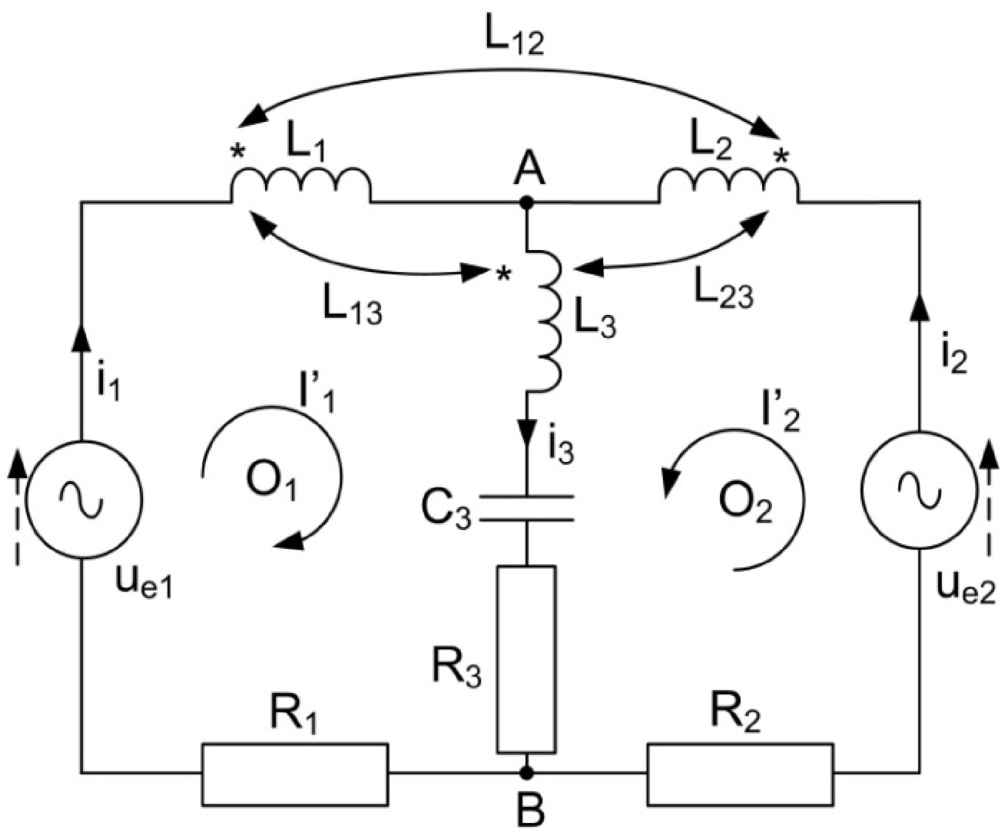
Circuite de curent alternativ
Partea IV

Metoda curentilor ciclici

Tema de casa – verificare rezultate, 22

$R_1=R_2=1\ \Omega$; $R_3=3\ \Omega$; $\omega L_1=\omega L_2=4\ \Omega$; $\omega L_3=1\ \Omega$; $\omega L_{12}=4\ \Omega$; $\omega L_{13}=\omega L_{23}=2\ \Omega$;

$\frac{1}{\omega C_3}=9\ \Omega$; $\underline{U}_{e1}=3+4j$; $\underline{U}_{e2}=4+3j$.



Să se determine curenții folosind:

a) Metoda curentilor ciclici;

b) Să se verifice rezultatele prin bilanțul puterilor.

$R_1 = R_2 = 1\,\Omega$

$R_3 = 3\,\Omega$

$\omega L_1 = \omega L_2 = 4\,\Omega$

$\omega L_3 = 1\,\Omega$

$\omega L_{12} = 4\,\Omega$

$\omega L_{13} = \omega L_{23} = 2\,\Omega$

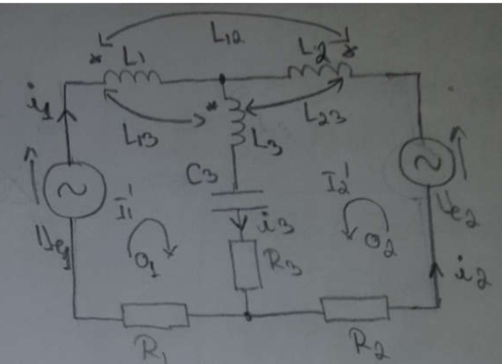
$\frac{1}{\omega C_3} = 9\,\Omega$

$\underline{U}_{e_1} = 3+4i$

$\underline{U}_{e_2} = 4+3i$

a) curenții

b) verificare



a) $\begin{cases} \underline{Z}_{11}\cdot\underline{I}_1' + \underline{Z}_{12}\cdot\underline{I}_2' = \underline{U}_{e_1}' \\ \underline{Z}_{21}\cdot\underline{I}_1' + \underline{Z}_{22}\cdot\underline{I}_2' = \underline{U}_{e_2}' \end{cases}$

