CURS 11_12

Circuite trifazate

CIRCUITE TRIFAZATE

Prin sistem trifazat se înţelege orice sistem format din 3 mărimi sinusoidale:

$$\begin{cases} a_1(t) = \sqrt{2} A_1 \sin(\omega t + \varphi_1) \\ a_2(t) = \sqrt{2} A_2 \sin(\omega t + \varphi_2) \\ a_3(t) = \sqrt{2} A_3 \sin(\omega t + \varphi_3) \end{cases}$$

Cazurile particulare:

referitor la <u>valorile eficace</u>:

Sistem **simetric** dacă $A_1 = A_2 = A_3 = A$

Dacă cele 3 mărimi sunt oarecare atunci sistemul se cheamă sistem **nesimetric**.

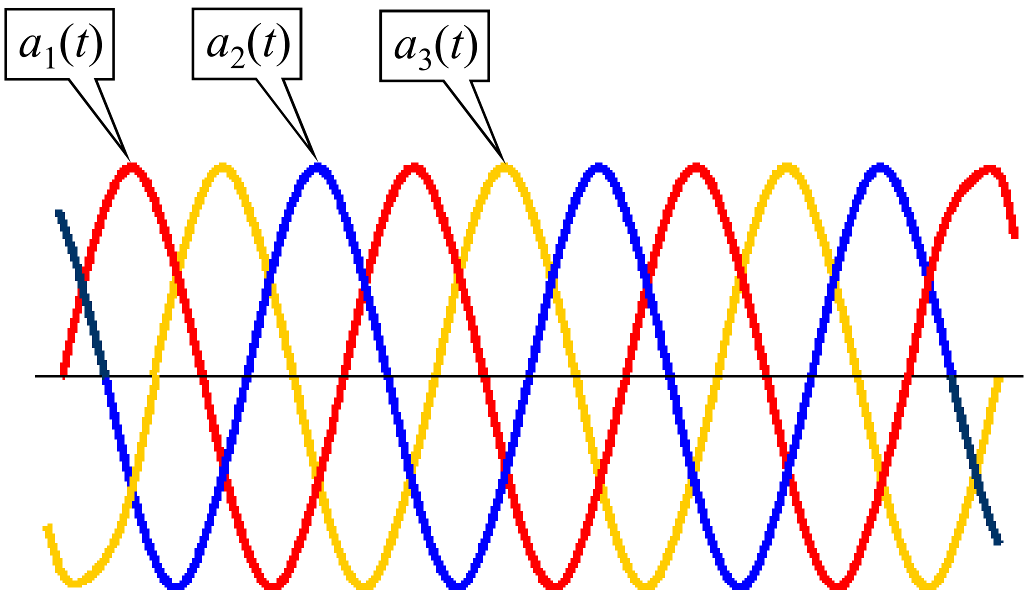
referitor la <u>defazaje</u>

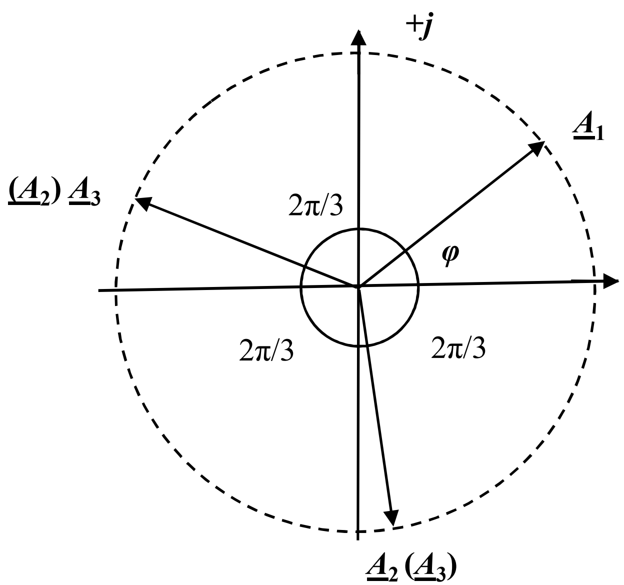
$\varphi, \varphi - \frac{2\pi}{3}, \varphi - \frac{4\pi}{3}$ sistem trifazat **de succesiune directă**

$\varphi, \varphi + \frac{2\pi}{3}, \varphi + \frac{4\pi}{3}$ sistem trifazat **de succesiune inversă**

Considerăm un sistem trifazat simetric direct:

$$\begin{cases} a_1(t) = \sqrt{2}A\sin(\omega t + \varphi) \rightarrow \underline{A}_1 = Ae^{j\varphi} \\ a_2(t) = \sqrt{2}A\sin\left(\omega t + \varphi - \dfrac{2\pi}{3}\right) \rightarrow \underline{A}_2 = Ae^{j\left(\varphi - \frac{2\pi}{3}\right)} = Ae^{j\varphi}e^{-j\frac{2\pi}{3}} \\ a_3(t) = \sqrt{2}A\sin\left(\omega t + \varphi - \dfrac{4\pi}{3}\right) \rightarrow \underline{A}_3 = Ae^{j\left(\varphi - \frac{4\pi}{3}\right)} = Ae^{j\varphi}e^{-j\frac{4\pi}{3}} \end{cases}$$





Pentru simplificarea scrierii se introduce un numar complex notat cu "**a**" de modul 1 şi argument $\frac{2\pi}{3}$.

$$a = e^{j\frac{2\pi}{3}} = \cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

a = **operator de rotaţie** datorită faptului că prin înmulţirea oricărui numar complex cu el rezultă un numar complex având argumentul majorat cu $\frac{2\pi}{3}$ adică rotit în sens trigonometric cu $\frac{2\pi}{3}$

$$\begin{cases} \underline{A}_1 = Ae^{j\varphi} \\ \underline{A}_2 = Ae^{j\varphi}e^{-j\frac{2\pi}{3}} = a^2\underline{A}_1 \\ \underline{A}_3 = Ae^{j\varphi}e^{-j\frac{4\pi}{3}} = a\underline{A}_1 \end{cases}$$

$$\begin{cases} \underline{A}_1 = Ae^{j\varphi} \\ \underline{A}_2 = Ae^{j\varphi}e^{-j\frac{2\pi}{3}} = a^2\underline{A}_1 \\ \underline{A}_3 = Ae^{j\varphi}e^{-j\frac{4\pi}{3}} = a\underline{A}_1 \end{cases}$$

$$\begin{cases} \underline{A}_1 \\ \underline{A}_2 = a^2\underline{A}_1 \\ \underline{A}_3 = a\underline{A}_1 \end{cases}$$

sistem trifazat simetric direct

$$\begin{cases} \underline{A}_1 \\ \underline{A}_2 = a\underline{A}_1 \\ \underline{A}_3 = a^2\underline{A}_1 \end{cases}$$

sistem trifazat simetric invers

$$a = e^{j\frac{2\pi}{3}} = \cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

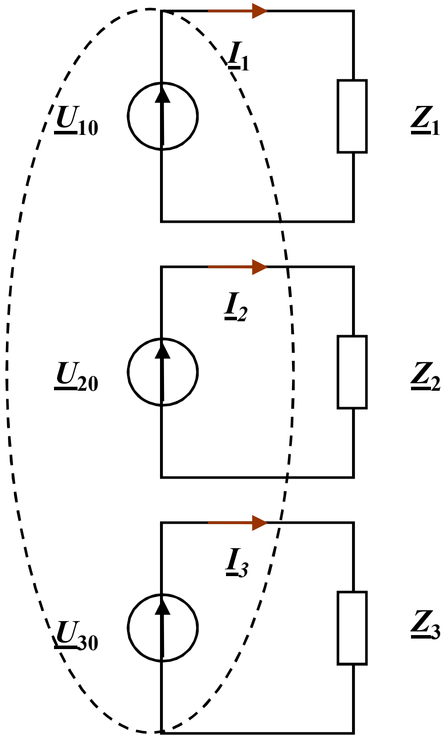
$$a^2 = e^{j\frac{4\pi}{3}}$$

$$1 + a + a^2 = 0 \Rightarrow \underline{A}_1 + \underline{A}_2 + \underline{A}_3 = (1 + a + a^2)\underline{A}_1 = 0$$

$$\underline{A}_1 + \underline{A}_2 + \underline{A}_3 = 0$$

Pentru un sistem trifazat simetric direct sau invers la orice moment suma celor trei mărimi sinusoidale este identic nulă.

Sisteme de tensiuni şi curenţi în reţele trifazate. Conexiuni în reţele trifazate.



$$\begin{cases} \underline{Z}_1 = \underline{Z}_2 = \underline{Z}_3 = \underline{Z} \\ \underline{Z}_1 \neq \underline{Z}_2 \neq \underline{Z}_3 \end{cases}$$

