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2017 MCM/ICM Summary Sheet

Optimizing the Airport Security Screening : A GSPN and Simulation-Based Approach

The U.S. Transportation Security Agency (TSA) aims to protect the air public with heightened security measures. However, enhanced security should not come at the heavy cost of travelers' waiting time. Therefore, we try to optimize the security schemes, increasing the checkpoint throughput and reducing the wait time variance.

We then modeling the passenger flow through single channel security checkpoint, for the regular passengers and the pre-checked passengers, respectively. Invoking the Generalized Stochastic Petri Net (GSPN) model, we are able to spot the bottlenecks within single channel by calculating the number of tokens under steady state.

While our GSPN model provides us valuable insights into the structure of single channel security checkpoint, it fails to give us a whole picture. Then we carefully adjust the structure of the whole system, which finally leads us to another nice improvement method.

To verify our theoretical results, we establish a simulation-based model which is completed in Python. And the simulation results agree with our expectancy. Besides, we are surprised to find out that, the average waiting time and variance are significantly improved by adjusting the ratio between regular passenger channels and pre-check passenger channels, while other methods that make modifications within the single channel structure only improve the system performance by certain amount. Moreover, adjusting the channel ratio do not cost extra.

Finally, we use variable-controlling method to perform the sensitivity analysis of GSPN model, the result shows that our model is not sensitive to all parameters and has strong stability. As for our simulation-based model, we assume the input, which is different traveler style, contains three parameters: walking delay, passengers per group and percentage of groups, which will be discussed at length later. As we expected, our improved model are less sensitive to the variation of parameters, although passengers per group seems to have the biggest impact on our improvement.

Contents

1 Introduction

The tragedy of 9/11 has left a deep scar on the American people. Shortly after the terrorist attack, the U.S. Transportation Security Agency (TSA) was created with the mission to protect the air public.[3] Despite the courteous and professional services provided by the TSA officers, passengers are frustrated to find themselves trapped in the long waiting lines at the checkpoints, especially during the spring and summer travel season.[3] It's partly due to the current tight security schemes, which consists of identity check, millimeter wave (MW) scan for travelers, x-ray scan for their belongings and an additional inspection for suspicious passengers and baggage.

The dilemma of increased security and the resulting inconvenience to passengers has been a long-existing problem for many other airports. [2]

We first model the flow of passenger through the single channel checkpoint to determine the bottleneck of the screening process.

2 Problem Restatement

We are expected to develop one or two model to examine the current screening process and identify the bottlenecks.

Next, according to the bottlenecks above, we need to propose some modifications to improve passenger throughput and reduce variance in wait time.

Furthermore, to ensure that our modifications are globally valid, we have to consider cultural differences or traveler styles. So we need to show how the security system accommodate these differences.

Finally, we are expected to propose policy and procedural recommendations for the security managers based on our models.

3 GSPN model of Single channel security checkpoint

3.1 Model Overview

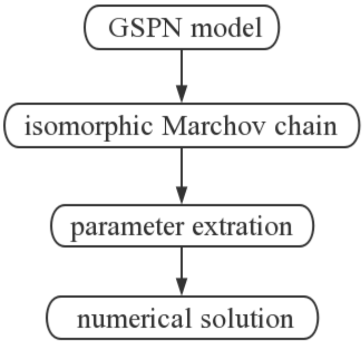


Figure 1 model overview

We first construct the GSPN model. Then, we convert it into isomorphic Marchov chain which allows us to apply linear algebra to get the solution. Most of the relevant parameters are extracted from the collected data with a few estimated. Based on the solution, we calculate the expected value of number of tokens under steady-state and identify the bottlenecks.

3.2 Assumptions

- **We take the single channel checkpoints as the cell structure** (shown in figure 2). The channel structures for regular and pre-checked passengers are the same but vary only in some parameters. All the checkpoints of the same kind have the same service efficiency and they equally share the service duty/ loads. The former assumption is approximately true in reality and it can greatly simplify our discussion. The latter condition is readily achieved since passengers always choose a relatively short line.
- **The service time of each screening stage has a negative exponential distribution** with rate parameter μ, where 1/μ is the mean service time. The memoryless property of the probability distribution function allows us to characterize discrete-state stochastic processes in continuous time, which otherwise is extremely difficult. And the given sampled data agrees well with the exponential fitting.
- **Passengers arrive at the airports with sufficient ~~time budget~~.** That is they won't miss their flights and will go through the whole screening process. Besides, all passengers are civilized thus there's no line cutting.
- **The duration of passengers' moving from one screening stage to another is neglected.** Compared with the time wasted in lines, such duration is very small.

For sake of clarity and brevity, we define:

- Passengers that pass the MW scan without alarming the machine as **clear passengers**
- Passengers that fail to pass the MW scan as **suspicious passengers**
- Baggage that pass the X-ray scan without alarming the machine as **clear baggage**
- Baggage that fail to pass the X-ray scan as **suspicious passenger**s

3.3 GSPN Model

Figure 2 gives a graphical representation of our GSPN model, in which the circles denote places and boxes denote transition. There are ten places (P = $\{p_1,\ p_2\ldots,\ p_{10}\}$) in our model, which represent different stages of the passenger/baggage, and ten transitions (T = $\{t_1,\ t_2\ \ldots,t_{10}\}$), which represent a specific activity as listed in table 1. We assume that initially only p1 contains a token, which represent a passenger about to start his/her journey of security screening with his/her baggage.

