Monte Carlo, Session 2

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Exercise 1

Ex1.cpp (run from main.cpp, to get the randoms.h connected correctly.)

The distribution obtained from LCG are presented in Figure 1.

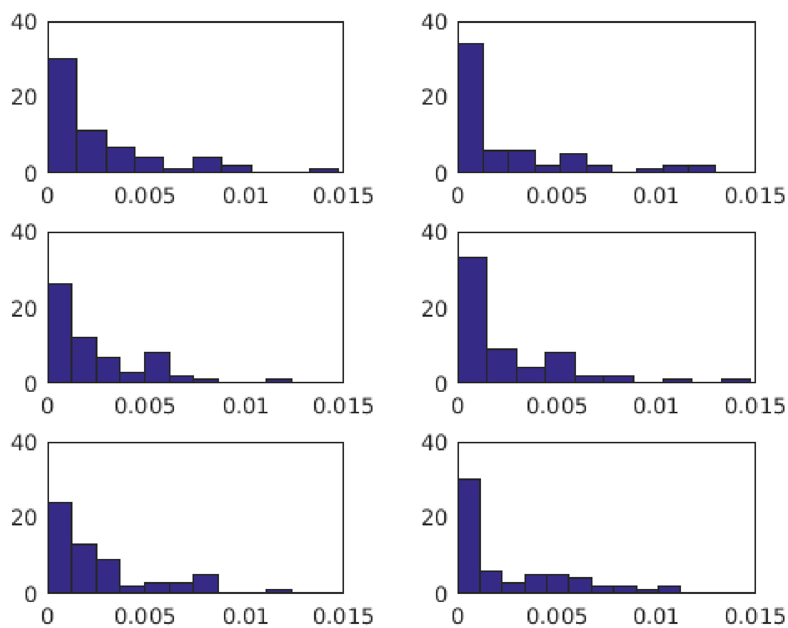

Figure 1: Ex1, LCG.

The distribution obtained from PM are presented in Figure 2.

The distribution obtained from MT are presented in Figure 3.

Additionally, ex1.m produces that the average Chi2 value for LCG is 0.1525, for PM is 59.599 and for MT is 58.31.

The degree of freedom is number of bins minus one, so it is 59 in our case. Then for α=0.05 the lower tail is 39.662. PM and MT are bigger, but LCG fails since it is lower.

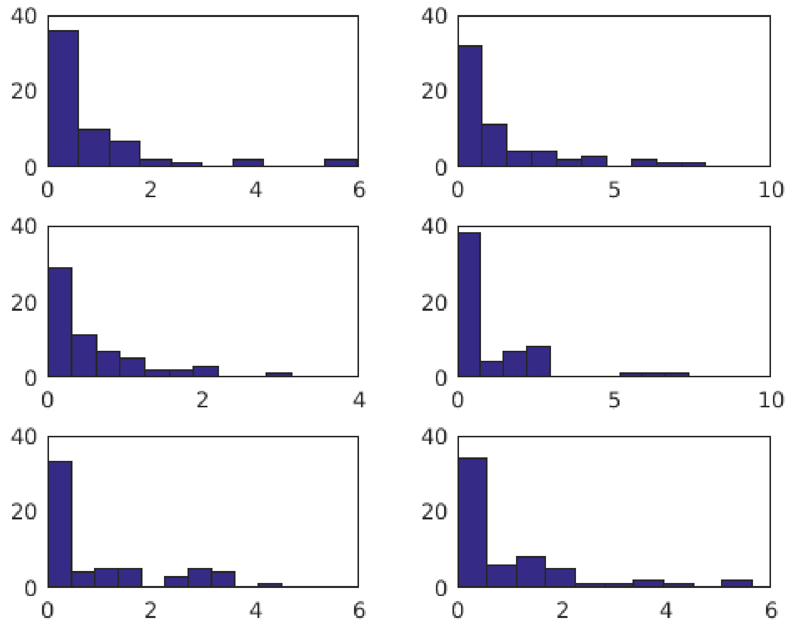

Figure 2: Ex1, PM.

Exercise 2

Ex2.cpp

Generates data to subfolder data. N is 10^7 and k is 10^6.

At the start lets visualize all calculated C_k values in Figure 4.

As it can be seen, no too clear pattern observed, so lets take a look at histograms presented in Figure 5.

Theoretically, it is assumed to see rising around abs(x)=1 and lowering around 0. However, even due to usage of the course generators, onliest difference noticed is that LCG is more evenly distributed than the others.

