Университет ИТМО
МФ КТиУ, Ф ПИиКТ

Лабораторная работа №5
Дисциплина «Вычислительная математика»

Интерполяция функции

Выполнил
Аскаров Эмиль Рамилевич

Преподаватель:
Машина Екатерина
Алексеевна

г. Санкт-Петербург
2024 г.

Вычислительная реализация задачи

x	1.10	1.25	1.40	1.55	1.70	1.85	2.00
y	0.2234	1.2438	2.2644	3.2984	4.3222	5.3516	6.3867

n = 6

Таблица конечных разностей:

x_i	1.10	1.25	1.40	1.55	1.70	1.85	2.00
y_i	0.2234	1.2438	2.2644	3.2984	4.3222	5.3516	6.3867
Δy_i	1.0204	1.0206	1.034	1.0238	1.0294	1.0351	-
$\Delta^2 yi$	0.0002	0.0134	-0.0102	0.0056	0.0057	-	-
$\Delta^3 yi$	0.0132	-0.0236	0.0158	0.0001	-	-	-
$\Delta^4 yi$	-0.0368	0.0394	-0.0157	-	-	-	-
$\Delta^5 yi$	0.0762	-0.0551	-	-	-	-	-
$\Delta^6 yi$	-0.1313	-	-	-	-	-	-

$$\Delta^k y_i = \Delta^{k-1} y_{i+1} - \Delta^{k-1} y_i$$

$X_1 = 1.121$

$x_0 <= X_1 <= x_1$ => первая интерполяционная формула Ньютона

h = 1.25 – 1.10 = 0.15

t = (x – x0) / h = (x – 1.10) / 0.15

$$N_n(\mathrm{x}) = y_0 + t\Delta y_0 + \frac{\mathrm{t(t-1)}}{2!}\Delta^2 y_0 + \frac{\mathrm{t(t-1)(t-2)}}{3!}\Delta^3 y_0 + \frac{\mathrm{t(t-1)(t-2)(t-3)}}{4!}\Delta^4 y_0 + \frac{\mathrm{t(t-1)(t-2)(t-3)(t-4)}}{5!}\Delta^5 y_0 + \frac{\mathrm{t(t-1)(t-2)(t-3)(t-4)(t-5)}}{6!}\Delta^6 y_0$$

x = 1.121=> t = (1.121 – 1.10) / 0.15 = 0.14

$$N_6(1.121) = 0.2234 + 0.14 * 1.0204 + \frac{0.14(0.14-1)}{2}0.0002 + \frac{0.14(0.14-1)(0.14-2)}{6}0.0132 + \frac{0.14(0.14-1)(0.14-2)(0.14-3)}{24}(-0.0368) + \frac{0.14(0.14-1)(0.14-2)(0.14-3)(0.14-4)}{120}0.0762 + \frac{0.14(0.14-1)(0.14-2)(0.14-3)(0.14-4)(0.14-5)}{720}(-0.1313)$$

=0.2234+0.142856+0+0.00049+0.00098+0.00156+0.00219 = **0.3715**

$X_2 = 1.482$
$a = x_3 = 1.55$
$x_2 < X_2 < x_3 \Rightarrow X_2 < a \Rightarrow$ вторая интерполяционная формула Гаусса

$h = 1.25 – 1.10 = 0.15$
$t = (x – a) / h = (x – 1.55) / 0.15$

Таблица конечных разностей:

i	-3	-2	-1	0	1	2	3
x_i	1.10	1.25	1.40	1.55	1.70	1.85	2.00
y_i	0.2234	1.2438	2.2644	3.2984	4.3222	5.3516	6.3867
Δy_i	1.0204	1.0206	1.034	1.0238	1.0294	1.0351	-
$\Delta^2 yi$	0.0002	0.0134	-0.0102	0.0056	0.0057	-	-
$\Delta^3 yi$	0.0132	-0.0236	0.0158	0.0001	-	-	-
$\Delta^4 yi$	-0.0368	0.0394	-0.0157	-	-	-	-
$\Delta^5 yi$	0.0762	-0.0551	-	-	-	-	-
$\Delta^6 yi$	-0.1313	-	-	-	-	-	-

$$G_n(x) = y_0 + t\Delta y_{-1} + \frac{t(t+1)}{2!}\Delta^2 y_{-1} + \frac{t(t+1)(t-1)}{3!}\Delta^3 y_{-2} + \frac{t(t+2)(t+1)(t-1)}{4!}\Delta^4 y_{-2} + \frac{t(t+2)(t+1)(t-1)(t-2)}{5!}\Delta^5 y_{-3} + \frac{t(t+3)(t+2)(t+1)(t-1)(t-2)}{6!}\Delta^6 y_{-3}$$

