Lucrare de control

subiectul B

rândul R1

(5) Sîrbu Alexandrea-Paul 216

of 1p

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1. Determinați inf, sup, min, max pt mulțimea A

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$$A = \left\{ \frac{p}{p+2} \mid p, 2 \in \mathbb{N}^* \right\}$$

2. Determinați mulțimea pct. limită

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$$x_n = \left(\frac{n+3}{n+1} \right)^{n \cdot \cos \frac{n\pi}{3}}$$

3. Calculați

0

$$\lim_{x \to 0} \frac{x^3 - tg^3 x}{x^5}$$

1. $A = \left\{ \frac{p}{p+2} \mid p, 2 \in \mathbb{N}^* \right\}$

$\frac{p}{p+2} < \frac{p+2}{p+2}$

$p < p+2 \Rightarrow 2 > 0 \Rightarrow \frac{p}{p+2}$ - subunitară

$p, 2 \in \mathbb{N}^* \Rightarrow \frac{p}{p+2} > 0 \Rightarrow \frac{p}{p+2} \in [0, 1]$

$p = 2 = 1 \Rightarrow \frac{p}{p+2} = \frac{1}{2}$

$\frac{p}{p+2} > \frac{1}{2} \Rightarrow \frac{p}{p+2} - \frac{1}{2} > 0$

$\frac{2p - p - 2}{2(p+2)} > 0$

Fie $1 - \inf A = \sup A$

$\frac{p}{p+2} > 1 \Rightarrow p > p+2 \Rightarrow 2 < 0$ (F) $\Rightarrow (\forall) x \in A, x \leq 1$

$\frac{p-2}{2(p+2)} > 0$

$(\forall) \varepsilon > 0, \exists y \in A$ a.î. $y > M - \varepsilon$

Fie $x \in A, x = \frac{p}{p+2}$

$x > 1 - \varepsilon$

$\frac{p}{p+2} > 1 - \varepsilon \Rightarrow p > p + 2 - \varepsilon(p+2)$

$2 < \varepsilon(p+2)$

$\Rightarrow 1 = \sup A$ $p, 2 \in \mathbb{N}^*$ ~~$\min A$~~

Fie $0 = \inf A \Rightarrow \begin{cases} (i)\ (\forall) x \in A,\ x \geq 0 \\ (ii)\ (\forall)\varepsilon > 0\ \exists\ y \in A \text{ a.i. } y < \varepsilon \end{cases}$

(i) - trivial

$\frac{p}{p+q} \geq 0 \Rightarrow \begin{matrix} p \geq 0 \\ p \geq 0 \\ p+q > 0 \\ p, q \in \mathbb{N} \end{matrix} \Rightarrow \frac{p}{p+q} \geq 0$

ii') $(\forall)\varepsilon > 0,\ \exists\ y \in A$ a.i. $y < \varepsilon$

$\frac{p}{p+q} < \varepsilon \Rightarrow p < \varepsilon(p+q)$

$p+q \in \mathbb{N}^*$ $\Rightarrow A)$

$\Rightarrow 0 = \inf A$ ~~$\min A$~~

$\inf A = 0$

$\sup A = 1$ ✓

$\min A$ ~~$\max A$~~ ?

3. $\ell = \lim_{x\to 0} \frac{x^3 - \operatorname{tg}^3 x}{x^5} = \lim_{x\to 0}\left(\frac{x^3}{x^5} - \frac{\operatorname{tg}^3 x}{x^5}\right)$

$= \lim_{x\to 0}\left(\frac{1}{x^2} - \frac{\operatorname{tg}^3 x}{x^3}\cdot\frac{1}{x^2}\right) = \lim_{x\to 0}\left(\frac{1}{x^2} - \left(\frac{\operatorname{tg} x}{x}\right)^3\cdot\frac{1}{x^2}\right)$

$\lim_{x\to 0}\frac{\operatorname{tg} x}{x} \overset{\left[\frac{0}{0}\right]}{\underset{l'H}{=}} \lim_{x\to 0}\frac{\frac{1}{\cos^2 x}}{1} = 1$

$\ell = \lim_{x\to 0}\left(\frac{1}{x^2} - 1\cdot\frac{1}{x^2}\right) = \lim_{x\to 0}\frac{0}{x^2} = 0$

$\lim_{x\to 0}\frac{1}{x^2}\left(1 - \left(\frac{\operatorname{tg} x}{x}\right)^3\right) \overset{\left[\frac{0}{0}\right]}{\underset{l'H}{=}}$

$= \lim_{x\to 0}\frac{-3\left(\frac{\operatorname{tg} x}{x}\right)^2\cdot\left(\frac{x}{\cos^2 x} - \operatorname{tg} x\right)}{x^2}$

x

$= \lim_{x\to 0} -\frac{3}{x}\left(\frac{\operatorname{tg} x}{x}\right)^2\cdot\left(\frac{x}{x^2\cos^2 x} - \frac{\operatorname{tg} x}{x^2}\right)$