Exercise 3

The function is presented in Figure 6. Basing on the image, the first box is when y is from 0 to 7. For the combined method lets use y from 0 to 3 while x from 0 to 4, and then the same y from 0 to 7 box.

Then, the program outputs the Figure 7. As it can be seen, the combined method is roughly twice better. (Better with smaller b, but becomes proportionally less effective when b¿4, as it should according to definition)

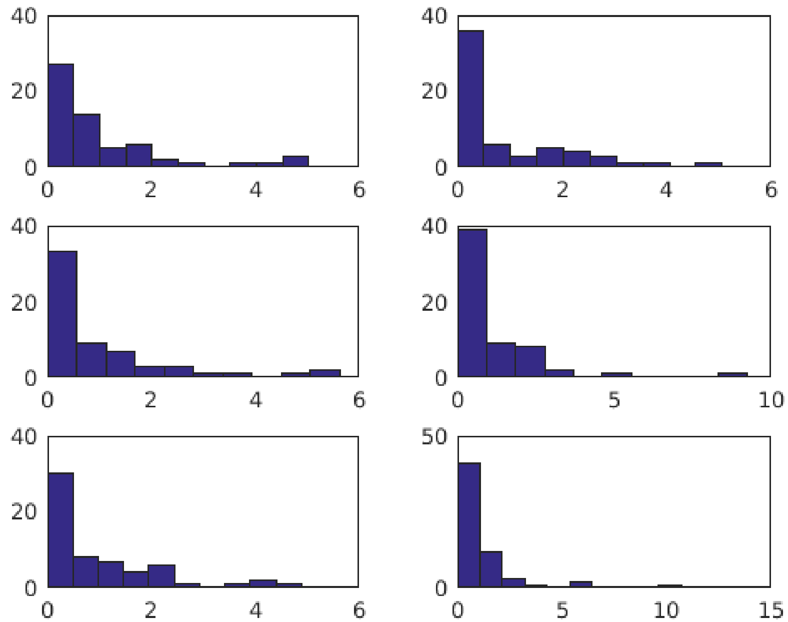

Figure 3: Ex1, MT.

Exercise 4

Ex4, Inversion method

$$f(x) = \frac{10}{\pi\gamma} * \frac{\gamma^2}{x^2 + \gamma^2},\ where\ \gamma = 2 \tag{1}$$

$$\int_0^x f(x)dx = \Big|_0^t \frac{10}{\pi\gamma} * \gamma * tan^{-1}\left(\frac{x}{\gamma}\right) = \frac{10}{\pi} * tan^{-1}\left(\frac{x}{\gamma}\right) = F(x) \tag{2}$$

Now lets check that F(∞) is 1.

$$F(x \to \infty) = *...wolframalpha...* = \frac{10}{\pi}\frac{1}{2}\sqrt{\frac{1}{\gamma^2}}\gamma\pi = 5 \tag{3}$$

Since we can produce random number from 0 to 1, we want F to be 1. Thus we scale the F function with $\pi/2$.

$$s = \frac{F}{5} \tag{4}$$

Now lets input F(x) from Equation 2, and solve x as the function of s.

