

Quadrotor

Dynamics equation:

$$\ddot{x} = -\frac{f_t}{m}[\sin \phi \sin \psi + \cos \phi \cos \psi \sin \theta] \quad (1)$$

$$\ddot{y} = -\frac{f_t}{m}[\cos \phi \sin \psi \sin \theta + \cos \psi \sin \phi] \quad (2)$$

$$\ddot{z} = g - \frac{f_t}{m}[\cos \phi \cos \theta] \quad (3)$$

$$\ddot{\phi} = \frac{I_y - I_z}{I_x} \dot{\theta} \dot{\psi} + \frac{\tau_x}{I_x} \quad (4)$$

$$\ddot{\theta} = \frac{I_z - I_x}{I_y} \dot{\phi} \dot{\psi} + \frac{\tau_y}{I_y} \quad (5)$$

$$\ddot{\psi} = \frac{I_x - I_y}{I_z} \dot{\phi} \dot{\theta} + \frac{\tau_z}{I_z} \quad (6)$$

where

x, y, z are linear coordinates in reference frame (z is a vertical coordinate)

ϕ, θ, ψ are roll, pitch and yaw angles

f_t is a thrust

τ_x, τ_y and τ_z are force momentums

m is a quadrotor mass

g is a free fall acceleration

I_x, I_y, I_z are inertia momentums along respective axis of quadrotor

l is the distance between any rotor and the center of the quadrotor

b is the thrust coefficient

d is the drag factor

Dependence between forces and velocities:

$$f_t = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \quad (7)$$

$$\tau_x = bl(\Omega_3^2 - \Omega_1^2) \quad (8)$$

$$\tau_y = bl(\Omega_4^2 - \Omega_2^2) \quad (9)$$

$$\tau_z = d(\Omega_1^2 + \Omega_4^2 - \Omega_2^2 - \Omega_3^2) \quad (10)$$

where Ω is a velocity

$$\Omega_1 = \sqrt{\frac{f_t - \frac{b}{d}\tau_z - \frac{2}{l}\tau_x}{4b}} \quad (11)$$

$$\Omega_2 = \sqrt{\frac{f_t + \frac{b}{d}\tau_z - \frac{2}{l}\tau_y}{4b}} \quad (12)$$

$$\Omega_3 = \sqrt{\frac{f_t - \frac{b}{d}\tau_z + \frac{2}{l}\tau_x}{4b}} \quad (13)$$

$$\Omega_4 = \sqrt{\frac{f_t + \frac{b}{d}\tau_z + \frac{2}{l}\tau_y}{4b}} \quad (14)$$