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Editor's Introduction

Christophe Granet continues his series on designing dual-reflector antennas with this column. His earlier articles were published in this column in April, 1998: "Designing Axially Symmetric Cassegrain and Gregorian Dual-Reflector Antennas;" June, 1998: "Minimum Blockage in Dual-Reflector Antennas While Taking into Account the Phase-Center of the Feed;" December, 1999: "Displaced-Axis Dual Reflector Antennas" (four types); and December, 2001: "Dragonian Dual-Reflector." I invited Christophe to write this article to find the parameters of dual-offset Cassegrain and Gregorian reflectors. All 12 designs include the Mizugutch feed-axis rotation to reduce cross polarization. You will want to make this one part of your tools for the design of dual reflectors.

The article continues the series by presenting the geometry of the reflector system in all possible combinations of parameters, and gives the equations to calculate every other dimension needed for construction. I wrote these equations into a series of *FORTRAN* subroutines, and will send them to you by e-mail if you request them by e-mail. I still have the series of subroutines that I wrote for his earlier articles in this column.

Designing Classical Offset Cassegrain or Gregorian Dual-Reflector Antennas from Combinations of Prescribed Geometric Parameters

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1. Introduction

This paper proposes a simple procedure for the design of classical offset Cassegrain or Gregorian dual-reflector antennas from combinations of prescribed geometric parameters. This procedure has already been applied to classical Cassegrain and Gregorian antennas [1, 2], to classical displaced-axis Cassegrain and Gregorian antennas [3], and to classical offset Dragonian antennas [4]. The antenna systems can be fully characterized by 21 parameters, of which only five need to be provided by the antenna designer [5], as the remaining 16 parameters can be derived in closed form using the procedure described here. In this paper, we assume that the main reflector (MR) has a circular aperture, while the subreflector (SR) has an elliptical aperture.

All the antenna geometries presented here satisfy the Mizugutch condition [6], which is the geometric-optics condition for zero cross-polarized radiation. Note that this procedure is very close to the one used for offset Dragonian systems [4], but all the relevant information is repeated here for completeness.

2. Design Parameters

Two types of offset dual-reflector systems are considered here, the classical offset-Cassegrain system (see Figure 1), and the classical offset-Gregorian system (see Figure 2). The 21 design parameters are listed in Table 1 (see also Figures 1 to 5).

As in [4] and [5], the angles are considered positive (negative) if they correspond to a counterclockwise (clockwise) rotation about the y axis, with the exception of θ_e, which is always positive. Another important parameter is σ, which, as in [1, 2, 5], is

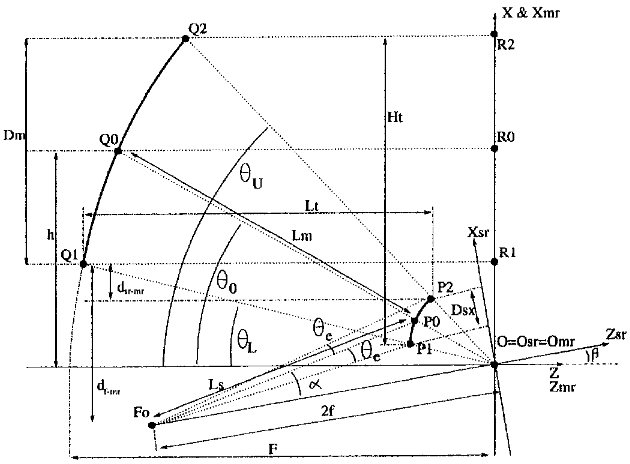

Figure 1. The offset Cassegrain geometry.

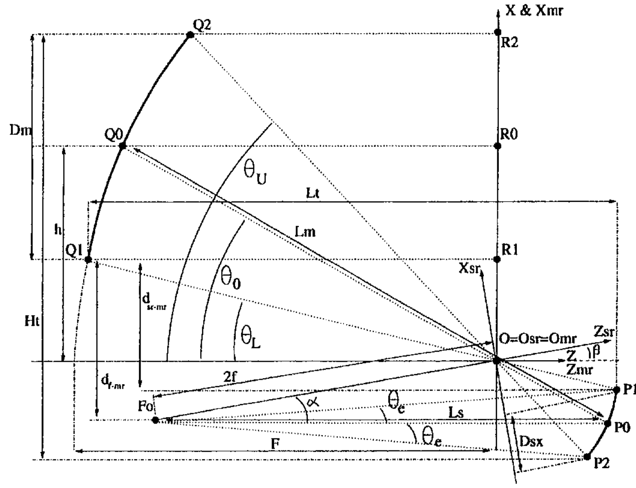

Figure 2. The offset Gregorian geometry.

such that $\sigma = -1$ for a Cassegrain system, and $\sigma = +1$ for a Gregorian system.

3. Geometry of the Antenna

3.1 Main Reflector

The main-reflector profile, $z_{mr}(x_{mr}, y_{mr})$, is paraboloidal, and depends on the real parameter F. It is of the form

Table 1. Design parameters.