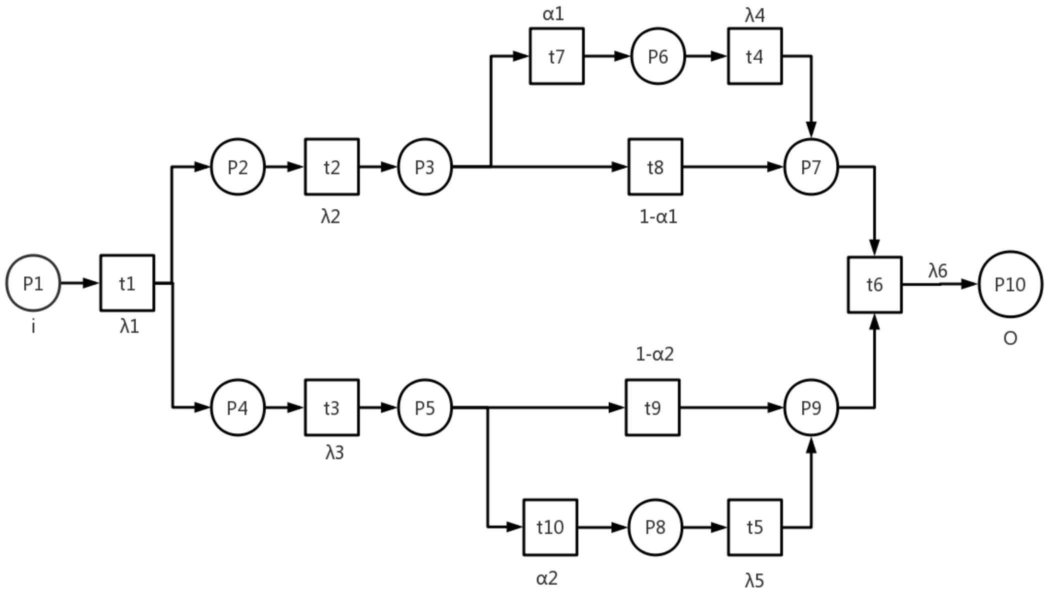


Figure 2 GSPN model

To describe the dynamics of the system, timing specifications are added to the basic structure. $\{\lambda_1,\lambda_2,...\}$ are the transition rates of the timed transitions, which are reversely proportional to the corresponding delay time, while the immediate transitions have a delay of null. Following the rule of transition and firing [1], when there's a token in p_3, there will be a new token either in p_6 or p_7 via the firing of immediate transition of t_7 or t_8, with possibility of alpha1 and 1-alpha1 respectively. To form a strongly connected network, we virtually connect the p_{10} to p_1 and associate it with a transition rate λ.

The steady-state number of the tokens contained in places is used to characterize the congestion level of each stage, which is of great interest to us.

Table 1 Notations of Places

Symbols	Definition
p_1	Passenger arriving at the checkpoint with his/her baggage, ready for ID check
p_2	Passenger ready for the MW scan
p_3	Passenger having passed the MW scan
p_4	Baggage ready for the X-ray scan
p_5	Baggage having passed the X-ray scan
p_6	Suspicious passenger that need additional inspection
p_7	Passengers waiting to collect the baggage
p_8	Suspicious baggage that need additional inspection
p_9	Baggage waiting to be collected
p_{10}	Passengers having finished the whole screening process

Table 2 Notations of Transitions

Symbols	Definition
t_1	ID checking
t_2	MW scan for clear passengers
t_3	X-ray scan for clear baggage
t_4	Additional inspection for suspicious passengers
t_5	Additional inspection for suspicious baggage
t_6	Passengers' collecting their baggage
t_7	Passengers' being flagged suspicious
t_8	Passengers' being flagged clear
t_9	Baggage's being flagged clear
t_{10}	Baggage's being flagged suspicious

3.4 Converted Isomorphic Marchov Chain

Due to the memoryless property of the exponential distribution of timed transition delays, GSPN is isomorphic to Continuous Time Markov chain (CTMC). [1] Figure 3 shows the converted Marchov chain, where $M_1, M_2, \dots, M_{10}$ denote tangible markings.

