1 Transformata Laplace

1. Să se determine imaginea prin transformata Laplace a următoarelor funcţii original

   (a) $f(t) = \sin^2 6t$
   (b) $f(t) = \cos^2 3t$
   (c) $f(t) = \sin 3t \cdot \sin 4t$
   (d) $f(t) = \cos 2t \cdot \cos 6t$
   (e) $f(t) = \cos 3t \cdot \sin 4t$
   (f) $f(t) = e^{2t} \sin 3t$
   (g) $f(t) = e^{-5t} \cos 3t$
   (h) $f(t) = e^{-2t} sh4t$
   (i) $f(t) = e^{-t} t^4$
   (j) $f(t) = e^{4t} ch2t$
   (k) $f(t) = e^{-3t} t^3$
   (l) $f(t) = \sin(5t - 10)$
   (m) $f(t) = \cos(4t + 8)$
   (n) $f(t) = sh(3t + 7)$
   (o) $f(t) = ch(2t - 5)$
   (p) $f(t) = t \cdot sh3t$
   (q) $f(t) = t^2 \sin t$
   (r) $f(t) = t^3 e^{2t}$
   (s) $f(t) = \frac{sht}{t}$
   (t) $f(t) = \frac{1-cht}{t}$
   (u) $f(t) = \frac{\sin t}{t}$
   (v) $f(t) = \frac{1-\cos t}{t}$

2. Să se determine funcţia original a următoarelor imagini prin transformata Laplace

   (a) $F(s) = \frac{s+3}{s^2+4}$
   (b) $F(s) = \frac{3s-5}{s^2+6s+5}$
   (c) $F(s) = \frac{2s+7}{s^2(s^2-9)}$
   (d) $F(s) = \frac{2s+7}{s^2(s^2+9)}$
   (e) $F(s) = \frac{2s+5}{s^2-4)(s^2-8s+12)}$
   (f) $F(s) = \frac{s^2+4}{(s-2)(s+4)(s-3)}$
   (g) $F(s) = \frac{2s^2-3s+1}{(s^2+1)(s^2+4)}$

3. Să se se rezolve următoarele probleme Cauchy folosind transformata Laplace:

   (a) $x'' + 3x' + 2x = 20e^{3t}, x(0) = 0, x'(0) = 0$
   (b) $x'' - x = 2\cos t, x(0) = 0, x'(0) = 1$
   (c) $x'' + 2x' - 3x = t, x(0) = 1, x'(0) = 0$
   (d) $x'' + x = \sin t, x(0) = 0, x'(0) = 0$
   (e) $x'' - 3x' + 2x = 4 + e^{3t}, x(0) = 1, x'(0) = -1$
   (f) $x'' + x = 2, x(0) = 0, x'(0) = 3$
   (g) $x'' - 4x = 2\sin t, x(0) = 1, x'(0) = -1$

(h) $x'' - x' - 2x = 6e^{2t}, x(0) = 2, x'(0) = 3$

4. Utilizând transformata Laplace să se rezolve următoarele ecuţii integrale:

   (a) $x(t) = e^t - 2\int\limits_0^t x(\tau)\cos(t-\tau)d\tau$

   (b) $x(t) - 2\int\limits_0^t x(\tau)d\tau = \frac{1}{9}(1 - \cos 3t)$

   (c) $x(t) - \int\limits_0^t x(\tau)ch2(t-\tau)d\tau = 1 - t + 2t^2$

   (d) $\int\limits_0^t x(\tau)d\tau + 2x(t) - e^{2t} = 0$

   (e) $x(t) - \int\limits_0^t x(\tau)\sin 3(t-\tau)d\tau = t\cos 3t$

   (f) $x(t) - \int\limits_0^t x(\tau)\sin 2(t-\tau)d\tau = 2\cos 2t + t$

   (g) $x(t) - 4\int\limits_0^t x(\tau)\sin 2(t-\tau)d\tau = 3\cos 2t$