$\underline{Z}_{11} = R_1 + i\omega L_1 + i\omega L_3 + \frac{1}{i\omega C_3} + R_3 + 2i\omega L_{13} =$

$= 1+3+\cancel{4i}+\cancel{i}-\cancel{9i}+\cancel{4i} \Rightarrow \underline{Z}_{11} = 4$

$\underline{Z}_{12} = \underline{Z}_{21} = i\omega L_3 + \frac{1}{i\omega C_3} + R_3 + i\omega L_{31} +$

$+ i\omega L_{32} + i\omega L_{12} = \cancel{i} - \cancel{9i} + 3 + \cancel{2i} + \cancel{2i} + \cancel{4i} \Rightarrow \underline{Z}_{12} = 3 = \underline{Z}_{21}$

$\underline{Z}_{22} = R_2 + R_3 + i\omega L_2 + i\omega L_3 + \frac{1}{i\omega C_3} + 2i\omega L_{23} =$

$= 1+3+\cancel{4i}+\cancel{i}-\cancel{9i}+\cancel{4i} = 4$

$\underline{U}_{e_1}' = \underline{U}_{e_1} = 3+4i$

$\underline{U}_{e_2}' = \underline{U}_{e_2} = 4+3i$

$\begin{cases} 4\underline{I}_1' + 3\underline{I}_2' = 3+4i \;|\cdot(-4) \\ 3\underline{I}_1' + 4\underline{I}_2' = 4+3i \;|\cdot 3 \end{cases} \Leftrightarrow \begin{cases} -16\underline{I}_1' - 12\underline{I}_2' = -12-16i \\ 9\underline{I}_1' + 12\underline{I}_2' = 12+9i \end{cases}$ (+)

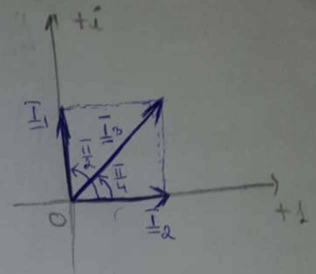
$-7\underline{I}_1' \;/\; = \;/\; -7i \Rightarrow$

$\Rightarrow \boxed{\underline{I}_1' = i} \Rightarrow \boxed{\underline{I}_2' = \frac{3+4i-4i}{3} = 1}$

$\underline{I}_1 = \underline{I}_1' = i \Rightarrow |\underline{I}_1| = 1$

$\underline{I}_2 = \underline{I}_2' = 1 \Rightarrow |\underline{I}_2| = 1$

$\underline{I}_3 = \underline{I}_1' + \underline{I}_2' = 1+i \Rightarrow |\underline{I}_3| = \sqrt{2}$



$$\boxed{\begin{aligned} i_1 &= \sqrt{2}\sin\left(\omega t + \tfrac{\pi}{2}\right)\,[A] \\ i_2 &= \sqrt{2}\sin\omega t\,[A] \\ i_3 &= 2\sin\left(\omega t + \tfrac{\pi}{4}\right)\,[A] \end{aligned}}$$

b) $\underline{S}_G = \underline{U}_{e_1}\cdot\underline{I}_1^* + \underline{U}_{e_2}\cdot\underline{I}_2^* = (3+4i)\cdot(-i) + 4+3i = -\cancel{3i}+4+4-\cancel{3i} = 8$

$\Rightarrow \begin{cases} P_G = 8\,W \\ Q_G = 0\,VAr \end{cases}$

$P_\Sigma = R_1 I_1^2 + R_2 I_2^2 + R_3 I_3^2 = 1+1+3\cdot 2 = 8\,W$

$Q_\Sigma = \omega L_1\cdot I_1^2 + \omega L_2 I_2^2 + I_3^2\left(\omega L_3 - \frac{1}{\omega C_3}\right) + 2\omega L_{13}\cdot Re\{\underline{I}_1\cdot\underline{I}_3^*\}$

$+ 2\omega L_{23}\cdot Re\{\underline{I}_2\cdot\underline{I}_3^*\} + 2\omega L_{12}\cdot Re\{\underline{I}_1\cdot\underline{I}_2^*\} =$

$= 4+4+2(1-9)+2\cdot 2\cdot(1)+2\cdot 2\cdot 1+2\cdot 4\cdot 0 =$

$= 8-16+4+4 = 16-16 = 0\,VAr$

$\underline{S}_\Sigma = 8$

$\underline{S}_G = \underline{S}_\Sigma$

Metoda curenţilor de ochi (ciclici)

Pentru reducerea numãrului de ecuaţii necesare pentru rezolvarea unei reţele, se utilizeazã o schimbare de variabilã în sistemul obţinut prin teoremele lui Kirchhoff. În locul curenţilor reali din laturi se introduc niște necunoscute fictive numite curenţi ciclici (de ochi) asociate fiecărui ochi independent al reţelei.

Sensurile curenţilor ciclici se aleg în mod arbitrar. Folosind aceste necunoscute, numărul de ecuaţii independente se reduce de la l la o.

