# 3. GRAPHICAL REPRESENTATIONS IN MATLAB

**Objectives of the paper:**

- Recapitulation of some types of coordinate systems,
- Fixing knowledge regarding the creation of 2D graphs in different coordinate systems and of 3D graphs using the Matlab programming environment,
- by studying some examples and solving some problems.

It is recommended to go through Annex M3 before studying paragraphs 3.1 and3.2.

## 3.1. Elements regarding graphical representations in Matlab

### *Plane graphic representations*

Matlab allows you to create both plane (2D) and spatial (3D) graphical representations. Graphs of functions and polygonal surfaces can be represented in the plane (2D). Graphical representations of functions can be made in several types of coordinate systems: Cartesian, polar, logarithmic, semi-logarithmic. Some of the functions for the **2D graphical representations** in Matlab are shown in the table 3.1.:

***Table 3.1.*** *Matlab functions for 2D graphic representations*

| Function | Employment |
|---|---|
| `fill` | graphic representation of polygonal surfaces |
| `line` | graphic representation of polygonal lines |
| `loglog` | graphs in logarithmic X-Y coordinates (base 10) |
| `plot` | graphs in X-Y cartesian (linear) coordinates |
| `polar` | graphs in polar coordinates |
| `semilogx, semilogy` | graphs in semi-logarithmic X-Y coordinates (base10) |

To define line types, marker types (representation symbols) and colors, the variants shown in the table 3.2 are available.

### *Spatial graphical representations*

In Matlab space curves, surfaces and 3-dimensional bodies can be graphically represented in 3D.Some of the **3D graphic representation** functions are shown in the table 3.3.:

### *Auxiliary functions for graphical representations*

In the case of graphical representations, different properties of the representation mode can be set, coordinate system axis axes can be controlled, texts can be placed on the graph, etc. In Table 3.4. some of the auxiliary Matlab functions for graphical representations are mentioned:

***Table 3.2.*** *Representation symbols for line types, markers, and colors*

| **Line types** | | **Marker types** | | **Colors** | |
|---|---|---|---|---|---|
| **Line types** | **Symbol** | **Marker types** | **Symbol** | **Color** | **Symbol** |
| continuous | - | asterisk | * | white | w |
| dashed | -- | circle | o | blue | b |
| dot-dashed | -. | hexagon | h | cyan | c |
| points | : | square | s | yellow | y |
| | | pentagon | p | magenta | m |
| | | plus | + | black | k |
| | | point | . | red | r |
| | | rhombus | d | green | g |
| | | triangle | v, ^, <, > | | |
| | | x | x | | |

***Table 3.3.*** *Matlab functions for 3D graphic representations*

| **Function** | **Employment** |
|---|---|
| *plot3* | representing curves in space |
| *mesh* | graphical representation of 3D surfaces as a mesh |
| *surf, surfl* | graphical representation of filled surfaces |
| *fill3* | spatial graphic representation of polyhedra |
| *cylinder, sphere, ellipsoid* | graphical representation of three-dimensional bodies |

***Table 3.4.*** *Auxiliary Matlab functions for graphical representations*

| **Function** | **Employment** |
|---|---|
| *meshgrid* | defining the 3D surface representation range as a point network |
| *grid* | overlay a grid of lines on the graph |
| *axes, axis* | controlling the appearance and determining the length of coordinate system axis representation units |
| *subplot* | dividing the graphical window into multiple graphical regions |
| *hold* | keeping the current graph and its properties |
| *colormap* | setting or returning the color matrix used for 3D graphic representations |
| *shading* | setting the shading mode for surfaces in 3D space |
| *title* | inserting a title for the graphic representation |
| *xlabel, ylabel, zlabel* | inserting coordinate system axis labels |
| *gtext* | place text on the graph at the selected mouse position |

## 3.2. Examples

**Example 3.1:** Represent graphically the functions f1, f2:[-2,6]→***R***, definedby the relations: $f_1(x) = \dfrac{1}{\sin(x)+2}$ and $f_2(x) = \dfrac{\cos(x)}{e^{\frac{x}{2}}}$.

*Solution.*: The steps of the 2D graphic representation are:

- rendering the representation domain (interval) by defining a vector with a linear step:

```
>>x=-2:0.1:6;
```

- defining the function/ functions, using array operators:

```
>>f1=1./(sin(x)+2);f2=cos(x)./exp(x/2);
```

- drawing the graph/ graphs and, possibly, specification of certain properties of the graphical representation:

```
>>plot(x,f1,'b-',x,f2,'ro')
```

- setting other proprieties for the graphical representation (graphic representation title, axis labels, text on the graph, etc.):

```
>> title('2D graphs'), xlabel('x'), ylabel('y')
>> grid; gtext('Graphs for 2 functions')
```

The result of executing the sequence of commands is illustrated in the figure 3.1.