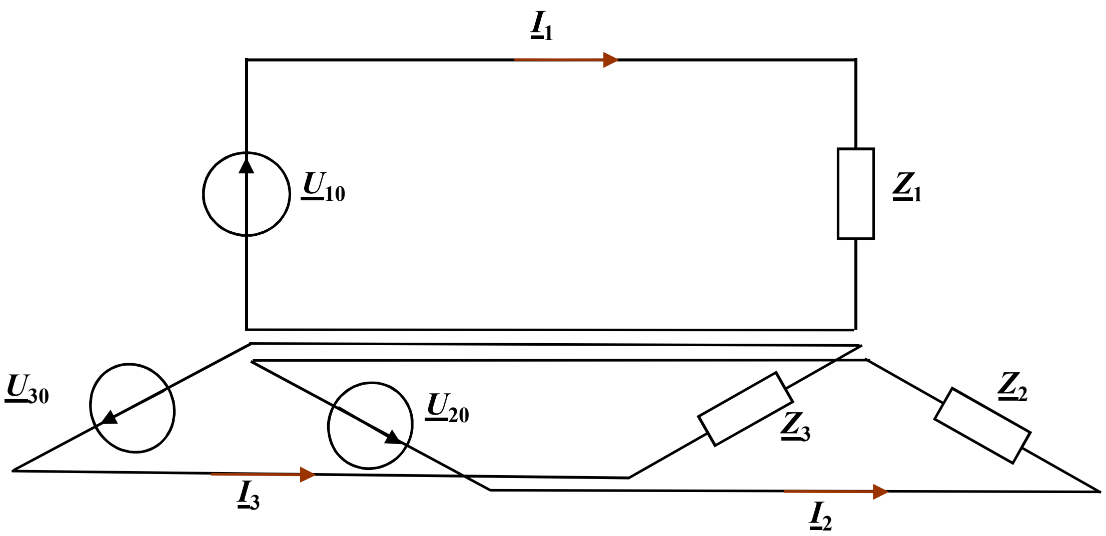
$$\underline{U}_{10} = \underline{U}$$

$$\underline{U}_{20} = a^2 \underline{U}$$

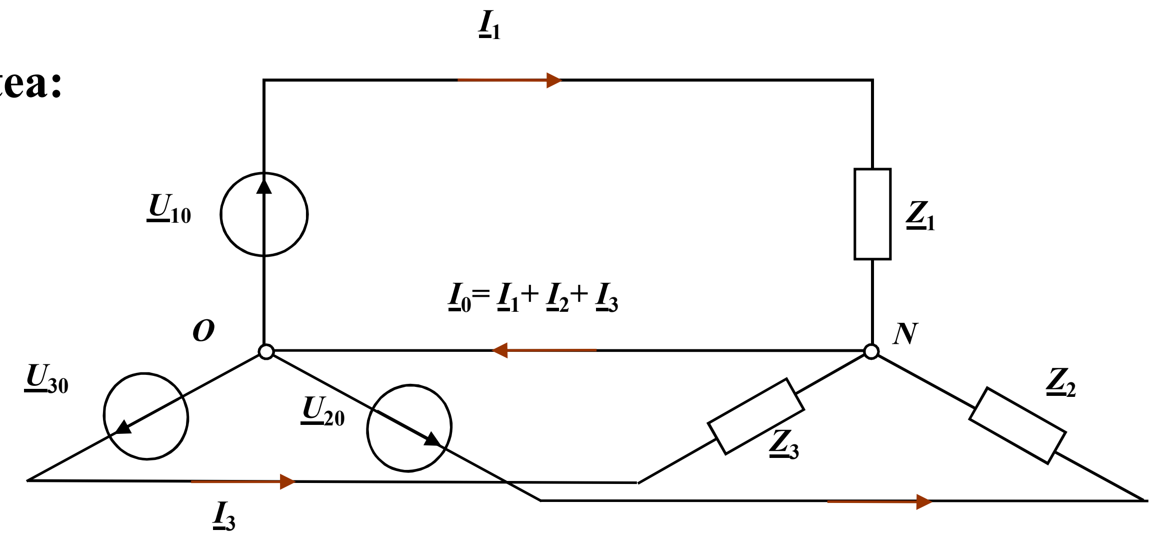
$$\underline{U}_{30} = a\underline{U}$$

Ansamblul celor 3 surse poartă denumirea de **generator** iar ansamblul celor 3 impedanţe poartă denumirea generică de **receptor**. Dacă $\underline{Z}_1 = \underline{Z}_2 = \underline{Z}_3 = \underline{Z}$ se spune că receptorul este echilibrat. În caz contrar receptorul este dezechilibrat.

Fiecare dintre cele 3 circuite poartă denumirea de **fază**.



Conexiune stea:

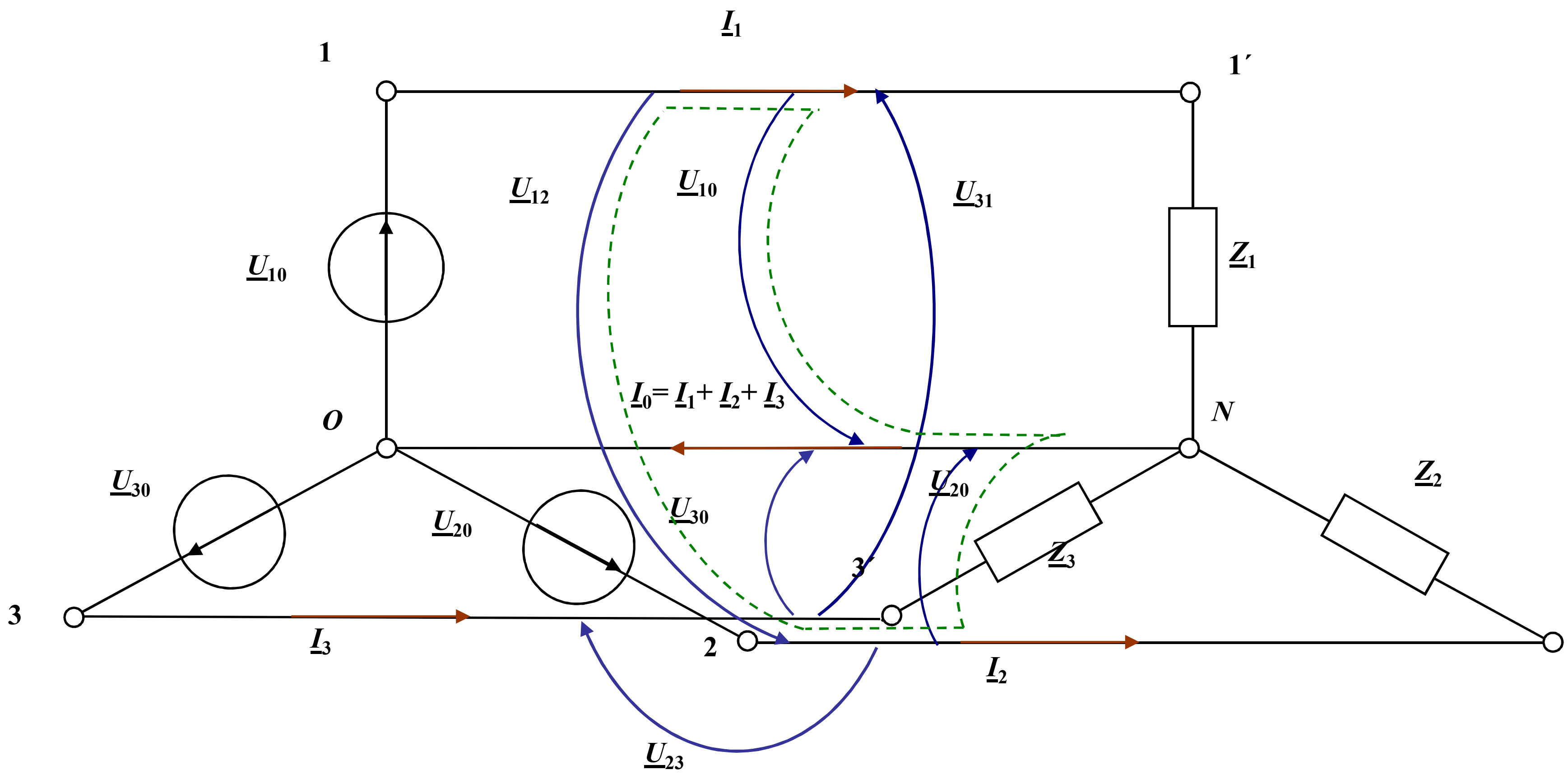


N = **punctul de nul al receptorului**

0 = **punctul de nul al generatorului**

Conductorul care leagă cele 2 puncte = **conductor de nul** al reţelei sau **nul de lucru** al reţelei.

Reţea trifazată cu conexiune stea cu conductor neutru.



Conductoarele 1-1', 2-2', 3-3' formează linia de alimentare trifazată iar curenţii care străbat aceste conductoare poartă denumirea de **curenţi de linie**.

Conductoarele: 1'-N, 2'-N, 3'-N = faze ale receptorului

Conductoarele: 1-0, 2-0, 3-0 = faze ale generatorului

Curenţii care străbat conductoarele de fază = **curenţi de fază**.

Curenţii de linie se notează cu "I_l".

Curenţii de fază se notează cu "I_f".

În cazul unei reţele cu **conexiune stea** se observă că curenţii de linie sunt egali cu curenţii de fază.

$$\boxed{I_l = I_f}$$

Tensiuni de fază = tensiuni măsurate între un conductor de linie şi conductorul de nul: U_f

$$\underline{U}_{10}, \underline{U}_{20}, \underline{U}_{30}$$

Tensiuni de linie = tensiuni măsurate între 2 conductoare de linie: U_l

$$\underline{U}_{12}, \underline{U}_{23}, \underline{U}_{31}$$

Din teorema a doua a lui Kirchhoff rezultă: $\underline{U}_{12} + \underline{U}_{20} - \underline{U}_{10} = 0$

Analog, rezultă relaţiile de legătură între tensiunile de linie şi cele de fază:

$$\begin{cases} \underline{U}_{12} = \underline{U}_{10} - \underline{U}_{20} \\ \underline{U}_{23} = \underline{U}_{20} - \underline{U}_{30} \\ \underline{U}_{31} = \underline{U}_{30} - \underline{U}_{10} \end{cases}$$

$$\underline{U}_{12} + \underline{U}_{23} + \underline{U}_{31} = 0$$

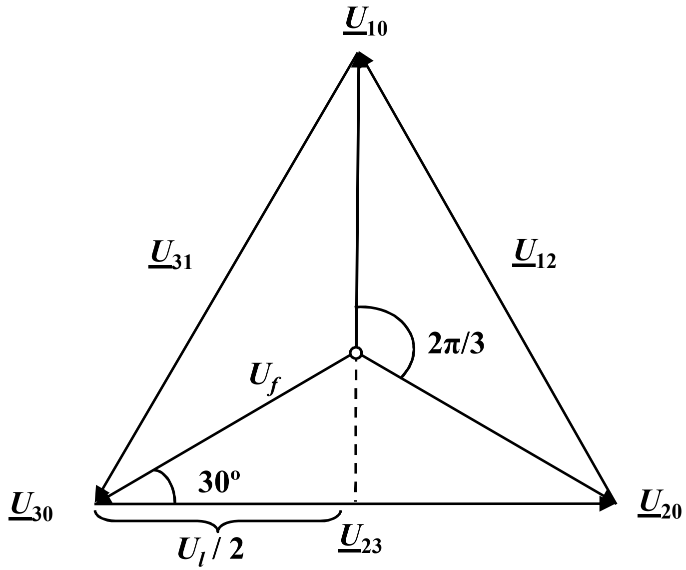
Suma tensiunilor de linie <u>este identic nulă</u> în orice moment.

Presupunem că tensiunile de fază formează un sistem trifazat simetric direct sau invers:

În acest caz, fazorii tensiunilor de linie formează un triunghi echilateral:

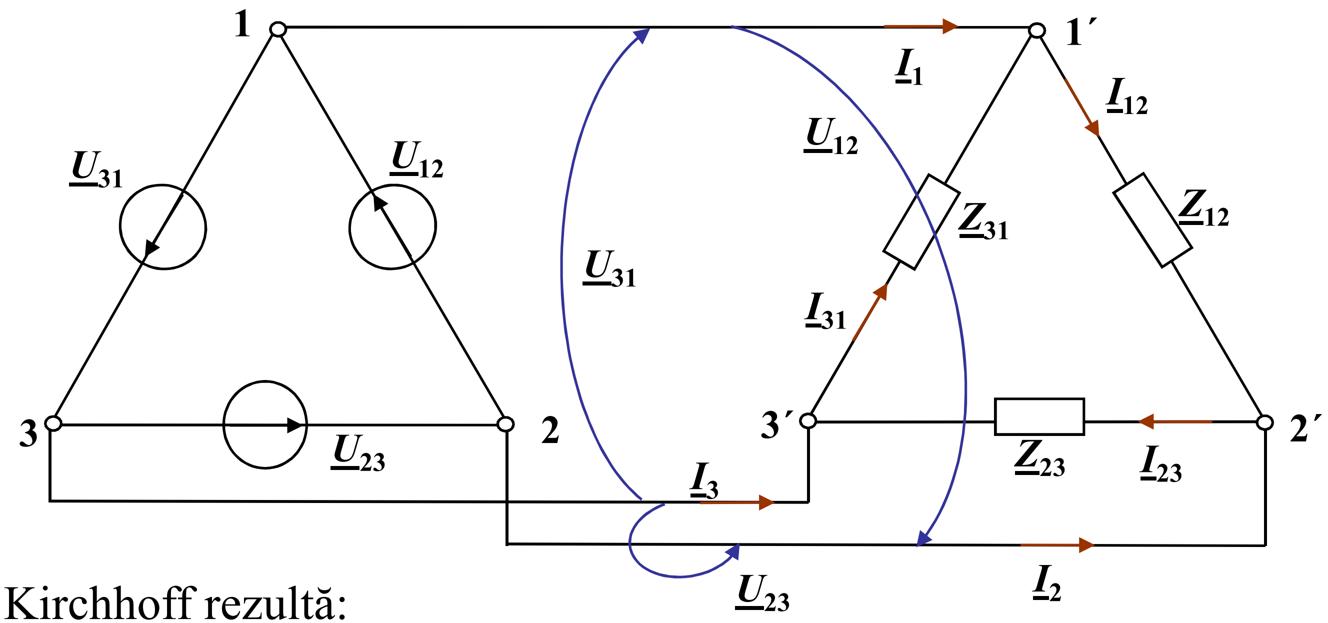
$$\cos 30^o = \frac{\frac{U_l}{2}}{U_f} \qquad \frac{\sqrt{3}}{2} = \frac{U_l}{2U_f}$$

$$\boxed{U_l = \sqrt{3}\, U_f}$$



a doua relaţie care caracterizează conexiunea stea; relaţia este valabilă doar în cazul unui sistem trifazat simetric.

Conexiune triunghi



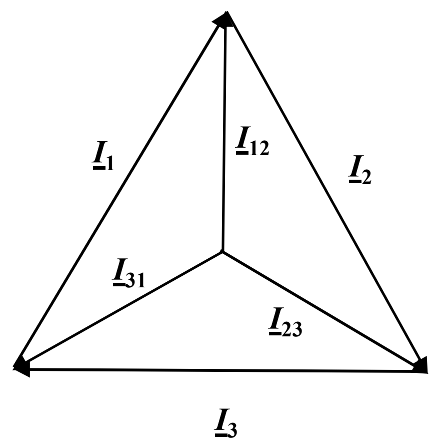
Din teorema întâi a lui Kirchhoff rezultă:

$$\begin{cases} \underline{I}_1 = \underline{I}_{12} - \underline{I}_{31} \\ \underline{I}_2 = \underline{I}_{23} - \underline{I}_{12} \\ \underline{I}_3 = \underline{I}_{31} - \underline{I}_{23} \end{cases} \qquad \underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$$

Suma curenţilor de linie <u>este</u> <u>identic nulă</u> în orice moment.

Presupunem că <u>sistemul curenţilor</u> de fază este <u>simetric direct sau invers</u>. Atunci:

$$\boxed{I_l = \sqrt{3}\, I_f} \qquad \boxed{U_l = U_f}$$



Calculul reţelelor trifazate

A calcula o reţea trifazată = a determina curenţii prin cele 3 faze cunoscând tensiunile aplicate şi impedanţele receptorului.

Referitor la tensiunile aplicate pot intreveni următoarele două situaţii:

- sistem de tensiuni trifazat **simetric**, direct sau invers;
- sistem de tensiuni trifazat **nesimetric**, direct sau invers.

Pentru fiecare din cele două situaţii anterioare:

- Receptorul poate fi: **echilibrat** sau **dezechilibrat**
- Conexiunea poate fi: **stea** sau **triunghi**

Sistem simetric, receptor echilibrat, conexiune stea

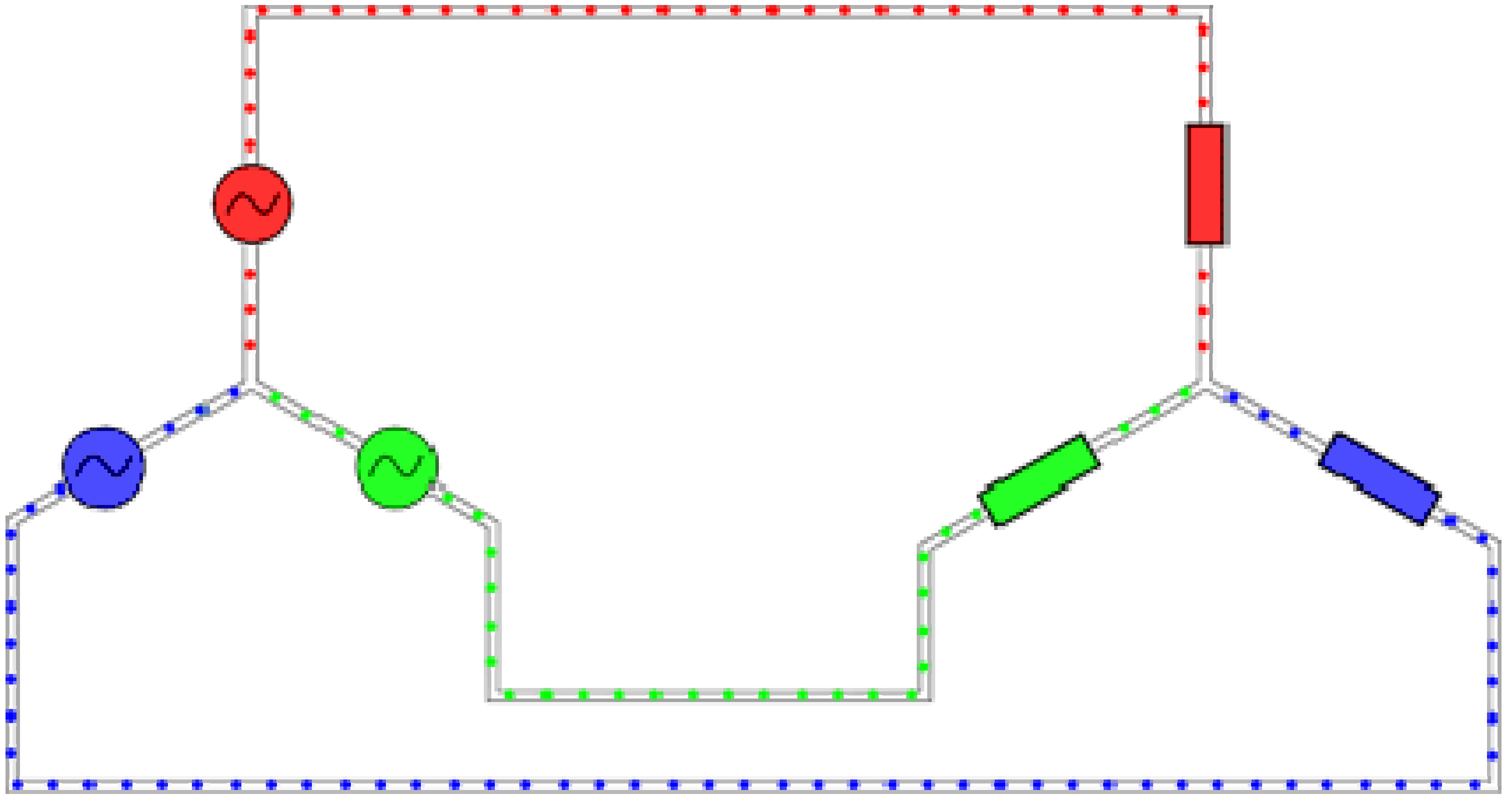
În acest caz cele 3 faze sunt identice şi pentru rezolvarea relaţiei este suficientă determinarea curentului pentru una dintre faze (de regulă, prima fază).

Pentru celelalte faze, curenţii vor avea aceeaşi valoare dar vor fi defazaţi cu $\frac{2\pi}{3}$ în ordinea specificată de sistemul de tensiuni aplicat.

Dacă se aplică un sistem de tensiune simetric direct, curenţii vor forma de asemenea un sistem direct.

În cazul în care conexiunea este triunghi, se face o transfigurare la stea şi se reduce modul de rezolvare la situaţia anterioară.

Linie de alimentare trifazată



Generator trifazat
simetric

Receptor trifazat
echilibrat

	L1	L2	L3	Neutral	Ground / Protective Earth
United States (common practice)	Black	Red	Blue	White or Gray	Green, Green/yellow striped or a bare copper wire
United States (alternative practice)	Brown	Orange (Delta) or Violet (Wye)	Yellow	Gray or White	Green
Canada (mandatory)	Red	Black	Blue	White	Green (or bare copper)
Canada (isolated three-phase installations)	Orange	Brown	Yellow	White	Green
European Union, and all countries who use European CENELEC standards April 2004 (IEC 60446), Hong Kong from July 2007	Brown	Black	Grey	Blue	Green/yellow striped
Older European (IEC 60446, varies by country)	Black or brown	Black or brown	Black or brown	Blue	Green/yellow striped
UK until April 2006, Hong Kong until April 2009, South Africa, Malaysia	Red	Yellow	Blue	Black	Green/yellow striped (green on installations approx. before 1970)
Pakistan	Red	Yellow	Blue	Black	Green
India	Red	Yellow	Blue	Black	Green
Australia and New Zealand (per AS/NZS 3000:2007 Figure 3.2)	Red	White (prev. yellow)	Dark Blue	Black	Green/yellow striped (green on very old installations)
People's Republic of China (per GB 50303-2002 Section 15.2.2)	Yellow	Green	Red	Light Blue	Green/yellow striped