Se rezolvă următorul sistem de ecuaţii:

$$\sum_{j=1}^{o} \underline{Z}_{ij} \cdot \underline{I}'_{j} = \underline{U}'_{ei} \qquad i = 1,\ldots\ldots,o$$

$$\begin{cases} \underline{Z}_{11} \cdot \underline{I}'_{1} + \underline{Z}_{12} \cdot \underline{I}'_{2} = \underline{U}'_{e1}; \\ \underline{Z}_{21} \cdot \underline{I}'_{1} + \underline{Z}_{22} \cdot \underline{I}'_{2} = \underline{U}'_{e2}. \end{cases}$$

Metoda curenţilor de ochi (ciclici)

$\underline{Z}_{ii}$ – impedanta complexa proprie a ochiului *i*, egala ca <u>suma</u> a impedantelor proprii ale laturii ochiului *i*. La aceasta suma se mai adauga si impedanta mutuala dintre bobinele laturilor ochiului *i*.

$$\underline{Z}_{ii} = +\sum_{k \in Oi} (R_k + j\omega L_k + \frac{1}{j\omega C_k}) \pm 2\sum j\omega L_{ks}$$

L_{ks} – poate fi pozitiv sau negativ. Se ia cu semnul (+) daca curentul ciclic prin cele 2 bobine *k* si *s* este orientat la fel fata de bornele polarizate ale bobinelor k si s, iar cu semnul (-) in caz contrar.

$\underline{Z}_{ij}$ – impedanta complexa proprie a laturilor comune ochiului *i* si ochiului *j*. Se ia cu semnul (+) daca curentii ciclici trec in celasi sens prin latura comuna si cu semnul (-) in cand sunt sensuri contrare.

$$\underline{Z}_{ij} = \pm \sum_{k \in Oi} \left(R_k + j\omega L_k + \frac{1}{j\omega C_k}\right) \pm \sum_{\substack{k \in Oi \\ s \in Os}} j\omega L_{ks}$$

L_{ks} – suma impedantelor mutuale intre bobinele dintre ochiul *i* si ochiul *j*. Sensul depinde de sensul curentului ciclic fata de bornele polarizate ale bobinelor cuplate magnetic.

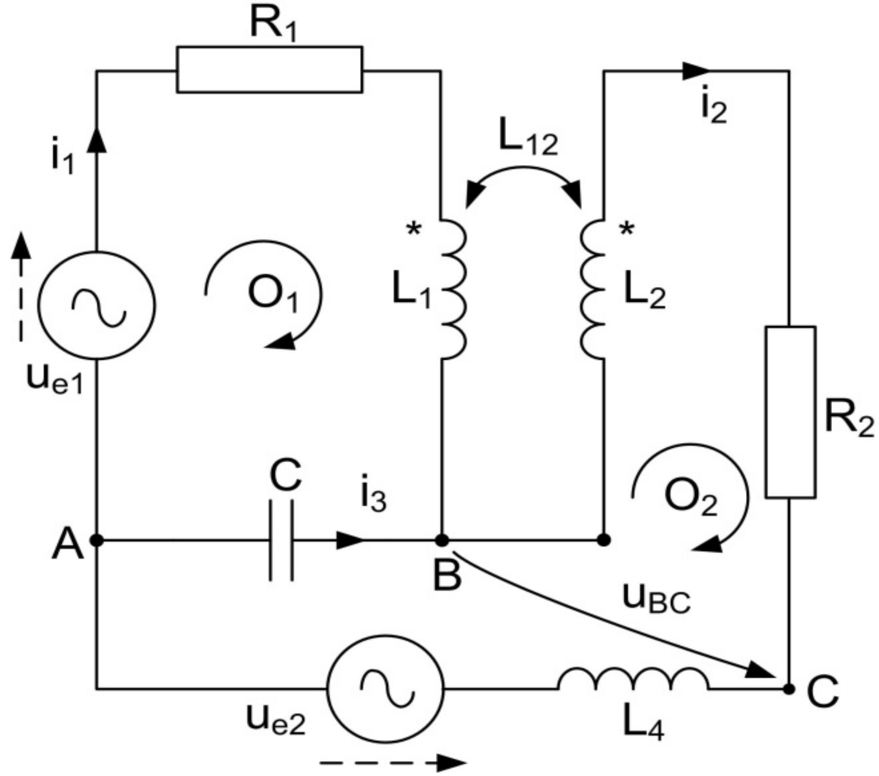
U'_{ei} – tensiunea electromotoare proprie ochiului *i*, se calculeazã prin suma algebrica a tensiunilor electromotoare ce aparţin ochiului *i*. Se ia cu (+) daca curentul ciclic are acelasi sens cu t.e.m si cu (-) in caz contrar.

Dupa rezolvarea sistemului ➔ curentii ciclici. Curentii reali din laturi se determina facand suma algebrica a curentilor ciclici ce trec prin latura respectiva. Curentii ciclici se iau cu (+) daca au acelasi sens cu curentul real prin latura si cu (-) in caz contrar.