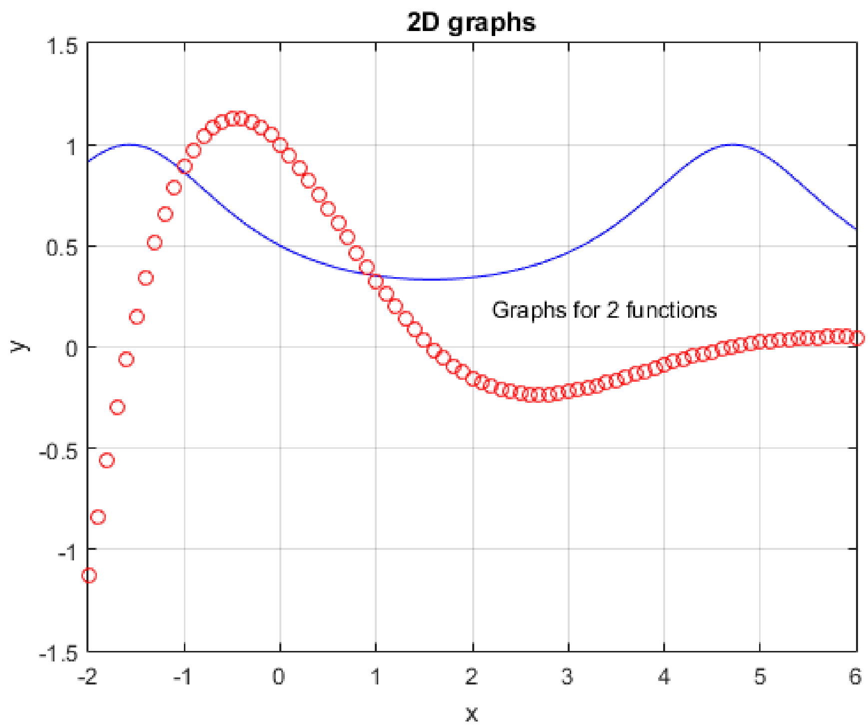



***Fig.3.1.*** *Graphs for the two functions from example 3.1.*

*Comments:* 1. For graphical representation of functions in Matlab it is necessary to define the representation domain by points. In graphical representations in Matlab two consecutive points of the graph are joined together with the segment between them. The distance between any two consecutive points of the graph must be small enough for the graphic representation to be correct.

At the same time, a too little distance requires more time to compute. So a compromise solution must always be found. In figure 3.2. the same graphs are presented, but for a 5 times larger representation vector.

2. Representing multiple graphs of functions in the same Matlab graph can be done by enumerating the functions (in the form of: variable, function, and optionally color, marker, and/ or line type, for each function) in the same *plot* function call, or in different calls of the *plot* function, but in the second case provided the *hold on* command is given before or immediately after the first call and after the last call the *hold off* command is given. For example, the same graph could be obtained with the following sequence of commands:

```
>> plot(x,f1,'b-')
>> hold on
>> plot(x,f2,'ro')
>> hold off
```

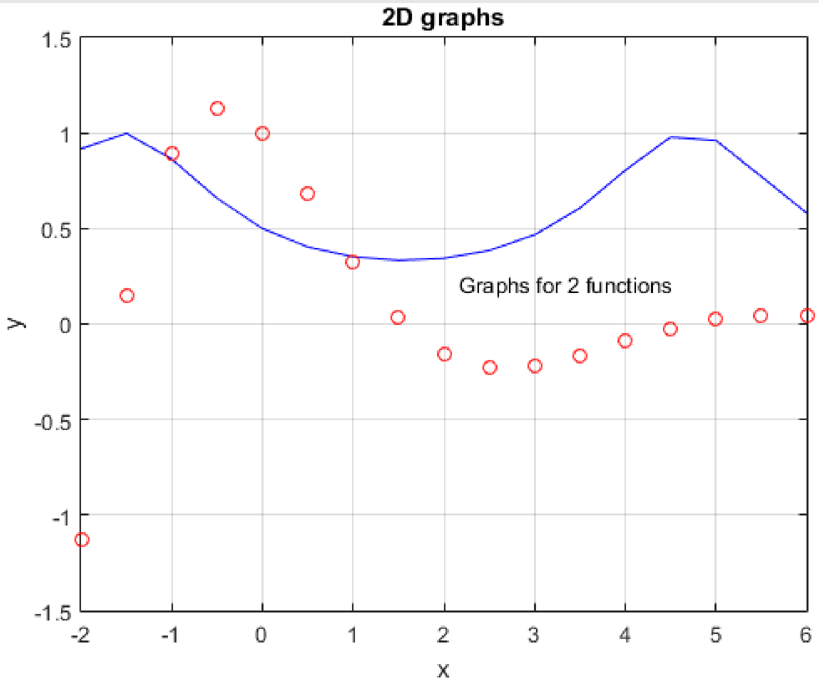



***Fig.3.2.** Graphs for the functions from example 3.1. with a step of 0.5*

**Example 3.2**: Write a Matlab function that receives as arguments another function f and the ends of a closed interval [a, b] on which this function is defined, and represents its graph in semi-logarithmic coordinates, for both cases, the function f on the interval [a, b].If the graphical representation in at least one of the two cases cannot be performed, an error message will be displayed.

*Solution*:

- In this case, the step of defining the function that is intended to be represented graphically is either omitted if an already defined function is chosen or is performed separately by defining it in a function file.
- In order to be able to track the source code of the program, it is necessary to consult the Matlab help for the functions *min* and *feval*.
- The source code of the required Matlab function, called `grafic_log`, is:

```
function grafic_log(f,a,b)
if a>b
    disp('Void Interval (a>b) !')
    return
end
n=100; % number of points for the representation of the interval-1
step=(b-a)/n; % representation step
% [a,b]interval
if a==b  x=a;
else  x=a:step:b;
end
if a<=0 | min(feval(f,x))<=0
    disp(['At least one of the graphs in semilogarithmic '...
        'coordinates cannot be represented!'])
    return
end
% semilogarithmic coordinates on axis x
subplot(2,1,1); semilogx(x,feval(f,x))
% semilogarithmic coordinates on axis y
subplot(2,1,2); semilogy(x,feval(f,x))
```

*Comments:* 1. In order for the vector x = a: step: b to also touch the right end, the step must be a submultiple of the interval length.

2. In the representation of the two graphs, it was chosen to display them in the same graphical window. It was divided into two subfields located one below the other.In the upper subfield the function in semi-logarithmic coordinates on the x-axis will be graphically represented, and in the lower subfield in semi-logarithmic coordinates on y-axis.

3. Obviously it is necessary to test the a and b values in the program to know if they form a proper interval (a <= b) and the values of the function f and of the interval [a, b] in order to graphically represent the function in the two semi-logarithmic coordinate systems (f must be strictly positive, and the interval must contain strictly positive values).