Parameter	Description
Dm	Diameter of the MR aperture when projected on the xy plane.
F	Focal length of the MR.
h	Offset of the MR, i.e., the distance between the point Q_0 on the MR and the MR coordinate system z_{mr} axis.
θ_0	Offset angle of the MR.
θ_U	Offset angle of the top of the MR.
θ_L	Offset angle of the bottom of the MR.
β	Tilt angle between the SR coordinate system z_{sr} axis and the MR coordinate system z_{mr} axis.
θ_e	Angle between the z_f axis and the edge of the SR.
e	SR eccentricity.
a	SR surface parameter.
f	SR surface parameter (half the interfocal distance, $f = ae$).
Ds_x	Major axis of the SR elliptical aperture taken parallel to the x_{sr} axis.
Ds_y	Major axis of the SR elliptical aperture taken parallel to the y_{sr} axis.
α	Tilt angle between the SR coordinate system z_{sr} axis and the feed coordinate system z_f axis.
Ls	Following the central ray, Ls is the distance between the focal point F_0 and the point P_0 on the SR (distance between the feed and the SR).
Lm	Following the central ray, Lm is the distance between the SR point P_0 and the point Q_0 on the MR (distance between the SR and the MR).
d_{sr-mr}	Minimum vertical distance (along the x axis) between the SR edges (P_1, P_2) and the MR edges (Q_1).
d_{f-mr}	Minimum vertical distance (along the x axis) between the feed (F_0) and the MR edge (Q_1).
Lt	Maximum length (along the z axis) of the two-reflector combination.
Ht	Maximum vertical length (along the x axis) of the two-reflector combination.
C_{sr}	Point expressed in the SR coordinate system defining the center of the SR elliptical aperture.

$$z_{mr}\left(x_{mr}, y_{mr}\right)=\frac{x_{mr}^{2}+y_{mr}^{2}}{4F}-F$$

(in the (O,x,y,z) coordinate system). The main reflector has a circular aperture when projected on the (O,x,y) plane, which is centered on the point $(h,0,0)$. The equation of this aperture is

$$\frac{4(x-h)^2}{Dm^2}+\frac{4y^2}{Dm^2}=1.$$

Important relationships for a paraboloid are given in Appendix A, as Equations (35), (36), and (37), and in [5, 7, 8].

3.2 Subreflector

The subreflector profile, $z_{sr}\left(x_{sr}, y_{sr}\right)$, is either a convex hyperboloid (Cassegrain system, see Figure 3a) or an ellipsoid (Gregorian system, see Figure 3b). In both cases, it depends on the parameters *a* and *f*, and is of the form

$$z_{sr}\left(x_{sr}, y_{sr}\right)=a\sqrt{1+\frac{x_{sr}^{2}+y_{sr}^{2}}{f^{2}-a^{2}}}-f$$

(in the $\left(O,x_{sr},y_{sr},z_{sr}\right)$ coordinate system). Important relationships for a convex hyperboloid are given in Appendix A, as Equations (30), (32), (33), and (34), and in [5, 7, 8].

The subreflector rim is the intersection of the cone with a semi-angle θ_e, originating from F_0, and the surface of the concave hyperboloid defined by *a* and *f*. In the $\left(F_0,x_f,y_f,z_f\right)$ coordinate system (see Figure 3), the unit vectors on the sides of the cone with a semi-angle θ_e are

$$\vec{n}_{rim}\left|\begin{array}{c}\sin(\theta_e)\cos(\phi)\\ \sin(\theta_e)\sin(\phi),\\ \cos(\theta_e)\end{array}\right.$$

where ϕ is the azimuthal angle.

Now, we want to express these unit vectors in the $\left(O,x_{sr},y_{sr},z_{sr}\right)$ coordinate system. We need to rotate by $-\alpha$ around the y_f axis (since by convention, the rotation angle is positive for rotations from the z_f axis towards the x_f axis, and α is negative). The rotation matrix to apply is therefore

$$|M|=\begin{vmatrix}\cos(\alpha) & 0 & \sin(\alpha)\\ 0 & 1 & 0\\ -\sin(\alpha) & 0 & \cos(\alpha)\end{vmatrix},$$

so

$$\underbrace{\vec{n}_{rim}}_{in\ O,x_{sr},y_{sr},z_{sr}}=|M|\underbrace{\vec{n}_{rim}}_{in\ F_0,x_f,y_f,z_f}.$$

This leads to

$$\underbrace{\vec{n}_{rim}}_{in\ O,x_{sr},y_{sr},z_{sr}}=\begin{vmatrix}n_{rim,x}=\cos(\alpha)\sin(\theta_e)\ \cos(\phi)+\sin(\alpha)\cos(\theta_e)\\ n_{rim,y}=\sin(\theta_e)\sin(\phi)\\ n_{rim,z}=-\sin(\alpha)\sin(\theta_e)\cos(\phi)+\cos(\alpha)\cos(\theta_e)\end{vmatrix},$$

so each point on the subreflector rim will be expressed as

$$\underbrace{P_{rim}}_{in\ O,x_{sr},y_{sr},z_{sr}}=\begin{vmatrix}P_{rim,x}=\left[\cos(\alpha)\sin(\theta_e)\cos(\phi)+\sin(\alpha)\cos(\theta_e)\right]\rho_{sr}\\ P_{rim,y}=\left[\sin(\theta_e)\sin(\phi)\right]\rho_{sr}\\ P_{rim,z}=\left[-\sin(\alpha)\sin(\theta_e)\cos(\phi)+\cos(\alpha)\cos(\theta_e)\right]\rho_{sr}-2f\end{vmatrix},$$

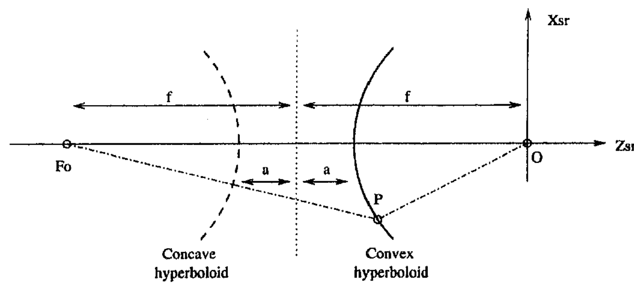

Figure 3a. The geometry of the subreflector for a convex hyperboloid.