Sistem simetric, receptor dezechilibrat, conexiune stea

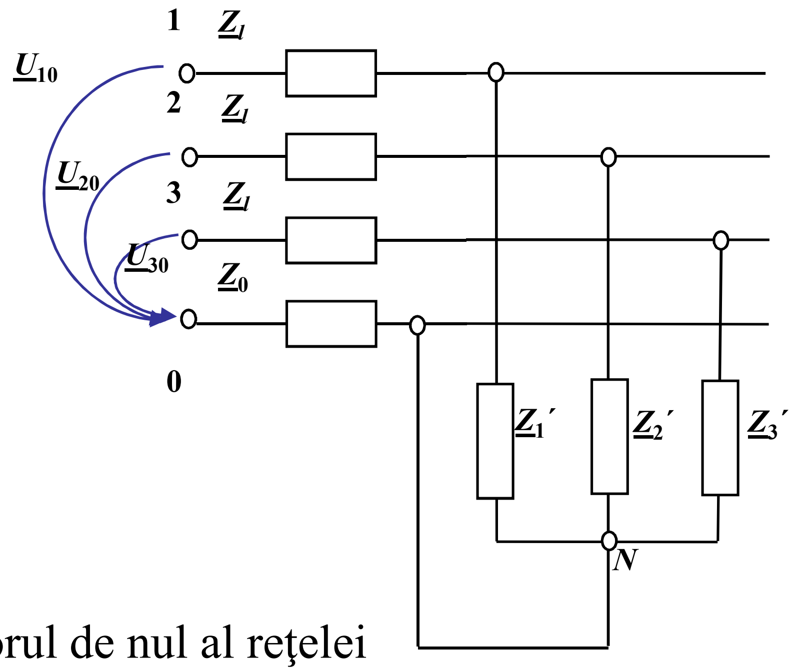
Impedanţa liniei de alimentare:

$$\underline{Z}_l = R_l + jX_l$$

$$R_l = l \cdot r \qquad X_l = l \cdot x$$

r – rezistenţă specifică

x – reactanţă specifică



$$\begin{cases} \underline{U}_{10} = \underline{U} \\ \underline{U}_{20} = a^2\underline{U} \\ \underline{U}_{30} = a\underline{U} \end{cases}$$

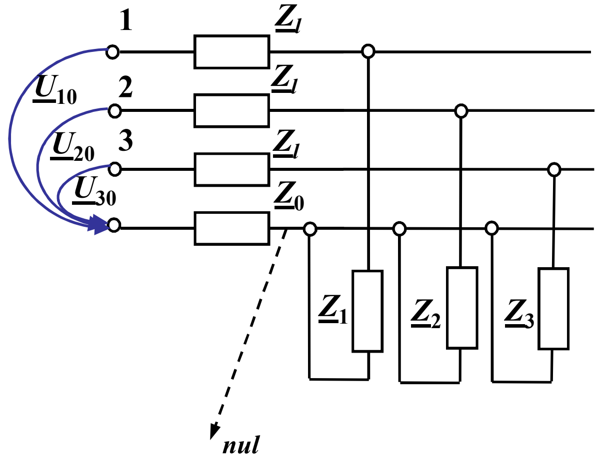
$$U_l = \sqrt{3}\, U_f$$

Dacă legătura de la N la conductorul de nul al reţelei este realizată: => conexiune stea cu fir neutru

Dacă nu: => conexiune stea fără fir neutru.

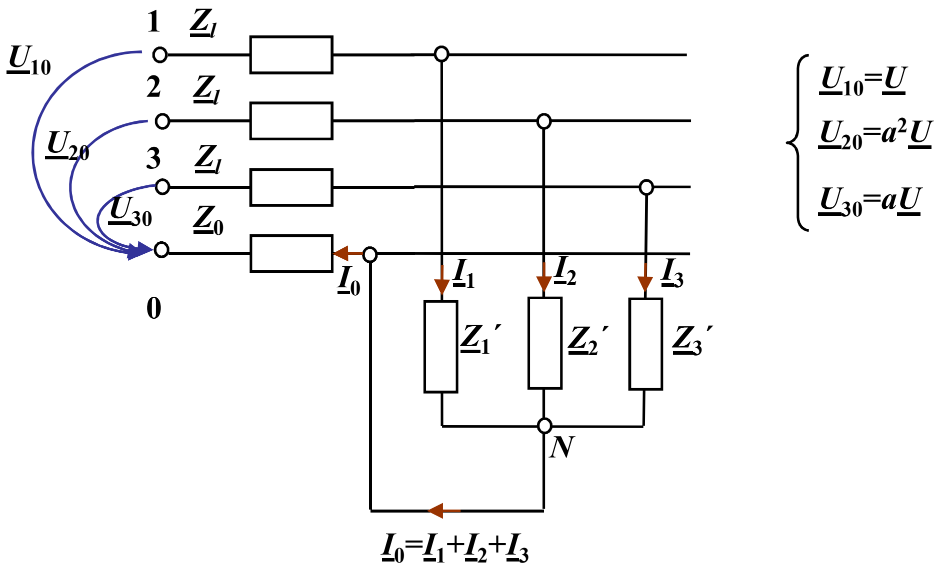
Un receptor monofazat se conectează între o fază şi conductorul de nul:

Dacă sunt mai multe receptoare monofazate ele se leagă astfel încât să se echilibreze sistemul.

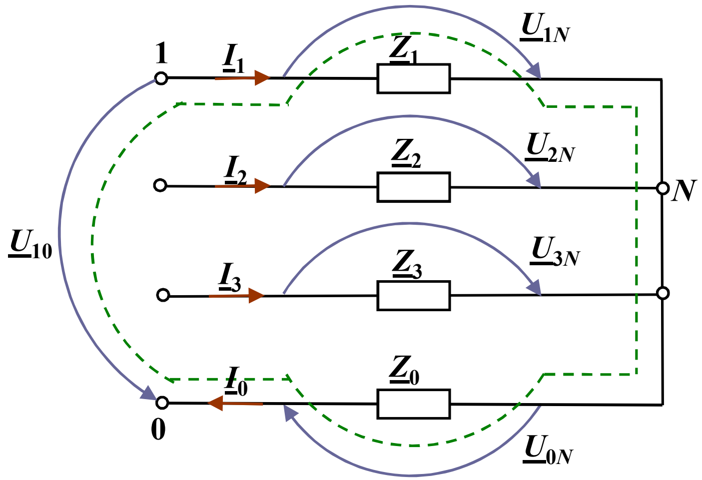


$$\begin{cases} \underline{U}_{10} = \underline{U} \\ \underline{U}_{20} = a^2\underline{U} \\ \underline{U}_{30} = a\underline{U} \end{cases}$$

Presupunem cazul în care legătura de la punctul de nul al receptorului la conductorul de nul al reţelei este realizată:



Schema echivalentă simplificată:

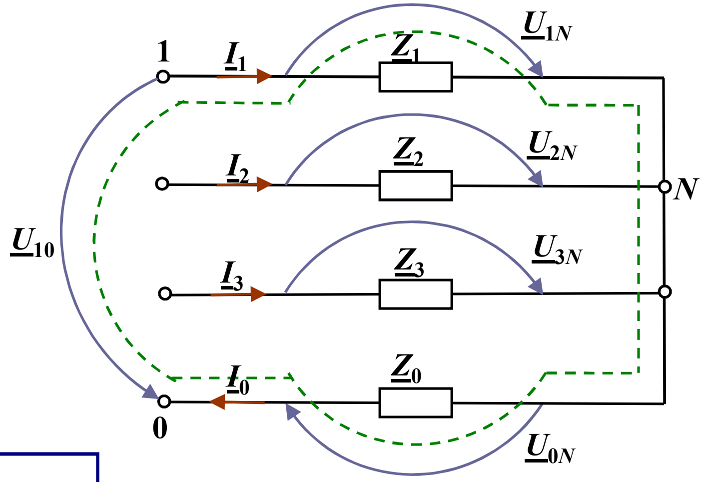


(1)

$$\boxed{\begin{cases} \underline{I}_1 = \underline{Y}_1 \underline{U}_{1N} \\ \underline{I}_2 = \underline{Y}_2 \underline{U}_{2N} \\ \underline{I}_3 = \underline{Y}_3 \underline{U}_{3N} \\ \underline{I}_0 = \underline{Y}_0 \underline{U}_{N0} \end{cases}}$$

$$\underline{Z}_1 = \frac{1}{\underline{Y}_1} \qquad \underline{Z}_3 = \frac{1}{\underline{Y}_3}$$

$$\underline{Z}_2 = \frac{1}{\underline{Y}_2} \qquad \underline{Z}_0 = \frac{1}{\underline{Y}_0}$$



$$\begin{cases} \underline{U}_{10} = \underline{U}_{1N} + \underline{U}_{N0} \\ \underline{U}_{20} = \underline{U}_{2N} + \underline{U}_{N0} \\ \underline{U}_{30} = \underline{U}_{3N} + \underline{U}_{N0} \end{cases} \quad \Rightarrow$$

(2)

$$\boxed{\begin{cases} \underline{U}_{1N} = \underline{U}_{10} - \underline{U}_{N0} \\ \underline{U}_{2N} = \underline{U}_{20} - \underline{U}_{N0} \\ \underline{U}_{3N} = \underline{U}_{30} - \underline{U}_{N0} \end{cases}}$$

$$\begin{cases} \underline{U}_{10} = \underline{U} \\ \underline{U}_{20} = a^2 \underline{U} \\ \underline{U}_{30} = a \underline{U} \end{cases}$$

$$\underline{U}_{No} = ?$$

$$\underline{I}_0 = \underline{I}_1 + \underline{I}_2 + \underline{I}_3 \quad \Rightarrow \quad \underline{Y}_0 \underline{U}_{N0} = \underline{Y}_1 \left(\underline{U}_{10} - \underline{U}_{N0}\right) + \underline{Y}_2 \left(\underline{U}_{20} - \underline{U}_{N0}\right) + \underline{Y}_3 \left(\underline{U}_{30} - \underline{U}_{N0}\right)$$

(3)

$$\boxed{\underline{U}_{N0} = \frac{\underline{Y}_1 \underline{U}_{10} + \underline{Y}_2 \underline{U}_{20} + \underline{Y}_3 \underline{U}_{30}}{Y_1 + Y_2 + Y_3 + Y_0}}$$

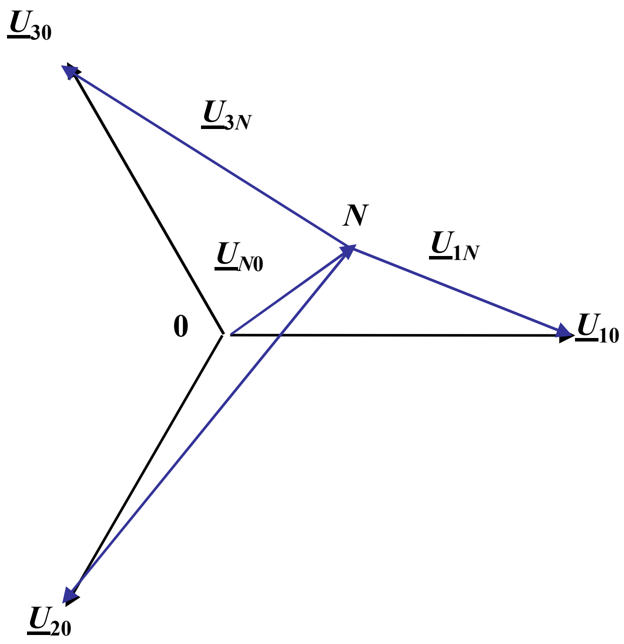
Procedura de rezolvare va fi următoarea:

• se determină $\underline{U}_{N0}$ cu relaţia (**3**)

• se calculează $\underline{U}_{1N}, \underline{U}_{2N}, \underline{U}_{3N}$ cu relaţiile (**2**)

• curenţii se calculează cu relaţiile (**1**)

• $\underline{U}_{N0}$ poartă denumirea de **deplasare a nulului**



$\underline{U}_{N0} \neq 0$ => nulul receptorului nu coincide cu nulul generatorului => cădere de tensiune între **0** şi ***N***.