- To test the program, the Matlab function `sin` is used together with the function:

```
function y=f(x)
y=sin(x)+2;
```

The call of the function `grafic_log` is done from the command line. The function *f* is forwarded as argument by the name of the function file in which it was defined. Here are the calls and their results in the form of responses in the command line, respectively the graphical representation in figure 3.3:

```
>>grafic_log('f',2,1)
Void Interval (a>b)!
>>grafic_log('f',-1,10)
At least one of the graphs in semilogarithmic coordinates cannot be represented!
>>grafic_log('sin',1,10)
At least one of the graphs in semilogarithmic coordinates cannot be represented!
>>grafic_log('f',1,10)
```

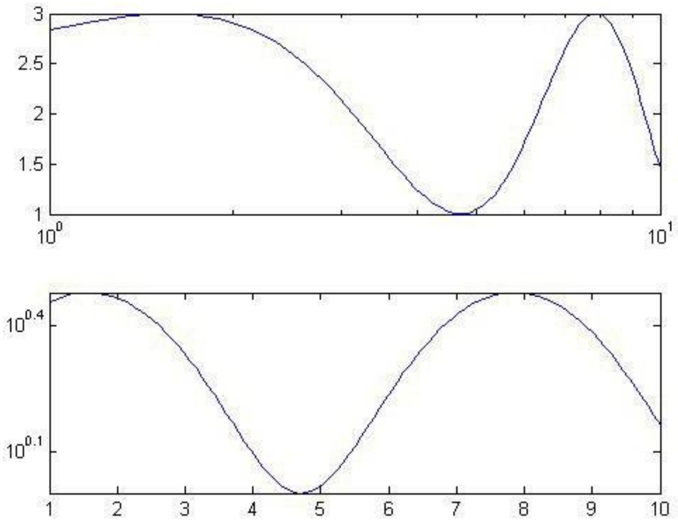



***Fig.3.3.*** *Graphs for the function from example 3.2. insemi-logarithmic coordinates*

**Example 3.3:** Graphically represent the triangle whose vertices have the coordinates (-1,3), (2,7) and (9,-4).

*Solution*: Solving the problem is based on using the function `line`, which receives as arguments the vector of all abscises and the vector of all the ordinates of the vertices (obviously in the same order):

```
>>line([-1 2 9 -1],[3 7 -4 3]);grid
```

In figure 3.4. the obtained graphic representation is displayed:

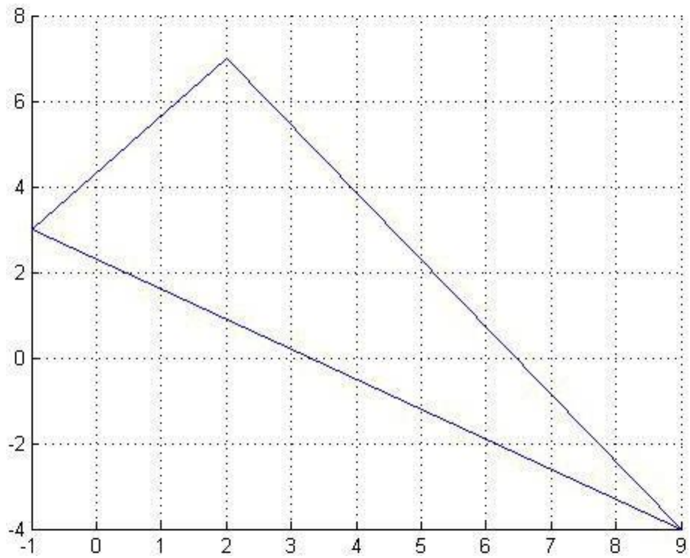



***Fig.3.4.*** *Graphic representation of the triangle from example 3.3.*

*Comment:* The Matlab function`line` joins two consecutive points from string of points with the segment determined by them. The triangle consisting of 3 segments requires four points, obviously the first and last point being the same.

**Example 3.4:** Graphically represent in three-dimensional space the 3D curve described by the following parametric equations: $x(t) = \ln(t^2 + 2)$, $y(t) = t\sin(t)$, $z(t) = -t - 1$, $t \in [-7,7]$.

*Solution*: The steps for graphically representing a 3D curve are:

- rendering the representation domain (interval) by defining a vector with a linear step:

```
>>t=-7:0.1:7;
```

- defining the functions corresponding to the coordinates:

```
>> x=log(t.^2+2); y=t.*sin(t);z=-t-1;
```

- drawing the graph and, possibly, specification of certain properties of the graphical representation:

```
>>plot3(x,y,z,'m')
```

- setting other proprieties for the graphical representation (axis labels, grid, etc.):

```
>> grid
>> xlabel('axis x'); ylabel('axis y'); zlabel('axis z');
```

The result of executing the sequence of commands is illustrated in figure 3.5.

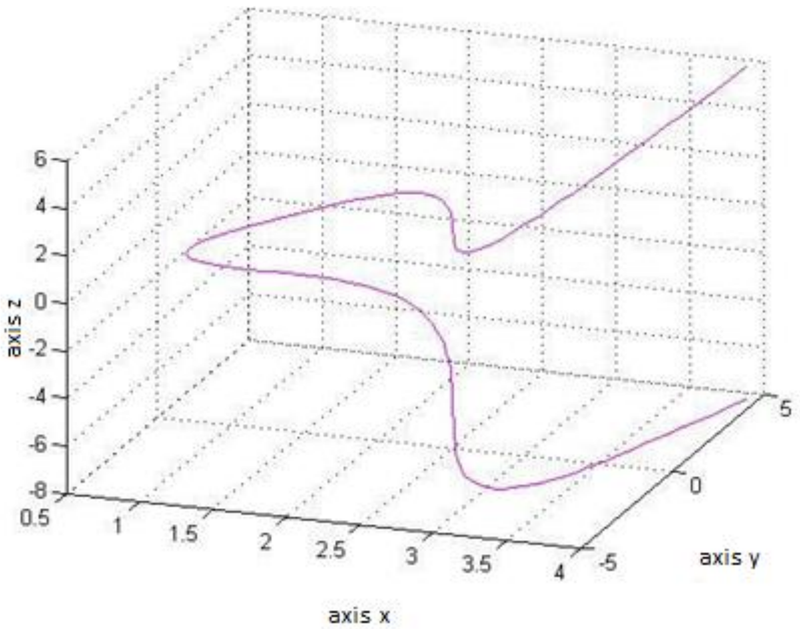



***Fig.3.5.*** *Graphic representation of the #D curve from example 3.4.*

*Comment:* Determine the direction of the curve in Figure 3.5.Hint: Use the Matlab `comet3` function.