1. Se dă schema electrică din fig. Se cunosc:

$u_{e1}=10\sqrt{2}\sin\omega t$ [V]; $u_{e2}=50\sqrt{2}\cos(\omega t-\frac{\pi}{2})$ [V]; f=50 Hz; $R_1=R_2=20\ \Omega$;

$L_1=L_{12}=\frac{100}{\pi}$ mH; $L_2=L_4=\frac{200}{\pi}$ mH; $C=\frac{1}{2\pi}$ mF.



Să se determine curenții folosind: Metoda curentilor ciclici;

a) Să se verifice rezultatele prin bilanțul puterilor;

b) Să se determine tensiunea U_{BC}.

Rezolvare:

$$\underline{U}_{e1}=10\,e^{j0}=10(\cos 0+j\sin 0)=10;$$

$$\underline{U}_{e2}=50\sqrt{2}\,\sin\left(\omega t-\frac{\pi}{2}\right)\rightarrow\underline{U}_{e2}=\ 50\,e^{j(-\frac{\pi}{2})}=50[\cos(-\frac{\pi}{2})+j\sin(-\frac{\pi}{2})]=-50j;$$

$$\omega=2\pi f=2\cdot\pi\cdot 50=100\pi; \qquad \omega L_1=\omega L_{12}=100\pi\cdot\frac{100}{\pi}\cdot 10^{-3}=10\ \Omega;$$

$$\omega L_2=\omega L_4=100\pi\cdot\frac{200}{\pi}\cdot 10^{-3}=20\ \Omega;$$

$$\frac{1}{\omega C}=\frac{1}{100\pi\cdot\frac{1}{2\pi}\cdot 10^{-3}}=20\ \Omega.$$

$$\underline{Z}_{11}=R_1+j\omega L_1+\frac{1}{j\omega C}=20-10j;$$

$$\underline{Z}_{12}=\underline{Z}_{21}=-\frac{1}{j\omega C}-j\omega L_{12}=10j;$$

$$\begin{cases}\underline{Z}_{11}\cdot\underline{I}_1'+\underline{Z}_{12}\cdot\underline{I}_2'=\underline{U}_{e1}';\\ \underline{Z}_{21}\cdot\underline{I}_1'+\underline{Z}_{22}\cdot\underline{I}_2'=\underline{U}_{e2}';\end{cases}$$

$$\underline{Z}_{22}=R_2+j\omega L_2+j\omega L_4+\frac{1}{j\omega C}=20+20j.$$

$$\underline{U}_{e1}'=\underline{U}_{e1}=10; \qquad \underline{U}_{e2}'=-\underline{U}_{e2}=50j.$$

u_{BC}

$$\begin{cases} (20-10j)\cdot \underline{I}_1^{'}+10j\cdot \underline{I}_2^{'}=10; \\ 10j\cdot \underline{I}_1^{'}+(20+20j)\cdot \underline{I}_2^{'}=50j. \end{cases} \rightarrow \underline{I}_1^{'}=1;\ \underline{I}_2^{'}=1+j.$$

$$\underline{I}_1=\underline{I}_1^{'}=1\rightarrow i_1=\sqrt{2}\sin \omega t\ [A]\ ;$$

$$\underline{I}_2=\underline{I}_2^{'}=1+j\rightarrow i_2=2\sin\left(\omega t+\frac{\pi}{4}\right)\ [A]\ ;$$

$$\underline{I}_3=\underline{I}_2^{'}-\underline{I}_1^{'}=j\rightarrow i_3=\sqrt{2}\sin\left(\omega t+\frac{\pi}{2}\right)\ [A]\ .$$

Pentru tensiunea u_{BC} se aplică teorema a II-a lui Kirchhoff ochiului BCB:

$$\underline{I}_2\cdot(R_2+j\omega L_2)-j\omega L_{21}\cdot\underline{I}_1-\underline{U}_{BC}=0\rightarrow\underline{U}_{BC}=30j;$$

$$u_{BC}=30\sqrt{2}\sin(\omega t+\frac{\pi}{2})\ [V].$$

Bilanțul puterilor:

$$\underline{S}_G = \underline{U}_{e1} \cdot \underline{I}_1^* - \underline{U}_{e2} \cdot \underline{I}_2^* = 10 \cdot 1 - (-50j)(1-j) = 60+50j;$$

$P_G = 60$ W; $Q_G = 50$ VAr;

$$P_Z = R_1 \cdot I_1^2 + R_2 \cdot I_2^2 = 20 \cdot 1 + 20 \cdot 2 = 60 \text{ W}.$$

