

Beispiel:
 Z -Werte unabhängig
 berechnen:
 $M(V) = E\left[\sum_{i=1}^V X_i\right] =$
 $\sum_{k=1}^{\infty} k \cdot P\left(\sum_{i=1}^k X_i = k\right)$

$M(V) = 1$
 $P(V=k) = \frac{1}{2^k}$

$X/V=k$	1	3	5
P	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

$M[X/V=k] = 1 + 3 + 5 = 9$

$X/V=k$	1	3	5
P	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

$M[X/V=k] = 1 + 3 + 5 = 9$

$P(Y=1) = 1/4$
 $P(Y=3) = 1/8$
 $P(Y=5) = 1/16$
 $P(Y=7) = 1/32$
 $P(Y=9) = 1/64$
 $P(Y=11) = 1/128$
 $P(Y=13) = 1/256$
 $P(Y=15) = 1/512$
 $P(Y=17) = 1/1024$
 $P(Y=19) = 1/2048$
 $P(Y=21) = 1/4096$
 $P(Y=23) = 1/8192$
 $P(Y=25) = 1/16384$
 $P(Y=27) = 1/32768$
 $P(Y=29) = 1/65536$
 $P(Y=31) = 1/131072$
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 $P(Y=49) = 1/67108864$
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$$\begin{aligned}
 &g) X(t_1, t_2) = \{X(t_1), X(t_2)\} = \\
 &= \{X(t_1) - \mu_1 X(t_1)\} \{X(t_2) - \mu_2 X(t_2)\} = \\
 &= \mu_1^2 + \mu_2^2 (V - \mu_2 V_3) \{X(t_2) - \mu_2 X(t_2)\} = \\
 &= \mu_1^2 V_3 + \mu_2^2 V_3 = t_1^2 t_2^2 (3 - 2 \cdot 3 \cdot 5 + 2 + 1 \cdot 3 \cdot 5) = \\
 &= 0.45 t_1^2 t_2^2, \text{ w.k.} \\
 &\mu_1 V_3 = \frac{3}{2} = 1.5 \\
 &\mu_2 V_3 = \frac{3}{4} \quad \text{w.k.} \quad \mu_2 V_3 = 3
 \end{aligned}$$

функціонал процесу
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доказ:
 $X(t) = Vt^2 + t$
 $t > 0$
 функція зростаюча і строго зростаюча
 $F(t_1, t_2) = P\{X(t_1) < X(t_2)\} = P\{Vt_1^2 + t_1 < Vt_2^2 + t_2\} = P\{V(t_2^2 - t_1^2) > t_2 - t_1\} = P\{V < \frac{t_2 - t_1}{t_2^2 - t_1^2}\} = P\{V < \frac{1}{t_2 + t_1}\}$

$$\begin{aligned}
 F(x) &= \begin{cases} \frac{x-a}{b-a}, & a < x < b \\ 0, & x \leq a \\ 1, & x \geq b \end{cases} \Rightarrow F(x_1, t_1; x_2, t_2) = \\
 &= P\{V < \frac{x_2 - t_2}{t_2^2 - t_1^2}\} = F\left(\frac{x_2 - t_2}{t_2^2 - t_1^2}\right) = \\
 &= \begin{cases} 0, & \text{w.k.} \left(\frac{x_2 - t_2}{t_2^2 - t_1^2}\right) < 0 \\ 1, & \text{w.k.} \left(\frac{x_2 - t_2}{t_2^2 - t_1^2}\right) \geq 3 \end{cases} \\
 &= \begin{cases} \frac{x_2 - t_2}{t_2^2 - t_1^2}, & 0 \leq \frac{x_2 - t_2}{t_2^2 - t_1^2} < 3 \\ 1, & \frac{x_2 - t_2}{t_2^2 - t_1^2} \geq 3 \end{cases}
 \end{aligned}$$

d) функція зростаюча і строго зростаюча.

$$\begin{aligned}
 F(x, t) &= P\{X(t) < x\} = P\{Vt^2 + t < x\} = \\
 &= P\{V < \frac{x-t}{t^2}\} = F\left(\frac{x-t}{t^2}\right) = \begin{cases} 0, & \frac{x-t}{t^2} < 0 \\ \frac{x-t}{t^2}, & 0 \leq \frac{x-t}{t^2} < 3 \\ 1, & \frac{x-t}{t^2} \geq 3 \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 f(x, t) &= \frac{\partial F(x, t)}{\partial x} \Rightarrow f(x, t) = \begin{cases} 0, & \frac{x-t}{t^2} < 0 \text{ or } \frac{x-t}{t^2} \geq 3 \\ \frac{1}{t^2}, & 0 \leq \frac{x-t}{t^2} < 3 \end{cases}
 \end{aligned}$$