**Example 3.5:** Graphically represent in three-dimensional space the classical „sombrero"(the surface described by the relationship: $\begin{cases} 1, & x = y = 0 \\ \dfrac{\sin(\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}}, & x, y \neq 0 \end{cases}$

*Solution*: The steps for the 3D graphic representation are the same as for the 2D case:

- rendering the representation domain:

```
>> x=-8:0.2:8; y=x;[x,y]=meshgrid(x,y);
```

- defining the function/ functions:

```
>> R=sqrt(x.^2+y.^2); S=sin(R);
>> [i,j]=find(R==0); R(i,j)=1; S(i,j)=1;
>> z=S./R;
```

- drawing the graph/ graphs and, possibly, specification of certain properties of the graphical representation(color, shading,etc):

```
>> surfl(x,y,z)
>> shading interp
>> colormap(flag)
```

The result of executing the sequence of commands is illustrated in figure 3.6.

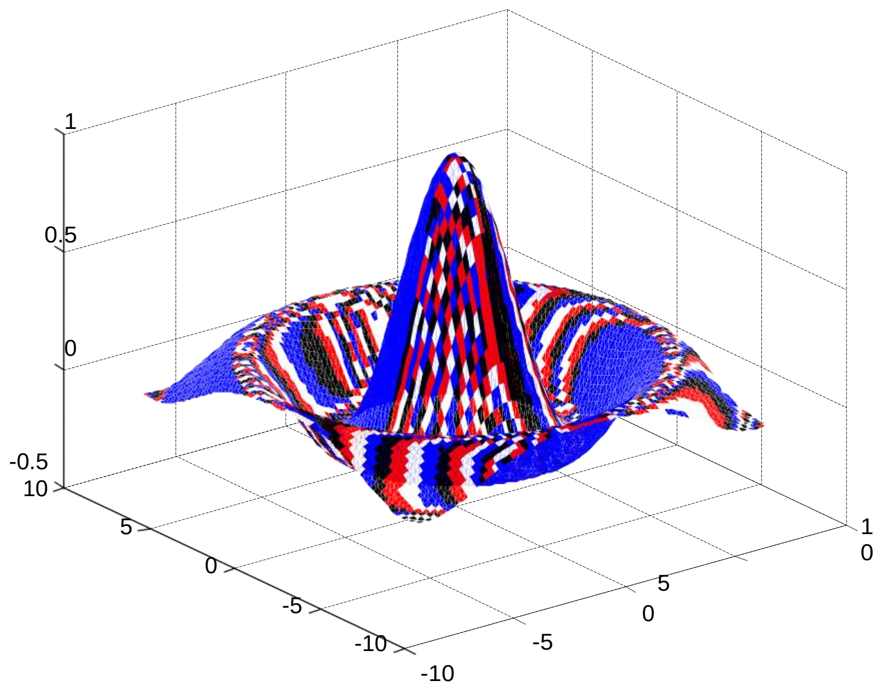



***Fig.3.6.*** *Graph for the „sombrero" function.*

*Comments:* 1. The representation domain of a surface is a net of points $\{(x_i, y_j)\}$, that usually is formed in the following way: the sets of points $\{x_i\}$ and $\{y_j\}$, are generated separately, then their Cartesian product is created using the Matlab function *meshgrid*.

2. To define the value of the function in the point (0,0), its position was searched in the R array with the Matlab function *find*, and then the value on the found position was set to 1, both in R and S arrays.

**Example 3.6:** Represent graphically a straight pyramid of height *h* having as base a regulated octagon inscribed in a circle with the radius *rc* and an ellipsoid with the semi-axes *rx*, *ry* and *rz*.

*Solution*: The pyramid will be represented by the Matlab function `cylinder` and the ellipsoid with one of the Matlab functions `sphere` or `ellipsoid`.

- The graphical representation of the pyramid, shown in figure 3.7.a, with rc=1.25 and h=3, is obtained as follows:

```
% the radius of the circle circumscribed to the base, pyramid hight
% and number of sides
rc=1.25; h=3; n=8;
%determining the coordinates of the surface of the pyramid with a
% height of 1
[xp,yp,z]=cylinder([rc 0],n);
% setting the required height
zp=h*z;
% graphical representation
surf(xp,yp,zp)
colormap(summer);axis('equal')
```

- The graphical representation of the ellipsoid is performed with both of the mentioned functions and is shown in the figures 3.7.b and 3.7.c (it was chosen as the center of the ellipsoid the coordinate point (3,-1,0), and for the semi-axes the values of 5,3,2 were considered):

```
% specifying the coordinates of the center
xc=3; yc=-1; zc=0;
% specifying the semi-axes
rx=5; ry=3; rz=2;
% graphical representation nwith the ellipsoid function
ellipsoid(xc,yc,zc,rx,ry,rz,30)
axis('equal'); colormap spring
pause

% graphical representationwith thesphere function

[x,y,z]=sphere(30);
xe=5*x+3; ye=3*y-1; ze=2*z;
surf(xe,ye,ze)
axis('equal');colormap(cool)
```