$x = 1.482 \Rightarrow t = (1.482 – 1.55) / 0.15 = -0.453$

$$G_6(x) = 3.2984 - 0.453 * 1.034 + \frac{-0.453(-0.453+1)}{2}(-0.0102) + \frac{-0.453(-0.453+1)(-0.453-1)}{6}(-0.0236) + \frac{-0.453(-0.453+2)(-0.453+1)(-0.453-1)}{24}(-0.0394) + \frac{-0.453(-0.453+2)(-0.453+1)(-0.453-1)(-0.453-2)}{120}0.0762 + \frac{-0.453(-0.453+3)(-0.453+2)(-0.453+1)(-0.453-1)(-0.453-2)}{720}(-0.1313)$$

= 3.2984 - 0.4684 + 0.00126 - 0.00141 - 0.00091 - 0.00087 + 0.00063 = **2.8287**

Программная реализация задачи

```
from functools import reduce
from math import factorial

from matplotlib import pyplot as plt


def calc_lagrange_polynomial(xs, ys):
    n = len(xs) - 1
    f = lambda x: sum([ys[i] *
                       reduce(lambda a, b: a * b,
                              [(x - xs[j]) / (xs[i] - xs[j])
                               for j in range(n + 1) if i != j])
                       for i in range(n + 1)])
    return f


def calc_newton_divided_difference_polynomial(xs, ys):
    div_difs = []
    div_difs.append(ys[:])
    n = len(xs) - 1
    for k in range(1, n + 1):
        new = []
        last = div_difs[-1][:]
        for i in range(n - k + 1):
            new.append((last[i + 1] - last[i]) / (xs[i + k] - xs[i]))
        div_difs.append(new[:])
    print("divided differences:")
    for row in div_difs:
        print(*map(lambda a: round(a, 5), row), sep='\t')
    print('-' * 30)
    f = lambda x: ys[0] + sum([
        div_difs[k][0] * reduce(lambda a, b: a * b,
                                [x - xs[j] for j in range(k)])
        for k in range(1, n + 1)])
    return f


def calc_newton_finite_difference_polynomial(xs, ys):
    fin_difs = []
    fin_difs.append(ys[:])
    n = len(xs) - 1
    for k in range(1, n + 1):
        last = fin_difs[-1][:]
        fin_difs.append(
            [last[i + 1] - last[i] for i in range(n - k + 1)])
    print("finite differences:")
    for row in fin_difs:
        print(*map(lambda a: round(a, 5), row), sep='\t')
    print('-' * 30)
    h = xs[1] - xs[0]
    f = lambda x: ys[0] + sum([
        reduce(lambda a, b: a * b,
               [(x - xs[0]) / h - j for j in range(k)])
        * fin_difs[k][0] / factorial(k)
        for k in range(1, n + 1)])
    return f


def calc_gauss_polynomial(xs, ys):
    n = len(xs) - 1
    alpha_ind = n // 2
    fin_difs = []
    fin_difs.append(ys[:])

    for k in range(1, n + 1):
        last = fin_difs[-1][:]
        fin_difs.append(
            [last[i + 1] - last[i] for i in range(n - k + 1)])
```

