The Cooper Union Department of Electrical Engineering
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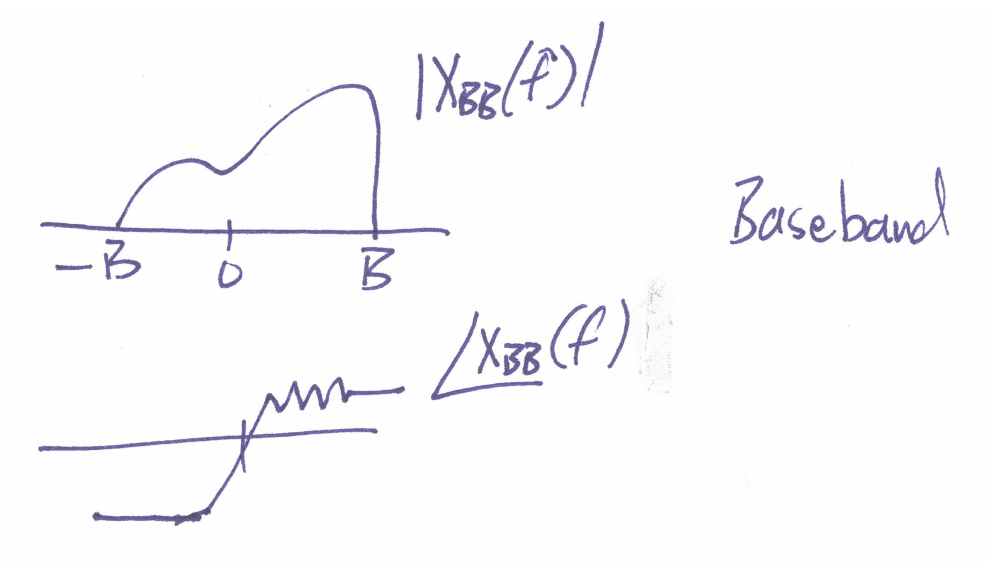
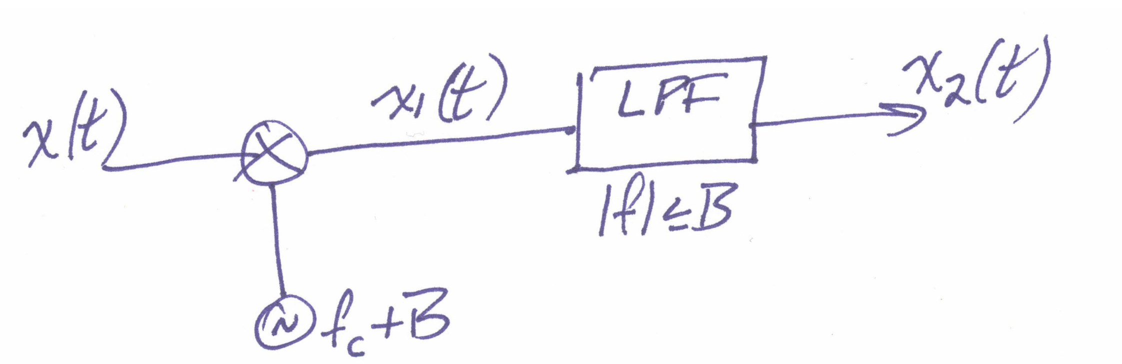
ECE300 Communication Theory
Problem Set IV: Analog Communications
October 16, 2023

1. Refer to the Figure below. The signal $x(t)$ is a bandpass signal at carrier frequency f_c, with baseband equivalent signal $x_{BB}(t)$ of bandwidth B, whose spectrum is shown. The signal $x(t)$ is mixed with an offset carrier at frequency $f_c + B$ to produce $x_1(t)$, which is then lowpass filtered (using an ideal filter with cutoff frequency B) to produce $x_2(t)$.

   Sketch the spectra (both magnitude and phase) for $x(t)$, $x_1(t)$ and $x_2(t)$. Note I am asking for the spectra of actual signals (not baseband equivalents). Comment on how $x_2(t)$ is related to the input $x(t)$. You **must** perform the entire problem *graphically*- do not attempt to write any analytic formulas.

   Now, *repeat* this problem for the case of the offset carrier $f_c - B$ instead.





2. A $10GHz$ carrier is frequency modulated by a $10MHz$ sinewave using a frequency sensitivity of $2.5MHz/volt$. For each of the following cases, determine (1)the amplitude of the modulating tone; (2)the approximate bandwidth via Carson's rule; (3)the approximate bandwidth via the universal curve. Summarize results in a table.

   (a) $\beta = 0.4$
   (b) $\beta = 2$
   (c) $\beta = 8$

3. A (real) message signal $m(t)$ with maximum amplitude m_0 and bandwidth B is passed through the modulation system shown in the Figure below. The message signal is added and subtracted from a carrier, that is:

