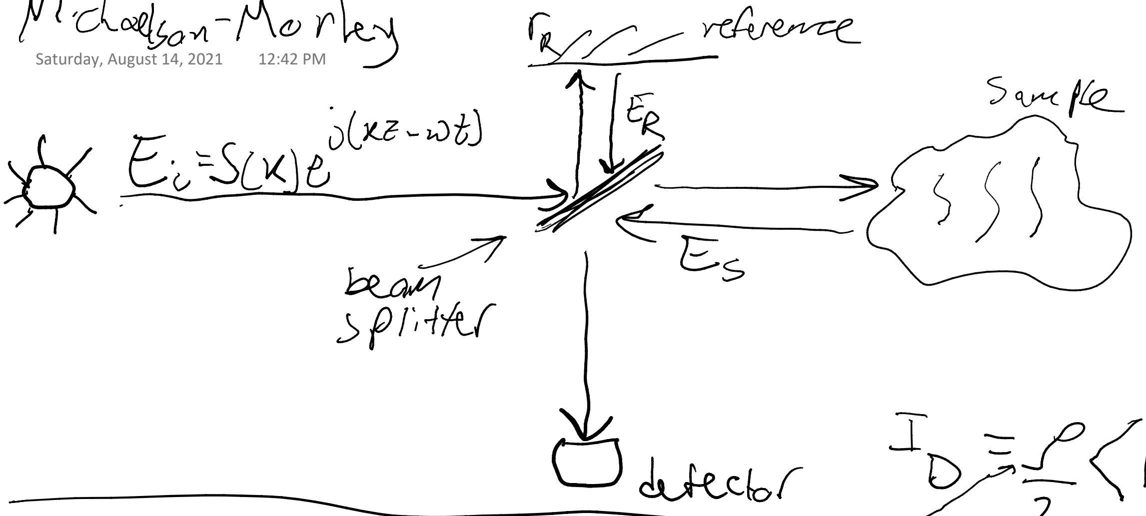
Michaelson-Morley

Saturday, August 14, 2021 12:42 PM



$$I_D = \frac{\rho}{2}\langle |E_R + E_s|^2 \rangle$$

responsivity

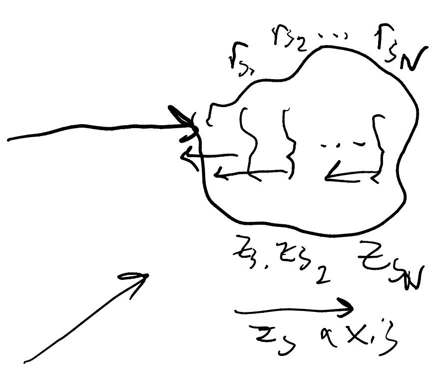
from passing thru beamsplitter again

S = spectrum of source

z here is OPL = length·n, K·OPL = phase change

$$\boxed{E_R = \frac{E_i}{\sqrt{2}} r_R e^{j2Kz_R}}$$

What is E_s? assume sample consists of N reflectors



$$E_s = \frac{E_i}{\sqrt{2}} \sum_{n=1}^{N} e^{j2kz_{s_n}} r_{s_n}$$

$$= \frac{E_i}{\sqrt{2}} \sum_n e^{j2kz_s} \delta(z_s - z_{s_n}) r_{s_n}$$

$$r_s(z_s) = \sum_n r_{s_n} \delta(z_s - z_{s_n})$$

$$\boxed{E_s = \frac{E_i}{\sqrt{2}} \left(e^{j2kz_s} \otimes r_s(z_s)\right)}$$

in cts case too, in lim $N \to \infty$

OCT goal

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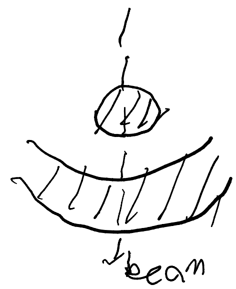
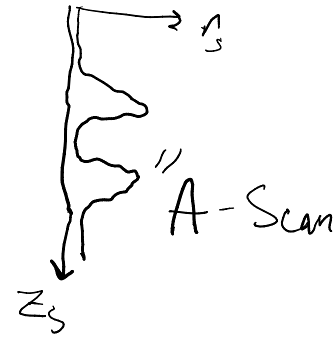
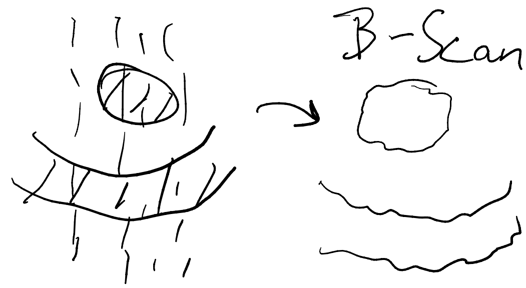
$\boxed{\text{Recover } r_s(z_s)}$. Result: object:



profile along beam:



repeat with beams across sample:



Can do in another dir. to get volume

$\widehat{I}_D$

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$|\cdot|^2 = (\cdot)(\cdot)^*$

$$I_D = \frac{\rho}{2}\left\langle |E_R + E_S|^2 \right\rangle$$

$$= \frac{\rho}{2}\left\langle \left| \frac{s}{\sqrt{2}} r_R e^{j(2kz_R - \omega t)} + \frac{s}{\sqrt{2}} \sum_n r_{s_n} e^{j(2kz_{s_n} - \omega t)} \right|^2 \right\rangle$$

$\leftarrow r^2$

$$= \frac{\rho}{4} S(k) \left(R_R + R_{s_1} + R_{s_2} + \dots R_{s_N} \right) \quad \text{(DC term)}$$

$$+ \frac{\rho}{4} S(k) \sum_n \sqrt{R_R R_{s_n}} \left(e^{j2k(z_R - z_{s_n})} + e^{-j2k(z_R - z_{s_n})} \right) \quad \text{(cross-corr)}$$

$$+ \frac{\rho}{4} S(k) \sum_{n \neq m} \sqrt{R_{s_n} R_{s_m}} \left(e^{j2k(z_{s_n} - z_{s_m})} + e^{-j2k(z_n - z_{s_m})} \right) \quad \text{(auto-corr)}$$

so

$$Re(I_D(k)) = \frac{\rho}{4} S(k) \left(R_R + R_{s_1} + R_{s_2} + \dots + R_{s_N} \right)$$

$$+ \frac{\rho}{2} S(k) \sum_n \sqrt{R_R R_{s_n}} \cos(2k(z_R - z_{s_n}))$$

$$+ \frac{\rho}{2} S(k) \sum_{n \neq m} \sqrt{R_{s_n} R_{s_m}} \cos(2k(z_{s_n} - z_{s_m}))$$

$R_R >> R_{s_i}$, so auto corr term small, DC largest

Cross-corr is the most important, as it encodes $r(z_s)$

One reflector

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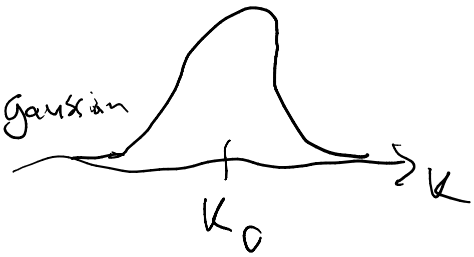
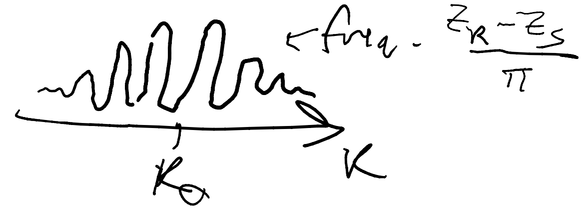
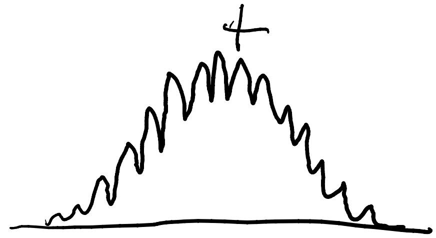
$$Re(I_D) = \frac{\rho}{4} S(k)(R_R + R_S) + \frac{\rho}{2} S(k)\sqrt{R_R R_S}\cos(2k(z_R - z_S))$$

Say $S \approx$ gaussian







You can find z_S by freq!

So freq. components in K represent distance in sample

Mag. at that component encodes r_S ($\sqrt{R_R R_S}$)

Three kinds of OCT

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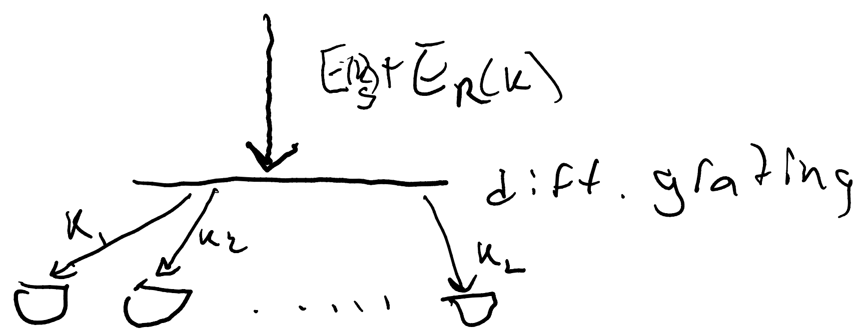
TDOCT time domain

SDOCT spectral domain

SSOCT swept source

will describe <u>SDOCT</u> first

in SDOCT, a <u>broadband</u> $S(k)$ is used, and all k values are recorded at once



$L \sim 2048$. "Line camera"

S(k)

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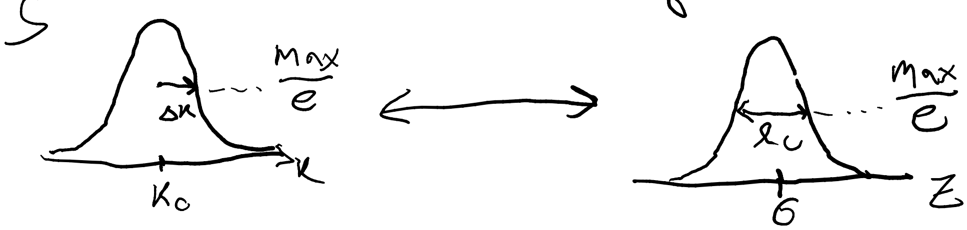
assume $S(k)$ gaussian around k_0, s.d. Δk "spectral bandwidth"

$$S(k) = \frac{1}{\Delta k\sqrt{\pi}} e^{-\left(\frac{(k-k_0)}{\Delta k}\right)^2} \xleftrightarrow{\mathcal{F}^{-1}} \gamma(z) = e^{-z^2 \Delta k^2}$$

$\gamma \leftarrow$ coherence function



l_c = coherence length

$$l_c = \frac{2\log 2}{\Delta k} = \boxed{\frac{2\log 2}{\pi}\frac{\lambda_0^2}{\Delta\lambda}}$$