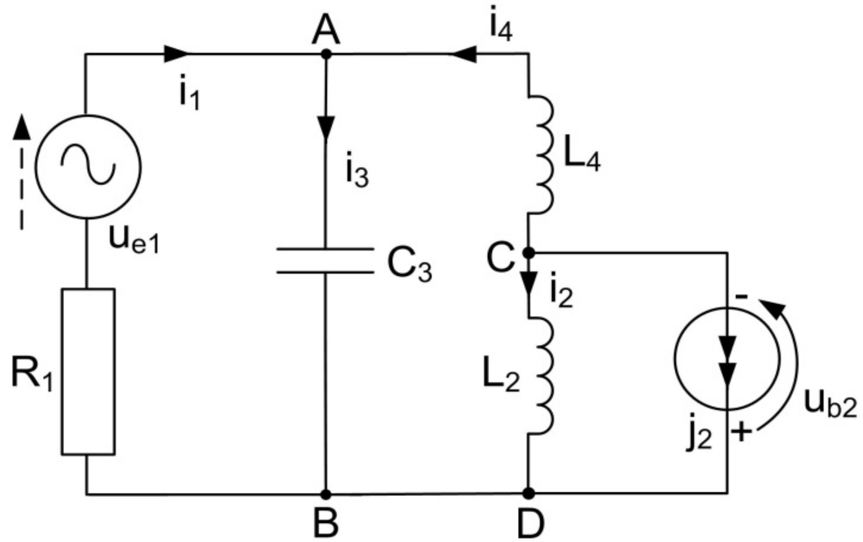
$$Q_Z = \omega L_1 \cdot I_1^2 + (\omega L_2 + \omega L_4) \cdot I_2^2 - \frac{1}{\omega C} \cdot I_3^2 - 2\omega L_{12} \cdot \mathrm{Re}\left\{\underline{I}_1 \cdot \underline{I}_2^*\right\};$$

$$Q_Z = 10 \cdot 1 + 40 \cdot 2 - 20 \cdot 1 - 20 \cdot \mathrm{Re}\{1(1-j)\} = 50 \text{ VAr}.$$

2. Se dă schema electrică din fig. Se cunosc:

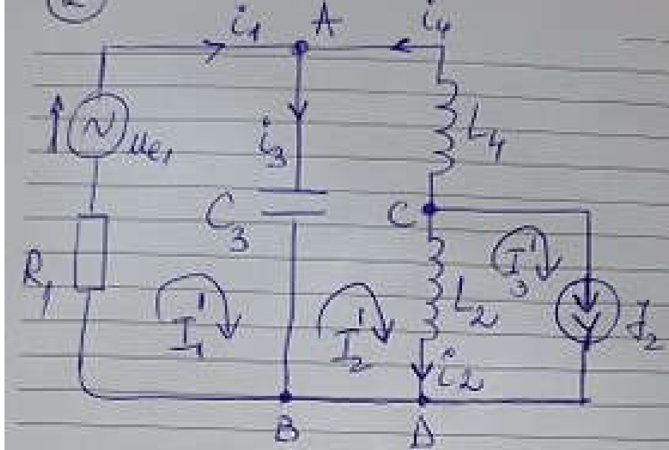
$R_1=10\ \Omega;\ \omega L_2=\omega L_4=5\ \Omega;\ ;\dfrac{1}{\omega C_3}=10\ \Omega;$

$u_{e1}=100\sqrt{2}\sin(100\pi t)[V];\ j_2(t)=20\sqrt{2}\sin\left(100\pi t+\dfrac{\pi}{2}\right)[A].$



Să se determine curenții folosind: Metoda curentilor ciclici;
Să se verifice rezultatele prin bilanțul puterilor.

(2)



$U_{e1} = 100\,V$

$\underline{J}_2 = 20i\,A$

$$\begin{cases} \underline{Z}_{11}\cdot\underline{I}_1' + \underline{Z}_{12}\underline{I}_2' + \underline{Z}_{13}\underline{I}_3' = U_1' \\ \underline{Z}_{21}\underline{I}_1' + \underline{Z}_{22}\underline{I}_2' + \underline{Z}_{23}\underline{I}_3' = U_2' \end{cases}$$

$\underline{I}_3' = \underline{J}_2 = 20i\,A$

$\underline{Z}_{11} = R_1 + \frac{1}{i\omega C_3} = 10 - 10i$

$\underline{Z}_{12} = \underline{Z}_{21} = -\frac{1}{i\omega C_3} = 10i$

$\underline{Z}_{13} = 0$; $\underline{Z}_{22} = \frac{1}{i\omega C_3} + i\omega L_4 + i\omega L_2 = -10i + 5i + 5i = 0$

$\underline{Z}_{23} = -i\omega L_2 = -5i$

$U_{e1}' = U_{e1} = 100$

$U_{e2}' = 0$

$$\begin{cases} (10-10i)\underline{I}_1' + 10i\,\underline{I}_2' = 100 \quad /:10 \\ 10i\,\underline{I}_1' + 0\cdot\underline{I}_2' - 5i\cdot\underline{I}_3' = 0 \end{cases}$$

$10i\cdot\underline{I}_1' - 5i\cdot 20i = 0$

$10i\,\underline{I}_1' = -100 \Rightarrow \underline{I}_1' = -\frac{100}{10i} = 10i$

