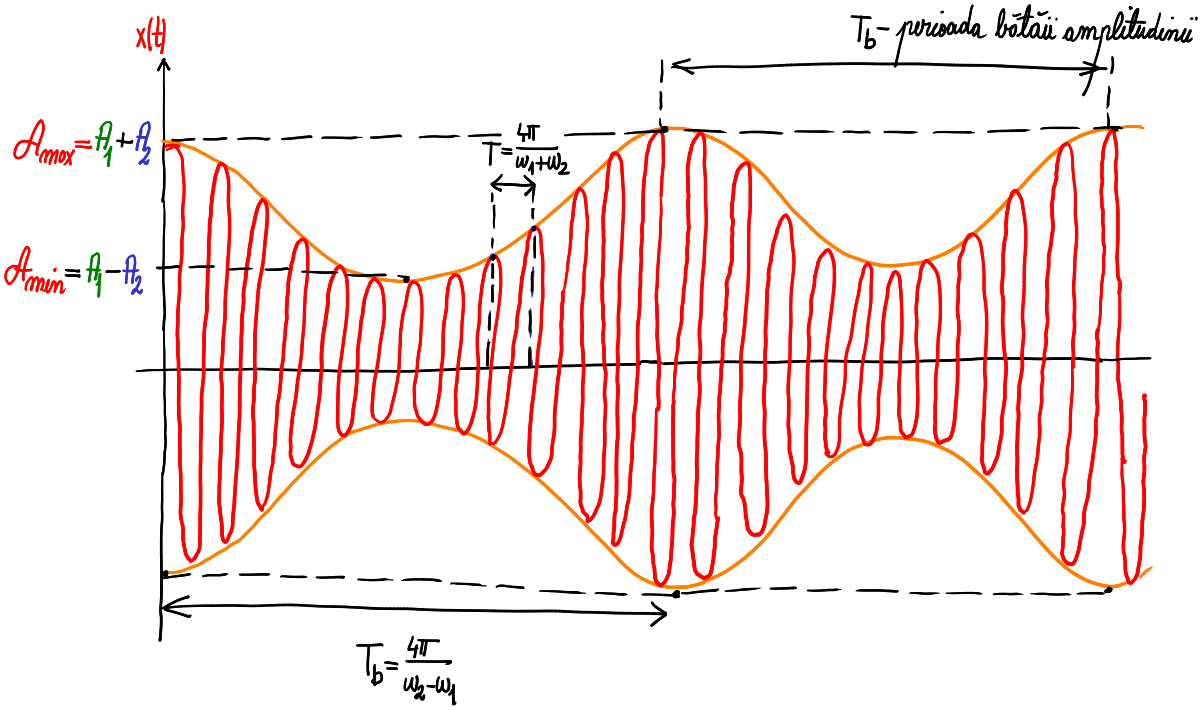
$\Rightarrow$ FENOMENUL BĂTĂILOR $\mathcal{A} \in [\mathcal{A}_{min}, \mathcal{A}_{max}]$

!Obs Amplitudine armonică cu perioada $T_b = \dfrac{2\pi}{\frac{(\omega_2-\omega_1)}{2}}$

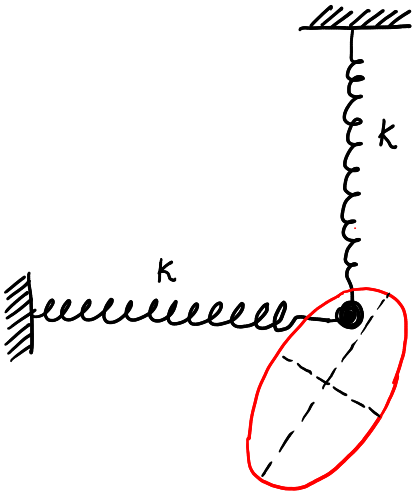
$\Rightarrow$ Perioada bătăii amplitudinii: $\boxed{T_b = \dfrac{4\pi}{\omega_2-\omega_1}}$

!Obs Perioada oscilației $x(t)$ este $T = \dfrac{2\pi}{\frac{(\omega_1+\omega_2)}{2}}$

$\Rightarrow$ Perioada oscilației: $\boxed{T = \dfrac{4\pi}{\omega_1+\omega_2}}$



COMPUNEREA OSCILAȚIILOR PERPENDICULARE DE FRECVENȚE EGALE $(\omega_1=\omega_2=\omega)$



$x = A_x \sin(\omega t+\varphi_{0x})$

$y = A_y \sin(\omega t+\varphi_{0y})$

$y(x)$

$\frac{x}{A_x} = \sin(\omega t+\varphi_{0x}) = \sin(\omega t)\cos\varphi_{0x} + \sin\varphi_{0x}\cos(\omega t) \quad | \cdot \cos\varphi_{0y}$