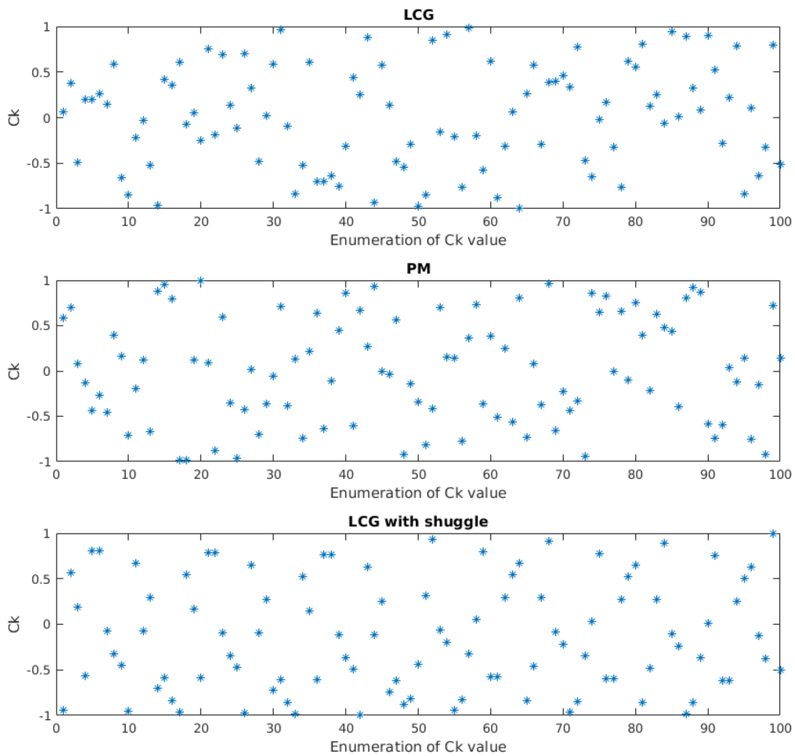

Figure 4: Ex1, C_k values from different generators

$$s = \frac{1}{5} * F = \frac{2}{\pi} * tan^{-1}\left(\frac{x}{\gamma}\right) \tag{5}$$

$$\implies x = \gamma * tan\left(\frac{s * \pi}{2}\right) \tag{6}$$

And after setting the $\gamma = 2$, we get:

$$x = 2 * tan\left(\frac{s * \pi}{2}\right) \tag{7}$$

where s is the random value from 0 to 1.

To evaluate the result, random numbers are generated and presented as histogram in Figure 8.

It can be noticed that the shape of Figure 8 is similar as the wanted distribution formula presented in Figure 9.

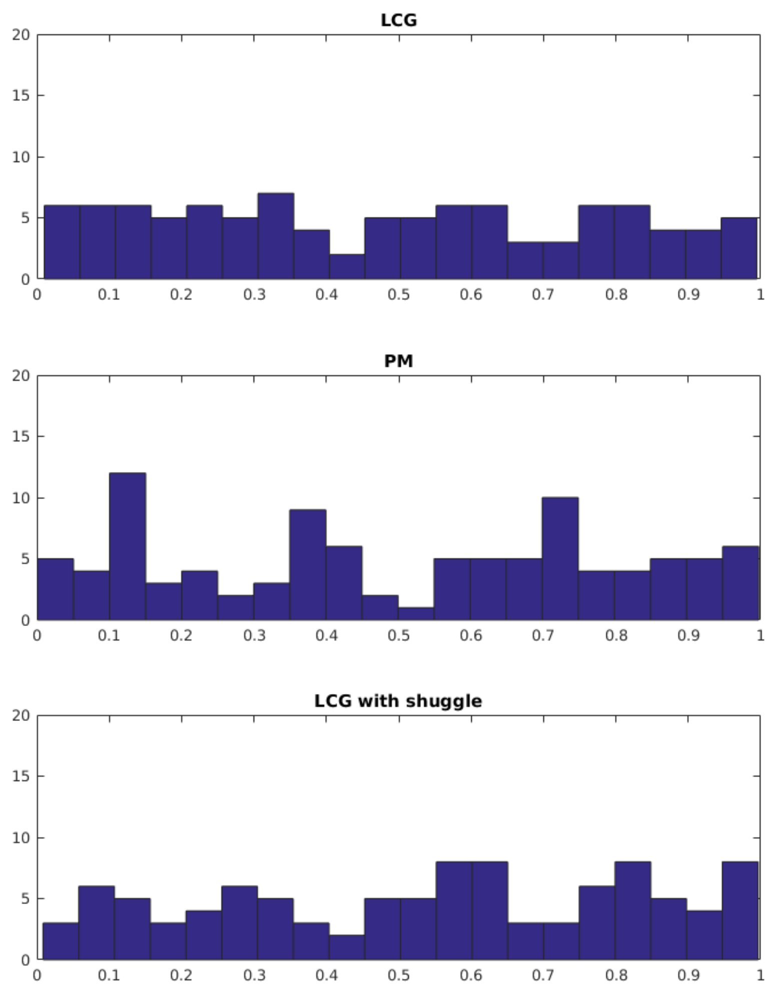

Figure 5: Ex1, abs(C_k) values in histogram basing on generator.

Note: Additionally, limited acceptable x-values to range [-10,10] as were required in the task. Note: Due to periodicity of tan function the values from 0 to 10 and from 0 to -10 are generated similarly with only the sign difference.

