Hypothesis tests for people who are over it

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Sampling distributions

Tests to include:

Z-test for population proportion

Suppose that X_i are independent and identically $\text{Normal}(\mu, \sigma^2)$.

Notation

Exact tests

One sample tests

name	known parameters	null hypothesis	test statistic	null distribution	p-value (two sided)
one sample z-test	μ unknown, σ^2 known	$H_0 : \mu = \mu_0$	$Z_{\text{obs}} = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$	N(0, 1)	$2 \cdot P(Z < -\lvert Z_{\text{obs}} \rvert)$
one sample t-test	μ, σ^2 unknown	$H_0 : \mu = \mu_0$	$T_{\text{obs}} = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$	t_{n-1}	$2 \cdot P(t_{n-1} < -\lvert T_{\text{obs}} \rvert)$
TODO	μ, σ^2 unknown	$H_0 : \sigma^2 = \sigma_0^2$	$TODO = \frac{(n-1) \cdot s^2}{\sigma_0^2}$	χ^2_{n-1}	TODO
sign test	none	$H_0 : m = m_0$	$B_{\text{obs}} = \sum_i I_{X_i > m_0}$	$\text{Binomial}(\sum_i I_{X_i \neq m_0}, \frac{1}{2})$	$2 \cdot \min(P(B \geq B_{\text{obs}}), P(B \leq B_{\text{obs}}))$

Two sample tests

where

name	known parameters	null hypothesis	test statistic	null distribution	p-value (two sided)
two sample z-test	μ_1, μ_2 unknown σ_1^2, σ_2^2 known	$H_0 : \mu_1 - \mu_2 = \delta_0$	$Z_{\text{obs}} = \frac{(\bar{x} - \bar{y}) - \delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$	N(0, 1)	$2 \cdot P(Z < -\lvert Z_{\text{obs}} \rvert)$
equal variance t-test	μ_1, μ_2 $\sigma_1^2 = \sigma_2^2$ unknown	$H_0 : \mu_1 - \mu_2 = \delta_0$	$T_{\text{obs}} = \frac{(\bar{x} - \bar{y}) - \delta_0}{s_{\text{pooled}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$	t_{n-1}	$2 \cdot P(t_{n_1+n_2-2} < -\lvert T_{\text{obs}} \rvert)$
welch's t-test	μ_1, μ_2 $\sigma_1^2 \neq \sigma_2^2$ unknown	$H_0 : \mu_1 - \mu_2 = \delta_0$	$T_{\text{obs}} = \frac{(\bar{x} - \bar{y}) - \delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$	t_ν	$2 \cdot P(t_\nu < -\lvert T_{\text{obs}} \rvert)$
F-test for equal variance	μ_1, μ_2 σ_1^2, σ_2^2 unknown	$H_0 : \sigma_1^2 = \sigma_2^2$	$F_{\text{obs}} = \frac{s_1^2}{s_2^2}$	F_{n_1-1, n_2-1}	TODO

$$s_{\text{pooled}} = \sqrt{\frac{(n_1 - 1) \cdot s_1^2 + (n_2 - 1) \cdot s_2^2}{n_1 + n_2 - 2}} \tag{1}$$

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}} \tag{2}$$

Paired two-sample tests

name	known parameters	null hypothesis	test statistic	null distribution	p-value (two sided)
paired z-test	μ unknown, σ^2 known	$H_0 : \mu = \mu_0$	$Z_{\text{obs}} = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$	N(0, 1)	$2 \cdot P(Z < -\lvert Z_{\text{obs}} \rvert)$
paired t-test	μ, σ^2 unknown	$H_0 : \mu = \mu_0$	$T_{\text{obs}} = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$	t_{n-1}	$2 \cdot P(t_{n-1} < -\lvert T_{\text{obs}} \rvert)$
paired sign test	TODO	TODO	TODO	TODO	TODO

Related to linear regression and ANOVA

- Tukey's HSD

name	known parameters	null hypothesis	test statistic	null distribution	p-value (two sided)
one-way anova / F-tests	μ unknown, σ^2 known	$H_0 : \mu = \mu_0$	$Z_{\text{obs}} = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$	N(0, 1)	$2 \cdot P(Z < -\lvert Z_{\text{obs}} \rvert)$

- Tests for coefficients (intercept and slope in simple linear regression)
- Overall F-test on linear regression coefficients

Nonparametric tests

- Kruskal-Wallis
- Rank-Sum
- Signed-Rank

Bootstrap and permutation (parametric, one and two sample, paired)