. So $l_c \propto \lambda_0^2$ short wavelength $\rightarrow$ low coh. length

$l_c \propto \Delta\lambda$ large BW $\rightarrow$ low coh length

SDOCT A-Scan

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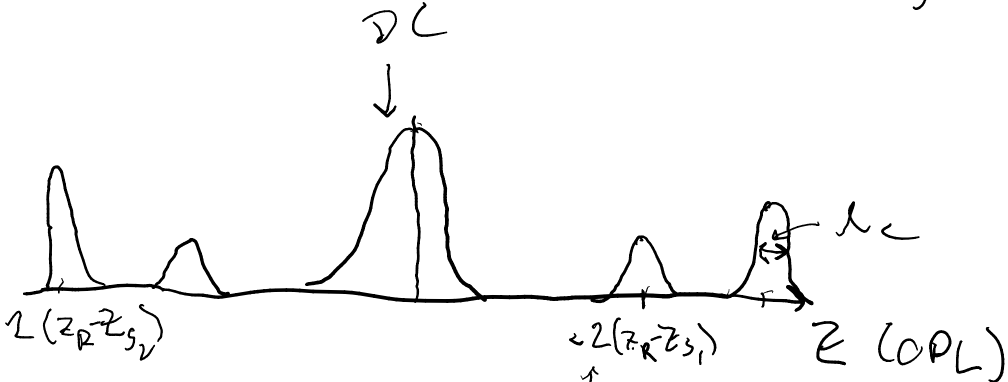
$$i_D(k) = \mathcal{F}^{-1}\{Re(I_D(k))\}(z) = \frac{\rho}{8}\gamma(z)(R_R + R_{S_1} + \dots + R_{S_n})$$

$$+ \frac{\rho}{4}\gamma(z) * \sum \sqrt{R_R R_{S_n}} \left(\delta(z+2(z_R - z_{S_n})) + \delta(z - 2(z_R - z_{S_n}))\right)$$

+ hopefully negligible



easily removed factor of 2, from fact that each arm is travelled twice (there and back)

a blurred depth profile, symm. about zero, with DC component

so axial resolution $\boxed{\Delta z = l_c = \frac{2\log 2}{\pi}\frac{\lambda_0^2}{\Delta\lambda}}$

note: Δx, lateral res., just based on how well the beam can be focused. i.e. DIFF limit $\boxed{\Delta x = 0.37\frac{\lambda_0}{NA}}$

Clear that lower $\lambda_0 \rightarrow$ better res., but less pen. depth ... fortunately less pen. depth unfortunately.

What we left out

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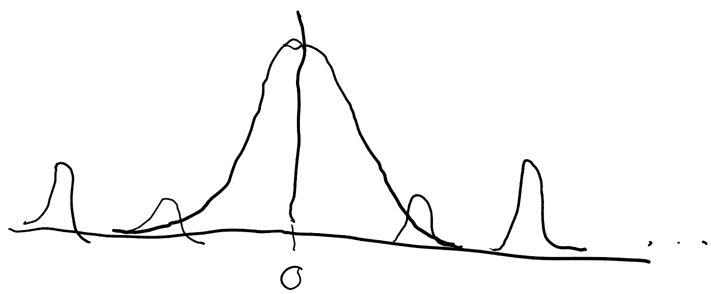
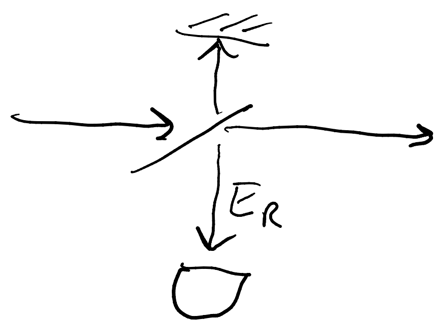


Same rolloff, but mag high so may interfere.

Solution: first scan:



, get only background.

Now you know Γ, so not only can you remove it via subtraction, but also deconvolve by it in z to get a signal more like $r(z_s)$.

We also left out that $\mathcal{F}$ is actually an FFT here,

[illegible]

So we need to window to avoid artifacts.

Lastly, nobody ever takes one A-Scan. Always average to get higher SNR

Pipeline

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1. Take B background scans and average them in K domain to get $b(k) \approx \frac{\rho}{4} R_R S(k)$. We usually also fit b to a polynomial to make it smooth.