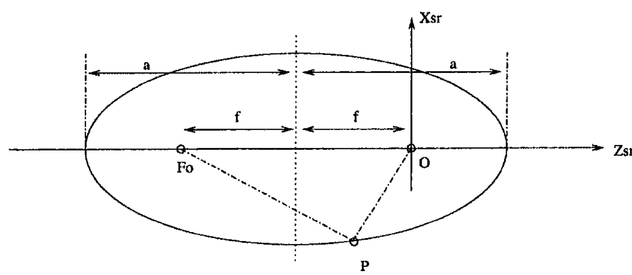

Figure 3a. The geometry of the subreflector for an ellipsoid.

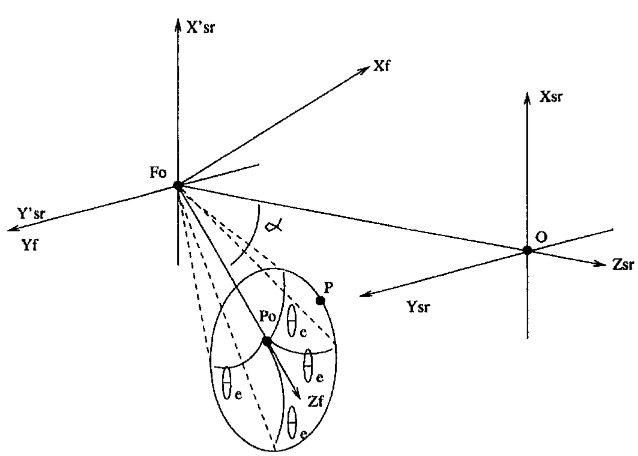

Figure 4. The feed coordinate system.

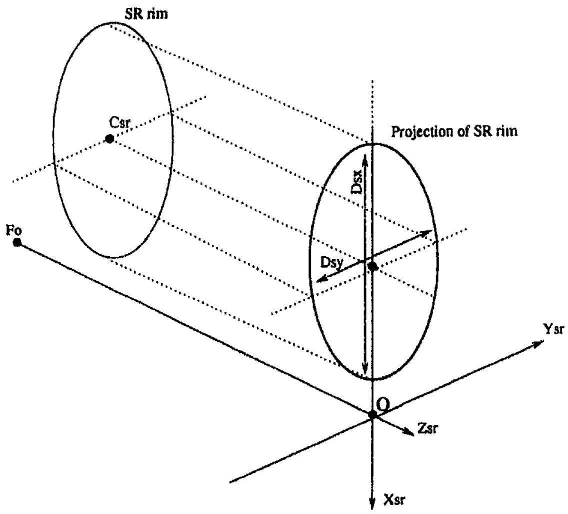

Figure 5. The subreflector rim shape.

with

$$\rho_{sr} = \frac{a\left(e^2-1\right)}{e\left[-\sin(\alpha)\sin(\theta_e)\cos(\phi)+\cos(\alpha)\cos(\theta_e)\right]-1}$$

(see [7]).

From [8], we know that the intersection of a reflector surface – generated from the rotation of a conic section about its focal axis – and a circular cone, with its tip on a focal point of the reflector surface, is always a planar curve, and this curve is an ellipse. This ellipse is defined in the (O, x_{sr}, y_{sr}) plane by (see Figure 5)

$$\frac{\left(x-x_{C_{sr}}\right)^2}{\left(\frac{Ds_x}{2}\right)^2}+\frac{y^2}{\left(\frac{Ds_y}{2}\right)^2}=1$$

(since $y_{C_{sr}}$ should always be 0).

Table 2. The coordinates of points.

Point	x	y	z
F_0	$-2f\sin(\beta)$	0	$-2f\cos(\beta)$
Q_0	h	0	$\frac{h^2}{4F}-F$
Q_1	$h-\frac{Dm}{2}$	0	$\frac{(2h-Dm)^2}{16F}-F$
Q_2	$h+\frac{Dm}{2}$	0	$\frac{(2h+Dm)^2}{16F}-F$
R_0	h	0	$Max\left(0, Z_{Q_0}, Z_{Q_1}, Z_{Q_2}\right)$
R_1	$h-\frac{Dm}{2}$	0	$Max\left(0, Z_{Q_0}, Z_{Q_1}, Z_{Q_2}\right)$
R_2	$h+\frac{Dm}{2}$	0	$Max\left(0, Z_{Q_0}, Z_{Q_1}, Z_{Q_2}\right)$
P_0	$\sigma\lVert OP_0\rVert\sin(\theta_0)$ (use Eq. 32)	0	$\sigma\lVert OP_0\rVert\cos(\theta_0)$ (use Eq. 32)
P_1	$\sigma\lVert OP_1\rVert\sin(\theta_L)$ (use Eq. 33)	0	$\sigma\lVert OP_1\rVert\cos(\theta_L)$ (use Eq. 33)
P_2	$\sigma\lVert OP_2\rVert\sin(\theta_U)$ (use Eq. 34)	0	$\sigma\lVert OP_2\rVert\cos(\theta_U)$ (use Eq. 34)

Table 3. Design procedure options.

Options	Parameter 1	Parameter 2	Parameter 3	Parameter 4	Parameter 5
No. 1	Dm	F	h	Ds_x	β
No. 2	Dm	F	h	Ls	β
No. 3	Dm	F	h	d_{f-mr}	β
No. 4	Dm	F	h	Lt	β
No. 5	Dm	F	h	Ht	β
No. 6	Dm	F	h	d_{sr-mr}	β
No. 7	Dm	θ_0	d_{f-mr}	Ls	β
No. 8	Dm	θ_0	θ_e	Ls	β
No. 9	Dm	θ_0	θ_e	Ds_x	β
No. 10	Dm	θ_0	θ_e	d_{sr-mr}	β
No. 11	Dm	θ_0	θ_e	Lt	β
No. 12	Dm	θ_0	θ_e	Ht	β

Table 4a. Design procedures No. 1 to No. 6.