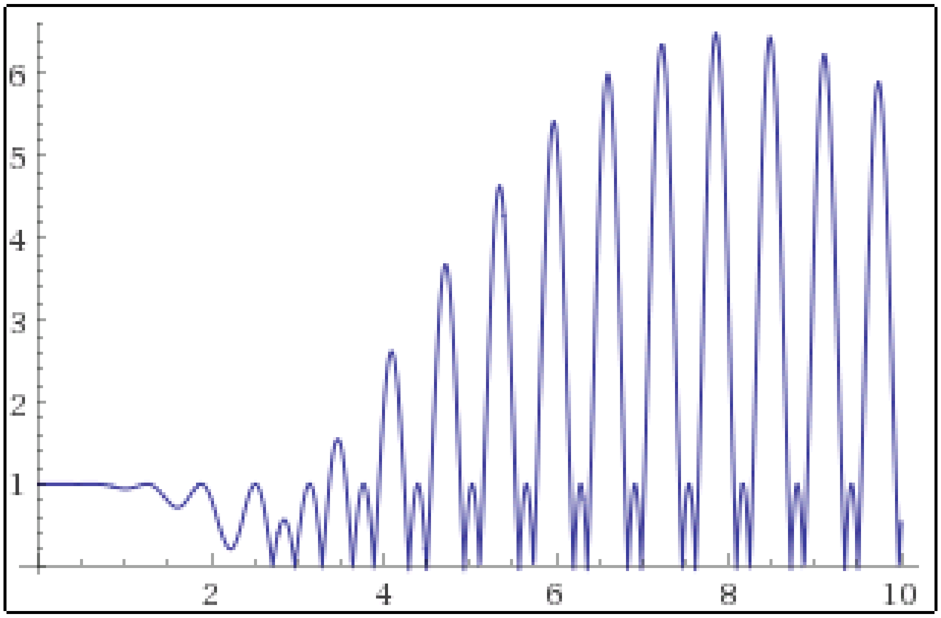

Figure 6: Ex3, the function.

Ex4, Combined method

As suggested, lets simply try some Gaussian functions to be the upper function to presented f(x).

Basing on Figure 9, it can be concluded that g(x):

$$g(x) = 2 * \sqrt{\frac{2}{\pi}} * e^{-\frac{x^2}{40}} \tag{8}$$

is suitable for upper limit prediction.

Now lets do same operations, as done in the inversion method. This will allow to generate the random numbers according to upper distribution, which will lead to better hit-miss ratio.

To solve this, let's lead it to Box-Muller example presented in the lecture. Firstly, lets redefine:

