

Pot sa fie absolut orice.

```
data Bool      = False | True
data Season    = Winter | Spring | Summer | Fall
data Shape     = Circle Float | Rectangle Float Float
data List a    = Nil | Cons a (List a)
data Nat       = Zero | Succ Nat
data Exp       = Lit Int | Add Exp Exp | Mul Exp Exp
data Tree a    = Empty | Leaf a | Branch (Tree a)(Tree a)
data Maybe a   = Nothing | Just a
data Pair a b  = Pair a b
data Either a b = Left a | Right b
```

Anotimpuri

```
data Season = Spring | Summer | Autumn | Winter
```

```
next :: Season -> Season
next Spring = Summer
next Summer = Autumn
next Autumn = Winter
next Winter = Spring
```

```
eqSeason :: Season -> Season -> Bool
eqSeason Spring Spring = True
eqSeason Summer Summer = True
eqSeason Autumn Autumn = True
eqSeason Winter Winter = True
eqSeason _      _      = False
```

```
showSeason :: Season -> String
Spring = " Spring "
Summer  = " Summer "
Autumn  = " Autumn "
Winter  = " Winter "
```

```
toInt :: Season -> Int
toInt Winter = 0
toInt Spring = 1
toInt Summer = 2
toInt Autumn = 3
```

```
fromInt :: Int -> Season
fromInt 0 = Winter
fromInt 1 = Spring
fromInt 2 = Summer
fromInt 3 = Autumn
```

```
next :: Season -> Season
next x = fromInt ((toInt x + 1) `mod` 4)
```

Cercuri si dreptunghiuri

```
type Radius = Float
type Width  = Float
type Height = Float
```

```
data Shape = Circle Radius
           | Rectangle Width Height
```

```
area :: Shape -> Float
area (Circle r)      = pi * r^2
area (Rectangle w h) = w * h
```

```
eqShape :: Shape -> Shape -> Bool
eqShape (Circle r)      (Circle r2)      = (r == r2)
eqShape (Rectangle w h) (Rectangle w2 h2) = (w == w2) &&
                                             (h == h2)
eqShape _                _                = False
```

```
isCircle :: Shape -> Bool
isCircle ( Circle r ) = True
isCircle _             = False
```

```
isRectangle :: Shape -> Bool
isRectangle ( Rectangle w h ) = True
isRectangle _                  = False
```

```
radius :: Shape -> Float
radius (Circle r) = r
```

```
width :: Shape -> Float
width (Rectangle w h) = w
```

```
height :: Shape -> Float
height (Rectangle w h) = h
```

```
area :: Shape -> Float
area (Circle r)      = pi * r^2
area (Rectangle w h) = w * h
```

```
area :: Shape -> Float
area s =
    if isCircle s then
        let
            r = radius s
        in
            pi * r^2
    else if isRectangle s then
        let
            w = width s
            h = height s
        in
            w * h
    else error " impossible "
```

Expresii

```

data Exp = Lit Int
         | Add Exp Exp
         | Mul Exp Exp

evalExp :: Exp -> Int
evalExp (Lit n)    = n
evalExp (Add e f)  = evalExp e + evalExp f
evalExp (Mul e f)  = evalExp e * evalExp f

showExp :: Exp -> String
showExp (Lit n)    = show n
showExp (Add e f)  = par (showExp e ++ "+" ++ showExp f)
showExp (Mul e f)  = par (showExp e ++ "*" ++ showExp f)

par :: String -> String
par s = "(" ++ s ++ ")"

e0, e1 :: Exp

e0 = Add (Lit 2) (Mul (Lit 3) (Lit 3))
e1 = Mul (Add (Lit 2) (Lit 3)) (Lit 3)

*Main> showExp e0
" (2+(3*3) )"

* Main> evalExp e0
11

*Main> showExp e1
" ((2+3) *3) "

* Main> evalExp e1
15