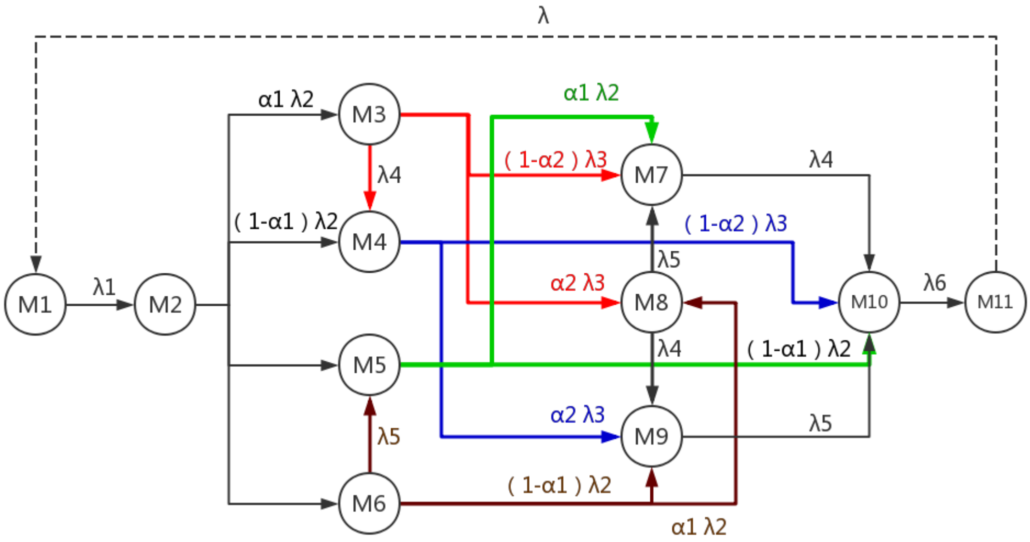


Figure 3 converted Marchov chain of markings

Let X be the steady-state distribution vector, which represent the probability distribution on tangible markings of GSPN.

$$\begin{cases} Q \cdot X = 0 \\ X \cdot \mathbf{1}^T = 1 \end{cases}$$

Q is the infinitesimal generator of the isomorphic CTMC.

The latter equation enforce the normalization condition.

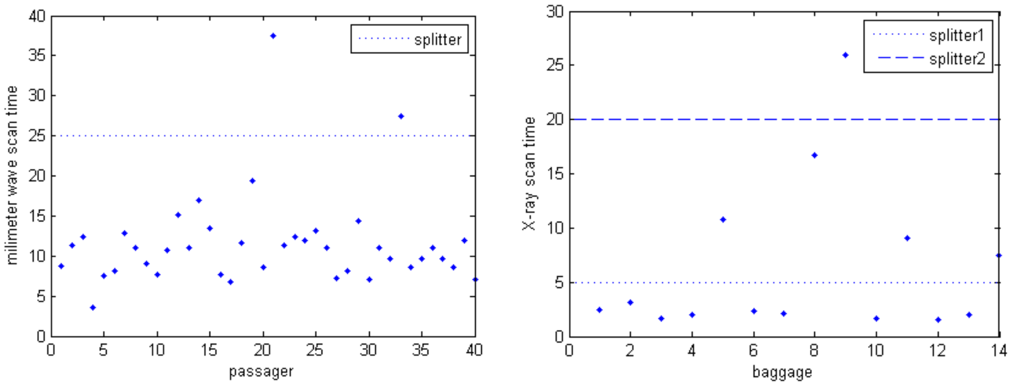
Evaluate the equation with the parameters extracted in section 3.3, we will have the following equations for pre-check channel

$$
\begin{cases}
1.155x_1 = x_{11} \\
2.986x_2 = 1.155x_1 \\
2.878x_3 = 0.016x_2 \\
2.663x_4 = 0.307x_2 + 0.216x_2 \\
0.324x_5 = 2.473x_2 + 0.144x_8 \\
0.467x_6 = 0.19x_2 \\
0.216x_7 = 2.473x_3 + 0.016x_5 + 0.144x_8 \\
0.36x_8 = 0.19x_3 + 0.016x_6 \\
0.144x_9 = 0.19x_4 + 0.307x_6 + 0.216x_7 \\
0.744x_{10} = 2.473x_4 + 0.307x_5 0.216x_7 + 0.144x_9 \\
\sum_1^n x_i = 1
\end{cases}
$$

3.5 Parameter Extractions

Most of the parameters are extracted from given sampled data and the rest are estimated.

Since there's no anomalous points among the sampled data of each screening stage, except for MW scan and X-ray scan (see Figure 4), we simply average the sampled delays as listed in Table 3.



a) b)

Figure 4 data distribution of a) millimeter wave scan and b) x-ray scan

We infer that the anomalous points in Figure 4 imply additional inspection for suspicious passengers, so we reasonably assume that the MW scan rejection probability is $\alpha_1 = 2/40 = 0.05$ (If a passenger is rejected by MW scan, he/she is flagged suspicious). Eliminating the anomalous points, we obtain the average delay for clear passengers to pass the MW scan is 10.463s.

Similar analysis is performed with X-ray scan. The sampled points of X-ray scan can be roughly divided into three groups from bottom to top. The group below is naturally assumed to be the case of clear baggage. If the rest of the points indicate suspicious baggage, the x-ray scan rejection probability would be 5/14=0.3571 (If a piece of baggage is rejected by X-ray scan, it is flagged suspicious), which is too high.

That is to say, the middle group is also clear baggage, though longer time is needed somehow. As such we obtain the X-ray scan rejection probability as 1/14=0.0714 and the average delay for clear baggage to pass the X-ray scan is 4.862s.

We get the average delay for passengers to collect their baggage through the equation below:

$T_{p7} = 28.6206\text{s} - \max(T_{p3}, T_{p4}) = 28.6206\text{s} - 11.2125\text{s} = 17.4082s$

Where 28.6206sthe average time to get the scanned property and T_{p3} is the delay of MW scan and T_{p4} is the delay of X-ray scan for pre-check channel.