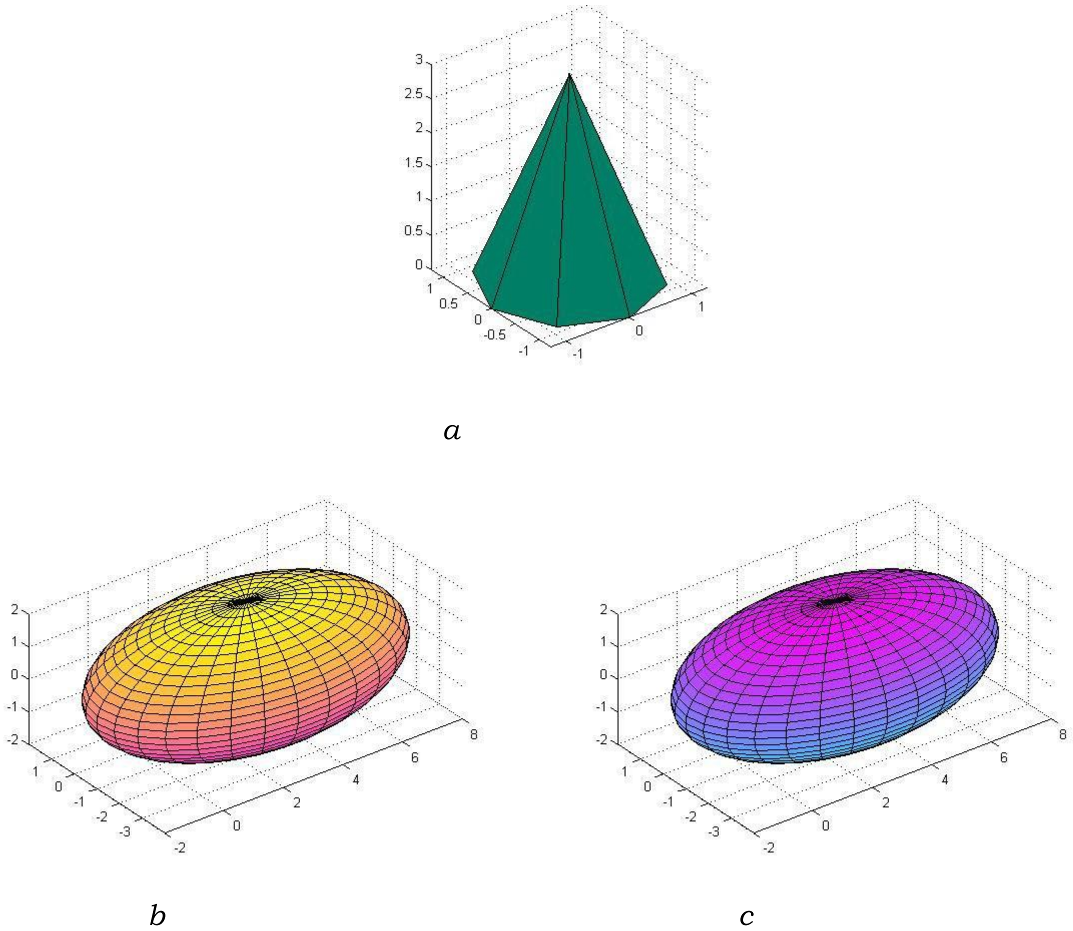



**Fig.3.7** *Graphic representation of 3D bodies: a - pyramid, b,c -ellipsoid*

## 3.3. Problems to solve

**P3.1.** Graphically represent the functions:

a) $y(t)=\begin{cases}\sin(5t), & if \quad \{-9\le t<-3\} \quad and \quad \{3\le t<9\}\\ \cos(t)-\cos(3)-\sin(15), & if \quad \{-3\le t<3\}\end{cases}$

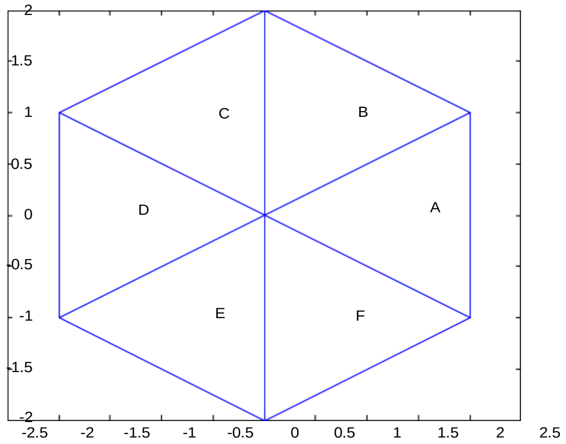
b) $f(t)=\sin(\pi t)\cos(\pi t)$, with magenta dashed line and $g(t)=\sin(\pi t+1)\cos(\pi t+1)$ with cyan square markers joined by continuous line, $t\in[-3,3]$.

**P3.2.** Represent in polar coordinates, $(r=f(t),\theta=t)$, the function:

$f(t)=\sqrt{\sin(t)+t^2},\quad t\in[0,6\pi].$

**P3.3.** Represent in logarithmic coordinates the function $f(t)=e^{3t},\ \ t\in[1,5]$.

**P3.4.** Given the hexagonal surface in the figure below, consisting of 6 regions: *A*,*B*,*C*,*D*,*E*,*F*. Write a program that takes as argument a strictly positive whole number *p* and graphically represents the hexagonal surface, colored according to the remainder *r* of the division *p* to 3 with *r* + 1 colors, alternating the colors.




**P3.5.** Graphically represent:

a) rectangle,

b) the rectangular surface with a color of choice, determined by points $A(5,4)$, $B(-7,4)$, $C(-7,-3)$,$D(5,-3)$.

**P3.6.** Represent graphically in the same plane a circle and an ellipse intersecting.

**P3.7.**Represent graphically the 3D spiral given by the relationships:

$x(t)=t,\quad y(t)=\sin(0.5t-3),\quad z9t)=\cos(0.5t),\quad t\in[-10\pi,10\pi].$

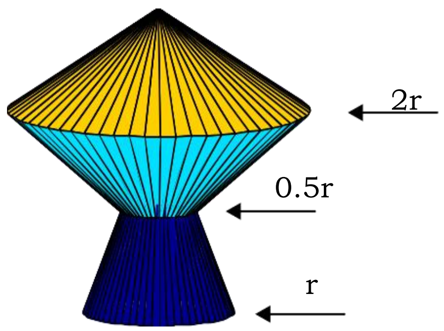
**P3.8.** Represent graphically, in turn, with `plot3`, `mesh`, `surf` and `surfl` the function:

$$z(x,y)=x^3-3xy^2,\quad x\in[-3,3],\quad y\in[-3,3].$$

**P3.9.** Represent graphically separately a truncated cone of rays $rc_1$=2, $rc_2$=1 and height $h$=3 and a pyramid of height $h$ having as base a regulated hexagon inscribed in a circle with the radius $rp$=3.

**P3.10.** Represent graphically a sphere with a radius of 6371, to resemble the Earth with meridians and parallels to 15 degrees longitude and latitude.