```

Expresii - forma infixata

```

data Exp = Lit Int
         | Exp `Add` Exp
         | Exp `Mul` Exp

evalExp :: Exp -> Int
evalExp (Lit n)    = n
evalExp (e `Add` f) = evalExp e + evalExp f
evalExp (e `Mul` f) = evalExp e * evalExp f

showExp :: Exp -> String
showExp (Lit n)    = show n
showExp (e `Add` f) = par (showExp e ++ "+" ++ showExp f)
showExp (e `Mul` f) = par (showExp e ++ "*" ++ showExp f)

par :: String -> String
par s = "(" ++ s ++ ")"

e0, e1 :: Exp

e0 = Lit 2 `Add` (Lit 3 `Mul` Lit 3)
e1 = (Lit 2 `Add` Lit 3) `Mul` Lit 3

*Main> showExp e0
" (2+(3*3) )"

* Main> evalExp e0
11

*Main> showExp e1
" ((2+3) *3) "

* Main> evalExp e1
15

```

Expresii - cu operatori

```
data Exp = Lit Int
         | Exp :+: Exp
         | Exp **: Exp
```

```
evalExp :: Exp -> Int
evalExp (Lit n)    = n
evalExp (e :+: f) = evalExp e + evalExp f
evalExp (e **: f) = evalExp e * evalExp f
```

```
showExp :: Exp -> String
showExp (Lit n)    = show n
showExp (e :+: f) = par (showExp e ++ "+" ++ showExp f)
showExp (e **: f) = par (showExp e ++ "*" ++ showExp f)
```

```
par :: String -> String
par s = "(" ++ s ++ ")"
```

```
e0, e1 :: Exp
```

```
e0 = Lit 2 :+: (Lit 3 **: Lit 3)
e1 = (Lit 2 :+: Lit 3) **: Lit 3
```

```
*Main> showExp e0
" (2+(3*3) ) "
```

```
* Main> evalExp e0
11
```

```
*Main> showExp e1
" ((2+3) *3) "
```

```
* Main> evalExp e1
15
```

Exercitii

Mere si portocale

```
-- The datatype 'Fruit'
data Fruit = Apple(String, Bool)
            | Orange(String, Int)

-- Some example Fruit
apple, apple', orange :: Fruit
apple  = Apple("Granny Smith", False) -- a Granny Smith
                                         apple with no worm
apple' = Apple("Braeburn", True)        -- a Braeburn apple
                                         with a worm
orange = Orange("Sanguinello", 10)     -- a Sanguinello
                                         with 10 segments

fruits :: [Fruit]
fruits = [Orange("Seville", 12),
          Apple("Granny Smith", False),
          Apple("Braeburn", True),
          Orange("Sanguinello", 10)]

-- This allows us to print out Fruit in the same way we
-- print out a list, an Int or a Bool.

instance Show Fruit where
    show (Apple(variety, hasWorm)) = "Apple(" ++ variety
                                     ++ ", " ++ show hasWorm ++ ")"
    show (Orange(variety, segments)) = "Orange(" ++ variety
                                         ++ ", " ++ show segments ++ ")"

isBloodOrange :: Fruit -> Bool
isBloodOrange (Orange (b, c)) = if b `elem` ["Tarocco",
                                              "Moro", "Sanguinello"]
    then True
    else False
isBloodOrange _ = False
```

```
getNumberOfSlices :: Fruit -> Int
getNumberOfSlices (Apple (b, c)) = 0
getNumberOfSlices (Orange (b, c)) = c

bloodOrangeSegments :: [Fruit] -> Int
bloodOrangeSegments xs = foldl (+) 0 (getNumberOfSlices
                                     `map` (filter isBloodOrange xs))

-- [Orange("Seville", 12),
--   Orange("Moro", 11),
--   Apple("Granny Smith", False),
--   Apple("Braeburn", True),
--   Orange("Sanguinello", 10)]

isAppleWithWorm :: Fruit -> Bool
isAppleWithWorm (Apple (_, True)) = True
isAppleWithWorm _ = False

worms :: [Fruit] -> Int
worms xs = length (filter isAppleWithWorm xs)
```

Expresii

The following data type represents arithmetic expressions over a single variable:

```
data Expr = X                      -- variable
          | Const Int              -- integer constant
          | Expr :+: Expr          -- addition
          | Expr :-: Expr          -- subtraction
          | Expr *: Expr           -- multiplication
          | Expr :/: Expr          -- integer division
          | IfZero Expr Expr Expr -- conditional
                                   expression
```

``IfZero p q r`` represents the expression that would be written in Haskell as `if p == 0 then q else r`.

