The Cooper Union Department of Electrical Engineering
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ECE300 Communication Theory
Problem Set II: Special Systems & Filter Design
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1. A discrete-time ARMA$(2,2)$ process has PSD given by:

$$S(\omega) = \frac{(5 - 4\cos\omega)^2}{13 + 12\cos\omega}$$

   Use spectral factorization to find the innovations filter $H(z)$ (assuming unit variance innovations signal).

   Remark: As a shortcut, $z = e^{j\omega}$ yields $z^{-1} = e^{-j\omega}$ and hence make the direct substitution $\cos\omega \to \frac{z+z^{-1}}{2}$. I would advise not multiplying out the squaring factor in the numerator, since the goal is to find the poles and zeros. Simplify to a ratio of polynomials (i.e., positive powers of z), and you can use MATLAB to find the poles and zeros; *do not forget to check for any poles or zeros at* ∞ or 0! After you find the poles and zeros of H, remember to check for a possible scaling factor. [**Hint:** Plug in $z = 1$ corresponding to $\omega =?$ to find the scaling factor]

2. In this problem you will be using MATLAB tools for analog and digital IIR filter designs: *buttord, butter, cheb1ord, cheby1, cheb2ord, cheby2, ellipord, ellip.* They all have similar syntax, but you must be careful of slight differences. I expect you to review the documentation to learn how to use these functions properly for this problem. Your results should look reasonable; if they don't, it is likely you made a mistake in calling the functions.

   The following are specifications for a *bandstop* filter:

   - Passband below $9MHz$ and above $12.5MHz$, stopband from $9.5MHz$ to $12MHz$.
   - Passband variation $1.5dB$, stopband attenuation $40dB$.
   - For the digital filters, assume a sampling rate of $40MHz$.

   For every case:

   (a) Find the filter order (defined as the degree of the denominator polynomial, not the order of the underlying prototype).

   (b) Obtain pole-zero plots. Be careful of scaling the axes for the analog case. You can use *zplane* for both analog and digital cases.

   (c) Plot the magnitude and phase responses the usual way; you can use *freqs,freqz* to compute the frequency responses, but do the plots "by hand". The frequencies should be 1000 points from DC to 20MHz, and the frequency axis should be labeled in MHz in your plots. You should put the magnitude (in dB) and phase (unwrapped, in degrees) responses as subplots in the same figure; scale the vertical axis for the magnitude plot for the range -50 to $+2dB$.

(d) In each case, we would expect the peak passband gain to be $0dB$ and the passband edges (at $9MHz$ and $12.5MHz$) to be exactly at $-1.5dB$, but the stopband edges may be an overdesign (i.e., the actual gain at the edges $9.5MHz$ and $12MHz$ may be below $-40dB$). Verify all of these, and in particular report out the gain as the stopband edges.

3. In this problem you will be using MATLAB tools for digital FIR filter design to achieve the same specifications as the previous problem.

   - Kaiser window design: *kaiserord* to estimate filter order and other parameters; *kaiser* to design the window; *fir1* to determine the FIR filter.
   - Equiripple FIR filter design: *firpmord* to estimate parameters and *firpm* to generate the filter. [Here, *'pm'* refers to the Parks-McClellan algorithm that is used in the design process]

   (a) Read the MATLAB documentation to learn how to use these functions to perform the designs. For the equiripple FIR filter design, you will need to derive tolerances (deviations) $\delta_{pass}, \delta_{stop}$ on a linear scale so that:

   $$\begin{aligned} \text{in the passband} &: \quad 1-\delta_{pass} \le |H(\omega)| \le 1+\delta_{pass} \\ \text{in the stopband} &: \quad |H(\omega)| \le \delta_{stop} \end{aligned}$$

   (here $|H|$ is *not* in decibels). So, as a first step, derive $\delta_{pass}, \delta_{stop}$ from r_p, r_s.

   (b) Now obtain the two FIR filters. In each case, determine the filter length; obtain a stem plot of the filter coefficients; obtain a pole-zero plot for the filter; and plot the frequency response in the same manner as you did in the previous problem.

   (c) The function *firpmord* returns a set of weights that are used in the design. As discussed in lecture, the weight for a band is inversely proportional to the tolerance (deviation) in the band (on a linear scale). Examine the weights for the passband and stopband, and compare with the equivalent tolerances (on a linear scale), to verify this. Specifically, there is a pair of weights and deviations for the passband and stopband respectively; check that $W_{pass}/W_{stop} = \delta_{stop}/\delta_{pass}$.

   (d) Unlike for the analog and digital IIR filter designs, here the FIR filters may not have a peak gain of 0dB in the passband. Find the maximum and minimum gains in the passband, and check that they differ by 1.5dB (if not, how close are they?) Also, look at the peak gains in the stopband and compare with the specification (that require this to be $\le -30dB$).

   (e) Do either of your FIR filters fail to meet the specifications exactly? I'm not asking you to fix this, but if you were to try to fix this, what would you do?