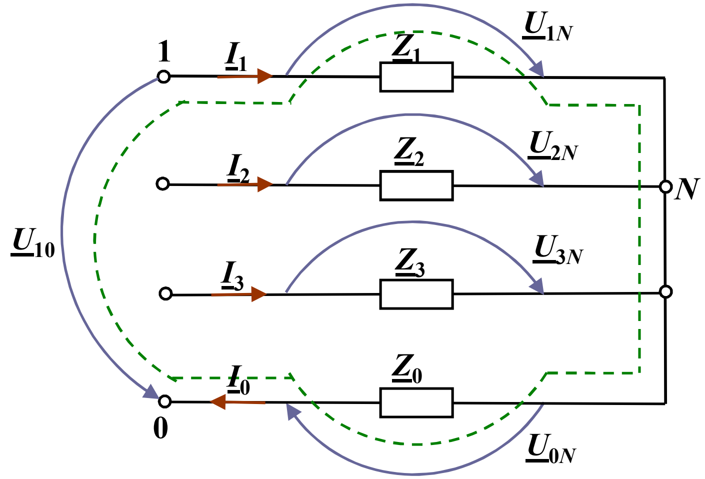
Deplasarea nulului trebuie să fie cât mai mică.

$\underline{U}_{1N}, \underline{U}_{2N}, \underline{U}_{3N}$: formează un sistem trifazat nesimetric

Dacă receptorul este dezechilibrat tensiunile care se aplică pe fazele receptorului nu mai sunt egale => o proastă funcţionare a receptorului.

Cu cât $\underline{U}_{N0}$ este mai mare cu atât nesimetria sistemului de tensiuni U_{1N}, U_{2N}, U_{3N} este mai mare.

Calculul puterilor.



Se însumează puterile complexe pe fiecare fază:

$$\underline{S} = \underline{U}_{1N}\underline{I}_1^* + \underline{U}_{2N}\underline{I}_2^* + \underline{U}_{3N}\underline{I}_3^* + \underline{U}_{N0}\underline{I}_0^* \qquad \underline{I}_0 = \underline{I}_1 + \underline{I}_2 + \underline{I}_3$$

$$\underline{S} = \underbrace{\left(\underline{U}_{1N} + \underline{U}_{N0}\right)}_{\underline{U}_{10}}\underline{I}_1^* + \underbrace{\left(\underline{U}_{2N} + \underline{U}_{N0}\right)}_{\underline{U}_{20}}\underline{I}_2^* + \underbrace{\left(\underline{U}_{3N} + \underline{U}_{N0}\right)}_{\underline{U}_{30}}\underline{I}_3^*$$

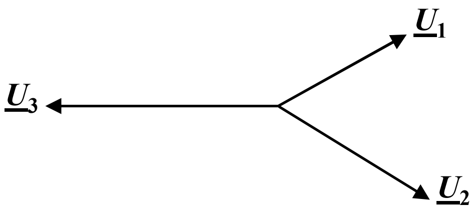
(4) $$\boxed{\underline{S} = \underline{U}_{10}\underline{I}_1^* + \underline{U}_{20}\underline{I}_2^* + \underline{U}_{30}\underline{I}_3^*}$$

Relaţia (**4**) este valabilă şi în cazul în care legătura la nul este întreruptă deci şi în cazul conexiunii în triunghi.

Receptor echilibrat, sistem de tensiuni nesimetric

Metoda de rezolvare în această situaţie este **metoda componentelor simetrice**

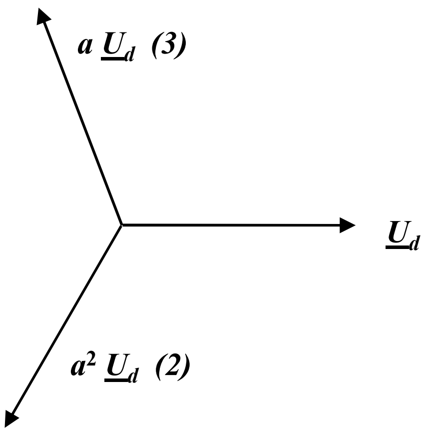
Orice sistem trifazat nesimetric poate fi descompus în 3 sisteme trifazate simetrice.

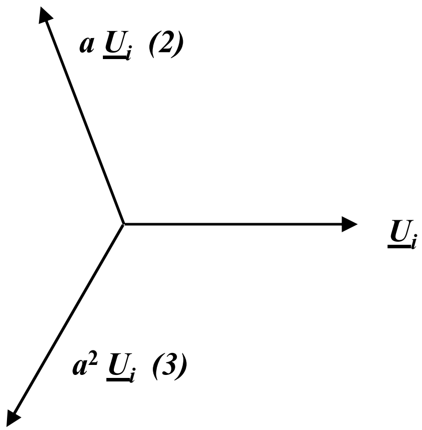


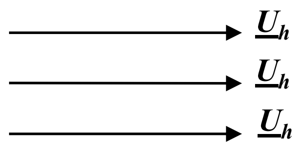
1 Sistem simetric direct $\underline{U}_d, \quad a^2\underline{U}_d, \quad a\underline{U}_d$

2 Sistem simetric invers $\underline{U}_i, \quad a\underline{U}_i, \quad a^2\underline{U}_i$

3 Sistem omopolar $\underline{U}_h, \quad \underline{U}_h, \quad \underline{U}_h$







$$\begin{cases} \underline{U}_1 = \underline{U}_h + \underline{U}_d + \underline{U}_i \\ \underline{U}_2 = \underline{U}_h + a^2\underline{U}_d + a\underline{U}_i \\ \underline{U}_3 = \underline{U}_h + a\underline{U}_d + a^2\underline{U}_i \end{cases}$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} = a^4 + a + a - a^2 - a^2 - a^2 = 3\left(a - a^2\right) = 3\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2} + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = 3\sqrt{3}\,j \neq 0$$

$$\underline{U}_1 + \underline{U}_2 + \underline{U}_3 = 3\underline{U}_h + \underline{U}_d\underbrace{\left(1 + a^2 + a\right)}_{0} + \underline{U}_i\underbrace{\left(1 + a + a^2\right)}_{0}$$

$$\boxed{\underline{U}_h = \frac{1}{3}\left(\underline{U}_1 + \underline{U}_2 + \underline{U}_3\right)}$$

$$\left\{\begin{aligned} \underline{U}_1 &= \underline{U}_h + \underline{U}_d + \underline{U}_i \\ \underline{U}_2 &= \underline{U}_h + a^2\underline{U}_d + a\underline{U}_i \\ \underline{U}_3 &= \underline{U}_h + a\underline{U}_d + a^2\underline{U}_i \end{aligned}\right. \qquad \left|\begin{matrix} 1 \\ a \\ a^2 \end{matrix}\right.$$

$$\underline{U}_1 + a\underline{U}_2 + a^3\underline{U}_3 = \underline{U}_h\underbrace{\left(1 + a + a^2\right)}_{0} + \underline{U}_d\underbrace{\left(1 + a^3 + a^3\right)}_{3} + \underline{U}_i\underbrace{\left(1 + a^2 + a^4\right)}_{0}$$

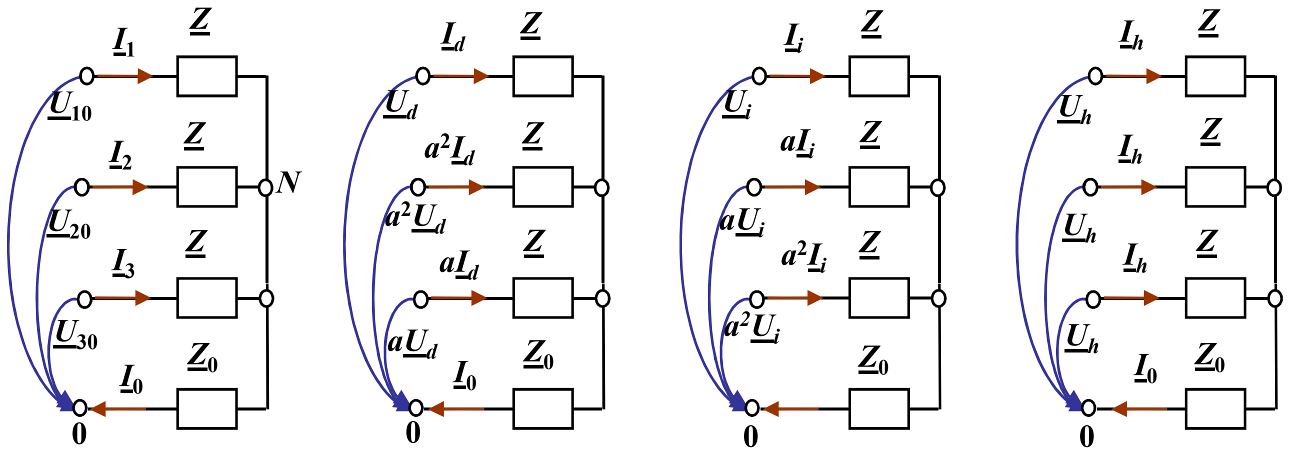
$$\boxed{\underline{U}_d = \frac{1}{3}\left(\underline{U}_1 + a\underline{U}_2 + a^2\underline{U}_3\right)}$$

$$\left\{\begin{aligned} \underline{U}_1 &= \underline{U}_h + \underline{U}_d + \underline{U}_i \\ \underline{U}_2 &= \underline{U}_h + a^2\underline{U}_d + a\underline{U}_i \\ \underline{U}_3 &= \underline{U}_h + a\underline{U}_d + a^2\underline{U}_i \end{aligned}\right. \qquad \left|\begin{matrix} 1 \\ a^2 \\ a \end{matrix}\right.$$

$$\underline{U}_1 + a^2\underline{U}_2 + a\underline{U}_3 = \underline{U}_h\underbrace{\left(1 + a^2 + a\right)}_{0} + \underline{U}_d\underbrace{\left(1 + a^4 + a^2\right)}_{0} + \underline{U}_i\underbrace{\left(1 + a^3 + a^3\right)}_{3}$$

$$\boxed{\underline{U}_i = \frac{1}{3}\left(\underline{U}_1 + a^2\underline{U}_2 + a\underline{U}_3\right)}$$

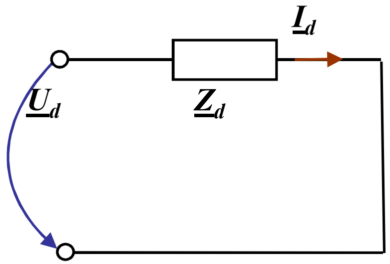
Se consideră un receptor echilibrat cu conexiune stea cu conductor neutru <u>fără</u> <u>cuplaje magnetice</u> între faze.