**P3.11.** Write a program that receives as argument a real number $r$ and performs the following graphical representation (if $r$ is negative or null, for the representation it is implicitly considered $r$ = 1):



# ANNEX M3. ELEMENTS REGARDING PLANE AND SPATIAL GRAPHICALREPRESENTATIONS

## M3.1. Types of coordinate systems

### a. Cartesian coordinates

Let *xOy* be a plane Cartesian coordinate system. Let *P* be a point on the plane with the coordinates *xp* on the axis *Ox* and *yp* on the axis *Oy*. The *xp* coordinate is also called ***the abscissa of the point P***, and the axis *Ox* is called ***the axis* of the abscissas**, and *yp* is also called ***the ordinate of the point P***, and the axis Oy is called ***the axis of the ordinates***. It will be noted *P*(*xp*,*yp*). Cartesian coordinates are also called ***linear coordinates***.
The axes *Ox* and *Oy* divide the plane into four regions, called ***open quadrants***:

- quadrant I is the set of points that have both coordinates strictly positive;
- quadrant II is the set of points that have strictly negative abscissas and strictly positive ordinates;
- quadrant III is the set of points that have both coordinates strictly negative;
- quadrant IV is the set of points that have strictly positive abscissas and strictly negative ordinates.

A system of Cartesian coordinates in space is noted as *xOyz*. The positions of the point *P* in three-dimensional space is given by the three coordinates, *xp* on the *Ox* axis, *yp* on the *Oy* axis and *zp* on the *Oz* axis. It will be noted *P*(*xp*,*yp*,*zp*).

### b. Polar coordinates

Let *xOy* be a plane Cartesian coordinate system and *P*(*xp*,*yp*) a point in the plane different than the origin *O* of the system. Let *r* be the distance from *P* to *O* and $\theta$ the angle formed in trigonometric sense by the line (*OP* with *Ox* axis, $\theta\in[0,2\pi)$. The numbers *r* and $\theta$ are called ***the polar coordinates of the point P***. It will be noted $P(r,\theta)$. *r* is called ***polar radius of P***, and $\theta$ ***polar argument of P***.

The relation between the Cartesian coordinates and the polar coordinates of P are expressed by the relations:

$$r=\sqrt{xp^2+yp^2},$$

$$\cos(\theta)=\frac{xp}{\sqrt{xp^2+yp^2}},\quad \sin(\theta)=\frac{yp}{\sqrt{xp^2+yp^2}},\quad \theta\in[0,2\pi)$$

### c. Logarithmic coordinates

***The logarithmic coordinates*** are the expression of the coordinates of a point on a ***logarithmic scale***, that is, as logarithms in a specified base *b* of the Cartesian coordinates of the specified point. Since the logarithm can only be calculated for strictly positive values, the only points that can be represented in logarithmic coordinates are those in the quadrant I. Thus, if *xOy* is a Cartesian coordinate system and *P*(*xp*,*yp*) a point in the quadrant I, then the logarithmic coordinates of the point are $x=\log_b(xp)$ and $y=\log_b(yp)$, namely $xp=b^x$ şi $yp=b^y$.

**d. Semi-logarithmic coordinates**

**Semi-logarithmic coordinates** represents a pair of coordinates, one of which is a Cartesian coordinate (linear) and the other a logarithmic coordinate. If the logarithmic coordinate corresponds to the x axis, then the name used is **semi-logarithmic coordinates on the x axis**. Analogously, if the logarithmic coordinate corresponds to the y axis, then the name used is **semi-logarithmic coordinates on the y axis**.

## M3.2. Geometric figures in plane

**a. The line**

Let $xOy$ be a Cartesian coordinate system. Any line parallel to $Ox$ is called a ***horizontal line***. Any line parallel to $Oy$ is called a ***vertical line***. Any line that is neither horizontal nor vertical is called an ***oblique line***. The tangent of the angle formed by an oblique line with the Ox axis (angle in the set $\left(0,\frac{\pi}{2}\right) \cup \left(\frac{\pi}{2},\pi\right)$) is called ***the slope of the oblique line*** and is marked with $m$.

***Equation of an oblique line determined by a point and a slope***

Let $d$ be an oblique line with a slope $m$ and $P(xp, yp)$ a point of the straight $d$. Then the equation of the line $d$ is:

$$y - yp = m(x - xp)$$

***Equation of the line determined by two distinct points***

Let $d$ be a line and $P(xp, yp)$ and $R(xr, yr)$ two distinct points on the line. Then the equation of the line $d$ is:

$x = xp$, *when the line is vertical*

$y = yp$, *when the line is horizontal*

$\dfrac{x - xp}{xr - xp} = \dfrac{y - yp}{yr - yp}$, *when the line is oblique*

***General Cartesian equation of the line***

Let $d$ be a line. The general Cartesian equation of the line $d$ has the implicit form:

$$ax + by + c = 0, \quad with \quad a,b,c \in \mathbf{R}, \quad a^2 + b^2 \neq 0$$

**b. The circle**

The locus of all the points in the plane equally spaced from a given point is called a ***circle***. The given point is called ***the center of the circle***, and the distance from it to any of the circle points is called ***the radius of the circle***.

Let $xOy$ be a Cartesian coordinate system, and $\mathscr{C}$ a circle with the center $C(xc, yc)$ and radius $r$. The equations of the circle $\mathscr{C}$ are:

- ***the implicit circle equation:*** $(x - xc)^2 + (y - yc)^2 = r^2$
- ***the explicit circle equations:*** $y = yc \pm \sqrt{r^2 - (x - xc)^2}, \quad x \in [xc - r, xc + r]$
- ***the parametric circle equations:*** $\begin{cases} x = xc + r\cos(\theta) \\ y = yc + r\sin(\theta) \end{cases}, \quad \theta \in [0, 2\pi)$

The set of points whose distance to $C$ is strictly less than $r$ is called ***the interior of the circle***. The reunion between the circle and its interior is called ***disc with the center C and radius r.***

**c. The ellipse**

The locus of all the points in the plane that have the property that the sum of their distances at two fixed points is constant is called an ***ellipse***. The two fixed points are called ***the focal points of the ellipse***. The distance between the two focal points is called ***the focal distance*** and the distances from a certain point *P* of the ellipse to the two focal points are called ***the focal radii of the point P***.

Let *F* and *F'* be the two focal points, *C* the middle of the segment [*FF'*], *A* and *A'* the intersection points of the line *FF'* with the ellipse, *B* and *B'* the intersection of the line perpendicular to *FF'* in *C* with the ellipse, *a* the distance *CA* and *b* the distance *CB*. *C* is ***the center of symmetry of the ellipse***, and *AA'* and *BB'* are ***the axis of symmetry of the ellipse***. *a* and *b* are called ***the semi-axes of the ellipse***.