We think pre-check can expedite the screening process mainly because passengers need not remove their shoes. Hence, the relatively small average delay of MW scan is that of the pre-checked passengers.

Consulting data sources and real life experiences, we give the estimates of missing parameter as following.

Table 3 Timing specifications of GSPN for
a) pre-checked passengers b) regular passengers

a)

Transition rate	**Delay**	**Value**	**Definitions**
λ_{p1}	T_{p1}	9.1895	Passengers' arrival interval
λ_{p2}	T_{p2}	11.2125	Delay of ID check
λp_3	T_{p3}	10.463	Delay of MW scan
λ_{p4}	T_{p4}	4.862	Delay of X-ray scan
λ_{p5}	T_{p5}	60	Delay of additional inspection for suspicious passengers
λ_{p6}	T_{p6}	90	Delay of additional inspection for suspicious baggage
λ_{p7}	T_{p7}	17.4082	Delay for passengers to collect the baggage

b)

Transition rate	**Delay**	**Value**	**Definitions**
λ_{r1}	T_{r1}	12.9457	passengers' arrival interval
λ_{r2}	T_{r2}	11.2125	Delay of ID check
λ_{r3}	T_{r3}	40	Delay of MW scan
λ_{r4}	T_{r4}	4.862	Delay of X-ray scan
λ_{r5}	T_{r5}	60	Delay of additional inspection for suspicious passengers
λ_{r6}	T_{r6}	90	Delay of additional inspection for suspicious baggage
λ_{r7}	T_{r7}	17.4082	Delay for passengers to collect the baggage

Table 4 Probability specifications of GSPN for both channels

Symbol	**Value**	**Definition**
α_1	0.05	MW scan rejection probability
α_2	0.0714	X-ray scan rejection probability

The transition rate λ_i in Table 3 is given by:

$$\frac{\lambda_i}{\lambda} = \frac{T}{T_i}$$

3.6 Results analysis

The expected value of number of tokens in each place p_j is given by:

$$\mathrm{E}\left[N(\mathrm{p}_j)\right] = 1 \cdot P\{A_j\}$$

Where A_j is the subset of the reachability set (RS) of our GSPN model, for which there is a token in p_j (p_j happens to be either 1 or 0 in our cases), and $P\{A_j\}$ is the probability of the subset. As we mentioned before, it characterizes the congestion level of each screening stage.

Table 5 Performance indexes for a) pre-check channel and b) regular channel

a)

$\mathbf{p}_j$	$\mathbf{E}[\boldsymbol{N}(\mathbf{p}_j)]$	$\mathbf{t}_j$	$\mathbf{U}(\mathbf{t}_j)$
p_1	0.2406	t_1	0.2406
p_2	0.2245	t_2	0.2245
p_3	0	t_3	0.1043
p_4	0.1043	t_4	0.0644
p_5	0	t_5	0.1379
p_6	0.0644	t_6	0.1161
p_7	0.2732		
p_8	0.1379		
p_9	0.3199		
p_{10}	0.1972		

b)

$\mathbf{p}_j$	$\mathbf{E}[\boldsymbol{N}(\mathbf{p}_j)]$	$\mathbf{t}_j$	$\mathbf{U}(\mathbf{t}_j)$
p_1	0.1251	t_1	0.1251
p_2	0.4462	t_2	0.4462
p_3	0	t_3	0.0542
p_4	0.0542	t_4	0.0335
p_5	0	t_5	0.0717
p_6	0.0335	t_6	0.1942
p_7	0.2508		
p_8	0.0717		
p_9	0.6046		
p_{10}	0.1444		

We find out that that p_7 and p_9 contains more tokens when the system reaches steady state. That is to say, they're more likely to be the bottlenecks. It suggests that passengers have to wait for their scanned property or the other way around.

4 Simulation-based Model

4.1 Model Overview

We write a simulation program in Python to verify our GSPN model and to test our proposals. We adopt two performance indexes to evaluate our proposals: passenger throughput and variance in wait time. Since our GSPN model deals with single channel, it can't give us a whole picture, (though it provide us with valuable insights). To be close to reality, we simulate a (3+1) channel system. The simulation results show that:

- Increased efficiency of the additional inspection for pre-check channel
- Increased efficiency of the MW scan for regular channel
- (2+2)-channel structure

can significantly improve the system performance.

4.2 Simulation description

The whole security screening system is assumed to be comprised of many identical 3+1 channel set. Thus, we focus on the basic component of 3+1 channel set, of which every channel share the common structure as shown in Figure 5. The pre-check and regular channel only differs in the ***service time*** (here we borrow the familiar concept from the queuing theory. In our cases, it actually implies execution time) of each screening stage. We assume that passengers go to either pre-check or regular channel with a probability value of 45% and 55% respectively.

Structure of single channel

- ***Service buffer*s** are marked with numbers from Ⅰ to Ⅴ, standing for the waiting lines at different screening stages. They record the numbers of passengers being held up in this section.
- The boxes are ***service points*** which allows only one passenger / one piece of baggage each time. Each is assigned with a **constant service time** for sake of simplicity.
- The arrow-headed lines indicate a fast shift to the exit with no delay.
- The blue bar represent the ***arrival gate***. We assume, that the **passengers' arrival interval approximately has an exponential distribution** as we can observe from the sampled data.