1. Write a function ``eval :: Expr -> Int -> Int``, which given an expression and the value of the variable ``X`` returns the value of the expression. For example:

```
eval (X :+: (X *: Const 2)) 3      = 9
eval (X :/: Const 3) 7             = 2
eval (IfZero (X :-: Const 3) (X :/: X) (Const 7)) 3 = 1
eval (IfZero (X :-: Const 3) (X :/: X) (Const 7)) 4 = 7
eval (Const 15 :-: (Const 7 :/: (X :-: Const 1))) 0 = 22
```

but both of the following should produce a divide-by-zero exception:

```
eval (Const 15 :-: (Const 7 :/: (X :-: Const 1))) 1
eval (X :/: (X :-: X)) 2
```

2. Write a function ``protect :: Expr -> Expr`` that protects against divide-by-zero exceptions by "guarding" all uses of division with a test for a zero-valued denominator. In this case the result should be ``maxBound`` (the maximum value of type ``Int``, which is platform dependent). Do not attempt to simplify the result by omitting tests that appear to be unnecessary. For example:

```
protect (X :+: (X *: Const 2)) = (X :+: (X *: Const 2))
```

```
eval (protect (X :+: (X *: Const 2))) 3 = 9
```

```
protect (X :/: Const 3)
      = IfZero (Const 3) (Const maxBound) (X :/:
                                             Const 3)
```

```
eval (protect (X :/: Const 3)) 7 = 2
```

```
eval (protect (X :/: (X :-: X))) 2 = maxBound
```

Afisarea

```
showExpr :: Expr -> String
showExpr X      = "X"
showExpr (Const n) = show n
showExpr (p :+: q) = "(" ++ showExpr p ++ "+" ++ showExpr q ++ ")"
showExpr (p :-: q) = "(" ++ showExpr p ++ "-" ++ showExpr q ++ ")"
showExpr (p *: q) = "(" ++ showExpr p ++ "*" ++ showExpr q ++ ")"
showExpr (p :/: q) = "(" ++ showExpr p ++ "/" ++ showExpr q ++ ")"
showExpr (IfZero p q r) = "(if " ++ showExpr p ++ " = 0
                           then " ++ showExpr q ++ "
                           else " ++ showExpr r ++
                           ")"
```

Evaluarea

```
eval :: Expr -> Int -> Int
eval X v          = v
eval (Const n) _  = n
eval (p :+: q) v   = (eval p v) + (eval q v)
eval (p :-: q) v   = (eval p v) - (eval q v)
eval (p :*: q) v   = (eval p v) * (eval q v)
eval (p :/: q) v   = (eval p v) `div` (eval q v)
eval (IfZero p q r) v = if (eval p v) == 0 then eval q v
                        else eval r v
```

Protectie pentru impartirea la 0

```
protect :: Expr -> Expr
protect X          = X
protect (Const n) = (Const n)
protect (p :+: q) = (protect p) :+: (protect q)
protect (p :-: q) = (protect p) :-: (protect q)
protect (p :*: q) = (protect p) :*: (protect q)
protect (p :/: q) = IfZero (protect q) (Const maxBound)
                  ((protect p) :/: (protect q))
protect (IfZero p q r) = IfZero (protect p) (protect q)
                              (protect r)
```

Arbori

Consider binary trees with `Int`-labelled nodes and leaves, defined as follows:

```
data Tree = Empty
          | Leaf Int
          | Node Tree Int Tree
```

and the following example of a tree:

```
t = Node (Node (Node (Leaf 1)
                    2
                    Empty)
        3
        (Leaf 4))
5
(Node Empty
 6
  (Node (Leaf 7)
    8
    (Leaf 9)))
```

Each label in a tree can be given an "address": this is the path from the root to the label, consisting of a list of directions:

```
data Direction = L | R
type Path = [Direction]
```

The empty path refers to the label at the root — in `t` above, the label `5`. A path beginning with L refers to a label in the left, or first, subtree, and a path beginning with R refers to the right, or second, subtree. Subsequent L/R directions in the list then refer to left/right subtrees of that subtree. So, for example, `[R,R,L]` is the address of the label 7 in `t`.