Parameter	Option 1	Option 2	Option 3	Option 4	Option 5	Option 6
Dm	Dm	Dm	Dm	Dm	Dm	Dm
F	F	F	F	F	F	F
h	h	h	h	h	h	h
θ_0	1 (1st)	1 (1st)	1 (1st)	1 (1st)	1 (1st)	1 (1st)
θ_U	2 (2nd)	2 (2nd)	2 (2nd)	2 (2nd)	2 (2nd)	2 (2nd)
θ_L	4 (3rd)	4 (3rd)	4 (3rd)	4 (3rd)	4 (3rd)	4 (3rd)
β	β	β	β	β	β	β
θ_e	7 (6th)	7 (6th)	7 (6th)	7 (6th)	7 (6th)	7 (6th)
e	5 (4th)	5 (4th)	5 (4th)	5 (4th)	5 (4th)	5 (4th)
a	18 (7th)	22 (7th)	17 (8th)	19 (7th)	20 (7th)	21 (7th)
f	15 (8th)	15 (8th)	16 (7th)	15 (8th)	15 (8th)	15 (8th)
Ds_x	Ds_x	27 (9th)	27 (9th)	27 (9th)	27 (9th)	27 (9th)
Ds_y	39 (16th)	39 (16th)	39 (16th)	39 (16th)	39 (16th)	39 (16th)
α	6 (5th)	6 (5th)	6 (5th)	6 (5th)	6 (5th)	6 (5th)
Ls	8 (9th)	Ls	8 (11th)	8 (11th)	8 (11th)	8 (11th)
Lm	9 (10th)	9 (10th)	9 (10th)	9 (10th)	9 (10th)	9 (10th)
d_{sr-mr}	11 (12th)	11 (12th)	11 (12th)	11 (12th)	11 (12th)	d_{sr-mr}
d_{f-mr}	10 (11th)	10 (11th)	d_{f-mr}	10 (13th)	10 (13th)	10 (12th)
Lt	12 (13th)	12 (13th)	12 (13th)	Lt	12 (14th)	12 (13th)
Ht	13 (14th)	13 (14th)	13 (14th)	13 (14th)	Ht	13 (14th)
C_{sr}	38 (15th)	38 (15th)	38 (15th)	38 (15th)	38 (15th)	38 (15th)

Table 4b. Design procedures No. 7 to No. 12.

Parameter	Option 7	Option 8	Option 9	Option 10	Option 11	Option 12
Dm	Dm	Dm	Dm	Dm	Dm	Dm
F	25 (6th)	26 (6th)	26 (4th)	26 (4th)	26 (4th)	26 (4th)
h	24 (5th)	23 (7th)	23 (5th)	23 (5th)	23 (5th)	23 (5th)
θ_0	θ_0	θ_0	θ_0	θ_0	θ_0	θ_0
θ_U	2 (7th)	3 (5th)	3 (3^{rd})	3 (3^{rd})	3 (3^{rd})	3 (3^{rd})
θ_L	4 (8th)	4 (8th)	4 (6th)	4 (6th)	4 (6th)	4 (6th)
β	β	β	β	β	β	β
θ_e	7 (9th)	θ_e	θ_e	θ_e	θ_e	θ_e
e	5 (1st)	5 (1st)	5 (1st)	5 (1st)	5 (1st)	5 (1st)
a	8 (3rd)	22 (3rd)	18 (7th)	21 (7th)	19 (7th)	20 (7th)
f	15 (4th)	15 (4th)	15 (8th)	15 (8th)	15 (8th)	15 (8th)
Ds_x	27 (11th)	27 (9th)	Ds_x	27 (9th)	27 (9th)	27 (9th)
Ds_y	39 (16th)	39 (16th)	39 (16th)	39 (16th)	39 (16th)	39 (16th)
α	6 (2nd)	6 (2nd)	6 (2nd)	6 (2nd)	6 (2nd)	6 (2nd)
Ls	Ls	Ls	8 (9th)	8 (10th)	8 (10th)	8 (10th)
Lm	9 (10th)	9 (10th)	9 (10th)	9 (11th)	9 (11th)	9 (11th)
d_{sr-mr}	11 (12th)	11 (11th)	11 (11th)	d_{sr-mr}	11 (12th)	11 (12th)
d_{f-mr}	d_{f-mr}	10 (12th)	10 (12th)	10 (12th)	10 (13th)	10 (13th)
Lt	12 (13th)	12 (13th)	12 (13th)	12 (13th)	Lt	12 (14th)
Ht	13 (14th)	13 (14^{th})	13 (14^{th})	13 (14^{th})	13 (14^{th})	Ht
C_{sr}	38 (15th)	38 (15^{th})	38 (15^{th})	38 (15^{th})	38 (15^{th})	38 (15^{th})

Table 5. Examples.