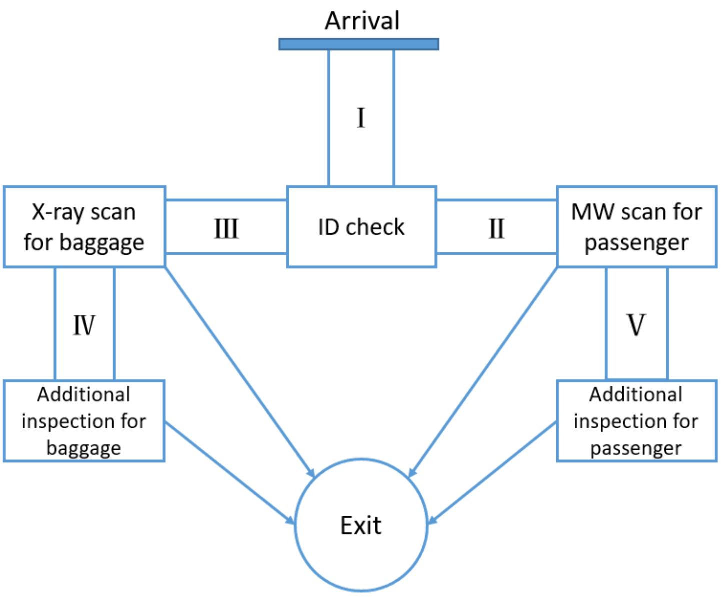


Figure 5 structure of single channel

Passenger

- Every passenger has a ***No.*** as index, and a ***walking delay*** as they transfer between ***service points,*** which allows us characterized different passengers later.
- Every passenger carries 5 independent ***queueing timers*** which record their delays at the five ***service buffer*** respectively, and 5 independent ***service timers*** which record their delays at the five ***service points*** respectively . and the queueing time will be added to the passenger's total waiting time.

In the simulation, time lapses in discrete steps.

At each time step, the passenger's states (his/her location and the timers on him/her) will be refreshed：

- For passengers at a ***service point***, he/she will be transferred to the next ***service buffer*** if it reaches the given service time; otherwise he/she will stay at the same service point.
- For passengers at ***service buffer***, he/she will stay, unless the ***service point*** ahead is empty and he/she will be sent there immediately.
- Once the passenger finished the whole process, he/she will be automatically added to the ***passenger list***.
- The ***service timers*** will increase by one unit, if the passenger stays at the corresponding ***service point***.
- The ***queueing timers*** will increase by one unit, if the passenger stays at the corresponding ***service buffer.***

4.3.1 Improvements within single channel

We half the delay of each screening stage and compare their influence on the system's performance as shown in Table 6. We find out that the average passenger waiting time and variance are more significantly improved when we reduce the additional baggage check time or additional person check time for pre-check passengers or the person check time for regular passengers.

Table 6

Improvement	average waiting time	variance in waiting time
Original scheme	2212.81376	11701350.09
Additional baggage check for pre-check passengers (90, 45)	**1984.6836**	**9401336.406**
Additional person check for pre-check passengers (60, 30)	1**965.90648**	**8877027.01**
Exit time of pre-check passengers(17,10)	2021.09484	9605495.489
MW scan for regular passengers(30,20)	**1881.0224**	**9518653.247**
Exit time of regular passengers(17,10)	2082.24612	10387801.05
channel ratio 2:2	1571.29652	1569509.849

Note: (m, n) means reduce the value from m to n

4.3.2 Improvements on Channel Ratio

Note that 45% of passenger are pre-checked which takes up nearly half of the passengers, while the channels for pre-checked passengers are much lesser than those for regular channels. Altering the ratio to be 2:2, we dramatically reduce the average waiting time and the mean variance in waiting time as shown in Table 7.

Table 7

Channel ratio	Mean waiting time	Variance in waiting time
3: 1	2212.81376	11701350.09
2: 2	1571.29652	1569509.849

5 Sensitivity Analysis

5.1 GSPN model

To keep our discussion simple, we just deal with one of the 9 parameter each time. By giving the parameter a perturbation, we shall find out how the number of tokens alter in each place. The result is shown in Figure 6 and Figure 7, where x-axis is the index of GSPN places and y-axis is the corresponding number of tokens under steady state. The perturbation of the parameter goes from 0.5 to 1.5 fold as the line color changes from dark to light.