2. Take M A-Scans, $I_m(K)$,

3. From $I_m(K)$, subtract $b(k)$

4. Multiply $(I_m - b(k))$ by a Hamming window (or some window) $H(k)$

5. Divide this signal by $b(k)$, $\tilde{I}_m = \frac{H(I_m - b)}{b}$; eq. to deconvolving by γ in time (or div. by S). Deblur

6. Take FFT.

7. Average

Oops...

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We missed something else!

Data may be collected in the λ domain

<u>not</u> the k domain.

Of course $I(\lambda) \longrightarrow I(k) = I(2\pi/\lambda)$

<u>but</u> even spacing in λ is not even spacing in

k!

Requires interpolation.

There are cute interpolation methods, e.g.

interpolation as a matrix multiplication.

SSOCT

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The difference here is only hardware.

Light source: very narrow band, controllable λ_0

$E_R(k_i) + E_S(k_i)$

$I_D(k_i)$

sweep over λ_0 to get

$I_D(k)$

then apply same principle: $r_s(2z) \approx \mathcal{F}^{-1}\{\mathrm{Re}(I_D)\}$

TDOCT

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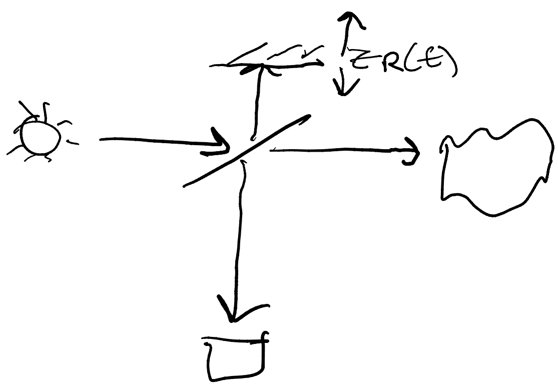
no spectrometer



movable reflector

$$I_D(z_R) = \frac{\rho}{4} S_0 (R_R + R_{S_1} + \dots + R_{S_N}) + \frac{\rho}{2} S_0 \sum_n \sqrt{R_R R_{S_n}}\, \gamma(z_R - z_{S_n}) \cos(2k_0(z_R - z_{S_n}))$$

+ autocorr

still, coherence length dets Δz

Quantization

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in SD-OCT: line camera has spectral width $\delta_r k$ (say)

in SS-OCT: swept source has FWHM $\delta_r k$ at each k_0

model this quantization error as gaussian blurring

w/ FWHM $\delta_r k$ $\left(sd = \frac{\delta_r k}{\sqrt{2}\log 2} \right) \rightarrow$

$$I_D(k) * e^{-\frac{4 \log 2 \, k^2}{(\delta_r k)^2}} \leftarrow \text{blurring}$$

$$\Big\downarrow \mathcal{F}^{-1}$$

$$i_D(\hat{z})\, e^{-\hat{z}^2 (\delta_r k)^2 / 4 \log 2}$$

(where $\hat{z} = 2z$ to account for the x2 A-Scan weirdness)

So quantization error is blurring in k, but leads to

$\sim e^{-z^2 \delta k^2}$ signal visibility loss in z,

$\rightarrow$ worse signal at deep z

$$\hat{z} \quad = \frac{2 \log 2}{\quad} = \frac{\log 2}{\quad} \frac{\lambda_0^{\,2}}{\quad}$$

$$\hat{z}_{6dB} = \frac{2\log 2}{\delta_r K} = \frac{\log 2}{\pi}\frac{\lambda_0^2}{\delta_r \lambda}$$, again want lower λ_0 for this

SDPM

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SDOCT lends itself nicely to Vibrometry through Spectral Domain Phase Microscopy, so long as the vibrations are at a sub-pixel scale. (eg. ~1–100 nm)