Parameter	Example 1	Example 2	Example 3	Example 4
σ	–1 (Cassegrain)	1 (Gregorian)	–1 (Cassegrain)	1 (Gregorian)
Dm (λ)	100.0	100.0	45.0	24.0
F (λ)	107.3	82.8	38.0	18.0
h (λ)	79.4	58.7	40.0	18.0
θ_0 (Deg.)	–40.608	–39.0356	–55.51708	–53.13010
θ_U (Deg.)	–62.1785	–66.5619	–78.8656	–79.61115
θ_L (Deg.)	–15.6018	–6.01468	–25.93417	–18.92464
β (Deg.)	10.1	5.4	6.0	5.6
θ_e (Deg.)	11.8767	11.9131	10.32476	11.50497
e (λ)	2.52016	0.492772	1.84393	0.54461
a (λ)	6.8966	28.6477	6.42302	21.04264
f (λ)	17.3805	14.1168	11.84361	11.46003
Ds_x (λ)	15.0	15.0	10.0	10.0
Ds_y (λ)	12.1380	16.7281	7.9488	11.9600
α (Deg.)	23.1295	–15.8030	20.03109	–18.83789
Ls (λ)	28.0096	41.2498	21.04870	30.54596
Lm (λ)	107.772	109.249	40.32365	34.03933
d_{sr-mr} (λ)	10.9297	10.2326	4.51682	9.20998
d_{f-mr} (λ)	35.4959	11.3570	19.97599	8.23661
Lt (λ)	95.539	97.1173	33.42990	26.86245
Ht (λ)	126.365	125.967	59.87143	43.92561
C_{sr} (λ)	$x_{C_{sr}} = 12.3933$ $y_{C_{sr}} = 0$ $z_{C_{sr}} = -8.6475$	$x_{C_{sr}} = -10.395$ $y_{C_{sr}} = 0$ $z_{C_{sr}} = 11.9214$	$x_{C_{sr}} = 8.17916$ $y_{C_{sr}} = 0$ $z_{C_{sr}} = -3.5292$	$x_{C_{sr}} = -9.1083$ $y_{C_{sr}} = 0$ $z_{C_{sr}} = 6.56358$
Geometry	See Figure 5a	See Figure 6a	See Figure 7a	See Figure 8a
Pattern	See Figure 5b	See Figure 6b	See Figure 7b	See Figure 8b

3.3 Coordinates of Some of the Useful Points

The coordinates of the points F_0, P_0, Q_0, R_0, P_1, Q_1, R_1, P_2, Q_2, and R_2 (of Figures 1 and 2) in the (O,x,y,z) coordinate system are given in Table 2. Note that the points R_0, R_1, and R_2 are located on an arbitrary plane, parallel to the (O,x,y) plane, located in front of the main reflector, and for convenience, this plane is defined as $z_{plane} = Max(0, Z_{Q_2})$.

4. Design Procedure

Table 3 gives the design-procedure options. It shows which input parameters can be used to design the antennas, while Tables 4a and 4b give the procedures for designing classical offset Cassegrain ($\sigma = -1$) or Gregorian ($\sigma = +1$) dual-reflector antennas. There are 12 options available to the antenna designer. From five initial parameters (see Table 3), the remaining 16 parameters can be derived in closed form, using the equations provided in Appendix A in the order specified in brackets (see Table 4a and 4b). All options include the main-reflector diameter, Dm, and β, the tilt angle between the main-reflector coordinate-system z_{mr} axis and the subreflector coordinate-system z_{sr} axis, as starting parameters.

It is important to note that you will have to check that the values of your input parameters do not lead to a non-physical design, i.e., check along the way that F, e, a, f, Ds_x, Ds_y, Ht, Lt, Ls, Lm, and h are positive. However, with practice, and using examples in the literature or in this paper, you will soon be able to tailor any geometry to your needs.

5. Examples

Four examples – two offset Cassegrain systems and two offset Gregorian systems – are presented in Table 5 and Figures 6 to 9. The values of the parameters are listed with many decimal points to allow you to check your software, if you decide to use the design procedures given in this paper. Another check can be to calculate the ray path and verify that Equation (31) is satisfied.

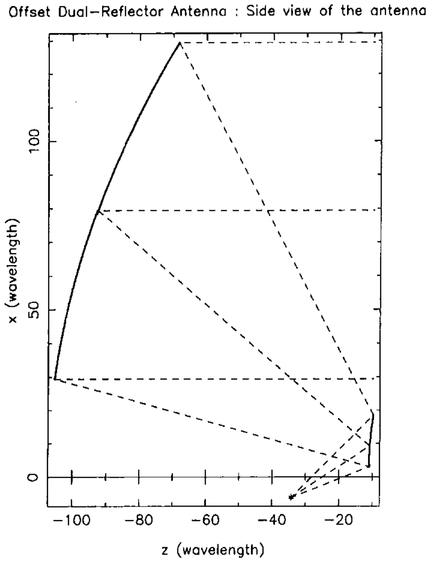

Figure 6a. The geometry of example No. 1.

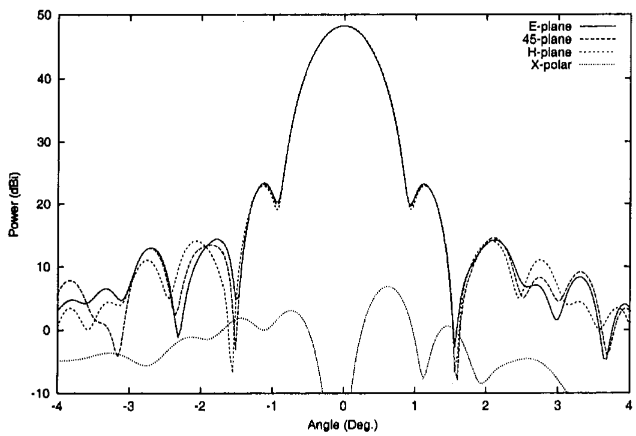

Figure 6b. The radiation pattern of example No. 1 with a theoretical Gaussian feed with a –12 dB taper at θ_e (gain = 48.3 dBi).

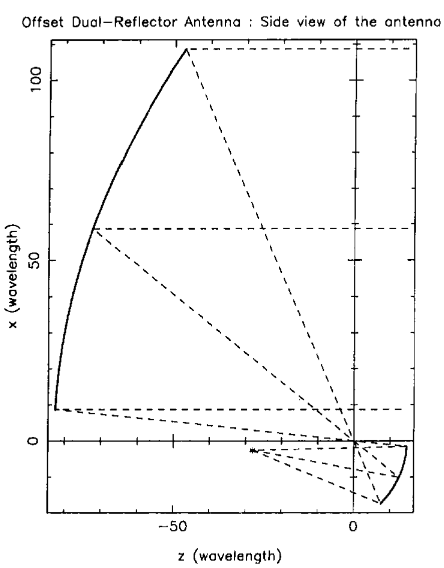

Figure 7a. The geometry of example No. 2.