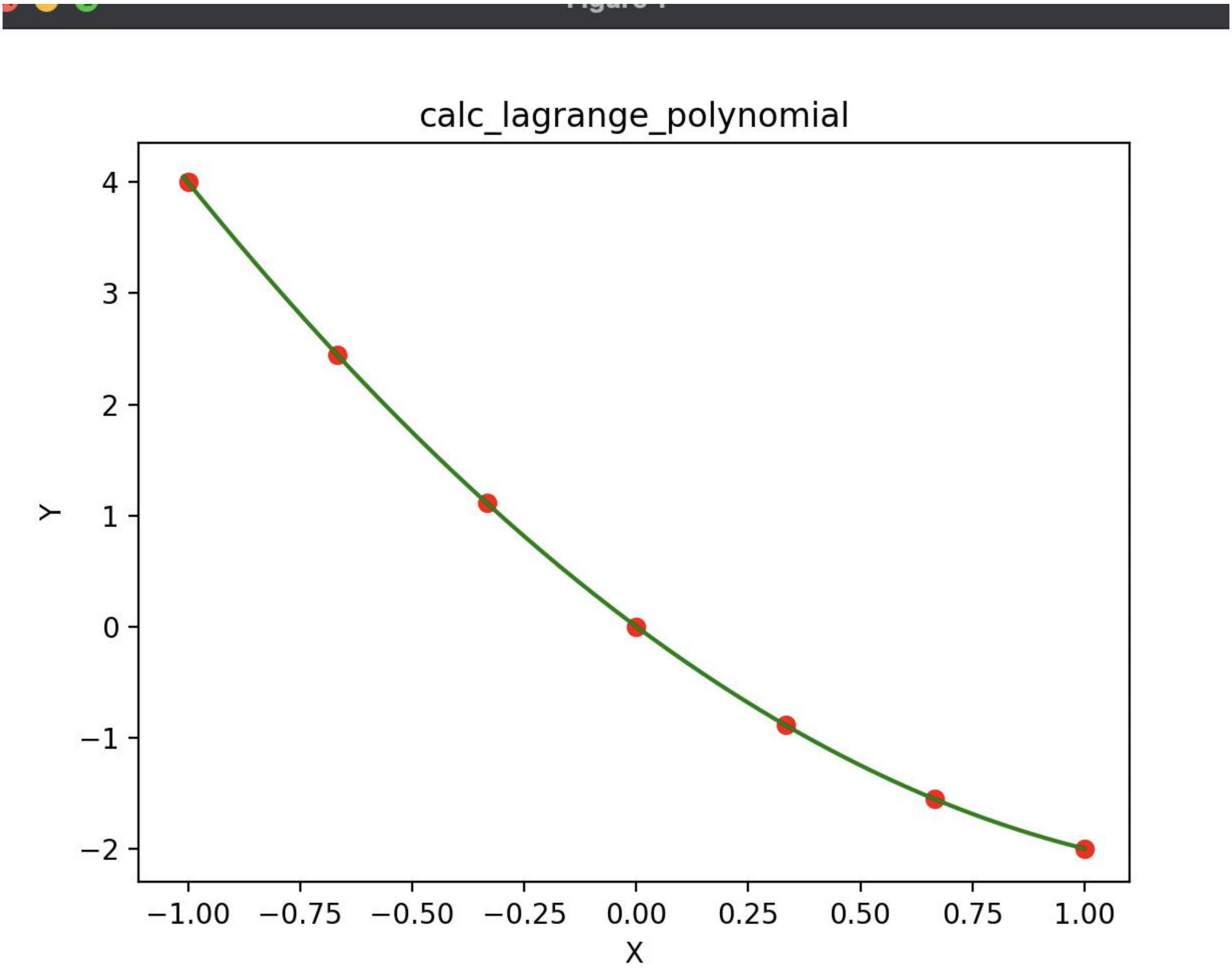
```
    h = xs[1] - xs[0]
    dts1 = [0, -1, 1, -2, 2, -3, 3, -4, 4]
    f1 = lambda x: ys[alpha_ind] + sum([
        reduce(lambda a, b: a * b,
               [(x - xs[alpha_ind]) / h + dts1[j] for j in range(k)])
        * fin_difs[k][len(fin_difs[k]) // 2] / factorial(k)
        for k in range(1, n + 1)])
    f2 = lambda x: ys[alpha_ind] + sum([
        reduce(lambda a, b: a * b,
               [(x - xs[alpha_ind]) / h - dts1[j] for j in range(k)])
        * fin_difs[k][len(fin_difs[k]) // 2 - (1 - len(fin_difs[k]) % 2)] / factorial(k)
        for k in range(1, n + 1)])
    return lambda x: f1(x) if x > xs[alpha_ind] else f2(x)


def calc_stirling_polynomial(xs, ys):
    n = len(xs) - 1
    alpha_ind = n // 2
    fin_difs = []
    fin_difs.append(ys[:])

    for k in range(1, n + 1):
        last = fin_difs[-1][:]
        fin_difs.append(
            [last[i + 1] - last[i] for i in range(n - k + 1)])

    h = xs[1] - xs[0]
    dts1 = [0, -1, 1, -2, 2, -3, 3, -4, 4]
    f1 = lambda x: ys[alpha_ind] + sum([
        reduce(lambda a, b: a * b,
               [(x - xs[alpha_ind]) / h + dts1[j] for j in range(k)])
        * fin_difs[k][len(fin_difs[k]) // 2] / factorial(k)
        for k in range(1, n + 1)])
    f2 = lambda x: ys[alpha_ind] + sum([
        reduce(lambda a, b: a * b,
               [(x - xs[alpha_ind]) / h - dts1[j] for j in range(k)])
        * fin_difs[k][len(fin_difs[k]) // 2 - (1 - len(fin_difs[k]) % 2)] / factorial(k)
        for k in range(1, n + 1)])
    return lambda x: (f1(x) + f2(x)) / 2


def calc_bessel_polynomial(xs, ys):
    n = len(xs) - 1
    alpha_ind = 0
    fin_difs = []
    fin_difs.append(ys[:])

    for k in range(1, n + 1):
        last = fin_difs[-1][:]
        fin_difs.append(
            [last[i + 1] - last[i] for i in range(n - k + 1)])

    h = xs[1] - xs[0]

    dts1 = [0, -1, 1, -2, 2, -3, 3, -4, 4, -5, 5]
    f = lambda x: (ys[alpha_ind] + ys[alpha_ind + 1]) / 2 + sum([
        reduce(lambda a, b: a * b,
               [(x - xs[alpha_ind]) / h + dts1[j] for j in range(k)])
        * fin_difs[k][len(fin_difs[k]) // 2] / factorial(2 * k) +
        ((x - xs[alpha_ind]) / h - 1 / 2) *
        reduce(lambda a, b: a * b,
               [(x - xs[alpha_ind]) / h + dts1[j] for j in range(k)])
        * fin_difs[k][len(fin_difs[k]) // 2] / factorial(2 * k + 1)
        for k in range(1, n + 1)])
    return f


def draw_plot(a, b, func, dx=0.01):
    xs, ys = [], []
```

```
    a -= dx
    b += dx
    x = a