$\frac{y}{A_y} = \sin(\omega t+\varphi_{0y}) = \sin(\omega t)\cos\varphi_{0y} + \sin\varphi_{0y}\cos(\omega t) \quad | \cdot \cos\varphi_{0x}$

$\ominus \quad \frac{x}{A_x}\cos\varphi_{0y} - \frac{y}{A_y}\cos\varphi_{0x} = \cos(\omega t)\cdot(\sin\varphi_{0x}\cos\varphi_{0y} - \sin\varphi_{0y}\cos\varphi_{0x})$

$= \cos(\omega t)\cdot[-\sin(\varphi_{0y}-\varphi_{0x})]$

$\Downarrow (\,)^2$

$\frac{x}{A_x} = \sin(\omega t+\varphi_{0x}) = \sin(\omega t)\cos\varphi_{0x} + \sin\varphi_{0x}\cos(\omega t) \quad | \cdot \sin\varphi_{0y}$

$\frac{y}{A_y} = \sin(\omega t+\varphi_{0y}) = \sin(\omega t)\cos\varphi_{0y} + \sin\varphi_{0y}\cos(\omega t) \quad | \cdot \sin\varphi_{0x}$

$\ominus \quad \frac{x}{A_x}\sin\varphi_{0y} - \frac{y}{A_y}\sin\varphi_{0x} = \sin(\omega t)\cdot(\sin\varphi_{0y}\cos\varphi_{0x} - \sin\varphi_{0x}\cos\varphi_{0y})$

$= \sin(\omega t)\cdot[\sin(\varphi_{0x}-\varphi_{0y})]$

$\Uparrow (\,)^2$

$\oplus$

$$\begin{cases} \frac{x^2}{A_x^2}\cos^2\varphi_{0y} - 2\frac{x}{A_x}\cdot\frac{y}{A_y}\cos\varphi_{0y}\cos\varphi_{0x} + \frac{y^2}{A_y^2}\cos^2\varphi_{0x} = \cos^2(\omega t)\sin^2(\varphi_{0y}-\varphi_{0x}) \\ \frac{x^2}{A_x^2}\sin^2\varphi_{0y} - 2\frac{x}{A_x}\frac{y}{A_y}\sin\varphi_{0y}\sin\varphi_{0x} + \frac{y^2}{A_y^2}\sin^2\varphi_{0x} = \sin^2(\omega t)\sin^2(\varphi_{0x}-\varphi_{0y}) \end{cases}$$

$$\left(\frac{x}{A_x}\right)^2 + \left(\frac{y}{A_y}\right)^2 - 2\frac{x}{A_x}\cdot\frac{y}{A_y}\cos(\varphi_{0y}-\varphi_{0x}) = \sin^2(\varphi_{0y}-\varphi_{0x})$$

ecuația generalizată a elipsei $y(x)$

$x = A_x \sin(\omega t)$

$y = A_y \sin(\omega t-\varphi)$

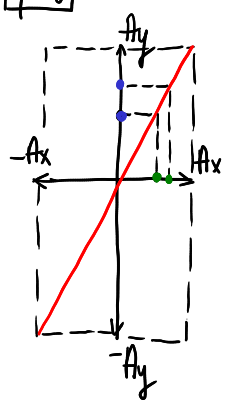
$\Rightarrow \quad \frac{x^2}{A_x^2} + \frac{y^2}{A_y^2} - 2\frac{xy}{A_xA_y}\cos\varphi = \sin^2\varphi$

Punctul material participă simultan la cele două mișcări.

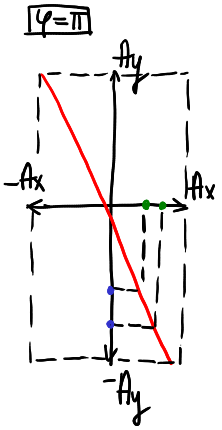
Ecuația parametrică a familiei de elipse care au același centru și care pot fi înscrise în dreptunghiul cu laturile $2A_x$ și $2A_y$.

Forma elipselor depinde de diferența de fază φ.