Verifica daca exista calea in arbore

```
present :: Path -> Tree -> Bool
present [] (Leaf n) = True
present [] (Node _ n _) = True
present (L:p) (Node t _ _) = present p t
present (R:p) (Node _ _ t) = present p t
present _ _ = False
```

Returneaza valoarea unui nod din arbore

```
label :: Path -> Tree -> Int
label [] (Leaf n) = n
label [] (Node _ n _) = n
label (L:p) (Node t _ _) = label p t
label (R:p) (Node _ _ t) = label p t
label _ _ = error "path absent"
```

Convertire arbore in drumuri

```
toFTree' :: Tree -> FTree
toFTree' (Leaf n) [] = n
toFTree' (Node t1 n t2) [] = n
toFTree' (Node t1 n t2) (L:p) = toFTree' t1 p
toFTree' (Node t1 n t2) (R:p) = toFTree' t2 p
toFTree' _ _ = error "path absent"
```

Construirea arborelui in oglinda

```
mirrorTree :: Tree -> Tree
mirrorTree Empty = Empty
mirrorTree (Leaf n) = Leaf n
mirrorTree (Node t1 n t2) = Node (mirrorTree t2) n
                             (mirrorTree t1)
```


Evaluarea adancimii celei mai indepartate frunze din arbore

```
leafdepth :: Tree -> Int
leafdepth Empty      = 0
leafdepth (Leaf n)    = 1
leafdepth (Node t t') | d == 0 && d' == 0 = 0
                      | otherwise       = 1 + max d d'
  where
    d = leafdepth t
    d' = leafdepth t'
```

Cea mai indepartata frunza din arbore

```
deepest2 :: Tree -> [Int]
deepest2 Empty      = []
deepest2 (Leaf x)    = [x]
deepest2 (Node t t') | d > d'      = deepest2 t
                      | d < d'      = deepest2 t'
                      | otherwise = deepest2 t ++ deepest2 t'
  where
    d = leafdepth t
    d' = leafdepth t'
```

Propoziti

```
data Prop = X
          | F
          | T
          | Not Prop
          | Prop :|: Prop
```

Aproximarea expresiei

```
showProp :: Prop -> String
showProp X      = "X"
showProp F      = "F"
showProp T      = "T"
showProp (Not p) = "(~" ++ showProp p ++ ")"
showProp (p :|: q) = "(" ++ showProp p ++ "|" ++ showProp
                        q ++ ")"
```

Evaluarea expresiei

```
eval :: Prop -> Bool -> Bool
eval X v      = v
eval F _      = False
eval T _      = True
eval (Not p) v = not (eval p v)
eval (p :|: q) v = (eval p v) || (eval q v)
```

Simplificarea expresiei

```
simplify :: Prop -> Prop
simplify X      = X
simplify F      = F
simplify T      = T
simplify (Not p) = negate (simplify p)
                    where
                        negate T = F
                        negate F = T
                        negate (Not p) = p
                        negate p = Not p
simplify (p :|: q) = disjoin (simplify p) (simplify q)
                    where
                        disjoin T p = T
                        disjoin F p = p
                        disjoin p T = T
                        disjoin p F = p
                        disjoin p q | p == q = p
                                    | otherwise =
                                        p :|: q
```

Expresii

```
data Expr = Var String
          | Expr :+: Expr
          | Expr **: Expr
```

Este Norm daca expresia este suma

```
isNorm :: Expr -> Bool
isNorm (a :+: b) = isNorm a && isNorm b
isNorm a         = isTerm a
```