$\underline{I}_0=\underline{I}_d(1+a^2+a)=0$ $\underline{I}_0=\underline{I}_i(1+a+a^2)=0$ $\underline{I}_0=3\underline{I}_h$

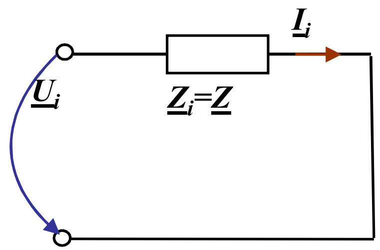
Se determină componentele simetrice ale curenţilor descompunând reţeaua în 3 sisteme (teorema superpoziţiei); sistemele fiind echilibrate curenţii formează sisteme trifazate simetrice.

$$\underline{I}_d=\frac{\underline{U}_d}{\underline{Z}}$$



$$\underline{Z}_d=\underline{Z}$$

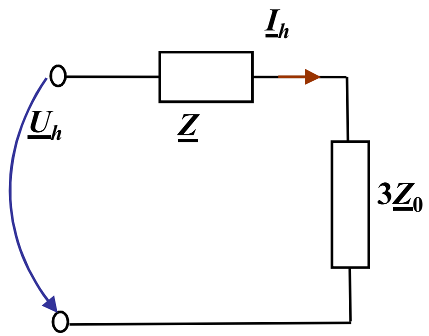
$$\underline{I}_i=\frac{\underline{U}_i}{\underline{Z}}$$



$$\underline{Z}_i=\underline{Z}$$

$$0 = \underline{Z}\,\underline{I}_h + \underline{Z}_0 3\underline{I}_h - \underline{U}_h$$

$$\underline{I}_h = \frac{\underline{U}_h}{\underline{Z} + 3\underline{Z}_0}$$



$$\underline{Z}_h = \frac{\underline{U}_h}{\underline{I}_h} = \underline{Z} + 3\underline{Z}_0$$

$$\underline{Z}_h = \underline{Z} + 3\underline{Z}_0$$

$\underline{I}_1$, $\underline{I}_2$, $\underline{I}_3$ se obţin cu relaţiile :

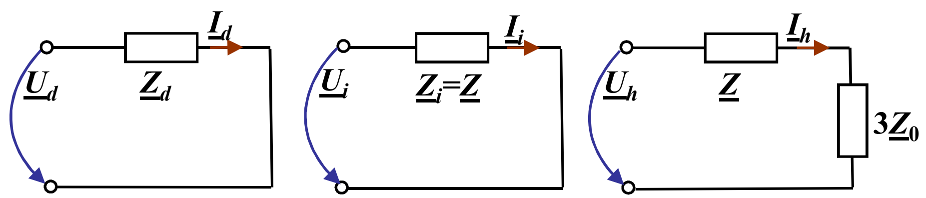
$$\begin{cases} \underline{I}_1 = \underline{I}_d + \underline{I}_i + \underline{I}_h \\ \underline{I}_2 = a^2 \underline{I}_d + a\underline{I}_i + \underline{I}_h \\ \underline{I}_3 = a\underline{I}_d + a^2 \underline{I}_i + \underline{I}_h \\ \underline{I}_0 = 3\underline{I}_h \end{cases}$$

Etapele de rezolvare:

1. Determinarea componentelor simetrice ale sistemului de tensiuni aplicat reţelei:

$$\underline{U}_h, \quad \underline{U}_d \quad ,\underline{U}_i$$

2. Determinarea componentelor simetrice ale curenţilor prin rezolvarea celor trei scheme monofazate:



$$\underline{Z}_d = \underline{Z} \qquad \underline{Z}_i = \underline{Z} \qquad \underline{Z}_h = \underline{Z} + 3\underline{Z}_0$$

3. Determinarea curenţilor cu relaţiile

$$\begin{cases} \underline{I}_1 = \underline{I}_d + \underline{I}_i + \underline{I}_h \\ \underline{I}_2 = a^2\underline{I}_d + a\underline{I}_i + \underline{I}_h \\ \underline{I}_3 = a\underline{I}_d + a^2\underline{I}_i + \underline{I}_h \\ \underline{I}_0 = 3\underline{I}_h \end{cases}$$

Puterea absorbită de la reţea exprimată în funcţie de componentele simetrice ale tensiunilor şi curenţilor

$$\left(a^2\right)^* = a$$

$$\left(a\right)^* = a^2$$

$$\underline{S} = \underline{U}_{10}\underline{I}_1^* + \underline{U}_{20}\underline{I}_2^* + \underline{U}_{30}\underline{I}_3^* =$$

$$= \left(\underline{U}_h + \underline{U}_d + \underline{U}_i\right)\underline{I}_1^* + \left(\underline{U}_h + a^2\underline{U}_d + a\underline{U}_i\right)\underline{I}_2^* + \left(\underline{U}_h + a\underline{U}_d + a^2\underline{U}_i\right)\underline{I}_3^*$$

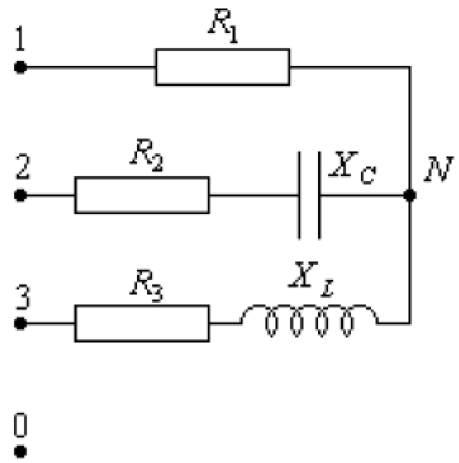
$$= \underline{U}_h\left(\underline{I}_1^* + \underline{I}_2^* + \underline{I}_3^*\right) + \underline{U}_d\left(\underline{I}_1^* + a^2\underline{I}_2^* + a\underline{I}_3^*\right) + \underline{U}_i\left(\underline{I}_1^* + a\underline{I}_2^* + a^2\underline{I}_3^*\right) =$$

$$= \underline{U}_h 3\underline{I}_h^* + \underline{U}_d 3\underline{I}_d^* + \underline{U}_i 3\underline{I}_i^* =$$

$$= 3\left(\underline{U}_d\underline{I}_d^* + \underline{U}_i\underline{I}_i^* + \underline{U}_h\underline{I}_h^*\right)$$

$$\boxed{\underline{S} = 3\left(\underline{U}_d\underline{I}_d^* + \underline{U}_i\underline{I}_i^* + \underline{U}_h\underline{I}_h^*\right)}$$

Probleme propuse:



Se dau:

$U_f = 120V$, $R_1 = 8\Omega$, $R_2 = R_3 = 1\Omega$,

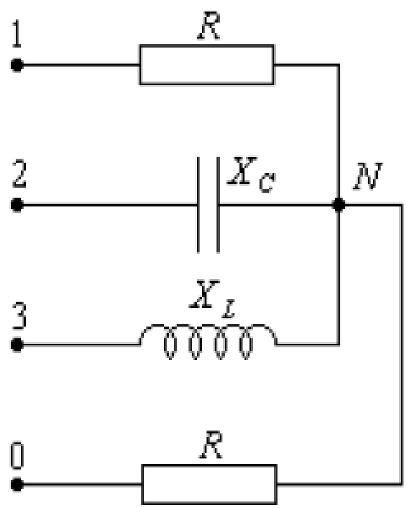
$|X_C| = \sqrt{3}\Omega$, $X_L = \sqrt{3}\Omega$

Se cer:

a) $\underline{U}_{1N}$, $\underline{U}_{2N}$, $\underline{U}_{3N}$

b) $\underline{I}_1$, $\underline{I}_2$, $\underline{I}_3$ şi $i_1(t)$, $i_2(t)$, $i_3(t)$

c) $\underline{S}$



Se dau:

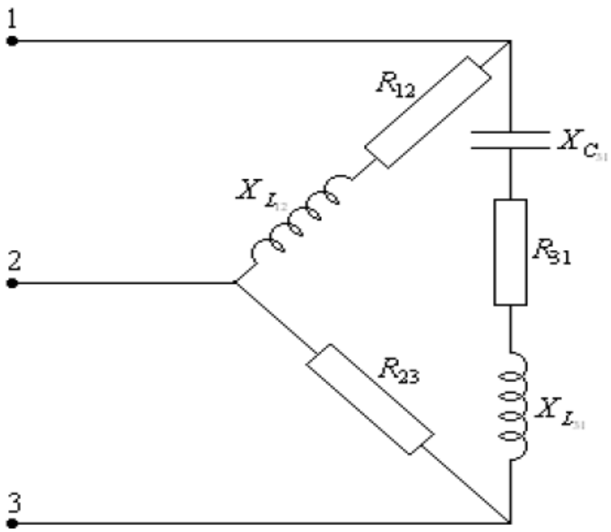
$U_f = 120V$, $R = 5\Omega$, $X_L = |X_C| = 5 \cdot \sqrt{3}\Omega$

Se cer:

a) $\underline{U}_{1N}$, $\underline{U}_{2N}$, $\underline{U}_{3N}$

b) $\underline{I}_1$, $\underline{I}_2$, $\underline{I}_3$ şi $i_1(t)$, $i_2(t)$, $i_3(t)$

c) $\underline{S}$



Se dau:

$U_l = 120V$, $R_{12} = R_{31} = 1\Omega$, $R_{23} = 2\Omega$,

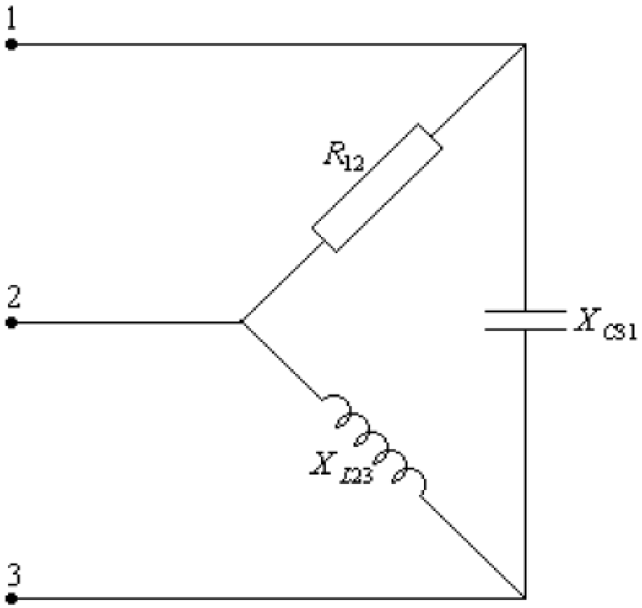
$X_{L_{12}} = \left|X_{C_{31}}\right| = \sqrt{3}\Omega$, $X_{L_{31}} = 2 \cdot \sqrt{3}\Omega$

Se cer:

a) $\underline{I}_1$, $\underline{I}_2$, $\underline{I}_3$

b) $\underline{I}_{12}$, $\underline{I}_{23}$, $\underline{I}_{31}$

c) $\underline{S}$



Se dau:

$U_l = 120V$, $R_{12} = 1\Omega$, $X_{L23} = \left|X_{C31}\right| = \sqrt{3}\Omega$.