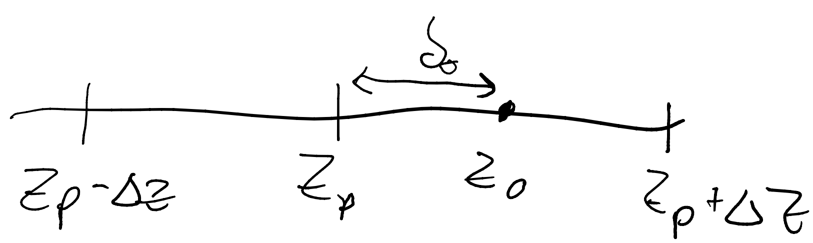
Say there is one reflector at z_0, contributing

$$I_0(k) = S(k)\, e^{j2k(z_0 - z_R)n}$$

(here, z is in m, not OPL, so n included)

Δz is pixel size

So z_0 shows up in pixel z_p, but is actually at $z_p + \delta_0$



Now say this reflector moves as a function of time, but never leaves the pixel. That is, $\delta = \delta_0 + \delta(t)$

$$\text{OPL}: (z_p + \delta_0 + \delta(t))n \quad , \text{ def. } \theta_0 = 2nK_0\delta_0$$

$$\theta(t) = \theta_0 + 2nK_0\delta(t)$$

$\theta(t) = \theta_0 + 2nk_0\delta(t)$

approx k as k_0, find

$$I_0(k) \approx S(k)e^{jknz_p}e^{j\theta(t)}\sqrt{R_R R_S}$$

$$Re(I_0) = S(k)\cos(2knz_p + \theta(t))\sqrt{R_R R_S}$$

$$\xrightarrow{\mathcal{F}^{-1}} \frac{1}{2}\left(e^{j\theta(t)}\gamma(2n(z-z_p)) + e^{-j\theta(t)}\gamma(-2n(z-z_p))\right)\sqrt{R_R R_S}$$

looking at pixel z_p, have $A(z_p) = \underbrace{\frac{\sqrt{R_R R_S}\,\gamma(2n(z-z_p))}{2}}_{\text{real}} e^{j\theta(t)}$

$$\text{So } \angle A(z_p) = \theta_0 + 2nk_0\delta(t)$$

$$\rightarrow \delta(t) = \frac{\theta - \theta_0}{2nk_0}$$

$$\text{or } \boxed{\frac{\theta}{2nk_0} = \text{Constant} + \delta(t)}$$

don't need to know const. to find vibration pattern

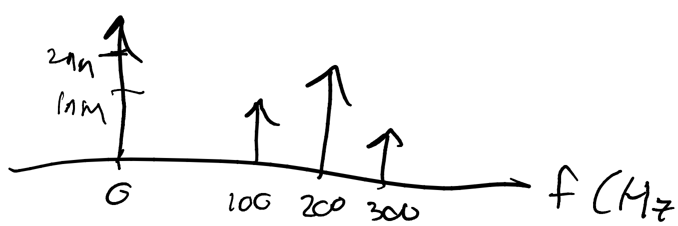
take $\mathcal{F}\left\{\frac{\theta}{2nk_0}\right\}$ and remove DC component to get

take $\mathcal{F}\left\{\frac{\cup}{2nk_0}\right\}$ and remove DC component to get
& vibration pattern

if interested in motion at pixel p,

$$D(t) = \frac{\lambda_0}{4\pi n} \arctan\left(\frac{Im(i(p))}{Re(i(p))}\right) = const + \delta(t)$$