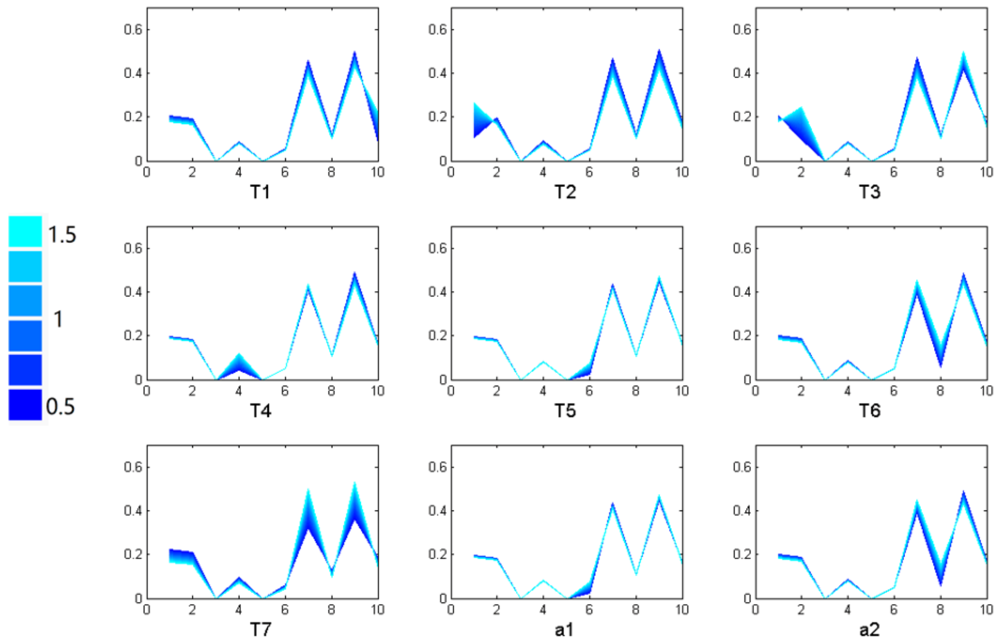


Figure 6 sensitivity analysis for the pre-check channel

For pre-check channel, we can observe that the number of tokens is slightly sensitive to T2, T3 and T7. Although the number of tokens might change a little in P1, P2, P7 or P9, P7 and P9 remains to be the bottlenecks.

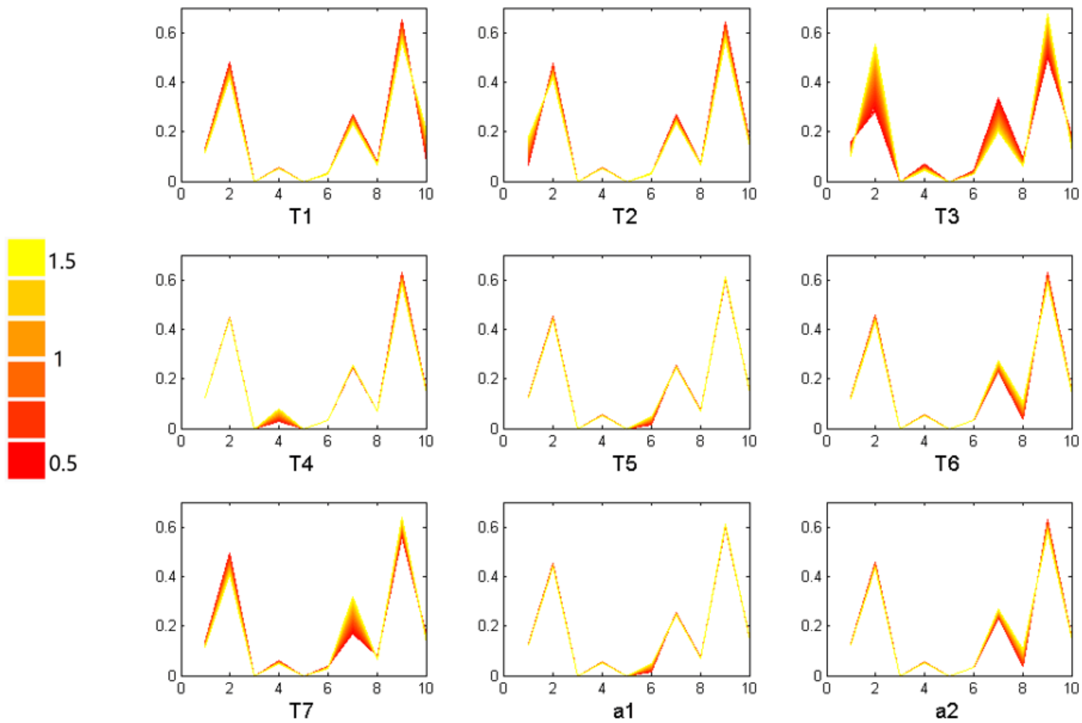


Figure 7 sensitivity analysis for the regular channel

As for the regular case, the number of tokens is relatively sensitive to T3 and T7. With T3 increased, tokens in P2 or P9 increase remarkably while that in P7 drops. Anyway, the bottlenecks remain unchanged.

In short, our model is not very sensitive to the variation of parameters.

5.2 Simulation-based Model

In the overview we have already defined three variables:

- Passenger walking delay
- Passengers per group
- Proportion of each group.

These three variables are determined by the following assumptions:

(1) Every passenger has various delays due to his/her personal reasons. Our previous assumption states that there' no cut-in in our system, thus the delay might accumulate.