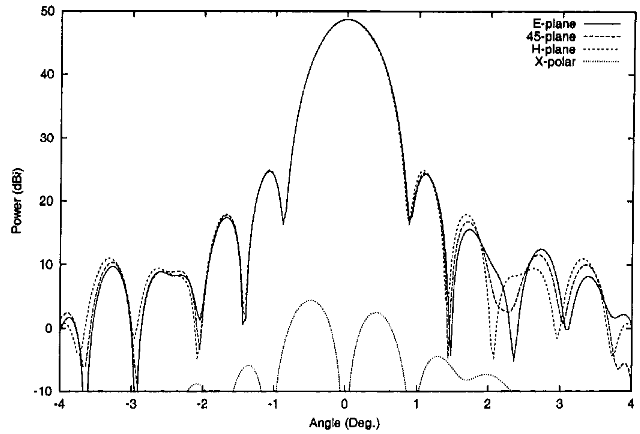

Figure 7b. The radiation pattern of example No. 2 with a theoretical Gaussian feed with a –12 dB taper at θ_e (gain = 48.7 dBi).

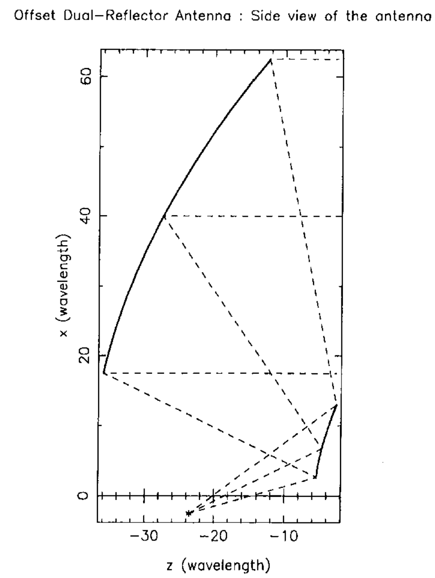

Figure 8a. The geometry of example No. 3.

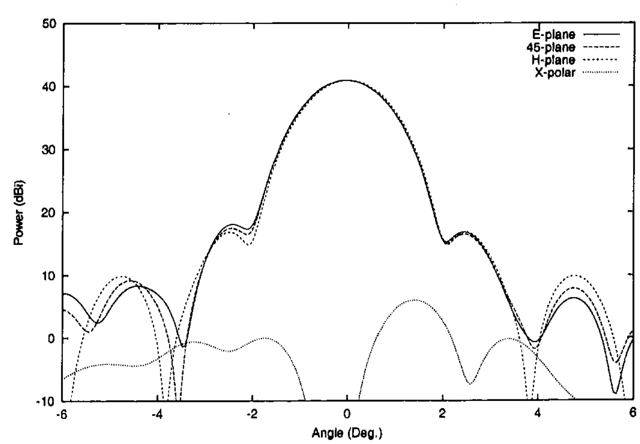

Figure 8b. The radiation pattern of example No. 3 with a theoretical Gaussian feed with a –12 dB taper at θ_e (gain = 40.9 dBi).

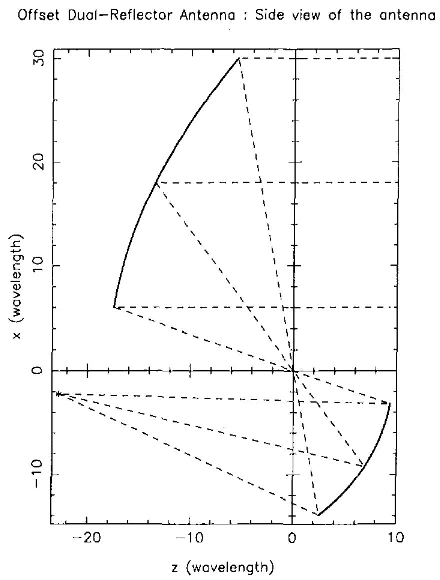

Figure 9a. The geometry of example No. 4.

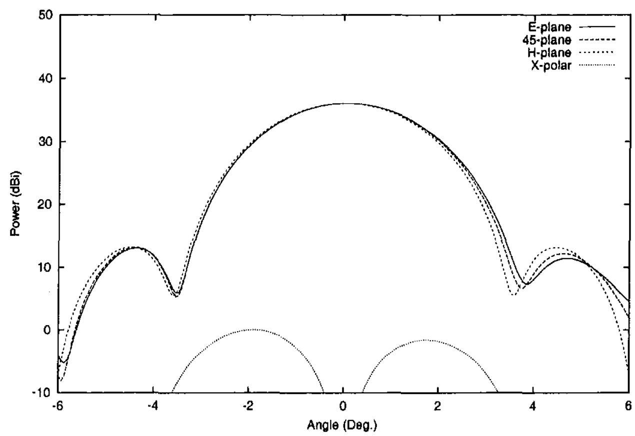

Figure 9b. The radiation pattern of example No. 4 with a theoretical Gaussian feed with a –12 dB taper at θ_e (gain = 36.1 dBi).

6. Conclusions

We have provided antenna designers with an easy-to-follow procedure to design classical offset Cassegrain or Gregorian dual-reflector antennas, from combinations of prescribed geometric parameters.