$$\mathcal{D}(f) = \mathcal{F}\{D(t)\}(f)$$



big structure moving subpixel

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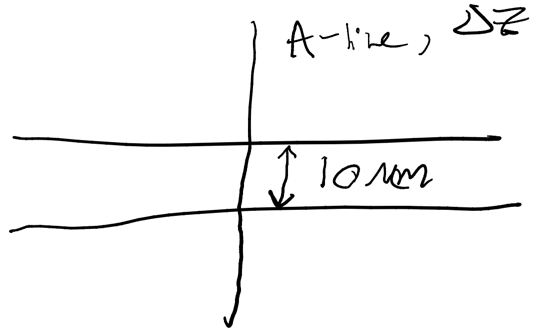
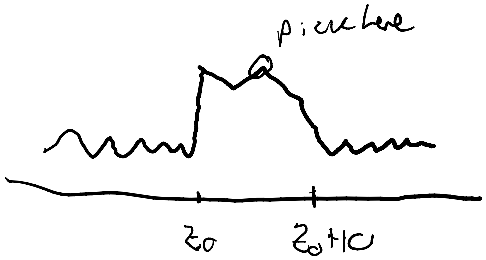
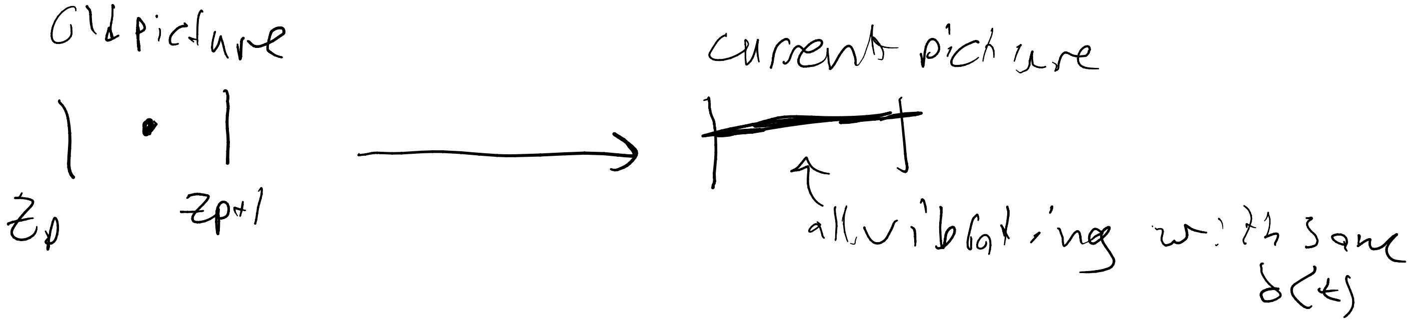


← Speaker diaphragm

moves on order of 10-100nm





Consider first, N reflectors, then for $N \to \infty$

$$\theta_i = \theta_{0_i} + 2n k_0 \delta(t)$$

$$i_p(z) = \sum \frac{\sqrt{R_R R_{s_i}}}{2}\left(e^{j\theta_i(t)} \gamma(2n(z-z_p)) + e^{-j\theta_i(t)} \gamma(-2n(z-z_p))\right)$$

$$\rightarrow A(z_p) \propto \frac{1}{2}\gamma(2n(z-z_p)) \sum e^{j\theta_i(t)} \sqrt{R_R R_{s_i}}$$

$$\rightarrow A(z_p) \propto \frac{1}{2}\gamma(2n(z-z_p)) \sum e \qquad \sqrt{R_R R_{s_i}}$$

$$= \frac{1}{2}\gamma(2n(z-z_p))\; e^{j\delta(t)} \sum e^{j\theta_{0_i}} \sqrt{R_R R_{s_i}}$$

$$\angle A(z_p) = \angle\left(\sum e^{j\theta_{0_i}} \sqrt{R_R R_{s_i}}\right) + 2nk_0\,\delta(t)$$

$\uparrow$ const, say $\angle \sum e^{j\theta_{0_i}} \sqrt{R_R R_{s_i}} = \theta_0$

$$\rightarrow \boxed{\frac{\angle A(z_p)}{2nk_0} = \text{Const.} + \delta(t)}$$

~~So~~ yes! we can measure freq. resp. of speaker.