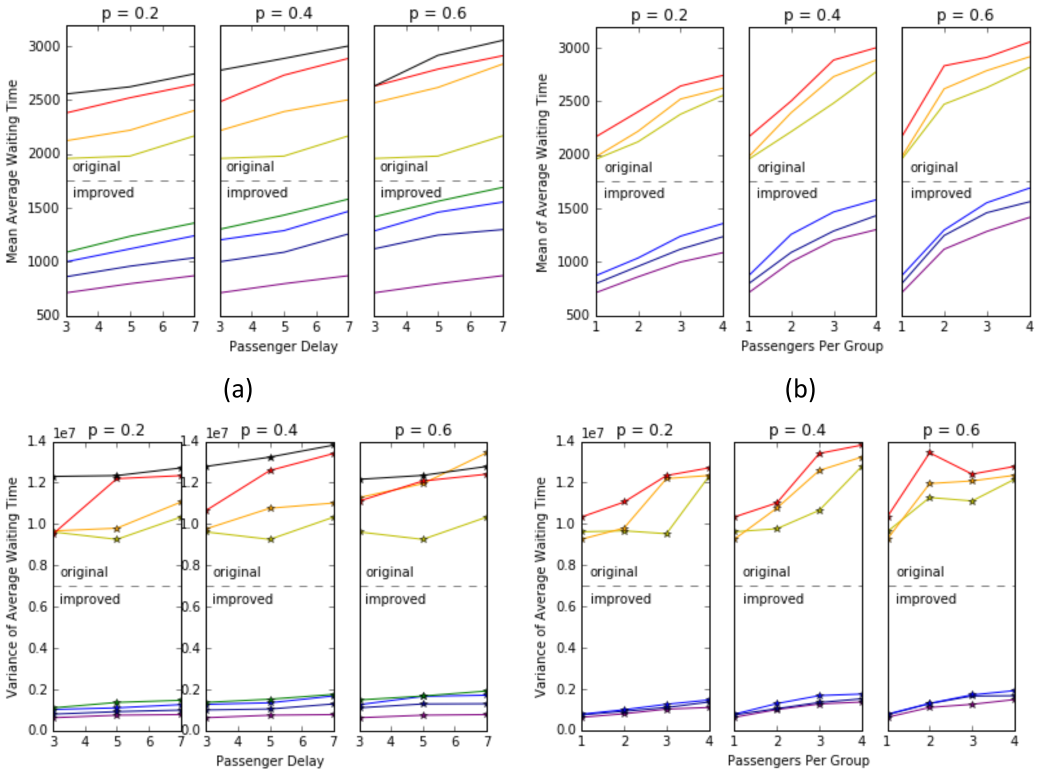
(2) Every group has one to four passengers, and each member in the group we assume that they arrive at the same time, and has to be pre-check passenger or regular passenger together.

(3) The value of the three variables is discrete

In order to evaluate how our model can accommodate those differences in a manner that expedites passenger throughput and reduces variance (sensitivity analysis), we need to compare the performance of both our improved model and the original model while changing all those three variables. However, because of the large state space produced by the combinations of different values of the three variables, we can't possibly enumerate all the situations even though the values of them are discrete. Thus, in order to observe the output differences more clearly, we use the control variable method to perform the sensitivity analysis.

The results are shown in Figure 8

- p means the proportion of the group;
- lines above the horizontal grey line are generated by the original model;
  lines below the grey line are generated by the improved model;
- in (a) and (c), different color means different type of group;
  in (b) and (d), different color means different delay time;
- outputs in (a) and (c) are mean of average waiting time;
  outputs in (b) and (d) are variance of average waiting time.
- The star-marked line stands for variance of average waiting time, the line with no markings stands for mean of average waiting time
- When we fix the value of delay/passengers per group, we mean that every passenger has the same delay, and every group has the same number of passengers

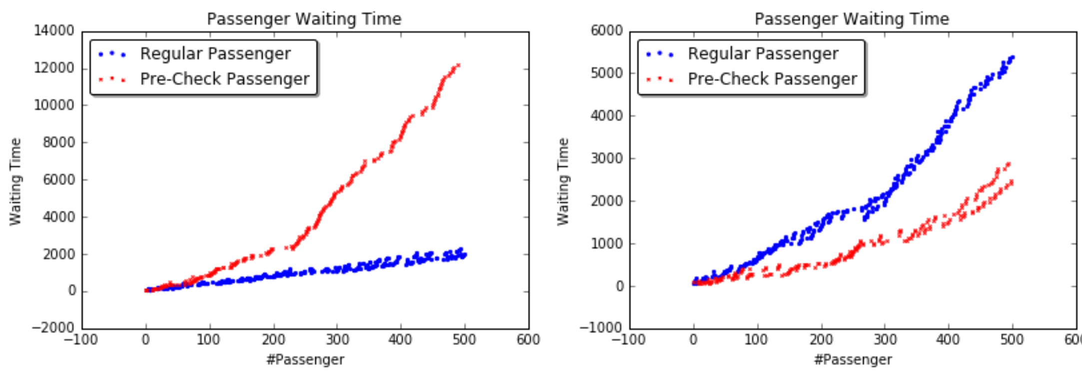


(a) (b)

(c) (d)

Figure 8 sensitivity analysis for the simulation-based model

We can notice that no matter how the percentage of the group change, the points which share the same value of the delay and passengers per group change seem to change little. Thus, we can infer that the output (Mean Average Waiting Time and Variance of Average Waiting Time) is not sensitive to the change of percentage of group. Therefore, we can just fix the percentage of the group and apply sensitivity analysis on the other two variables. We abstract four graphs from the Figure 9.