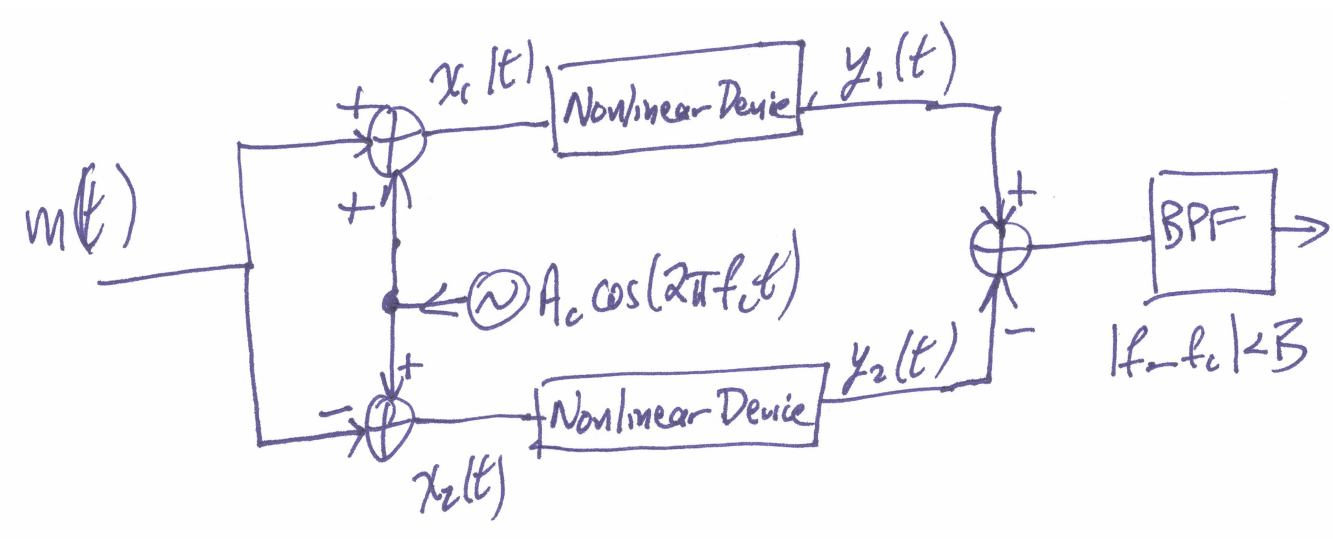
$$
\begin{aligned}
x_1(t) &= A_c \cos(2\pi f_c t) + m(t) \\
x_2(t) &= A_c \cos(2\pi f_c t) - m(t)
\end{aligned}
$$

The signals $x_1(t), x_2(t)$ are passed through identical nonlinear devices characterized as:

$$v_{out}(t) = a v_{in}(t) + b v_{in}^2(t)$$

where a, b are positive constants. The respective output signals $y_1(t), y_2(t)$ are subtracted, and the result is bandpass filtered at f_c. Assume $B \ll f_c$.

Find a simplified form for the output of the BPF, and identify the type of modulation formed. If AM, specify the modulation index. If not, specify the peak amplitude of the modulated signal.



4. A superheterodyne system downconverts an RF carrier at $850MHz$ to IF at $70MHz$.

   (a) Specify the frequency of the local oscillator used by the downconverter such that the image frequency is **greater than** $850MHz$.
   (b) In this case, what is the image frequency?

5. Let us examine the threshold effect that occurs with FM. We are looking at the effect of an additive noise term on the phase of a fixed amplitude carrier. At each time, the noise is $n = n_I + jn_Q$ where n_I, n_Q are iid $N\left(0, \frac{1}{2}\sigma^2\right)$, so that $E\left(|n|^2\right) = \sigma^2$.

   For any ϕ that is deterministic or, if random, is independent of n and uniform from 0 to 2π, the random quantities n and $ne^{-j\phi}$ have the same distribution (this is called the *isotropy* property of $n(t)$). So if we start (at baseband) with:

$$Ae^{j\phi(t)} + n(t)$$

   where $\phi(t)$ is the given phase signal for PM or FM, then we can cancel out the phase (multiply by $e^{-j\phi}$) and get:

$$A + n(t)$$

   Take $A = 1$, so the SNR is:

$$\sigma^2 = 10^{-SNR/10}$$

   Remark: The pdf of envelope $|A + n|$ obeys a *Rice* distribution (named after the engineer Steven O. Rice who studied it). The *Rician* pdf is:

$$f(x) = \frac{x}{\sigma^2} e^{-\frac{x^2+A^2}{2\sigma^2}} I_0\left(\frac{Ax}{\sigma^2}\right) \quad , \quad x > 0$$

   where $I_0(\cdot)$ is the modified 0^{th} order Bessel function of the first kind. We are not going to use this formula here; just giving it to you for fun :-D

   We are going to look at the envelope and phase of $A + n$. We are also particularly interested in cases the the magnitude of the phase error exceed 10°, i.e.:

$$abs\left(angle\left(A + n\right)\right) > 10 * pi/180$$

   (a) In MATLAB notation, set *SNR=10:2:30* (in decibels) and for each SNR generate $N = 10^7$ samples [**Note: first run your code with a smaller number, say 10^5; getting to 10^7 may be too much for your computer; don't store all the data for all values of SNR at once**] of the complex points $A + n$ as described above, and compute $x_{env} = |A + n|$ and $x_{ph} = angle\left(A + n\right)$.

      1. For the cases SNR=10,20,30 (only), plot the histograms of the envelope and the phase (six separate graphs). On the phase curves, demarcate the threshold value (the axis can be in radians, just place a colored vertical line or similar clear indicator at our threshold). Your histogram should use a *'Normalization'* of the type *'pdf'* (look up the documentation for the MATLAB function *histogram)* so the graph represents an estimate of the pdf.
      2. For each SNR, Use your random samples to estimate the probability the phase error x_{ph} exceeds $\pm 10^\circ$:

$$P_{TH}(SNR) = P\left(\text{phase error exceeds } 10^\circ\right) = P\left(|x_{ph}| > 10 * pi/180\right)$$

      which is a function of SNR. Create a plot of SNR in decibels versus $\log_{10} P_{TH}(SNR)$.

3. You should see a steeper rate of decrease of the log-probability for higher SNR. To see the reduction in probability of error given an increase of $2dB$ of SNR, compute (in MATLAB) *diff(log10(P_ TH))*. [Depending on how large your N is, you may get $-Inf$ or NaN in some positions] But you should see the rapid improvement as SNR increases. Eventually, 2dB increase in SNR yields multiple orders of magnitude improvement in this probability.