Se cer:

a) $\underline{I}_1$, $\underline{I}_2$, $\underline{I}_3$

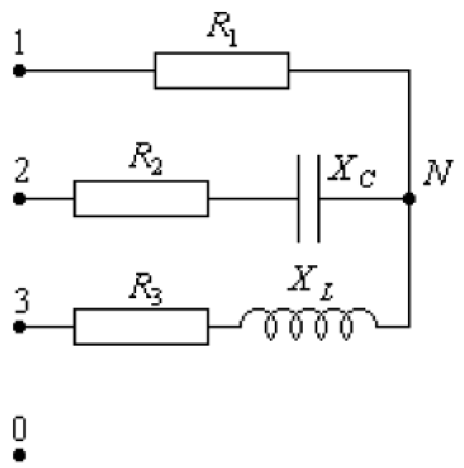
b) $\underline{I}_{12}$, $\underline{I}_{23}$, $\underline{I}_{31}$

c) $\underline{S}$

Subiecte examen

1. Conexiunea stea în reţele trifazate – desen, curenti de line/faza, tensiuni de line/faza.
2. Conexiunea triunghi în reţele trifazate – desen, curenti de line/faza, tensiuni de line/faza.

Rezolvare aplicatie 1:



Se dau:

$U_f = 120V$, $R_1 = 8\Omega$, $R_2 = R_3 = 1\Omega$,

$|X_C| = \sqrt{3}\Omega$, $X_L = \sqrt{3}\Omega$

Se cer:

a) $\underline{U}_{1N}$, $\underline{U}_{2N}$, $\underline{U}_{3N}$

b) $\underline{I}_1$, $\underline{I}_2$, $\underline{I}_3$ şi $i_1(t)$, $i_2(t)$, $i_3(t)$

c) $\underline{S}$

Circuitul fiind alimentat cu tensiuni ce formează un sistem simetric, rezultă în complex tensiunile de fază:

$$u_{10}(t) = U_f \cdot \sqrt{2} \cdot \sin(\omega \cdot t) \qquad \Rightarrow \qquad \underline{U}_{10} = U_f = 120V$$

$$u_{20}(t) = U_f \cdot \sqrt{2} \cdot \sin\left(\omega \cdot t - \frac{2 \cdot \pi}{3}\right) \qquad \Rightarrow \qquad \underline{U}_{20} = U_f \cdot \left(-\frac{1}{2} - j \cdot \frac{\sqrt{3}}{2}\right) = 120 \cdot \left(-\frac{1}{2} - j \cdot \frac{\sqrt{3}}{2}\right) V$$

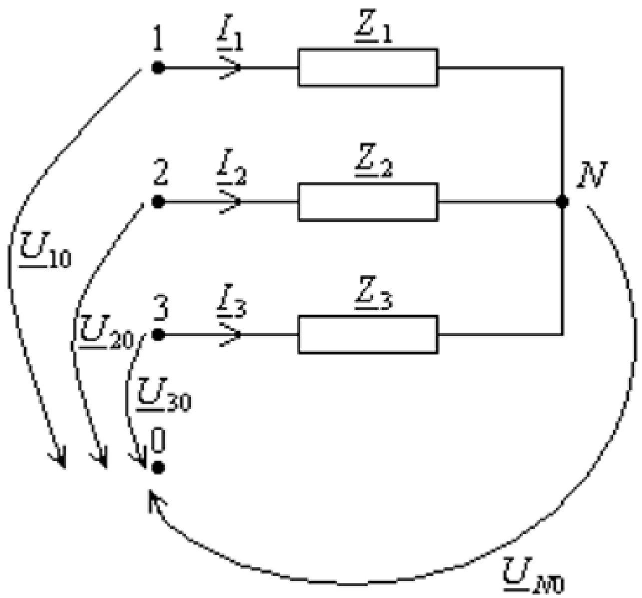
$$u_{30}(t) = U_f \cdot \sqrt{2} \cdot \sin\left(\omega \cdot t + \frac{2 \cdot \pi}{3}\right) \qquad \Rightarrow \qquad \underline{U}_{30} = U_f \cdot \left(-\frac{1}{2} + j \cdot \frac{\sqrt{3}}{2}\right) = 120 \cdot \left(-\frac{1}{2} + j \cdot \frac{\sqrt{3}}{2}\right) V$$

Calculăm impedanțele şi admitanțele fazelor:

$$\underline{Z}_1 = R_1 = 8\Omega \qquad \Rightarrow \underline{Y}_1 = \frac{1}{\underline{Z}_1} = \frac{1}{8}S$$

$$\underline{Z}_2 = R_2 - j \cdot X_C = 1 - j \cdot \sqrt{3}\Omega \qquad \Rightarrow \underline{Y}_2 = \frac{1}{\underline{Z}_2} = \frac{1}{1 - j \cdot \sqrt{3}} = \frac{1 + j \cdot \sqrt{3}}{4}S$$

$$\underline{Z}_3 = R_3 + j \cdot X_L = 1 + j \cdot \sqrt{3}\Omega \qquad \Rightarrow \underline{Y}_3 = \frac{1}{\underline{Z}_3} = \frac{1}{1 + j \cdot \sqrt{3}} = \frac{1 - j \cdot \sqrt{3}}{4}S$$



1. Calculăm deplasarea punctului neutru:

$$\underline{U}_{N0} = \frac{\underline{U}_{10}\cdot\underline{Y}_1+\underline{U}_{20}\cdot\underline{Y}_2+\underline{U}_{30}\cdot\underline{Y}_3}{\underline{Y}_1+\underline{Y}_2+\underline{Y}_3+\underline{Y}_N} =$$

$$= \frac{120\cdot\frac{1}{8}+120\cdot\left(-\frac{1}{2}-j\cdot\frac{\sqrt{3}}{2}\right)\cdot\frac{1+j\cdot\sqrt{3}}{4}+120\cdot\left(-\frac{1}{2}+j\cdot\frac{\sqrt{3}}{2}\right)\cdot\frac{1-j\cdot\sqrt{3}}{4}}{\frac{1}{8}+\frac{1+j\cdot\sqrt{3}}{4}+\frac{1-j\cdot\sqrt{3}}{4}} =$$

$$= \frac{15+15\cdot\left(-1-j\cdot\sqrt{3}\right)\cdot\left(1+j\cdot\sqrt{3}\right)+15\cdot\left(-1+j\cdot\sqrt{3}\right)\cdot\left(1-j\cdot\sqrt{3}\right)}{\frac{5}{8}} =$$

$$= \frac{8}{5}\cdot 15\cdot\left[1-\left(1+2\cdot j\cdot\sqrt{3}-3\right)-\left(1-2\cdot j\cdot\sqrt{3}-3\right)\right] = 120V$$

2. Calculăm tensiunile de fază la receptor:

$$\underline{U}_{1N} = \underline{U}_{10}-\underline{U}_{N0} = 120-120 = 0V$$

$$\underline{U}_{2N} = \underline{U}_{20}-\underline{U}_{N0} = 120\cdot\left(-\frac{1}{2}-j\cdot\frac{\sqrt{3}}{2}\right)-120 = 120\cdot\left(-\frac{3}{2}-j\cdot\frac{\sqrt{3}}{2}\right)V$$

$$\underline{U}_{3N} = \underline{U}_{30}-\underline{U}_{N0} = 120\cdot\left(-\frac{1}{2}+j\cdot\frac{\sqrt{3}}{2}\right)-120 = 120\cdot\left(-\frac{3}{2}+j\cdot\frac{\sqrt{3}}{2}\right)V$$

3. Calculăm curenții din fazele receptorului $\underline{I}_1$, $\underline{I}_2$, $\underline{I}_3$:

$$\underline{I}_1 = \underline{U}_{1N} \cdot \underline{Y}_1 = 0A$$

$$\underline{I}_2 = \underline{U}_{2N} \cdot \underline{Y}_2 = 120 \cdot \left(-\frac{3}{2} - j \cdot \frac{\sqrt{3}}{2} \right) \cdot \frac{1 + j \cdot \sqrt{3}}{4} = 15 \cdot \left(-3 - j \cdot \sqrt{3} \right) \cdot \left(1 + j \cdot \sqrt{3} \right) = -60 \cdot j \cdot \sqrt{3} A$$

$$\underline{I}_3 = \underline{U}_{3N} \cdot \underline{Y}_3 = 120 \cdot \left(-\frac{3}{2} + j \cdot \frac{\sqrt{3}}{2} \right) \cdot \frac{1 - j \cdot \sqrt{3}}{4} = 15 \cdot \left(-3 + j \cdot \sqrt{3} \right) \cdot \left(1 - j \cdot \sqrt{3} \right) = 60 \cdot j \cdot \sqrt{3} A$$

şi:

$$i_1(t) = 0A$$

$$i_2(t) = 60 \cdot \sqrt{6} \cdot \sin\left(\omega \cdot t - \frac{2 \cdot \pi}{3} \right)$$

$$i_3(t) = 60 \cdot \sqrt{6} \cdot \sin\left(\omega \cdot t + \frac{2 \cdot \pi}{3} \right)$$

$$P_c = \mathrm{Re}\{\underline{Z}_1\} \cdot I_1^2 + \mathrm{Re}\{\underline{Z}_2\} \cdot I_2^2 + \mathrm{Re}\{\underline{Z}_3\} \cdot I_3^2$$

$$Q_c = \mathrm{Im}\{\underline{Z}_1\} \cdot I_1^2 + \mathrm{Im}\{\underline{Z}_2\} \cdot I_2^2 + \mathrm{Im}\{\underline{Z}_3\} \cdot I_3^2$$