$\boxed{\underline{I}_1' = 10i\,A}$

$(1-i)\cdot\underline{I}_1' + i\,\underline{I}_2' = 10$

$10i(1-i) + i\,\underline{I}_2' = 10$

$i\,\underline{I}_2' = 10 - 10i(1-i) = \cancel{10} - 10i + \cancel{10i^2}$

$\underline{I}_2' = \frac{-10\cancel{i}}{\cancel{i}} = -10$

$\boxed{\underline{I}_2' = -10\,A}$

$\underline{I}_1 = \underline{I}_4' = 10i\ A$

$\underline{I}_2 = \underline{I}_2' - \underline{I}_3' = -10 - 20i\ A$

$\underline{I}_3 = \underline{I}_1' - \underline{I}_2' = 10i - 10\ A$

$\underline{I}_4 = -\underline{I}_2' = 10\ A$

$0 = -i\omega L_2 \cdot \underline{I}_2 - \underline{U}_{b_2}$

$\underline{U}_{b_2} = -i\omega L_2 \cdot \underline{I}_2 = -5i \cdot (-10 - 20i)$

$= 50i + 100i^2 = 50i - 100$

$\boxed{\underline{U}_{b_2} = 50i - 100\ V}$

$\underline{U}_1 \cdot \underline{I}_1^* + \underline{U}_2 \cdot \underline{I}_2^* = 100\,(-10i) + (50i - 100)(-20i)$

$= -1000i - 1000i^2 + 2000i$

$= 1000 + 1000i$

$P_G = 1000\,W$

$Q_G = 1000\,Var.$

$P_2 = R_1 I_1^2 = 10 \cdot 10^2 = 1000\,W$

$Q_2 = -\frac{1}{\omega C_3} I_3^2 + \omega L_4 I_4^2 + \omega L_2 \cdot I_2^2$

$= -10 \cdot (\sqrt{200})^2 + 5 \cdot 10^2 + 5 \cdot (\sqrt{500})^2$

$= -2000 + 500 + 2500$

$Q_2 = 1000\,Var.$

a) $i_1 = 10\sqrt{2}\sin\left(\omega t+\frac{\pi}{2}\right)$ [A]; $i_2 = 10\sqrt{10}\sin(\omega t-\mathrm{arctg}\,2)$ [A];

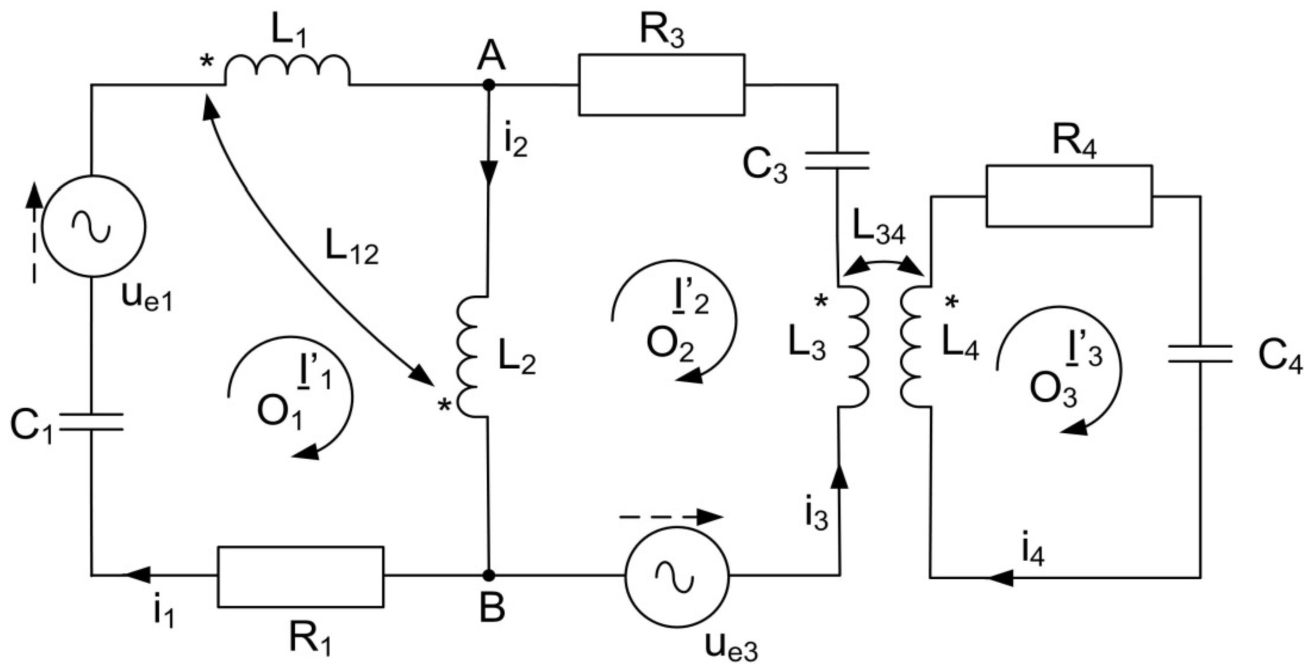
$i_3=20\sin\left(\omega t+\frac{\pi}{4}\right)$ [A]; $i_4=10\sqrt{2}\sin(\omega t)$ [A]; $u_{b2}=50\sqrt{10}\sin\left(\omega t-\mathrm{arctg}\,\frac{1}{2}\right)$ [V];

b) $P_G=P_Z= 1000$ W; $Q_G=Q_Z=1000$ VAr.