Este Term daca expresia este constanta sau produs

```
isTerm :: Expr -> Bool
isTerm (Var x)  = True
isTerm (a :+: b) = False
isTerm (a **: b) = isTerm a && isTerm b
```

Este normala daca expresia are proprietatea de distributivitate

```
norm :: Expr -> Expr
norm (Var v)  = Var v
norm (a :+: b) = norm a :+: norm b
norm (a **: b) = norm a *** norm b
               where
                 (a :+: b) *** c = (a ***
                                     c) :+: (b *** c)
                 a *** (b :+: c) = (a ***
                                     b) :+: (a *** c)
                 a *** b         = a **: b
```

Alte expresii

```
data Expr = X
          | Const Int
          | Neg Expr
          | Expr :+: Expr
          | Expr **: Expr
```

Transforma expresia intr-o aproximare matematica

```
showExpr :: Expr -> String
showExpr X          = "X"
showExpr (Const n) = show n
showExpr (Neg p)    = "(" ++ showExpr p ++ "-" ++ ")"
showExpr (p :+: q)  = "(" ++ showExpr p ++ "+" ++ showExpr
                        q ++ ")"
showExpr (p **: q)  = "(" ++ showExpr p ++ "*" ++ showExpr
                        q ++ ")"
```

Evalueaza expresia dupa un X dat

```
evalExpr :: Expr -> Int -> Int
evalExpr X v          = v
evalExpr (Const n) _ = n
evalExpr (Neg p) v    = - (evalExpr p v)
evalExpr (p :+: q) v = (evalExpr p v) + (evalExpr q v)
evalExpr (p **: q) v = (evalExpr p v) * (evalExpr q v)
```

Transforma expresia in Scriere Poloneza Inversa

```
rpn :: Expr -> [String]
rpn X          = ["X"]
rpn (Const n) = [show n]
rpn (Neg p)    = rpn p ++ ["-"]
rpn (p :+: q) = rpn p ++ rpn q ++ ["+"]
rpn (p **: q) = rpn p ++ rpn q ++ ["*"]
```

Evalueaza o expresie in Scriere Poloneza Inversa

```
evalrpn :: [String] -> Int -> Int
evalrpn s n = the (foldl step [] s)
    where
        step (x:y:ys) "+" = (y + x):ys
        step (x:y:ys) "*" = (y * x):ys
        step (x:ys) "-" = (-x):ys
        step ys "X"      = n:ys
        step ys m | all (\c -> isDigit c
|| c == '-') m = (read m :: Int):ys
                  | otherwise = error
                      "ill-formed RPN"

the :: [a] -> a
the [x] = x
the xs = error "ill-formed RPN"
```

Vectori

```
data Term = Vec Scalar Scalar
          | Add Term Term
          | Mul Scalar Term
```

Evaluarea expresiei

```
eva :: Term -> Vector
eva (Vec x y) = (x, y)
eva (Add t u) = add (eva t) (eva u)
eva (Mul x t) = mul x (eva t)
```

Printare expresia ca pereche vector

```
sho :: Term -> String
sho (Vec x y) = show (x, y)
sho (Add t u) = "(" ++ sho t ++ "+" ++ sho u ++ ")"
sho (Mul x t) = "(" ++ show x ++ "*" ++ sho t ++ ")"
```

Puncte

```
type Point = (Int, Int)
data Points = Rectangle Point Point
            | Union Points Points
            | Difference Points Points
```

Verificare daca un punct este intre 2 puncte

```
inPoints :: Point -> Points -> Bool
inPoints (x, y) (Rectangle (left,top) (right,bottom)) =
    left <= x && x <= right && top <= y && y <= bottom
inPoints p (Union ps qs) = inPoints p ps || inPoints p qs
inPoints p (Difference ps qs) = inPoints p ps && not
    (inPoints p qs)
```

Desenare puncte

```
showPoints :: Point -> Points -> [String]
showPoints (a, b) ps = [ makeline y | y <- [0..b] ]
    where
        makeline y = [ if
inPoints (x, y) ps then '*' else ' ' | x <- [0..a] ]
```