    while x <= b:
        xs.append(x)
        ys.append(func(x))
        x += dx
    plt.plot(xs, ys, 'g')


def main(xs, ys, x):
    methods = [calc_lagrange_polynomial,
               calc_newton_divided_difference_polynomial,
               calc_newton_finite_difference_polynomial,
               calc_gauss_polynomial,
               calc_stirling_polynomial,
               calc_bessel_polynomial]
    for method in methods:
        # для гаусса и стирлинга нечётное число узлов должно быть
        if (method is calc_gauss_polynomial or method is calc_stirling_polynomial) \
                and len(xs) % 2 == 0:
            continue
        # для бесселя чётное число узлов должно быть
        if method is calc_bessel_polynomial and len(xs) % 2 == 1:
            continue
        print(method.__name__)

        P = method(xs, ys)

        plt.title(method.__name__)

        draw_plot(xs[0], xs[-1], P)
        for i in range(len(xs)):
            plt.scatter(xs[i], ys[i], c='r')
        plt.xlabel("X")
        plt.ylabel("Y")
        plt.show()

        print(f'P({x}) = {P(x)}')
        print('-' * 60)


def read_number(s: str):
    while True:
        try:
            return float(input(s))
        except Exception:
            continue


if __name__ == '__main__':
    mode = read_number("Введите режим: ")
    if mode == 1:
        xs = list(map(float, input('input xs: ').split()))
        ys = list(map(float, input('input ys: ').split()))
        x = float(input('input x: '))
    elif mode == 2:
        with open('input.txt') as f:
            xs = list(map(float, f.readline().strip().split()))
            ys = list(map(float, f.readline().strip().split()))
            x = float(f.readline().strip())
    elif mode == 3:
        print('functions: ')
        print('1. x ^ 2 - 3 * x')
        print('2. x ^ 5')
        func_number = int(input('input 1 or 2: '))
        f = lambda x: x ** 2 - 3 * x if func_number == 1 else x ** 5
        n = int(input('input n: '))
        x0 = float(input('input first x: '))
        xn = float(input('input last x: '))
```

```
        h = (xn - x0) / (n - 1)
        xs = [x0 + h * i for i in range(n)]
        ys = list(map(f, xs))
        x = float(input('input x: '))
    else:
        # xs = [0.15, 0.2, 0.33, 0.47]
        xs = [0.15, 0.2, 0.25, 0.3]
        ys = [1.25, 2.38, 3.79, 5.44]
        x = 0.22
    # main(xs, ys, x)
    xs = sorted(xs)
    if len(set(xs)) != len(xs):
        print('Иксы должны быть разными')
    else:
        main(xs, ys, x)
```