3. Se dă schema electrică din fig. Se cunosc:

$R_1=R_3=2\ \Omega$; $R_4=5\ \Omega$; $L_1=L_{12}=L_4=\frac{20}{\pi}$ mH; $L_2=L_{34}=\frac{50}{\pi}$ mH; $L_3=\frac{10}{\pi}$ mH;

$C_1=\frac{10}{3\pi}$ mF; $C_3=\frac{2.5}{\pi}$ mF; $C_4=\frac{10}{7\pi}$ mF ; $\underline{U}_{e1}=50(1+j)$; $\underline{U}_{e3}=30(2+j)$; f=50Hz.



Să se determine curenții folosind: Metoda curentilor ciclici;
Să se verifice rezultatele prin bilanțul puterilor.

(3)

$$i\omega L_1 = i\omega L_{12} = i\omega L_4 = \frac{2\cancel{\pi}}{\cancel{\pi}} \cdot 10^{-3} \, i \cdot 100\pi$$

$$\underline{Z}_{L_1} = \underline{Z}_{L_{12}} = \underline{Z}_{L_4} = 2i$$

$$\underline{Z}_{L_2} = \underline{Z}_{L_{34}} = 5i$$

$$\underline{Z}_{L_3} = i$$

$$\underline{Z}_{C_1} = \frac{1}{i\omega \cdot C_1} = \frac{1}{i \cdot 100\pi \cdot \frac{10}{3\pi} \cdot 10^{-3}}$$

$$= \frac{3 \cdot 10^3}{i \cdot 10^3} = -3i$$

$$\underline{Z}_{C_3} = \frac{1}{i\omega C_3} = \frac{1}{i \cdot 100\pi \cdot \frac{2{,}5}{\pi} \, 10^{-3}}$$

$$= \frac{10^3}{250i} = -4i$$

$$\underline{U}_{e_1} = 50(1+i) \qquad \underline{Z}_{C_4} = -7i$$

$$\underline{U}_{e_2} = 30(2+i)$$

$$\begin{cases} \underline{Z}_{11}\underline{I}_1' + \underline{Z}_{12}\underline{I}_2' + \underline{Z}_{13}\underline{I}_3' = \underline{U}_{e_1}' \\ \underline{Z}_{21}\underline{I}_1' + \underline{Z}_{22}\underline{I}_2' + \underline{Z}_{23}\underline{I}_3' = \underline{U}_{e_2}' \\ \underline{Z}_{31}\underline{I}_1' + \underline{Z}_{32}\underline{I}_2' + \underline{Z}_{33}\underline{I}_3' = \underline{U}_{e_3}' \end{cases}$$

$$\underline{Z}_{11} = R_1 + i\omega L_1 + i\omega L_2 + \frac{1}{i\omega C_1} - 2i\omega L_{12}$$

$$= 2 + \cancel{2i} + \cancel{5i} - \cancel{3i} - \cancel{4i} = 2$$

$$\underline{Z}_{12} = \underline{Z}_{21} = -i\omega L_2 + i\omega L_{21} = -5i + 2i = -3i$$

$$\underline{Z}_{13} = \underline{Z}_{31} = 0$$

$$\underline{Z}_{22} = R_3 + i\omega L_2 + i\omega L_3 + \frac{1}{i\omega C_3}$$

$$= 2 + 5i + i - 4i = 2 + 2i$$

$$\underline{Z}_{23} = \underline{Z}_{32} = -i\omega L_{34} = -5i$$

$$\underline{Z}_{33} = R_4 + i\omega L_4 + \frac{1}{i\omega C_4} = 5 + 2i - 7i$$

$$= 5 - 5i$$

ELECTRO SISTEM
electro sistem grup

$$\underline{U}_{e1}' = \underline{U}_{e1}$$
$$\underline{U}_{e2}' = -\underline{U}_{e3}$$
$$\underline{U}_{e3}' = 0$$

$$\begin{cases} 2\underline{I}_1' - 3i\underline{I}_2' + 0\underline{I}_3' = 50(1+i) \\ -3i\underline{I}_1' + (2+2i)\underline{I}_2' - 5i\underline{I}_3' = -30(2+i) \\ 0\underline{I}_1' - 5i\underline{I}_2' + (5-5i)\underline{I}_3' = 0 \end{cases}$$

$$-5i\underline{I}_2' = -(5-5i)\underline{I}_3' \quad /:5$$

$$-i\underline{I}_2' = -(1-i)\underline{I}_3'$$

$$\underline{I}_2' = \frac{(1-i)\underline{I}_3'}{i}$$

$$\underline{I}_2' = -i(1-i)\underline{I}_3'$$
$$= (-i+i^2)\underline{I}_3'$$

$$\boxed{\underline{I}_2' = (-1-i)\underline{I}_3'}$$

$$\begin{cases} -3i\underline{I}_1' + (2+2i)(-1-i)\underline{I}_3' - 5i\underline{I}_3' = -30(2+i) \\ 2\underline{I}_1' - 3i(-1-i)\underline{I}_3' = 50(1+i) \end{cases}$$