7. Appendix A

$$\theta_0 = -2\arctan\left(\frac{h}{2F}\right) \tag{1}$$

$$\theta_U = -2\arctan\left(\frac{2h+Dm}{4F}\right) \tag{2}$$

$$\theta_U = 2\arctan\left[\frac{1+e}{1-e}\tan\left(\frac{\alpha-\sigma\theta_e}{2}\right)\right]+\beta \tag{3}$$

$$\theta_L = -2\arctan\left(\frac{2h-Dm}{4F}\right) \tag{4}$$

$$e = \frac{1-\sigma\sqrt{\dfrac{\tan\left(\dfrac{\beta}{2}\right)}{\tan\left(\dfrac{\beta-\theta_0}{2}\right)}}}{1+\sigma\sqrt{\dfrac{\tan\left(\dfrac{\beta}{2}\right)}{\tan\left(\dfrac{\beta-\theta_0}{2}\right)}}} \tag{5}$$

$$\alpha = 2\arctan\left[\frac{e+1}{e-1}\tan\left(\frac{\beta}{2}\right)\right] \tag{6}$$

$$\theta_e = -\sigma\left\{2\arctan\left[\frac{1-e}{1+e}\tan\left(\frac{\theta_U-\beta}{2}\right)\right]-\alpha\right\} \tag{7}$$

(note that σ is included to ensure that θ_e is always positive)

$$Ls = a\left[2+\frac{e^2-1}{e\cos(\beta-\theta_0)+1}\right] \tag{8}$$

$$Lm = -a\frac{e^2-1}{e\cos(\beta-\theta_0)+1}-\frac{h}{\sin(\theta_0)} \tag{9}$$

$$d_{f-mr} = h-\frac{Dm}{2}+2f\sin(\beta) \tag{10}$$

$$d_{sr-mr} = h-\frac{Dm}{2}-a\left(\frac{\sigma-1}{2}\right)\frac{(e^2-1)\sin(\theta_U)}{e\cos(\beta-\theta_U)+1} + a\left(\frac{\sigma+1}{2}\right)\frac{(e^2-1)\sin(\theta_L)}{e\cos(\beta-\theta_L)+1} \tag{11}$$

$$Lt = -a\left(\frac{\sigma+1}{2}\right)\frac{(e^2-1)\cos(\theta_L)}{e\cos(\beta-\theta_L)+1} + a\left(\frac{\sigma-1}{2}\right)\frac{(e^2-1)\cos(\theta_U)}{e\cos(\beta-\theta_U)+1}-\frac{(2h-Dm)^2}{16F}+F \tag{12}$$

$$Ht = h+\frac{Dm}{2}-a\left(\frac{\sigma-1}{2}\right)\frac{(e^2-1)\sin(\theta_L)}{e\cos(\beta-\theta_L)+1} + a\left(\frac{\sigma+1}{2}\right)\frac{(e^2-1)\sin(\theta_U)}{e\cos(\beta-\theta_U)+1} \tag{13}$$

$$Ht = Dm+d_{sr-mr}+Ds_x\cos(\beta) \tag{14}$$

$$f = ae \tag{15}$$

$$f = \frac{d_{f-mr}-h+\dfrac{Dm}{2}}{2\sin(\beta)} \tag{16}$$

$$a=\frac{f}{e} \tag{17}$$

$$a=\frac{-\sigma Ds_x}{\frac{\left(e^2-1\right)\sin(\beta-\theta_U)}{e\cos(\beta-\theta_U)+1}-\frac{\left(e^2-1\right)\sin(\beta-\theta_L)}{e\cos(\beta-\theta_L)+1}} \tag{18}$$

$$a=\frac{Lt+\frac{(2h-Dm)^2}{16F}-F}{\left(\frac{\sigma-1}{2}\right)\frac{\left(e^2-1\right)\cos(\theta_U)}{e\cos(\beta-\theta_U)+1}-\left(\frac{\sigma+1}{2}\right)\frac{\left(e^2-1\right)\cos(\theta_L)}{e\cos(\beta-\theta_L)+1}} \tag{19}$$

$$a=\frac{Ht-h-\frac{Dm}{2}}{\left(\frac{\sigma+1}{2}\right)\frac{\left(e^2-1\right)\sin(\theta_U)}{e\cos(\beta-\theta_U)+1}-\left(\frac{\sigma-1}{2}\right)\frac{\left(e^2-1\right)\sin(\theta_L)}{e\cos(\beta-\theta_L)+1}} \tag{20}$$

$$a=\frac{d_{sr-mr}-h+\frac{Dm}{2}}{\left(\frac{\sigma+1}{2}\right)\frac{\left(e^2-1\right)\sin(\theta_L)}{e\cos(\beta-\theta_L)+1}-\left(\frac{\sigma-1}{2}\right)\frac{\left(e^2-1\right)\sin(\theta_U)}{e\cos(\beta-\theta_U)+1}} \tag{21}$$

$$a=\frac{Ls}{2+\frac{e^2-1}{e\cos(\beta-\theta_0)+1}} \tag{22}$$

$$h=2F\tan\left(-\frac{\theta_0}{2}\right) \tag{23}$$

$$h=d_{f-mr}+\frac{Dm}{2}-2f\sin(\beta) \tag{24}$$

$$F=\frac{h}{2\tan\left(-\frac{\theta_0}{2}\right)} \tag{25}$$

$$F=\frac{Dm}{4\left[\tan\left(-\frac{\theta_U}{2}\right)-\tan\left(-\frac{\theta_0}{2}\right)\right]} \tag{26}$$

$$Ds_x=-\sigma a\left[\frac{\left(e^2-1\right)\sin(\beta-\theta_U)}{e\cos(\beta-\theta_U)+1}-\frac{\left(e^2-1\right)\sin(\beta-\theta_L)}{e\cos(\beta-\theta_L)+1}\right] \tag{27}$$