Let *xOy* be a Cartesian coordinate system, and (*xc*,*yc*) the coordinates of the centre of symmetry *C* of the ellipse. Next, it will be considered that the line *FF'* is parallel to the *Ox* axis. Let $\mathscr{E}$ be the ellipse with the center *C*(*xc*,*yc*) and semi-axes *a* and *b*. The equations of the ellipse $\mathscr{E}$ are:

- ***the implicit ellipse equation:*** $\dfrac{(x-xc)^2}{a^2}+\dfrac{(y-yc)^2}{b^2}=1$

- ***the explicit ellipse equations:*** $y = yc \pm b\sqrt{\left(1-\dfrac{(x-xc)^2}{a^2}\right)},\ \ x\in[xc-a, xc+a]$

- ***the parametric ellipse equations:*** $\begin{cases} x = xc + a\cos(\theta) \\ y = yc + b\sin(\theta)\end{cases},\ \ \theta\in[0,2\pi)$

**d. The hyperbola**

The locus of the planar points that have the property that the difference module of their distances to two fixed points is constant is called a ***hyperbola***. The two fixed points are called ***the focal points of the hyperbola***. The distance between the two focal points is called ***the focal distance*** and the distances from a certain point *P* of the hyperbola to the two focal points are called ***the focal radii of the point P***.

Let *F* and *F'* be the two focal points, *C* the middle of the segment [*FF'*], *A* and *A'* the intersection points of the line *FF'* with the hyperbola, *c* the distance *CF*, *a* the distance *CA* (*a*<*c*)and $b=\sqrt{c^2-a^2}$ . *C* is ***the center of symmetry of the hyperbola***, and *FF'* and the mediator of the segment [*FF'*] are ***the axis of symmetry of the hyperbola***. a and *b* are called ***the semi-axes of the hyperbola***.

Let *xOy* be a Cartesian coordinate system, and (*xc*,*yc*) the coordinates of the centre of symmetry *C* of the hyperbola. Next, it will be considered that the line *FF'* is parallel to the *Ox* axis. Let $\mathscr{H}$ be a hyperbola with the center *C*(*xc*,*yc*) and semi-axes *a* and *b*. The equations of the hyperbola $\mathscr{H}$ are:

- ***the implicit hyperbola equation:*** $\dfrac{(x-xc)^2}{a^2}-\dfrac{(y-yc)^2}{b^2}=1$

- ***the explicit hyperbola equations:***

$$y = yc \pm b\sqrt{\left(\frac{(x-xc)^2}{a^2}-1\right)},\ \ x\in(\infty, xc-a]\cup[xc+a,\infty)$$

The set of points of coordinates (*x*,*y*) that satisfy the equation:

$-\dfrac{(x-xc)^2}{a^2}+\dfrac{(y-yc)^2}{b^2}=1$ represent a hyperbola $\mathscr{H}'$ with the center *C*(*xc*,*yc*) and semi-axes *a* and *b*, for which the focal axis is parallel with the axis *Oy*.

The hyperbolae $\mathscr{H}$ and $\mathscr{H}''$ are called ***conjugated to each other hyperbolae***. A hyperbola with equal semi-axes is called ***a equilateral hyperbola***.

**d. The parabola**

The locus of all the points in the plane equally spaced from a fixed point and a fixed axis is called a ***parabola***. The fixed point is called ***the focus of the parabola***, and the fixed axis is called ***the directrix of the parabola***. The distance from a certain point *P* of the ellipse to the focal point is called ***the focal radius of the point P***. Let *F* be the focus, *A* the projection of the focus on the directrix of the parabola, *C* the intersection of the line *FA* with the parabola and *p* the distance between the focus and *A*. *C* is called the ***vertex of the parabola***. The line *AC* is called ***the axis of symmetry of the parabola.***

Let *xOy* be a Cartesian coordinate system, and (*xc*,*yc*) the coordinates of the vertex *C* of the parabola. Next, it will be considered that the line *AF* is parallel to the *Ox* axis. Let $\mathscr{P}$ be a parabola with the vertex *C*(*xc*,*yc*) and axis of symmetry *AF*. The equations of the parabola $\mathscr{P}$ are:

- ***the implicit parabola equation:*** $(y-yc)^2-2p(x-xc)=0$
- ***the explicit parabola equation:*** $y=yc\pm\sqrt{2p(x-xc)},\quad x\geq xc$

## M3.3. Geometric figures in space

**a. The line**

Let *xOyz* be a Cartesian coordinate system and *d* a line in the space structured by it.

***General Cartesian equations of the line***

Analytically, the line *d* is expressed as the intersection of two planes, i.e through the system of equations comprised of the equations of the two planes. Thus, if $\mathscr{P}_1$, of equation $a_1x+b_1y+c_1z+d_1=0$ and $\mathscr{P}_2$, of equation $a_2x+b_2y+c_2z+d_2=0$, are the two planes, then, the equations of the line*d* are:

$$\begin{cases}a_1x+b_1y+c_1z+d_1=0\\a_2x+b_2y+c_2z+d_2=0\end{cases},\quad a_1,\ldots,d_2\in\boldsymbol{R}$$

***The parametric equations of the line determined by two distinct points***

Let *P*(*xp*,*yp*,*zp*) and *R*(*xr*,*yr*,*zr*) be two distinct points on the line *d*. Then the parametric equations of the line *d* determined by the points *P* and *R* are:

$$\begin{cases}x=xp+k(xr-xp)\\y=yp+k(yr-yp),\quad k\in\boldsymbol{R}\\z=zp+k(zr-zp)\end{cases}$$

**b. The sphere**

The locus of all the points in space equidistant to a given point is called ***a sphere***. The give point is called ***the sphere center***, and the distance between it and any other point of the sphere is called ***the sphere radius***.