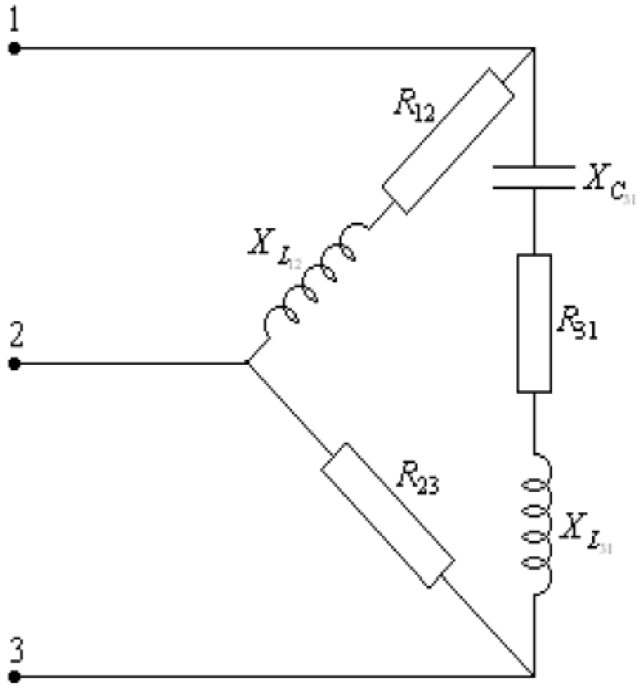
4. Verificăm bilanțul de puteri:

$$\begin{aligned}
\underline{S}_b &= \underline{U}_{10} \cdot \underline{I}_1^* + \underline{U}_{20} \cdot \underline{I}_2^* + \underline{U}_{30} \cdot \underline{I}_3^* = \\
&= 120 \cdot 0 + 120 \cdot \left(-\frac{3}{2} - j \cdot \frac{\sqrt{3}}{2} \right) \cdot \left(60 \cdot j \cdot \sqrt{3} \right) + 120 \cdot \left(-\frac{3}{2} + j \cdot \frac{\sqrt{3}}{2} \right) \cdot \left(-60 \cdot j \cdot \sqrt{3} \right) = \\
&= 0 + 60^2 \cdot \left(3 - j \cdot \sqrt{3} \right) + 60^2 \cdot \left(3 + j \cdot \sqrt{3} \right) = 6 \cdot 60^2 VA
\end{aligned}$$

$$\begin{aligned}
\underline{S}_c &= \underline{Z}_1 \cdot \underline{I}_1 \cdot \underline{I}_1^* + \underline{Z}_2 \cdot \underline{I}_2 \cdot \underline{I}_2^* + \underline{Z}_3 \cdot \underline{I}_3 \cdot \underline{I}_3^* = \\
&= 8 \cdot 0 \cdot 0 + \left(1 - j \cdot \sqrt{3} \right) \cdot \left(-60 \cdot j \cdot \sqrt{3} \right) \cdot \left(60 \cdot j \cdot \sqrt{3} \right) + \left(1 + j \cdot \sqrt{3} \right) \cdot \left(60 \cdot j \cdot \sqrt{3} \right) \cdot \left(-60 \cdot j \cdot \sqrt{3} \right) = \\
&= 0 + 3 \cdot 60^2 \cdot \left(1 - j \cdot \sqrt{3} \right) + 3 \cdot 60^2 \cdot \left(1 + j \cdot \sqrt{3} \right) = 6 \cdot 60^2 VA
\end{aligned}$$

Se observă că $\underline{S}_b = \underline{S}_c$ deci soluția este corectă.

Rezolvare aplicatie 3:



Se dau:

$U_l = 120V$, $R_{12} = R_{31} = 1\Omega$, $R_{23} = 2\Omega$,

$X_{L_{12}} = \left|X_{C_{31}}\right| = \sqrt{3}\Omega$, $X_{L_{31}} = 2 \cdot \sqrt{3}\Omega$

Se cer:

a) $\underline{I}_1$, $\underline{I}_2$, $\underline{I}_3$

b) $\underline{I}_{12}$, $\underline{I}_{23}$, $\underline{I}_{31}$

c) $\underline{S}$

Rezolvare:

Circuitul fiind alimentat cu tensiuni de linie ce formează un sistem simetric, rezultă în complex tensiunile de fază:

$$u_{12}(t) = U_l \cdot \sqrt{2} \cdot \sin(\omega \cdot t) \quad \Rightarrow \quad \underline{U}_{12} = U_l = 120V$$

$$u_{23}(t) = U_l \cdot \sqrt{2} \cdot \sin\left(\omega \cdot t - \frac{2 \cdot \pi}{3}\right) \quad \Rightarrow \quad \underline{U}_{23} = U_l \cdot \left(-\frac{1}{2} - j \cdot \frac{\sqrt{3}}{2}\right) = 120 \cdot \left(-\frac{1}{2} - j \cdot \frac{\sqrt{3}}{2}\right) V$$

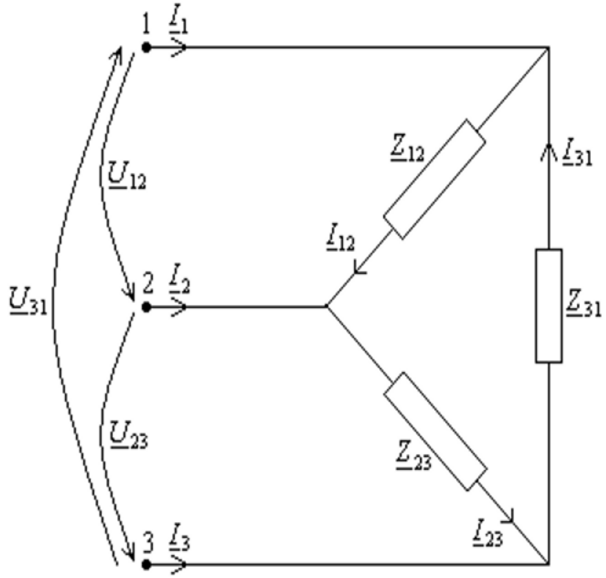
$$u_{31}(t) = U_l \cdot \sqrt{2} \cdot \sin\left(\omega \cdot t + \frac{2 \cdot \pi}{3}\right) \quad \Rightarrow \quad \underline{U}_{31} = U_l \cdot \left(-\frac{1}{2} + j \cdot \frac{\sqrt{3}}{2}\right) = 120 \cdot \left(-\frac{1}{2} + j \cdot \frac{\sqrt{3}}{2}\right) V$$

Calculăm impedanțele fazelor:

$$\underline{Z}_{12} = R_{12} + j \cdot X_{L_{12}} = 1 + j \cdot \sqrt{3}\Omega$$

$$\underline{Z}_{23} = R_{23} = 2\Omega$$

$$\underline{Z}_{31} = R_{31} + j \cdot \left(X_{L_{31}} - X_{C_{31}}\right) = 1 + j \cdot \left(2 \cdot \sqrt{3} - \sqrt{3}\right) = 1 + j \cdot \sqrt{3}\Omega$$



1. Calculăm curenții din fazele receptorului:

$$\underline{I}_{12} = \frac{\underline{U}_{12}}{\underline{Z}_{12}} = \frac{120}{1 + j \cdot \sqrt{3}} = \frac{120 \cdot \left(1 - j \cdot \sqrt{3}\right)}{4} = 30 \cdot \left(1 - j \cdot \sqrt{3}\right) A$$

$$\underline{I}_{23} = \frac{\underline{U}_{23}}{\underline{Z}_{23}} = \frac{120 \cdot \left(-\frac{1}{2} - j \cdot \frac{\sqrt{3}}{2}\right)}{2} = 30 \cdot \left(-1 - j \cdot \sqrt{3}\right) A$$

$$\underline{I}_{31} = \frac{\underline{U}_{31}}{\underline{Z}_{31}} = \frac{120 \cdot \left(-\frac{1}{2} + j \cdot \frac{\sqrt{3}}{2}\right)}{1 + j \cdot \sqrt{3}} = \frac{120 \cdot \left(-1 + j \cdot \sqrt{3}\right) \cdot \left(1 - j \cdot \sqrt{3}\right)}{4} = 30 \cdot \left(1 + j \cdot \sqrt{3}\right) A$$

2. Calculăm curenții de linie:

$$\underline{I}_1 = \underline{I}_{12} - \underline{I}_{31} = 30 \cdot \left(1 - j \cdot \sqrt{3}\right) - 30 \cdot \left(1 + j \cdot \sqrt{3}\right) = -60 \cdot j \cdot \sqrt{3} A$$

$$\underline{I}_2 = \underline{I}_{23} - \underline{I}_{12} = 30 \cdot \left(-1 - j \cdot \sqrt{3}\right) - 30 \cdot \left(1 - j \cdot \sqrt{3}\right) = -60 A$$

$$\underline{I}_3 = \underline{I}_{31} - \underline{I}_{23} = 30 \cdot \left(1 + j \cdot \sqrt{3}\right) - 30 \cdot \left(-1 - j \cdot \sqrt{3}\right) = 60 + 60 \cdot j \cdot \sqrt{3} A$$

3. Verificăm bilanțul de puteri:

$$\underline{S}_b = \underline{U}_{13} \cdot \underline{I}_1^* + \underline{U}_{23} \cdot \underline{I}_2^* = \left(-\underline{U}_{31}\right) \cdot \underline{I}_1^* + \underline{U}_{23} \cdot \underline{I}_2^* =$$

$$= -120 \cdot \left(-\frac{1}{2} + j \cdot \frac{\sqrt{3}}{2}\right) \cdot 60 \cdot j \cdot \sqrt{3} + 120 \cdot \left(-\frac{1}{2} - j \cdot \frac{\sqrt{3}}{2}\right) \cdot (-60) =$$

$$= 60^2 \cdot \left[\left(1 - j \cdot \sqrt{3}\right) \cdot j \cdot \sqrt{3} + \left(1 + j \cdot \sqrt{3}\right)\right] = 60^2 \cdot \left[j \cdot \sqrt{3} + 3 + 1 + j \cdot \sqrt{3}\right] = 2 \cdot 60^2 \cdot \left[2 + j \cdot \sqrt{3}\right] VA$$

$$\underline{S}_c = \underline{Z}_{12} \cdot \underline{I}_{12} \cdot \underline{I}_{12}^* + \underline{Z}_{23} \cdot \underline{I}_{23} \cdot \underline{I}_{23}^* + \underline{Z}_{31} \cdot \underline{I}_{31} \cdot \underline{I}_{31}^* =$$

$$= \left(1 + j \cdot \sqrt{3}\right) \cdot 30 \cdot \left(1 - j \cdot \sqrt{3}\right) \cdot 30 \cdot \left(1 + j \cdot \sqrt{3}\right) + 2 \cdot 30 \cdot \left(-1 - j \cdot \sqrt{3}\right) \cdot 30 \cdot \left(-1 + j \cdot \sqrt{3}\right) +$$

$$+ \left(1 + j \cdot \sqrt{3}\right) \cdot 30 \cdot \left(1 + j \cdot \sqrt{3}\right) \cdot 30 \cdot \left(1 - j \cdot \sqrt{3}\right) =$$

$$= 30^2 \cdot \left[\left(1 + j \cdot \sqrt{3}\right) \cdot 4 + 2 \cdot 4 + \left(1 + j \cdot \sqrt{3}\right) \cdot 4\right] = 8 \cdot 30^2 \cdot \left(2 + j \cdot \sqrt{3}\right) = 2 \cdot 60^2 \cdot \left(2 + j \cdot \sqrt{3}\right) VA$$

Se observă că $\underline{S}_b = \underline{S}_c$ deci soluția este corectă.