Other useful relationships are

$$\tan\left(\frac{\beta}{2}\right)=\left(\frac{e-1}{e+1}\right)^2\tan\left(\frac{\beta-\theta_0}{2}\right) \tag{28}$$

$$\tan(\alpha)=\frac{\left|e^2-1\right|\sin(\beta)}{\left(1+e^2\right)\cos(\beta)-2e} \qquad \text{(29) (see [6])}$$

$$\begin{aligned}\|F_0P_0\|+\sigma\|OP_0\|&=\|F_0P_1\|+\sigma\|OP_1\|\\&=\|F_0P_2\|+\sigma\|OP_2\|=2a\end{aligned} \tag{30}$$

$$\begin{aligned}\|F_0P_0\|+\|P_0Q_0\|+\|Q_oR_0\|&=\|F_0P_1\|+\|P_1Q_1\|+\|Q_1R_1\|\\&=\|F_0P_2\|+\|P_2Q_2\|+\|Q_2R_2\|\end{aligned} \tag{31}$$

$$\begin{aligned}\|OP_0\|&=-\sigma a\frac{e^2-1}{e\cos(\theta_0-\beta)+1}\\&=\sigma\left[\frac{h}{\sin(\theta_0)}+Lm\right]=\sigma(2a-Ls)\end{aligned} \tag{32}$$

$$\|OP_1\|=-\sigma a\frac{e^2-1}{e\cos(\theta_L-\beta)+1} \tag{33}$$

$$\|OP_2\|=-\sigma a\frac{e^2-1}{e\cos(\theta_U-\beta)+1} \tag{34}$$

$$\|OQ_0\|=\frac{2F}{1+\cos(\theta_0)} \tag{35}$$

$$\|OQ_1\|=\frac{2F}{1+\cos(\theta_L)} \tag{36}$$

$$\|OQ_2\|=\frac{2F}{1+\cos(\theta_U)} \tag{37}$$

Now, we define the other parameters of the ellipse defining the subreflector aperture. when projected on the (O,x_{sr},y_{sr}) plane. The "center" of the subreflector, expressed in the subreflector coordinate system, is

$$C_{sr}\left|\begin{aligned}x_{C_{sr}}&=\frac{\|F_0P_1\|\sin(\alpha+\theta_e)+\|F_0P_2\|\sin(\alpha-\theta_e)}{2}\\y_{C_{sr}}&=0\\z_{C_{sr}}&=-a\sqrt{1+\frac{x_{C_{sr}}^2}{f^2-a^2}}-f\end{aligned}\right. \tag{38}$$

and

$$Ds_y=Max\left[\frac{2a\left(e^2-1\right)\sin(\theta_e)\sin(\phi)}{e\left[-\sin(\alpha)\sin(\theta_e)\cos(\phi)+\cos(\alpha)\cos(\theta_e)\right]-1}\right] \quad \text{for } 0\le\phi\le 2\pi \tag{39}$$

7. References

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Ideas for Antenna Designer's Notebook

Ideas are needed for future issues of the Antenna Designer's Notebook. Please send your suggestions to Tom Milligan and they will be considered for publication as quickly as possible. Topics can include antenna design tips, equations, nomographs, or shortcuts, as well as ideas to improve or facilitate measurements.

Editor's Comments *Continued from page 73*

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5. If you want to search for an author or a word in the abstract or text of a publication, you do so from the same page used to access a particular issue of the publication (the page you are at when you have completed step 3). You can enter an author, an author and a year, a keyword, a phrase, or a Boolean expression into the search box. When you do so and click on "Search," if you haven't logged into your Web account, you will be prompted to so before the search is executed: see the discussion in step 4 regarding IEEE Web accounts.

Some comments about the syntax for searching are in order. If you enter several words with spaces between them, the search engine will find documents containing as many of these words as possible, and rank-order them by how many of the words each contains. To enter a phrase, enter the phrase in quotation marks. Thus, entering `coaxial waveguide` in the search box should return all documents containing both the words coaxial and waveguide. However, entering `"coaxial waveguide"` should return only those documents in which the actual phrase coaxial waveguide appears. Entering `"dual reflector" AND Cassegrain` should return documents containing both the phrase dual reflector and the word Cassegrain. This is an example of a Boolean search. A + sign in front of a word or phrase requires that it be present in the document; a – sign requires that it *not* be present. http://odysseus.ieee.org/ieeesearch/help/ contains more information on the search engine used on the IEEE Web site and its searching syntax.

The instructions provided with the *Digital Archive* are quite good, and should be sufficient: *read them!* In particular, you need to install the ASTAware search engine and the AP-S search index onto your hard drive, if you want to do full-text search of the *Archive*.

If you don't have the *Digital Archive*, you really need to order a copy (an order form appears in this issue). It's one of those things that you may not realize just how much you need until you have it handy. It's like having all of the AP-S publications available as a fully-searchable "ready reference" – in fact, that's exactly what it is!

From the Screen of Stone "Lite"

More on Graphics, Image Quality, and a Photo Printer

In my Editor's Comments in the last (April) issue, I commented on the image and graphics quality in the *Magazine*, and on my experiences with the Epson Stylus Photo 1270 color inkjet printer. A few of my comments there were based on some incorrect information I had received. The cover for the April issue (and, indeed, for most of the recent past issues of the *Magazine*) was, in fact, produced directly from an inkjet print made on the Epson printer. Furthermore, many of the black-and-white photos in the April issue were also produced directly from such inkjet prints. This includes all of those in Milton Punnett's Telstar article, and most of those in Steven Best's article on scale brass models. Given an adequate-quality digital image, this printer does produce truly publication-quality output.

Of course, technology doesn't stand still. Epson now has a new series of printer models in its "Photo" line. They use six ink colors, with each color a separately replaceable ink cartridge. I have not had an opportunity to try them, but the reports I've seen indicate that their output quality should equal or exceed the 1270, and they claim improved lifetimes for the prints.

On "Slow" *Word* Documents Containing Equations

I recently noticed that displaying the documents had become very slow when working with Microsoft *Word* documents con-

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