Let $xOyz$ be a Cartesian coordinate system and $\mathscr{S}$ a sphere of center $C(xc,yc,zc)$ and radius $r$. The equations of the sphere $\mathscr{S}$ are:

- ***the implicit sphere equation:*** $(x-xc)^2+(y-yc)^2+(z-zc)^2=r^2$
- ***the parametric sphere equations:*** $\begin{cases} x = xc + r\cos(\alpha)\cos(\beta) \\ y = yc + r\sin(\alpha)\cos(\beta), \quad \alpha\in[0,2\pi), \beta\in[-\pi,\pi] \\ z = zc + r\sin(\beta) \end{cases}$

The set of points whose distance to $C$ is strictly less than $r$ is called ***the interior of the sphere***. The reunion between the sphere and its interior is called ***a ball with the center C and radius r.***

**c. The ellipsoid**

An ***ellipsoid*** is a three-dimensional closed surface with the property that its intersection with any plane is an ellipse or a circle. An ellipsoid has three ***axes of symmetry*** that intersect in a point and which are perpendicular two by two. The intersection point is called ***the centre of symmetry***. Let *AA'*, *BB'* and *DD'* be the intersections of the three axes of symmetry with the ellipsoid, and *C* the center of symmetry. The distances *CA, CB* and *CD* are called ***the semi-axes of the ellipsoid*** and are noted with *a*, *b* and *c*.

Let $xOyz$ be a Cartesian coordinate system and $\mathscr{EL}$ an ellipsoid with the semi-axes $a$, $b$ and $c$, and the center of symmetry $C(xc,yc,zc)$. The equations of the ellipsoid $\mathscr{EL}$ are:

- ***the implicit ellipsoid equation:*** $\dfrac{(x-xc)^2}{a^2}+\dfrac{(y-yc)^2}{b^2}+\dfrac{(z-zc)^2}{c^2}=1$
- ***the parametric ellipsoid equations:*** $\begin{cases} x = xc + a\cos(\alpha)\cos(\beta) \\ y = yc + b\sin(\alpha)\cos(\beta), \quad \alpha\in[0,2\pi), \beta\in[-\pi,\pi] \\ z = zc + c\sin(\beta) \end{cases}$

**d. The prism**

Let $\mathscr{S}$ be a polygonal surface, included in a plane $\alpha$, $d$ a line that does not belong to the plane $\alpha$ and is not parallel to it, and $\alpha'$ a plane parallel to $\alpha$. For each point $P$ of the polygonal surface $\mathscr{S}$ let $P'$ be the intersection between the plane $\alpha'$ and the parallel to $d$ going through $P$. The reunion of all the segments $[PP']$, when $P$ passes through the surface $\mathscr{S}$, is called a ***prism***. Let $\mathscr{S}'$ be the set of all the points $P'$. $\mathscr{S}$ and $\mathscr{S}'$ are called ***the bases of the prism***. $\mathscr{S}$ and $\mathscr{S}'$ ' are congruent.

If the line $d$ is perpendicular to the plane $\alpha$, then the prism is ***a right prism***. A right prism whose base is a regular polygonal surface is called ***a regular prism***. A prism whose base is a parallelogram is called a ***parallelepiped.*** Aright parallelepiped is called ***a rectangular parallelepiped***. A rectangular parallelepiped that has only square-bound surfaces is called ***a cube***.

**e. The pyramid**

Let $\mathscr{S}$ be a polygonal surface, included in a plane $\alpha$, and $V$ a point that does not belong to the plane $\alpha$. The reunion of all the segments $[VP]$, when $P$ passes through the surface $\mathscr{S}$, is called a ***pyramid of apex V and base*** $\mathscr{S}$. The distance from the apex $V$ to the plane $\alpha$ is called ***the height of the pyramid***.

A pyramid whose base is a regular polygonal surface and for which the projection of $V$ on $\alpha$ is the center of $\mathscr{S}$ is called ***a regular pyramid***. A pyramid with a triangular surface as base is called ***a tetrahedron***.



Let $\alpha'$ be a plane parallel cu $\alpha$ that intersects the pyramid. Let $\mathcal{S}'$ be the intersection of the plane $\alpha'$ with the pyramid. $\mathcal{S}$ and $\mathcal{S}'$ are similar. The set of points of the pyramid between the planes $\alpha$ and $\alpha'$ together with the two surfaces $\mathcal{S}$ and $\mathcal{S}'$ is called ***a pyramidal frustum***. $\mathcal{S}$ are $\mathcal{S}'$ called ***the bases of the pyramidal frustum.*** The distance between the two planes is called ***the height of the pyramidal frustum***. A pyramidal frustum obtained from a regular pyramid is called ***a regular pyramid frustum***.

**f. The cylinder**

Let $\mathcal{D}$ be a disc, included in a plane $\alpha$, $d$ a line that does not belong to the plane $\alpha$ and is not parallel to it, and $\alpha'$ a plane parallel with $\alpha$. For each point $P$ of the disc $\mathcal{D}$ let $P'$ be the intersection between the plane $\alpha'$ and the parallel to $d$ going through $P$. The reunion of all the segments $[PP']$, when $P$ passes through the disc $\mathcal{D}$, is called ***a circular cylinder***. Let $\mathcal{D}'$ be the set of all the points $P'$. $\mathcal{D}$ and $\mathcal{D}'$ are called ***the bases of the circular cylinder***. $\mathcal{D}$ and $\mathcal{D}'$ have the same radius. If the line $d$ is perpendicular on the plane $\alpha$, then the cylinder is ***a right circular cylinder***.

**g. The cone**

Let $\mathcal{D}$ be a disc, included in a plane $\alpha$, and $V$ a point that does not belong to the plane $\alpha$. The reunion of all the segments $[VP]$, when $P$ passes through the disc $\mathcal{D}$, is called ***a circular cone of apex V and base*** $\mathcal{D}$. The distance from the apex $V$ to the plane $\alpha$ is called ***the height of the cone***. A cone for which the projection of $V$ on the base plane coincides with the base center is called ***a right cone***.

Let $\alpha'$ be a plane parallel cu $\alpha$ that intersects the cone. Let $\mathcal{D}'$ be the intersection of the plane $\alpha'$ with the cone. The set of points of the cone between the planes $\alpha$ and $\alpha'$ together with the two discs $\mathcal{D}$ and $\mathcal{D}'$ is called ***a conical frustum***. $\mathcal{D}$ and $\mathcal{D}'$ are called ***the bases of the conical frustum***. The distance between the two planes is called ***the height of the conical frustum***. A conical frustum obtained from aright circular cone is called a***right conical frustum***.