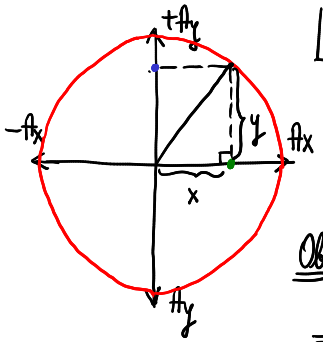
$\varphi = 0$



$\varphi = \pi$



$\varphi = \frac{\pi}{2}$



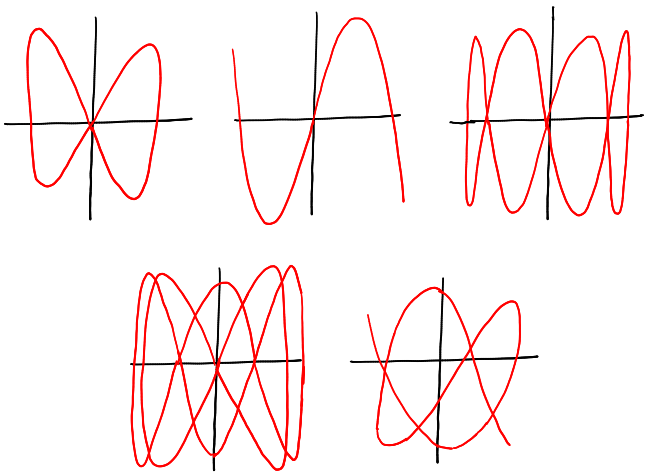
$\frac{x^2}{A_x^2} + \frac{y^2}{A_y^2} = 1$

Obs $A_x = A_y = A$

$\Rightarrow \boxed{x^2+y^2=A^2}$ ecuația cercului de rază A

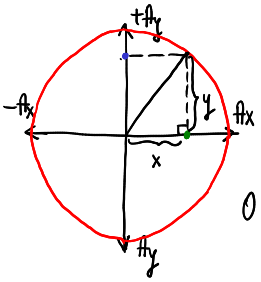
FIGURI LISSAJOUS

$\omega_x \neq \omega_y$

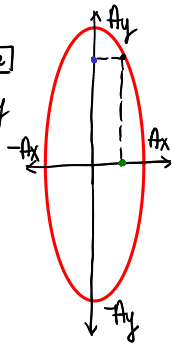


$\omega_x = \omega_y = \omega$

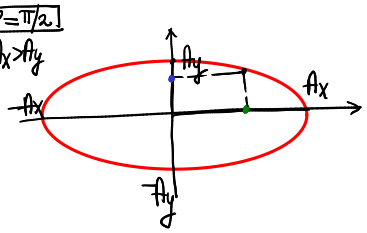
$\varphi = \frac{\pi}{2}$, $A_x = A_y$



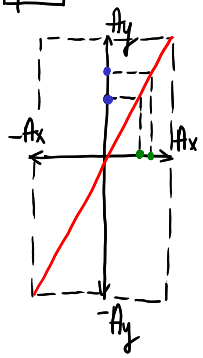
$\varphi = \pi/2$, $A_x < A_y$



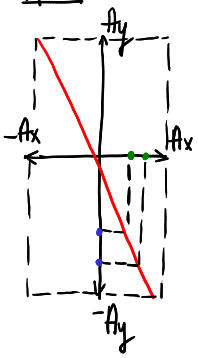
$\varphi = \pi/2$, $A_x > A_y$



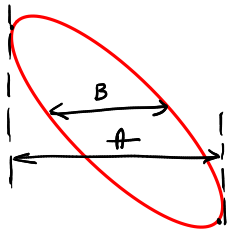
$\varphi = 0$



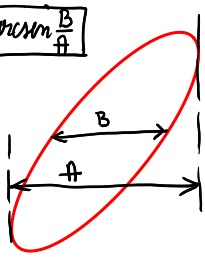
$\varphi = \pi$



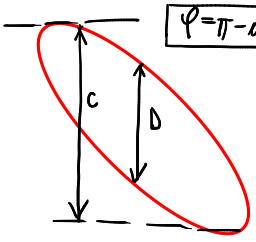
$\varphi = \pi - \arcsin \frac{B}{A}$



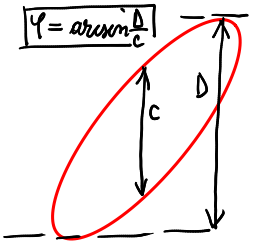
$\varphi = \arcsin \frac{B}{A}$



$\varphi = \pi - \arcsin \frac{D}{C}$

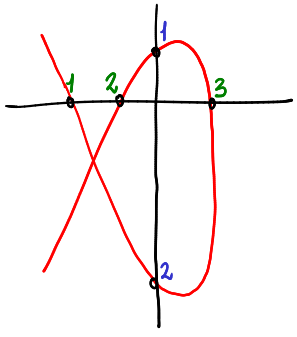


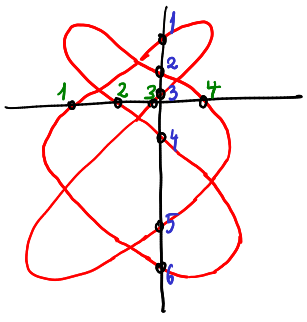
$\varphi = \arcsin \frac{D}{C}$



$\omega_x \neq \omega_y$

$\frac{\omega_x}{\omega_y} = \frac{3}{2}$





$\frac{\omega_x}{\omega_y} = \frac{4}{6}$