$$X = \frac{x}{\sqrt{20}} \tag{9}$$

Now it is almost the example presented in the lecture, except that

$$dX = \frac{dx}{\sqrt{20}} \tag{10}$$

And on page 25 from Lecture 3, the

$$f_\phi(\phi) = \frac{16}{2\pi} = \frac{8}{\pi} \tag{11}$$

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Ex3
b= 1
Basic method, hits: 141904
Basic method, misses:858096
Combined method, hits: 330386
Combined method, misses:669614
b= 2
Basic method, hits: 276869
Basic method, misses:1723131
Combined method, hits: 645557
Combined method, misses:1354443
b= 5
Basic method, hits: 415913
Basic method, misses:2584087
Combined method, hits: 906338
Combined method, misses:2093662
b= 10
Basic method, hits: 685891
Basic method, misses:3314109
Combined method, hits: 1237785
Combined method, misses:2762215
```

Figure 7: Ex3, the result.the combined method is roughly twice better. Better with smaller b, but becomes proportionally less effective when b¿4, as it should according to definition.

Therefore the task is to generate uniformly distributued u_1 and u_2. Then the angle can be obtained from:

$$\phi = \frac{\pi}{8} u_1 \tag{12}$$

$$r = \sqrt{-2log(1-u_2)} \tag{13}$$

$$y = r * sin\phi \tag{14}$$

$$X = r * cos\phi \tag{15}$$

$$\Longrightarrow x = X * \sqrt{20} = \sqrt{20} * r * cos\phi \tag{16}$$

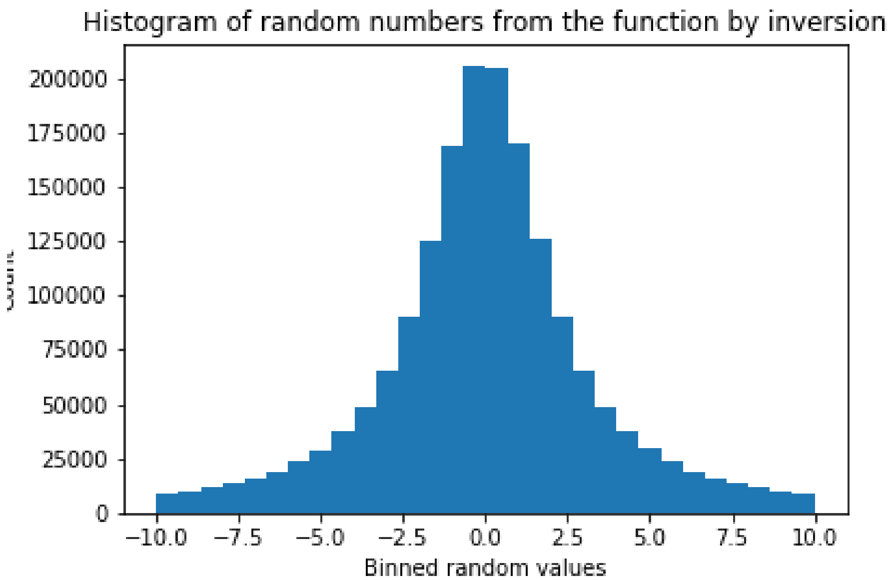

Figure 8: Histogram of random number that are generated using the inversion method.

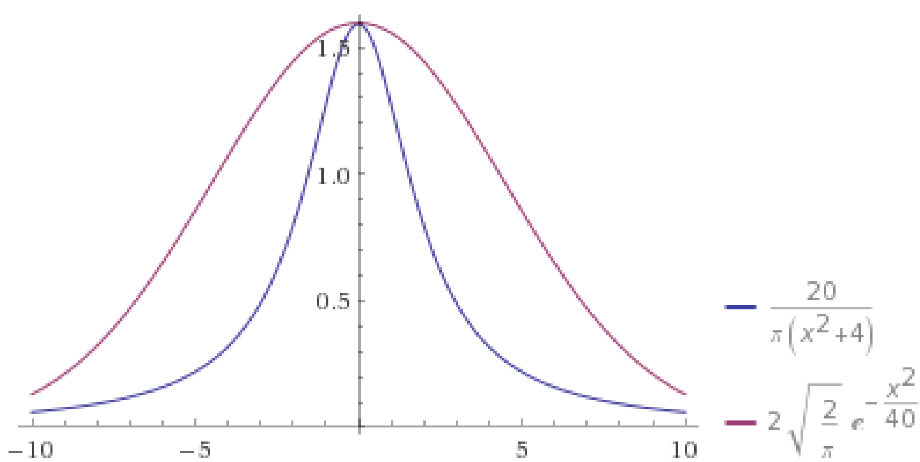

Figure 9: Simple iteractive attempt to predict the suitable Gaussian.

This formulas result in Figure 10. Overall shape is suitable, but for some reason centered at 5. Thus it is possible to conclude that I am having a scalar error somewhere, not sure where.

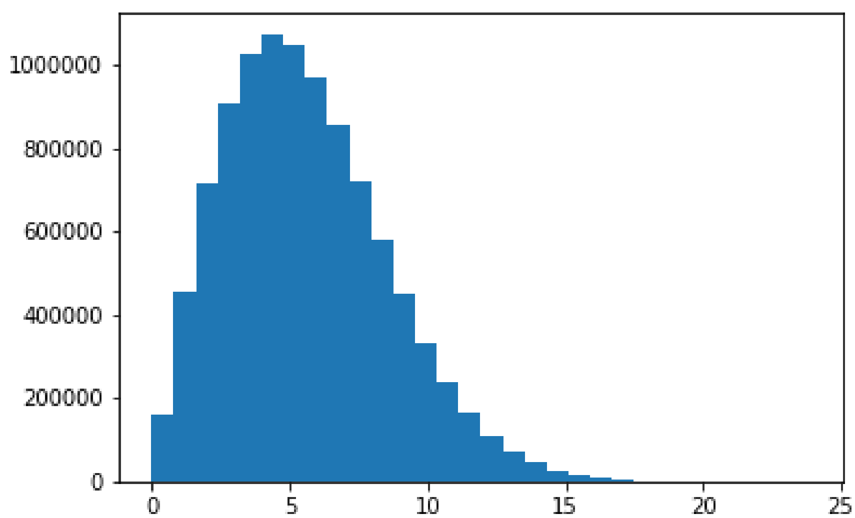

Figure 10: Histogram of random number that are generated using the combined method. (Gaussian with Box-Muller)