$\left(\frac{\operatorname{tg} x}{x}\right)^2 \to 1$

$\overset{\left[\frac{0}{0}\right]}{\underset{l'H}{=}} \lim_{x\to 0} -\frac{3}{x}\cdot\left(\frac{x}{x^2\cos^2 x} - \frac{\operatorname{tg} x}{x^2}\right) = 1$

2. $x_n = \left(\frac{n+3}{n+1}\right)^{n \cdot \cos\frac{n\pi}{3}}$

$n = 3k$

$x_{3k} = \left(\frac{3k+3}{3k+1}\right)^{3k \cdot \cos\frac{3k\pi}{3}} = \left(\frac{3k+3}{3k+1}\right)^{3k \cdot \cos k\pi}$

$= \left(\frac{3k+3}{3k+1}\right)^{3k \cdot (-1)^k}$

$k = 2p$

$x_{6p} = \left(\frac{6p+3}{6p+1}\right)^{6p}$

$\lim_{p\to\infty} \left(\frac{6p+3}{6p+1}\right)^{6p} = \lim_{p\to\infty} \left[\left(1 + \frac{2}{6p+1}\right)^{\frac{6p+1}{2}}\right]^{\frac{2}{6p+1} \cdot 6p}$

$= e^2$

$k = 2p+1$

$x_{6p+3} = \left(\frac{6p+6}{6p+4}\right)^{(6p+3)\cdot(-1)} = \left(\frac{6p+4}{6p+6}\right)^{6p+3}$

$\lim_{p\to\infty} \left(\frac{6p+4}{6p+6}\right)^{6p+3} = \lim_{p\to\infty} \left[\left(1 + \frac{-2}{6p+6}\right)^{\frac{6p+6}{-2}}\right]^{\frac{-2}{6p+6} \cdot 6p+3}$

$= e^{-2}$

$n = 3k+1$

$$x_{3k+1} = \left(\frac{3k+4}{3k+2}\right)^{(3k+1)\cdot\cos\frac{(3k+1)\pi}{3}}$$

$$= \left(\frac{3k+4}{3k+2}\right)^{(3k+1)\cdot\cos\left(k\pi+\frac{\pi}{3}\right)}$$

$$= \left(\frac{3k+4}{3k+2}\right)^{(3k+1)\cdot\left(-\cos\frac{\pi}{3}\right)(-1)^k\cdot\cos\frac{\pi}{3}}$$

$k = 2p$

$$x_{6p+1} = \left(\frac{6p+4}{6p+2}\right)^{(6p+1)\cdot\frac{1}{2}}$$

$$\lim_{p\to\infty}\left(\frac{6p+4}{6p+2}\right)^{(6p+1)\cdot\frac{1}{2}} = \lim_{p\to\infty}\left[\left(1+\frac{2}{6p+2}\right)^{\frac{6p+2}{2}}\right]\cdot\frac{2}{6p+2}\cdot\frac{6p+1}{2}$$

$$= e$$

$n = 2p+1$

$$x_{6p+4} = \left(\frac{6p+7}{6p+5}\right)^{(6p+4)\cdot\left(-\frac{1}{2}\right)} = \left(\frac{6p+5}{6p+7}\right)^{\frac{6p+4}{2}}$$

$$\lim_{p\to\infty}\left(\frac{6p+5}{6p+7}\right)^{\frac{6p+4}{2}} = \lim_{p\to\infty}\left(1+\frac{-2}{6p+7}\right)^{\frac{6p+7}{-2}\cdot\frac{-2}{6p+7}\cdot\frac{6p+4}{2}}$$

$$= e^{-1} = \frac{1}{e}$$

$n = 3k+2$

$$x_{3k+2} = \left(\frac{3k+5}{3k+3}\right)^{(3k+2)\cdot\cos\frac{(3k+2)\pi}{3}} = \left(\frac{3k+5}{3k+3}\right)^{(3k+2)\cdot(-1)^k\cos\frac{2\pi}{3}}$$

$$\cos\frac{2\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = -\frac{1}{2}$$

$k = 2p$

$$x_{6p+2} = \left(\frac{6p+5}{6p+3}\right)^{(6p+2)\cdot\left(-\frac{1}{2}\right)}$$

$$\lim_{p\to\infty} x_{6p+2} = \lim_{p\to\infty}\left(\frac{6p+5}{6p+3}\right)^{\frac{(6p+2)}{-2}} =$$

$$= \lim_{p\to\infty}\left[\left(1 + \frac{2}{6p+3}\right)^{\frac{6p+3}{2}}\right]^{\frac{2}{6p+3}\cdot\frac{6p+2}{-2}}$$

$$= e^{-1} = \frac{1}{e}$$

$k = 2p+1$

$$x_{6p+5} = \left(\frac{6p+8}{6p+6}\right)^{\frac{6p+5}{2}} =$$

$$\lim_{p\to\infty}\left[\left(1 + \frac{2}{6p+6}\right)^{\frac{6p+6}{2}}\right]^{\frac{2}{6p+6}\cdot\frac{6p+5}{2}} = e^1 = e$$

$$LIM(x_n) = \left\{\frac{1}{e^2}, \frac{1}{e}, e, e^2\right\}$$