$$\begin{cases} -3i\underline{I}_1' + \underline{I}_3'(-5i - \cancel{2} - 2i - 2i - \cancel{2i^2}) = -30(2+i) \\ 2\underline{I}_1' - 3i(-1-i)\underline{I}_3' = 50(1+i) \end{cases}$$

$$\begin{cases} -3i\underline{I}_1' + \underline{I}_3'(-9i) = -30(2+i) \quad /2 \\ 2\underline{I}_1' + \underline{I}_3'(3i-3) = 50(1+i) \quad /3i \end{cases}$$
$$(+)$$

$$\underline{I}_3'(-18i + 9i^2 - 9i) = -120 - 60i + 150i + 150i^2$$

$$\underline{I}_3'(-27i - 9) = -270 + 90i$$

$$\underline{I}_3' = \frac{-270 + 90i}{-27i - 9}$$

$$= \frac{(27i-9)(90i-270)}{810} = \frac{2430i^2 - 7290i - 810i + 2430}{810}$$

$$= \frac{-8100i}{810} = -10i \qquad \boxed{\underline{I}_3' = -10i}$$

$$2\underline{I}_1' + (3i-3)(-10i) = 50(1+i)$$

$$2\underline{I}_1' = 50+50i - (-30i^2+30i)$$

$$2\underline{I}_1' = 50+50i+30i^2-30i$$

$$\underline{I}_1' = \frac{20+20i}{2} = 10+10i$$

$$\underline{I}_2' = (-1-i)(-10i)$$

$$= 10i+10i^2 = 10i-10$$

$$\underline{I}_1' = 10+10i \text{ A}$$

$$\underline{I}_2' = 10i-10 \text{ A}$$

$$\underline{I}_3' = -10i \text{ A}.$$

$$\underline{I}_1 = \underline{I}_1' = 10+10i \text{ A}$$

$$\underline{I}_2 = \underline{I}_1' - \underline{I}_2' = 10+\cancel{10i}-\cancel{10i}+10 = 20 \text{ A}$$

$$\underline{I}_3 = -\underline{I}_2' = 10-10i \text{ A}$$

$$\underline{I}_4 = \underline{I}_3' = -10i \text{ A}$$

$$S_G = \underline{U}_{G1}\cdot\underline{I}_1^* + \underline{U}_{G3}\cdot\underline{I}_3^*$$

$$= 50(1+i)(10-10i) + 30(2+i)(10+10i)$$

$$= 500 - \cancel{500i} + \cancel{500i} - 500i^2 + 600 + 600i$$

$$+ 300i + 300i^2$$

$$= 1300 + 900i$$

$$P_G = 1300 \text{ W} \quad Q_G = 900 \text{ var.}$$

$$P_Z = R_1 I_1^2 + R_3 I_3^2 + R_4 I_4^2$$

$$= 2\cdot(\sqrt{200})^2 + 2\cdot(\sqrt{200})^2 + 5\cdot 10^2$$

$$= 400 + 400 + 500 = 1300 \text{ W}$$

$$Q_Z = \omega L_1 I_1^2 + \omega L_2 I_2^2 + \omega L_3 I_3^2 + \omega L_4 I_4^2 -$$

$$- \frac{1}{\omega C_1} I_1^2 - \frac{1}{\omega C_3} I_3^2 - \frac{1}{\omega C_4} I_4^2 - 2\omega L_{12}\cdot \mathrm{Re}\{\underline{I}_1\cdot\underline{I}_2^*\}$$

$$+ 2\omega L_{34}\cdot \mathrm{Re}\{\underline{I}_3\cdot\underline{I}_4^*\}$$

$$Q_Z = 2\cdot 200 + 5\cdot 400 + 1\cdot 200 + 2\cdot 100 -$$

$$- 3\cdot 200 - 4\cdot 200 - 7\cdot 100 - 2\cdot 2\cdot 200 +$$

$$+ 2\cdot 5\cdot 100$$

$$= 400 + 2000 + 200 + 200 - 600 - 800$$

$$- 700 - 800 + 1000$$

$$Q_Z = 900 \text{ var}$$

$i_1 = 20\sin\left(314t+\frac{\pi}{4}\right)$ [A]; $i_2 = 20\sqrt{2}\sin(314t)$ [A]; $i_3=20\sin\left(314t-\frac{\pi}{4}\right)$ [A];

$i_4=10\sqrt{2}\sin\left(314t-\frac{\pi}{2}\right)$ [A]; $\underline{I}'_1=10(1+j)$; $\underline{I}'_2=10(-1+j)$; $\underline{I}'_3=-10j$;

$P_G=P_Z=1300$ W; $Q_G=Q_Z=900$ VAr.