Тестовые данные

```
➜  lab5 git:(main) ✗ python3 main.py
3
Введите режим: functions:
1. x ^ 2 - 3 * x
2. x
input 1 or 2: 1
input n: 6
input first x: -1
input last x: 1
input x: 0
calc_lagrange_polynomial
```




```
------------------------------
P(1.121) = 0.37147968132678
------------------------------------------------------------
calc_gauss_polynomial
P(1.121) = 0.37147968132677844
------------------------------------------------------------
calc_stirling_polynomial
P(1.121) = 0.37147968132677844
------------------------------------------------------------
```

calc_lagrange_polynomial
P(0.0) = 5.551115123125783e-17
----
calc_newton_divided_difference_polynomial
divided differences:

```
4.0   2.16  0.64  -0.56  -1.44  -2.0
-4.6  -3.8  -3.0  -2.2  -1.4
1.0   1.0   1.0   1.0
-0.0  0.0   -0.0
```

```
0.0     -0.0
-0.0
------------------------------
P(0.0) = 0.0
------------------------------------------------------------
calc_newton_finite_difference_polynomial
finite differences:
4.0     2.16    0.64    -0.56   -1.44   -2.0
-1.84   -1.52   -1.2    -0.88   -0.56
0.32    0.32    0.32    0.32
-0.0    0.0     -0.0
0.0     -0.0
-0.0
------------------------------
P(0.0) = 0.0
------------------------------------------------------------
calc_bessel_polynomial
P(0.0) = 0.6499999999999995
------------------------------------------------------------
➜  lab5 git:(main) ✗ python3 main.py
Введите режим: 3
functions:
1. x ^ 2 - 3 * x
2. x
input 1 or 2: 1
input n: 7
input first x: -1
input last x: 1
input x: 0
calc_lagrange_polynomial
P(0.0) = 0.0
------------------------------------------------------------
calc_newton_divided_difference_polynomial
divided differences:
4.0     2.44444 1.11111 0.0     -0.88889        -1.55556        -2.0
-4.66667        -4.0    -3.33333        -2.66667        -2.0    -1.33333
1.0     1.0     1.0     1.0     1.0
-0.0    -0.0    0.0     0.0
-0.0    0.0     -0.0
0.0     -0.0
-0.0
------------------------------
P(0.0) = 0.0
------------------------------------------------------------
calc_newton_finite_difference_polynomial
finite differences:
4.0     2.44444 1.11111 0.0     -0.88889        -1.55556        -2.0
-1.55556        -1.33333        -1.11111        -0.88889        -0.66667        -0.44444
0.22222 0.22222 0.22222 0.22222 0.22222
```

0.0   0.0   -0.0   -0.0
-0.0   -0.0   0.0
-0.0   0.0
0.0

P(0.0) = -8.881784197001252e-16
----
calc_gauss_polynomial
P(0.0) = 0.0
----
calc_stirling_polynomial
P(0.0) = 0.0

Вывод

В ходе лабораторной работы я познакомился с интерполяцией функции разными методами (линейная, квадратичная, многочлен Лагранжа, многочлен Ньютона, многочлены Гаусса, Стирлинга и Бесселя).

Линейная и квадратичная интерполяция – простые методы, но неточные.

Многочлен Лагранжа – хороший метод, но много вычислений. Малая погрешность при небольших n, с изменением числа узлов все вычисления заново.

Многочлен Ньютона с разделёнными разностями – хороший метод. Используется для неравноотстоящих узлов. При добавлении новых узлов первые члены многочлена остаются неизменными.

Многочлен Ньютона с конечными разностями – хороший метод. Используется для равноотстоящих узлов. При добавлении новых узлов первые члены многочлена остаются неизменными. Есть формулы для интерполирования вперёд и назад. Можно использовать для экстраполирования (но будут бОльшие погрешности).