(a) original model (b) improved model

Figure 9 sensitivity analysis for the simulation-based model

If the “trend” of the lines changes dramatically, we say that the output is sensitive to the input. Now we can conclude that both Mean Average Waiting Time of the original model and the improved model are sensitive to the Passengers Per Group because the “trend” changes more dramatically, but less sensitive to the passenger delay because the "trend" changes more gently; Variance of Average Waiting Time of the original model is sensitive to both variables. However, Variance of Average Waiting Time is not sensitive to both variables; None of the model is sensitive to the change of the Percentage of Group

Further analysis

The sensitivity analysis above assumes that all passengers have the same delay and there are only two types of group. However, we've already mention that the actual world is more complex, but it's impossible to analyze the model sensitivity in all situations. Thus, we ran a sample scenario that is more similar to the actual world, where the delay of the passenger is random, and groups are also randomly generated (but passengers per group are at most 4 people), then we obtain the following result:

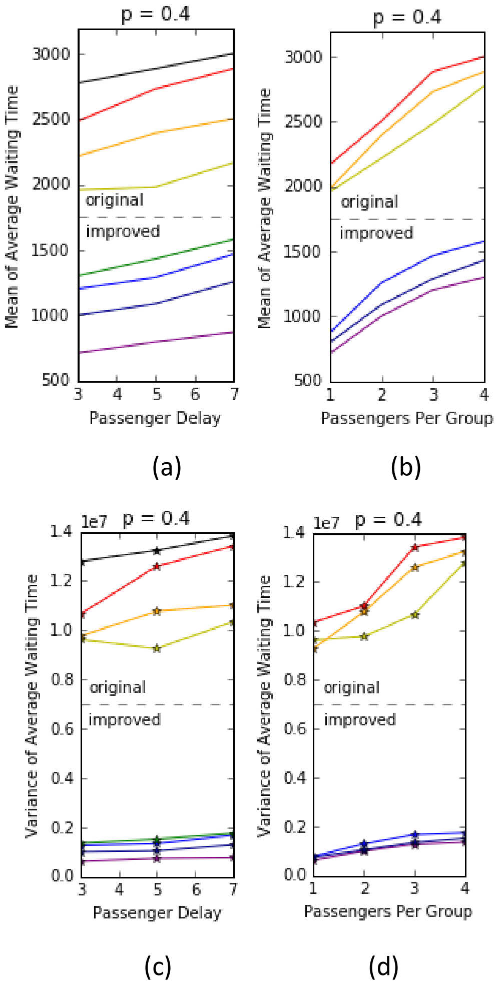


Figure 10 sensitivity analysis for the simulation-based model

Table 6 Performance Comparations

	Mean waiting time(s)	**Variance in waiting time(s^2)**
Original	[1959,3053]	[9.2598e+06,1.3824e+07]
Improved	[714,1689]	[6.2465e+05,1.9158e+06]

We can see that the improved model still has a much lower average waiting time and variance, and the fluctuation of its output are much smaller than the one of the original model. Through our sensitivity analysis, we can conclude that our model accommodates the differences caused by different traveler styles very well without compromising much performance.

6 Evaluation

Strengths:

- Our GSPN model is generally insensitive to parameter perturbations which allows more precision tolerance of the extracted parameters.
- Our simulation model is also not very sensitive to its parameters, thus our optimal system can well accommodate several kinds of passengers.

Weaknesses:

- GSPN can only deals with single channel.
- Our simulation model is based on many assumptions, which results in a discrepancy between the simulation results and reality.

Future work

We shall collect more statistics of different channels so that parameters like passengers' arrival intervals, MW scan rejection probability and X-ray scan rejection probability are close to the true value. Based on the collected data, we can also study the influence of the distribution of service time on our simulation model. In addition, we could model the passengers in more details.

A proposal letter to the security manager

Dear Sir / Madam:

We've heard that you've been caught in the dilemma between better securing passengers' safety and minimizing the inconvenience to them. Recently we made an investigation into the problem and constructed a model of the current security checking schemes. We believe our findings could help you solve the problem.

We identify the bottlenecks as following:

- Pre-checked passengers tend to linger at the exit of the checkpoint because they have to pack their luggage. (Put their electronics, medicine and other stuffs back, which are required to remove from their bags before the screening.)
- As for the regular passengers, they are more likely to stack up at the millimeter wave scan section, since they have to remove their shoes before the scan which apparently slow down the passenger flow, while their luggage might have already passed the X-ray scan causing a luggage jam there.
- The 3:1 ratio of pre-check to regular channels is not the best choice according to our simulation results.

Here, we would like to offer you some proposals to expedite the process and maximize passengers throughput :

- Additional staffing is needed to help passengers to fetch their belongings.
- Increase staffings at the millimeter scan section.
- Provide a larger area for more passengers to remove their shoes, jackets and belts.
- If there's a mess, collision or jam of luggage at the exit of the X-ray scan, staffs are expected to sort it out in time.
- When the pre-check channel is heavy loaded, one of the three regular channel should be altered to pre-check channel.
- Due to the fact that 45% of the passengers enroll in the pre-check program, we recommend the channel ratio to be 1:1.

We are proud that our proposed measures can accommodate different passengers. We've run simulations for four kinds of passengers: slow travelers, fast travelers, grouped travelers and single travelers respectively. The results suggests that the optimal system still works well with those passengers.

It would be our honor it you adopt our proposals. Thank you so much for your time and consideration.

Yours truly,

ICM team

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