**Butterworth approximation function zeros and poles computation**

As Butterworth transfer function has the unit numerator it does not have zeros. Therefore, only poles must be computed. Butterworth approximation poles are represented by complex conjugate pairs. So, we need to compute only half of them:

Where:

|  |  |
| --- | --- |
|  | * filter order |
|  | * number of the complex conjugate poles pairs |
|  | * if R=0 than filter transfer function doesn’t have real odd poles. If R = 1, then one real odd pole exists |
|  | * truncate function |
|  | * filter order |
|  | * number of the complex conjugate poles pairs |
|  | * if R=0 than filter transfer function doesn’t have real odd poles. If R = 1, then one real odd pole exists |
|  | * truncate function |

Complex conjugate poles pairs for Butterworth approximation are computed as follows:

Where:

Where:

|  |  |
| --- | --- |
|  | * stopband attenuation, Db |
|  | * stopband attenuation, p.u. |

**Chebyshev type I approximation function zeros and poles computation**

**Butterworth and Chebyshev type I digital lowpass filter computation**

Lowpass analogue prototype transfer function:

Where:

|  |  |
| --- | --- |
|  | * Laplace operator |
|  | * transfer function real odd pole |
|  | * transfer function i-th complex pole |
|  | * transfer function i-th complex conjugate pole |
|  | * the complex conjugate poles pairs number |
|  | * complex conjugate poles counter |
|  | * i-th section zero frequency gain |
|  | * real odd pole section zero frequency gain |

Cut-off frequency deformation:

Where:

|  |  |
| --- | --- |
|  | * cut-off frequency, Hz |
|  | * sampling frequency, Hz |

Frequency lowpass to lowpass transformation:

Bilinear transformation:

Where:

Thus, second order sections form:

**Butterworth and Chebyshev type I digital highpass filter computation:**

Lowpass analogue prototype transfer function:

Where:

|  |  |
| --- | --- |
|  | * Laplace operator |
|  | * transfer function real odd pole |
|  | * transfer function i-th complex pole |
|  | * transfer function i-th complex conjugate pole |
|  | * the complex conjugate poles pairs number |
|  | * complex conjugate poles counter |
|  | * i-th section zero frequency gain |
|  | * real odd pole section zero frequency gain |

Pass frequency deformation:

Where:

|  |  |
| --- | --- |
|  | * pass frequency, Hz |
|  | * sampling frequency, Hz |

Frequency lowpass to highpass transformation:

Bilinear transformation:

Where:

Thus, second order sections form:

**Butterworth and Chebyshev type I digital bandpass filter computation:**

Lowpass analogue prototype transfer function:

Where:

|  |  |
| --- | --- |
|  | * Laplace operator |
|  | * transfer function real odd pole |
|  | * transfer function i-th complex pole |
|  | * transfer function i-th complex conjugate pole |
|  | * the complex conjugate poles pairs number |
|  | * complex conjugate poles counter |
|  | * i-th section zero frequency gain |
|  | * real odd pole section zero frequency gain |

Bandpass frequencies deformation:

Where:

|  |  |
| --- | --- |
|  | * passband start frequency, Hz |
|  | * passband width frequency, Hz |
|  | * sampling frequency, Hz |

Frequency lowpass to bandpass transformation:

Where:

As the one second order section splits into two second order sections, we continue bilinear transformation for the one second order section without considering it’s gain:

Bilinear transformation of real odd pole second order section:

Thus, the second order sections form:

Where:

**Butterworth and Chebyshev type I digital bandstop filter computation:**

Lowpass analogue prototype transfer function:

Where:

|  |  |
| --- | --- |
|  | * Laplace operator |
|  | * transfer function real odd pole |
|  | * transfer function i-th complex pole |
|  | * transfer function i-th complex conjugate pole |
|  | * the complex conjugate poles pairs number |
|  | * complex conjugate poles counter |
|  | * i-th section zero frequency gain |
|  | * real odd pole section zero frequency gain |

Bandstop frequencies deformation:

Where:

|  |  |
| --- | --- |
|  | * passband start frequency, Hz |
|  | * passband width frequency, Hz |
|  | * sampling frequency, Hz |

Frequency lowpass to bandpass transformation:

Where:

As the one second order section splits into two second order sections, we continue bilinear transformation for the one second order section:

Real odd pole second order section:

Thus, the second order sections form:

Where: