Your First MLP Code

Recitation 1, part 1
Spring 2022

Overview

- Neural Networks
- Perceptrons
- Multilayer perceptrons
    - Forward Pass
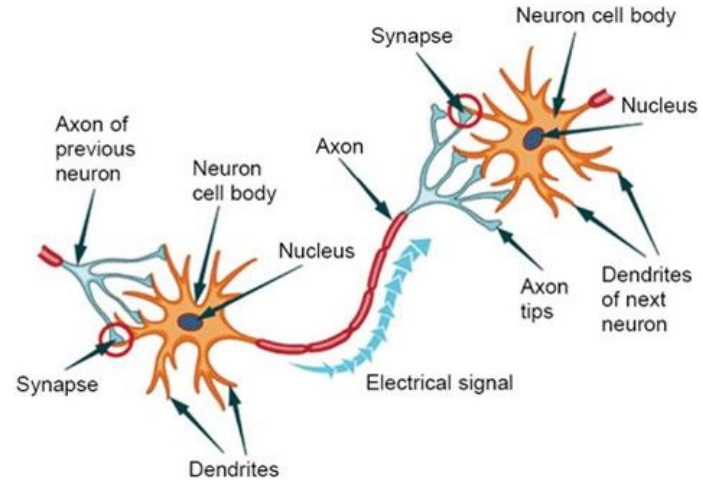   - Backpropagation
    - Update Weights

Neural Networks

- The brain, made up of connected neurons, are the inspirations for artificial neural networks.

Neural Networks

- A neuron is a node with many inputs and one output.
- A neural network consists of many interconnected neurons -- a "simple" device that receives data at the input and provides a response.
- Information are transmitted from one neuron to another by electrical impulses and chemical signals.

Perceptrons

- Perceptron is a single layer neural network.

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- The perceptron consists of 4 parts.

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- The perceptron consists of 4 parts.
    - Input values

x_1

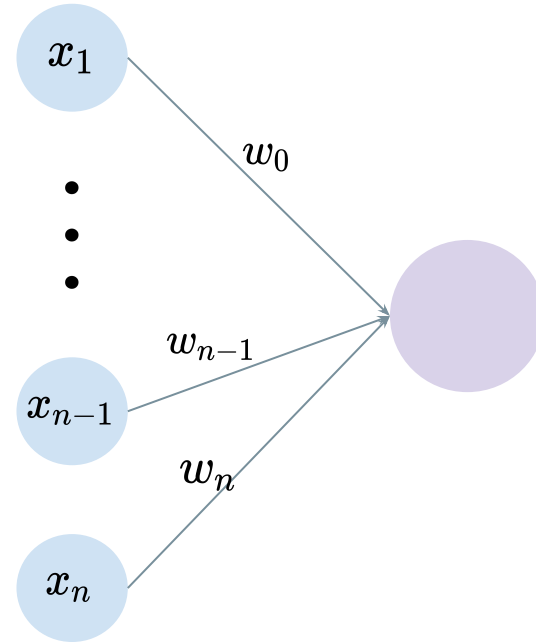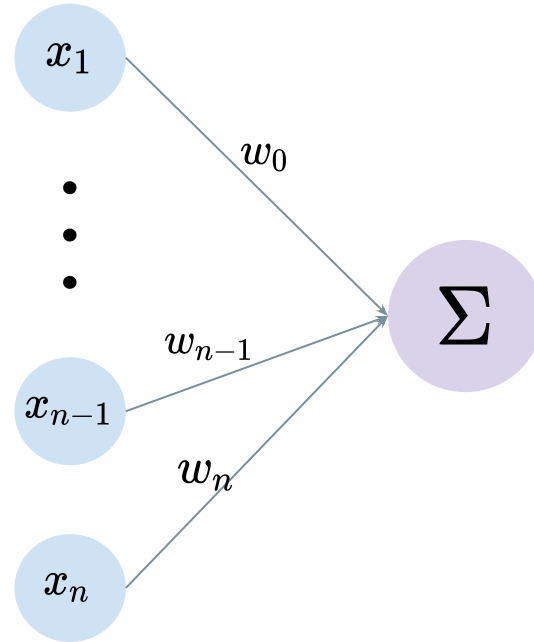$\bullet$
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$\bullet$

x_{n-1}

x_n

Perceptrons

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- The perceptron consists of 4 parts.
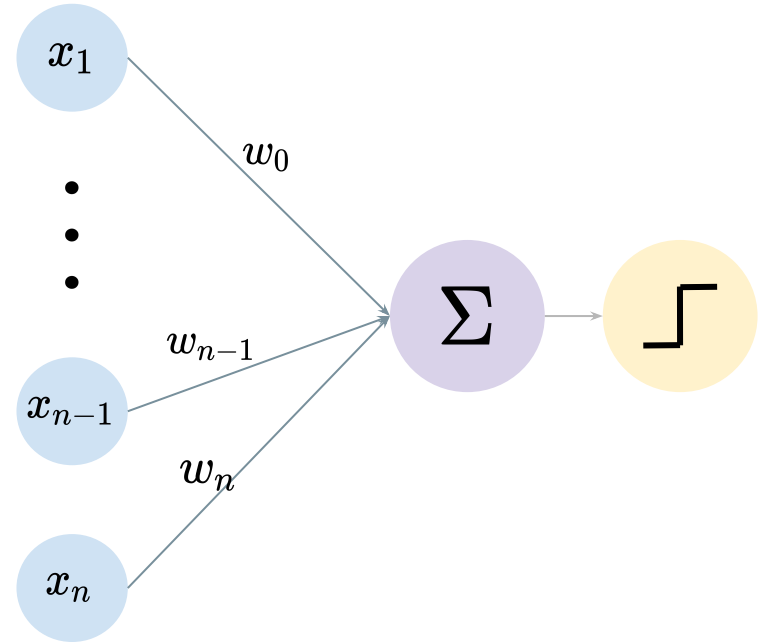 - Input values
  - Weights

Perceptrons

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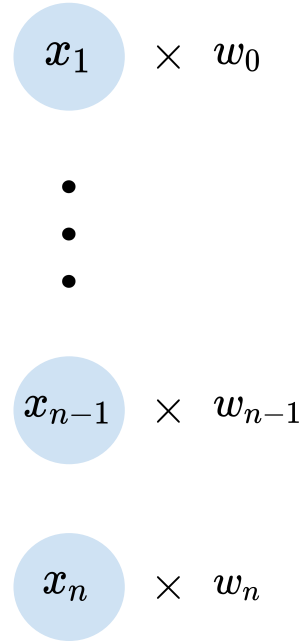 - Weighted sums

Perceptrons

- Perceptron is a single layer neural network.
- The perceptron consists of 4 parts.
    - Input values
    - Weights
    - Weighted sums
    - Threshold / Activation functions

x_1

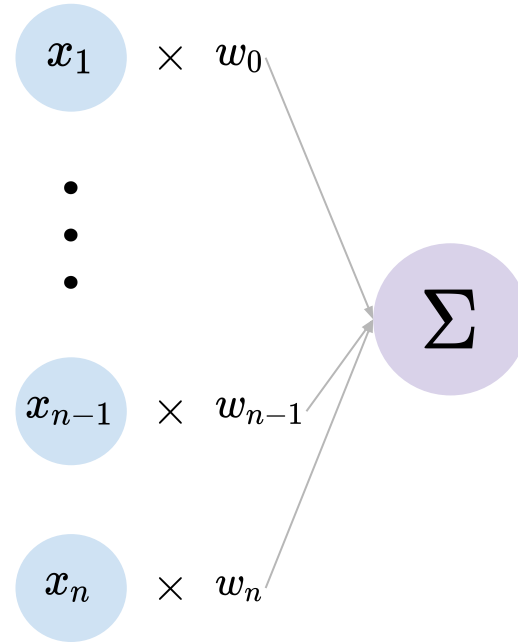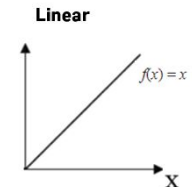x_{n-1}

x_n
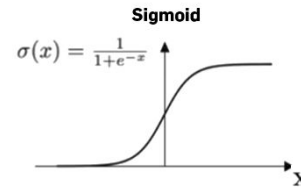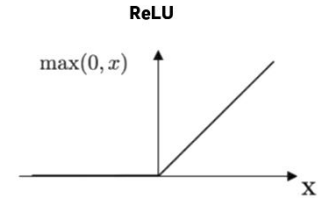
w_0

w_{n-1}

w_n

Σ

Perceptrons

- Perceptron is a single layer neural network.
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- The perceptron works on the following steps:
    - Multiply all inputs with their weights

x_1 $\times$ w_0

$\cdot$
$\cdot$
$\cdot$

x_{n-1} $\times$ w_{n-1}
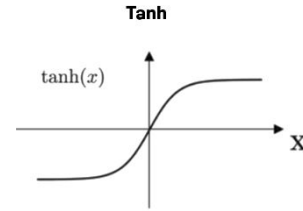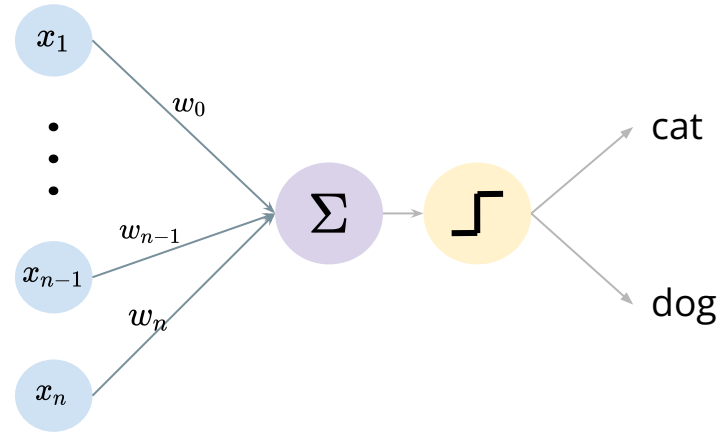
x_n $\times$ w_n

Perceptrons

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- The perceptron works on the following steps:
    - Multiply all inputs with their weights
    - Add all multiplies values → weighted sum

x_1 $\times$ w_0

$\bullet$
$\bullet$
$\bullet$

x_{n-1} $\times$ w_{n-1}
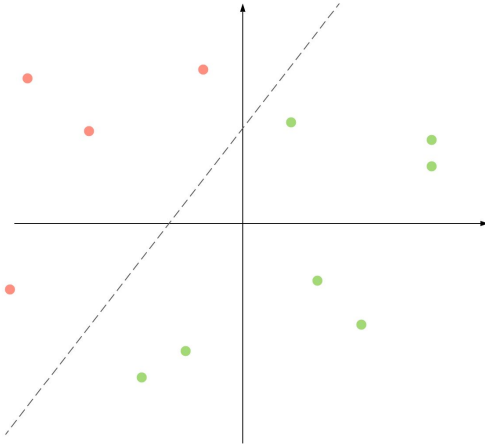
x_n $\times$ w_n

Σ

Perceptrons

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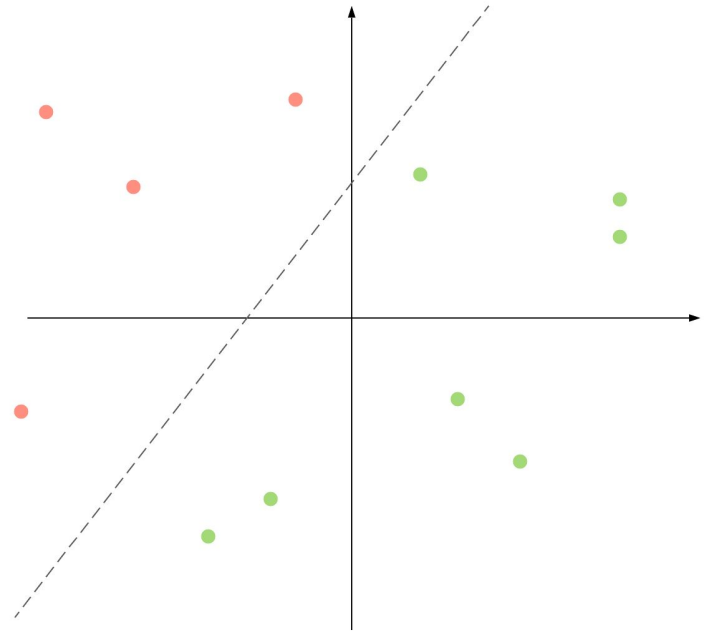 - Apply the weighted sum to activation function

Tanh

$\tanh(x)$

ReLU

$\max(0, x)$

Sigmoid

$\sigma(x) = \frac{1}{1+e^{-x}}$

Linear

$f(x) = x$

Perceptrons

- Perceptron is usually used to classify the data into two parts -- **Linear Binary Classifier**.

Perceptrons

- Perceptron is usually used to
  classify the data into two parts --
  Linear Binary Classifier.
  - **Weights** shows the **strength** of the
    particular node.
  - **Activation functions** are used to
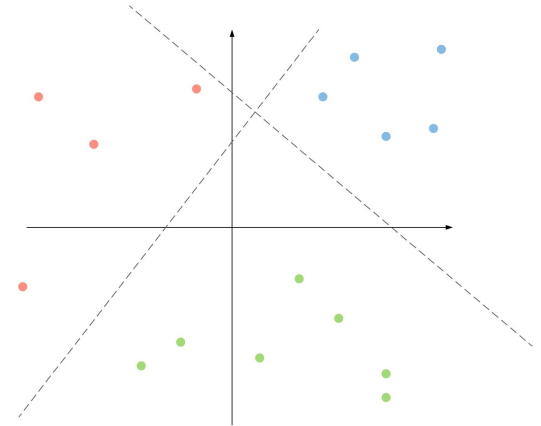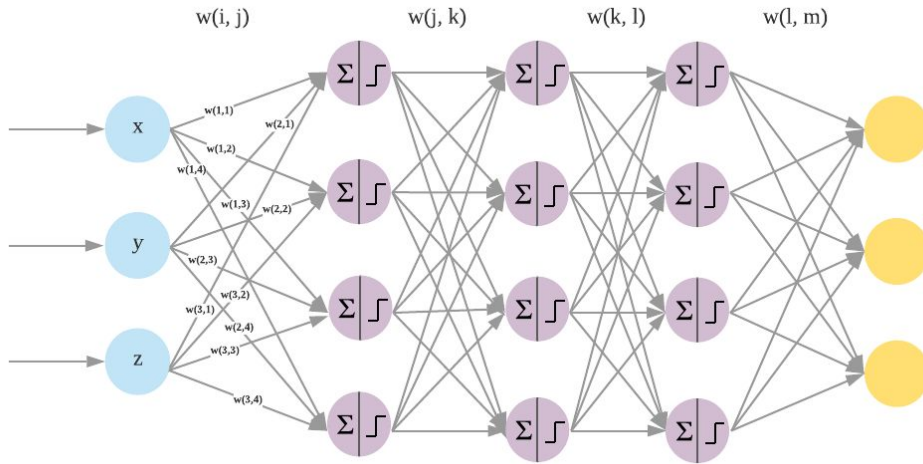   map the input between the required
    values

Multilayer Perceptrons

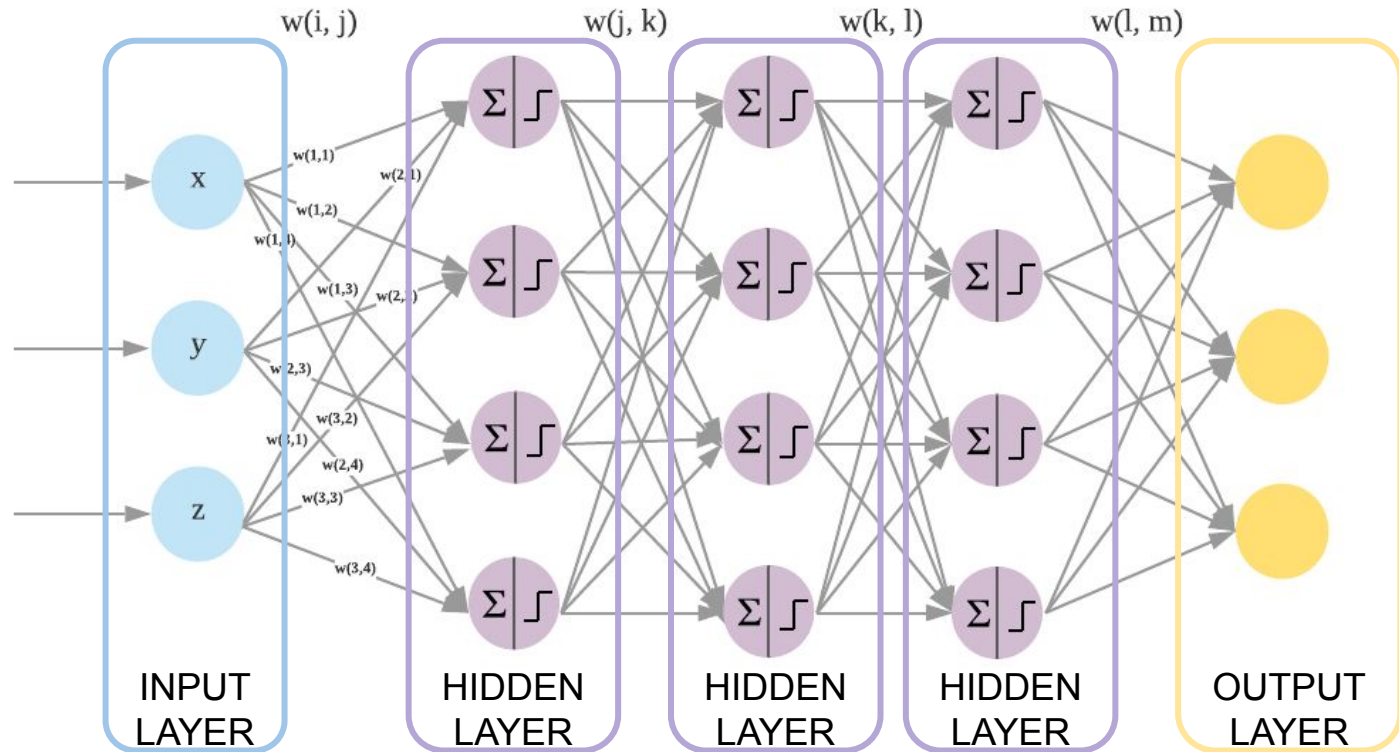**What if we want to be able to distinguish between more classes?**

Multilayer Perceptrons

What if we want to be able to distinguish between more classes?

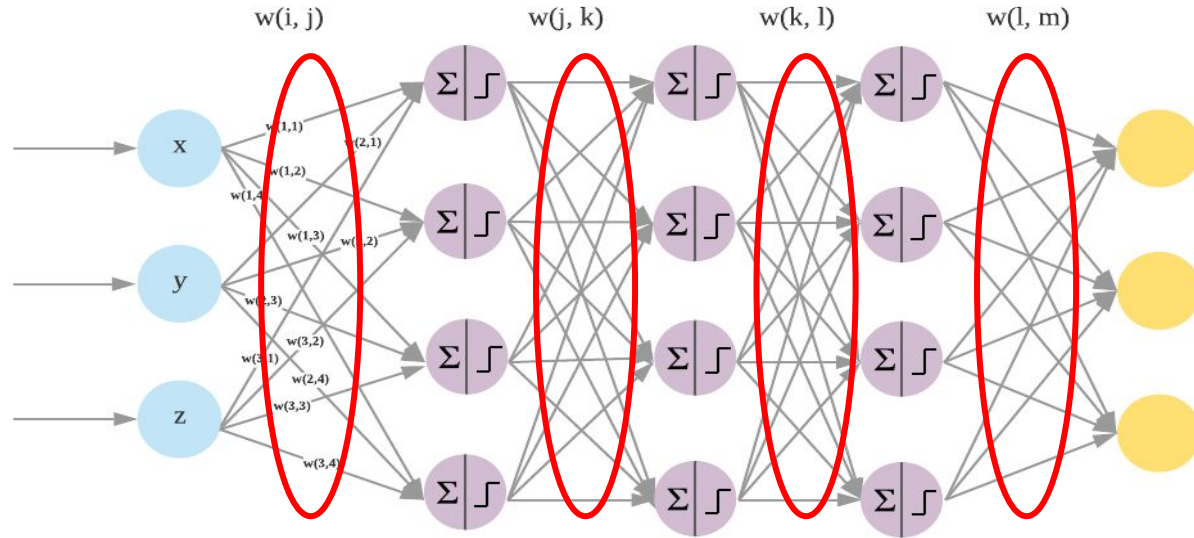- Introduce more perceptrons !

Multilayer Perceptrons

In order to correctly classify things, the network must be **learned**.

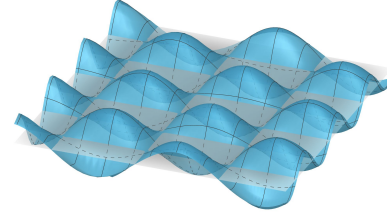# But first, **what** do we need to learn?
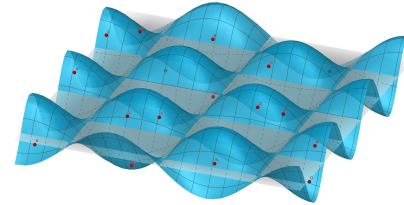
The parameters (or the weights)

How do we learn?

➔ Actual Function that we are trying to model:

◆ Note: We don't know the actual function.

➔ We only have several sample data points on this function.

➔ Our goal:

◆ **Estimate the function with the given samples.**

How do we learn?

➜ A measurement of **error**
  ◆ How much off is the **network output** with respect to the **desired output**

For each sample

MLP

Network Output

$$Loss(W) = \frac{1}{N} \sum_i div\left(f(X_i, W), d_i\right)$$

Desired Output

Number of samples

Divergence function

Sample value

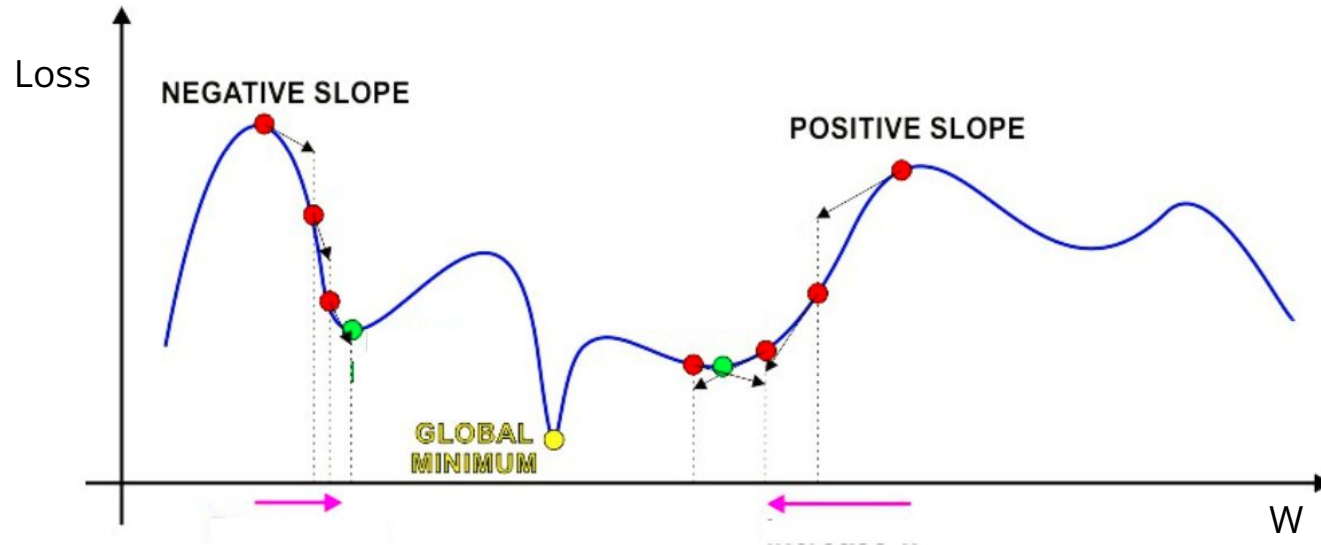Current weights of estimated function

➜ Our goal (more specifically):
  ◆ Minimize the loss

$$\hat{W} = \arg\min_W \ Loss(W)$$

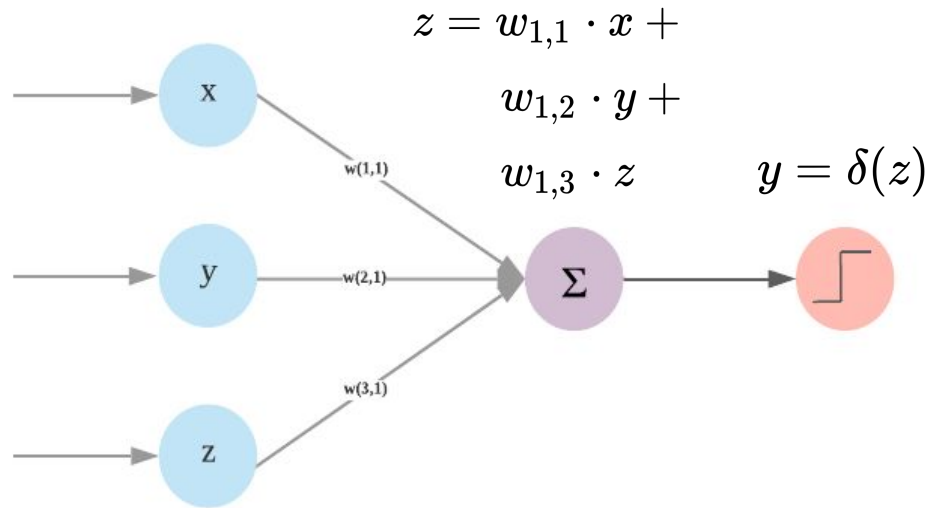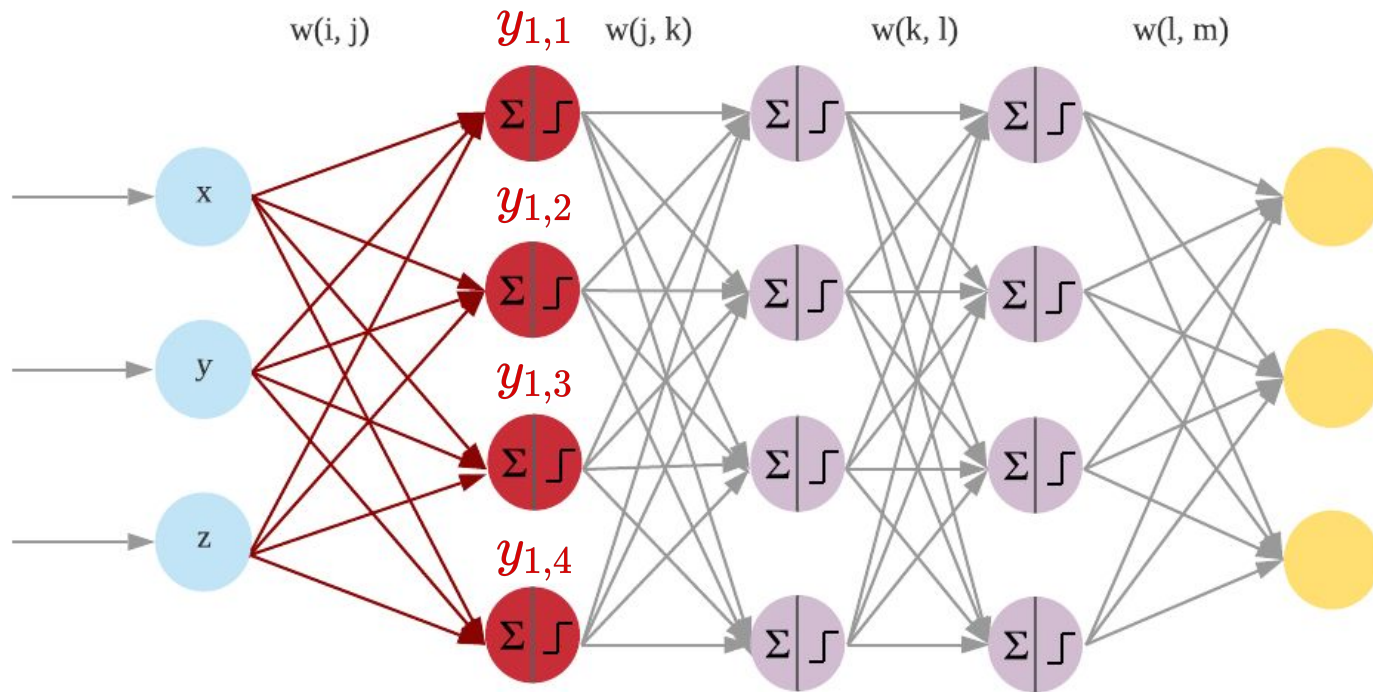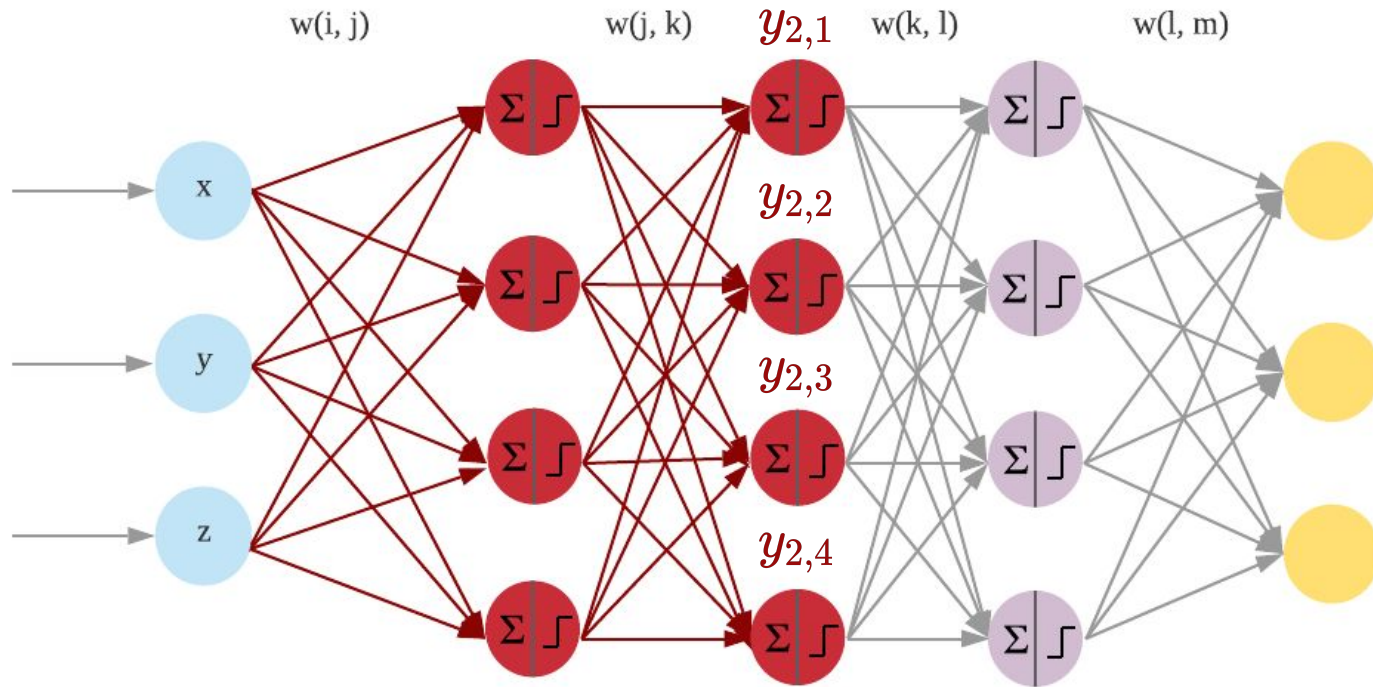# How do we learn?

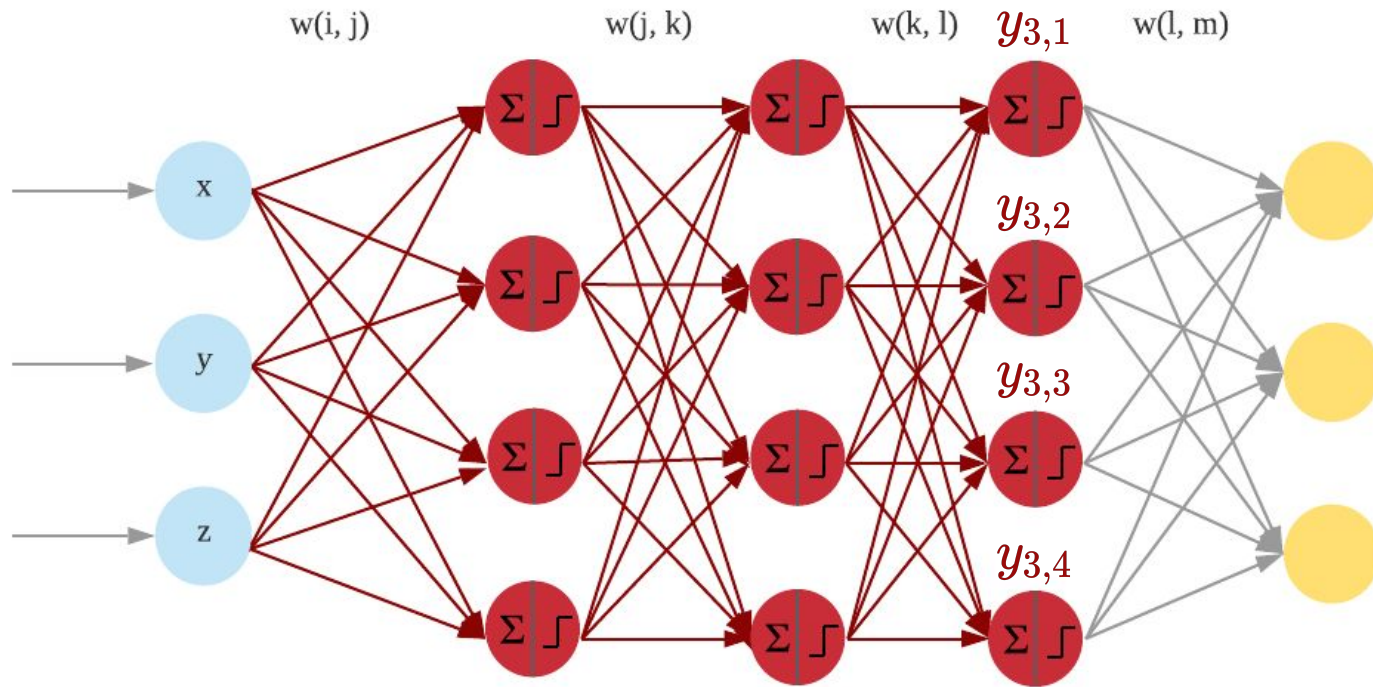➔ Gradient Descent

Forward Pass

- For each single perceptron

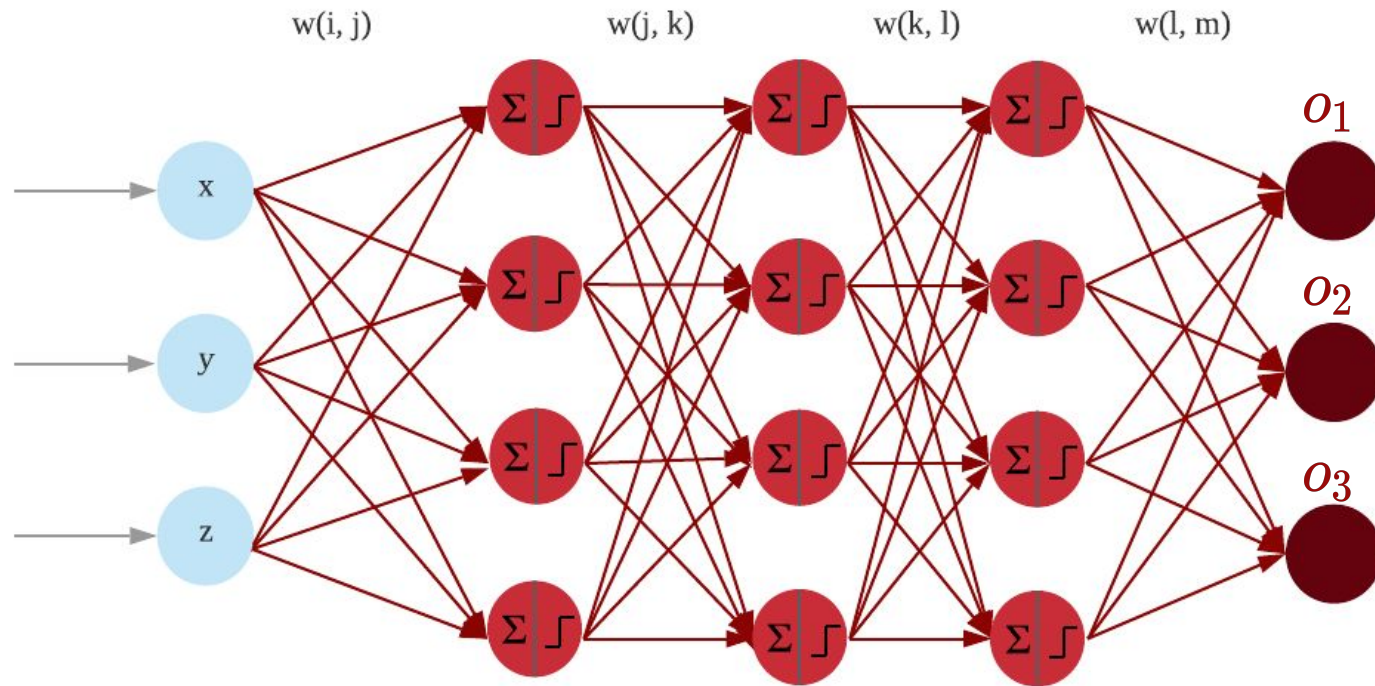$$z = w_{1,1} \cdot x +$$
$$w_{1,2} \cdot y +$$
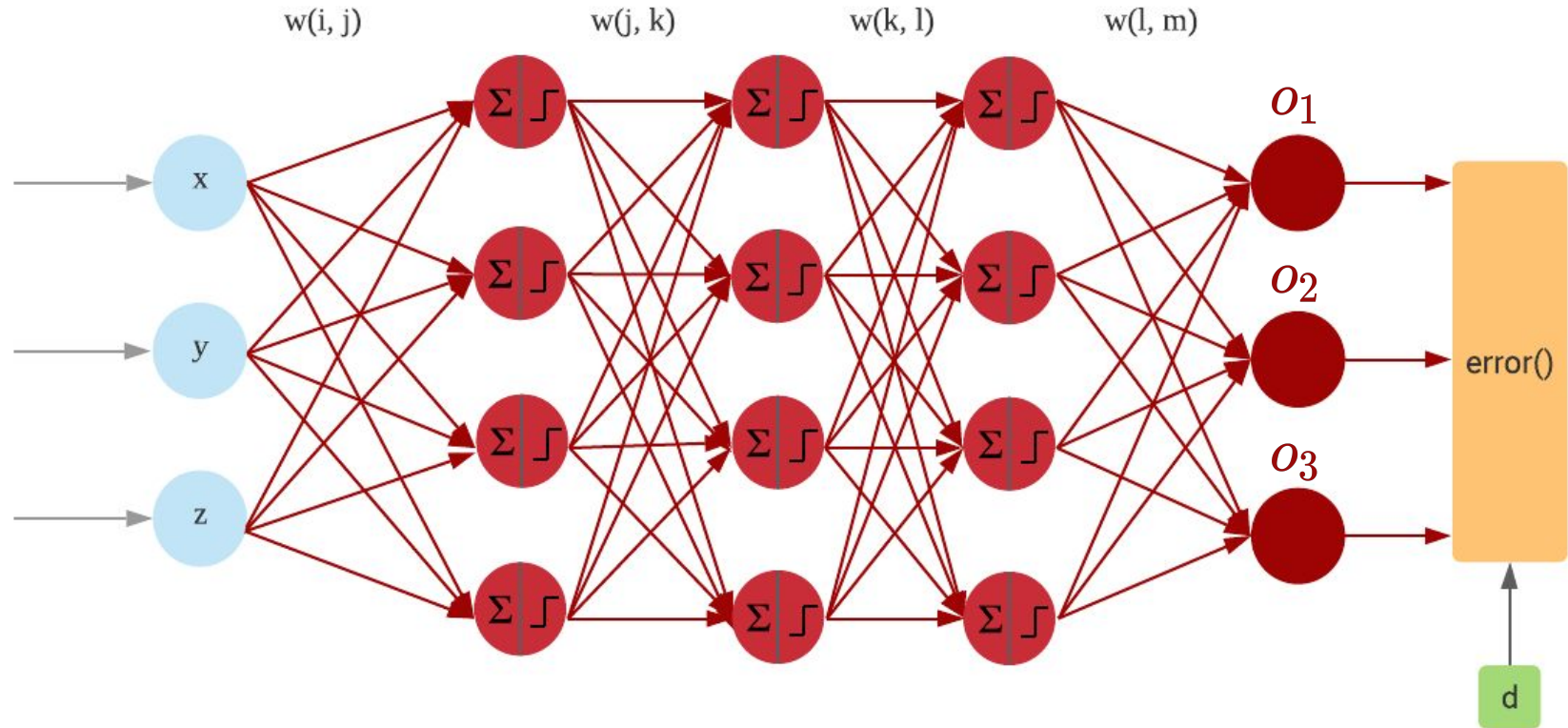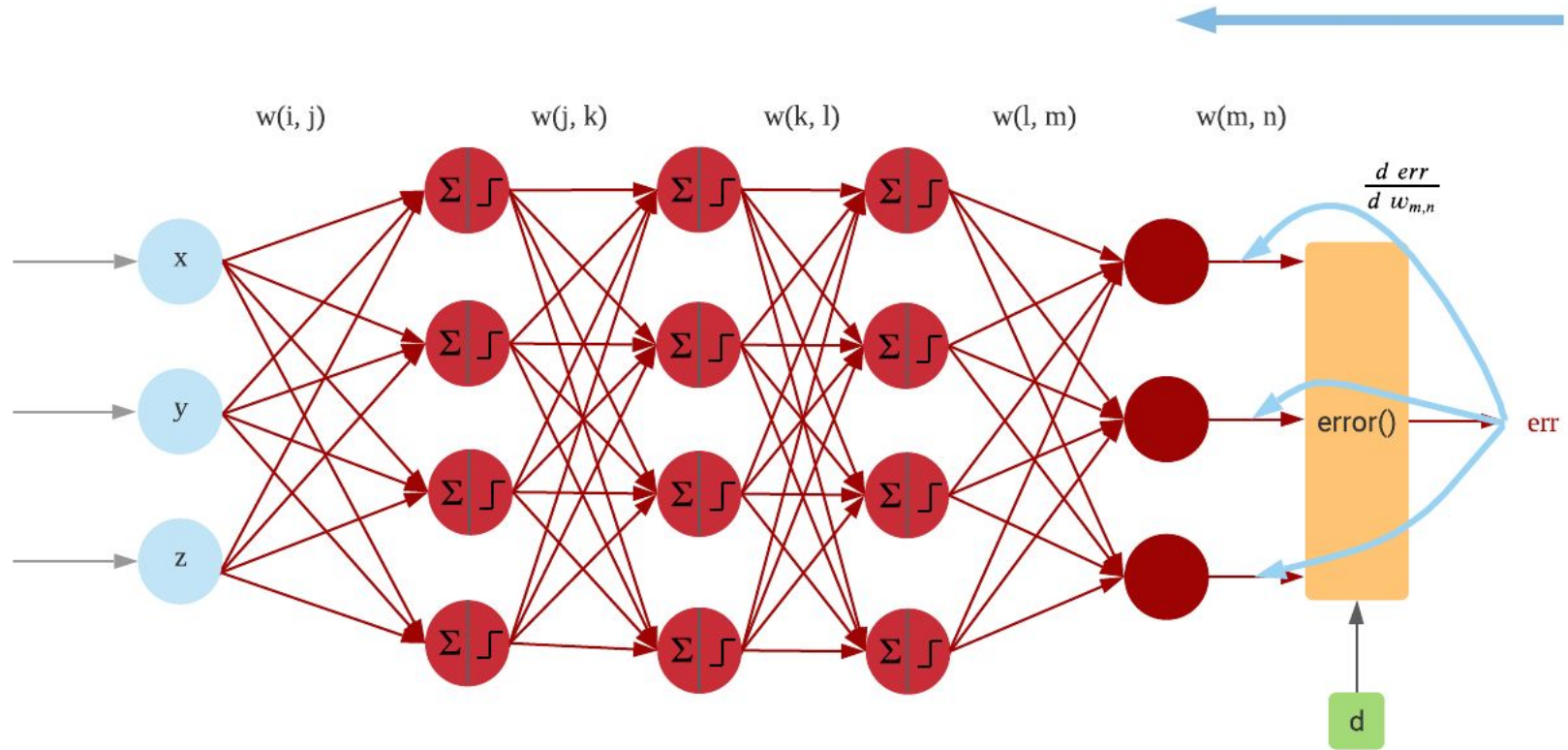$$w_{1,3} \cdot z \qquad y = \delta(z)$$

Forward Pass

Forward Pass

Forward Pass

Forward Pass

Error

Backpropagation

Backpropagation

Backpropagation

Backpropagation

w(i, j)   w(j, k)   w(k, l)   w(l, m)   w(m, n)

$$\frac{d\ err}{d\ o} \frac{d\ o}{d\ y_l}$$
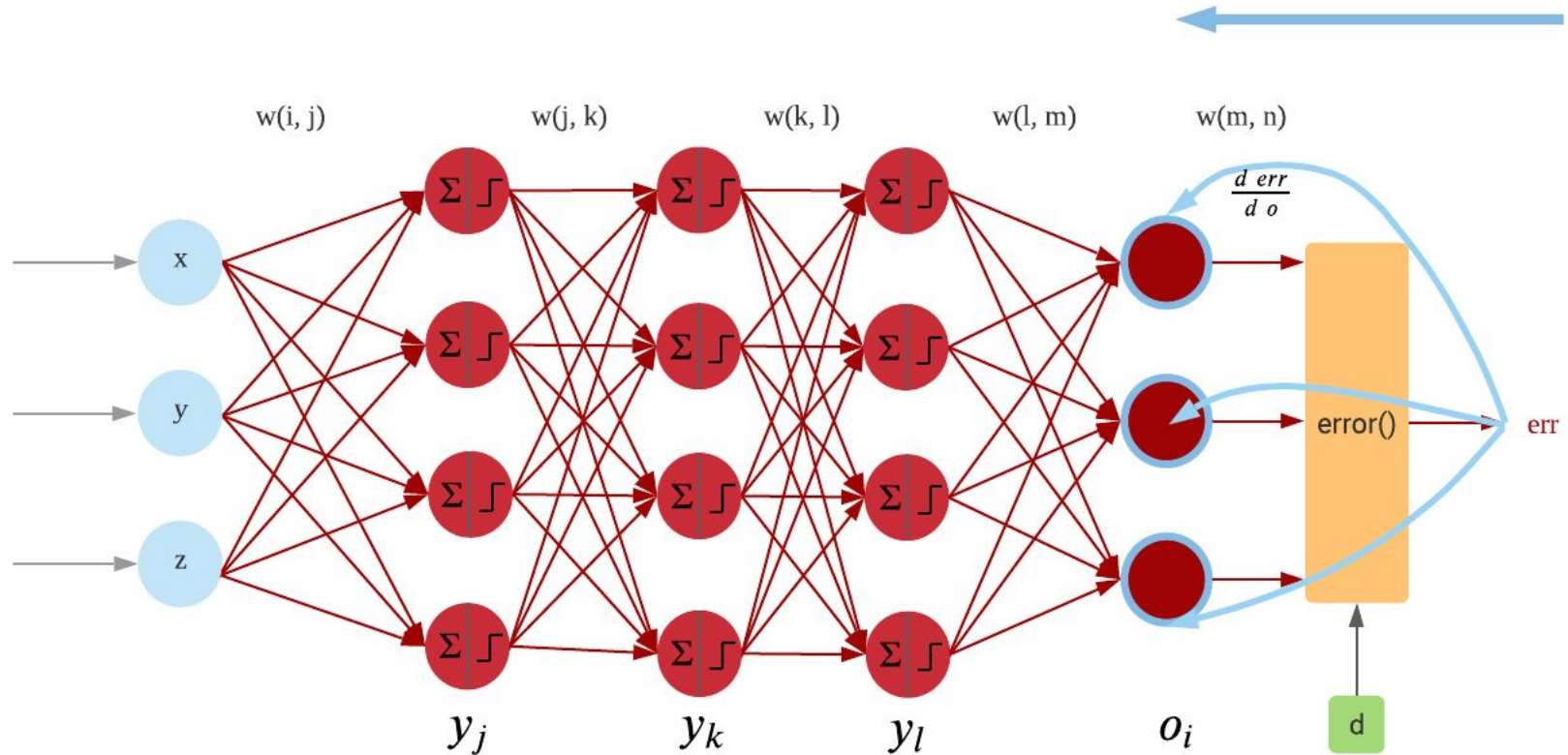
x

y

z

y_j y_k y_l o_i

error()   err

d
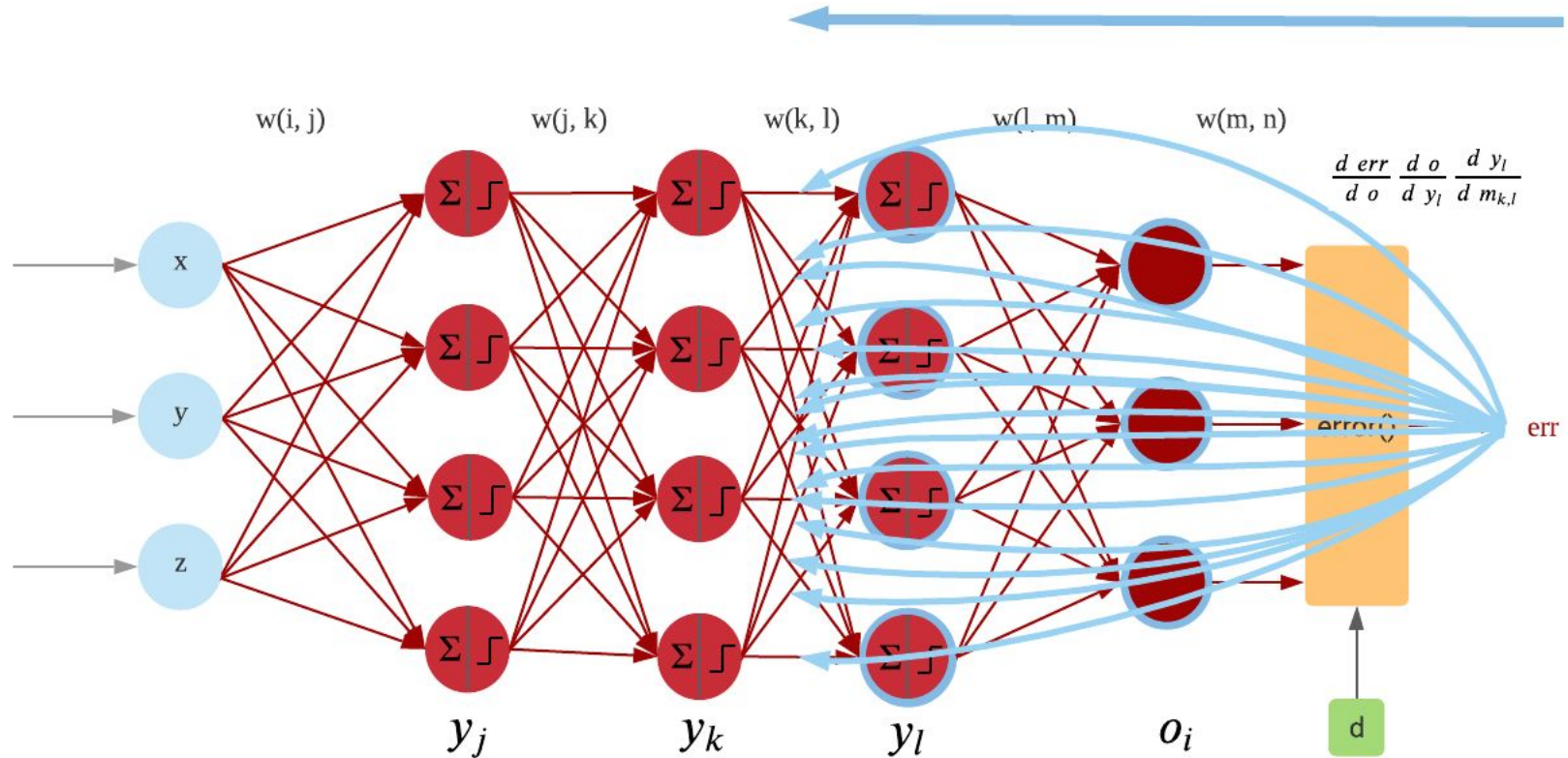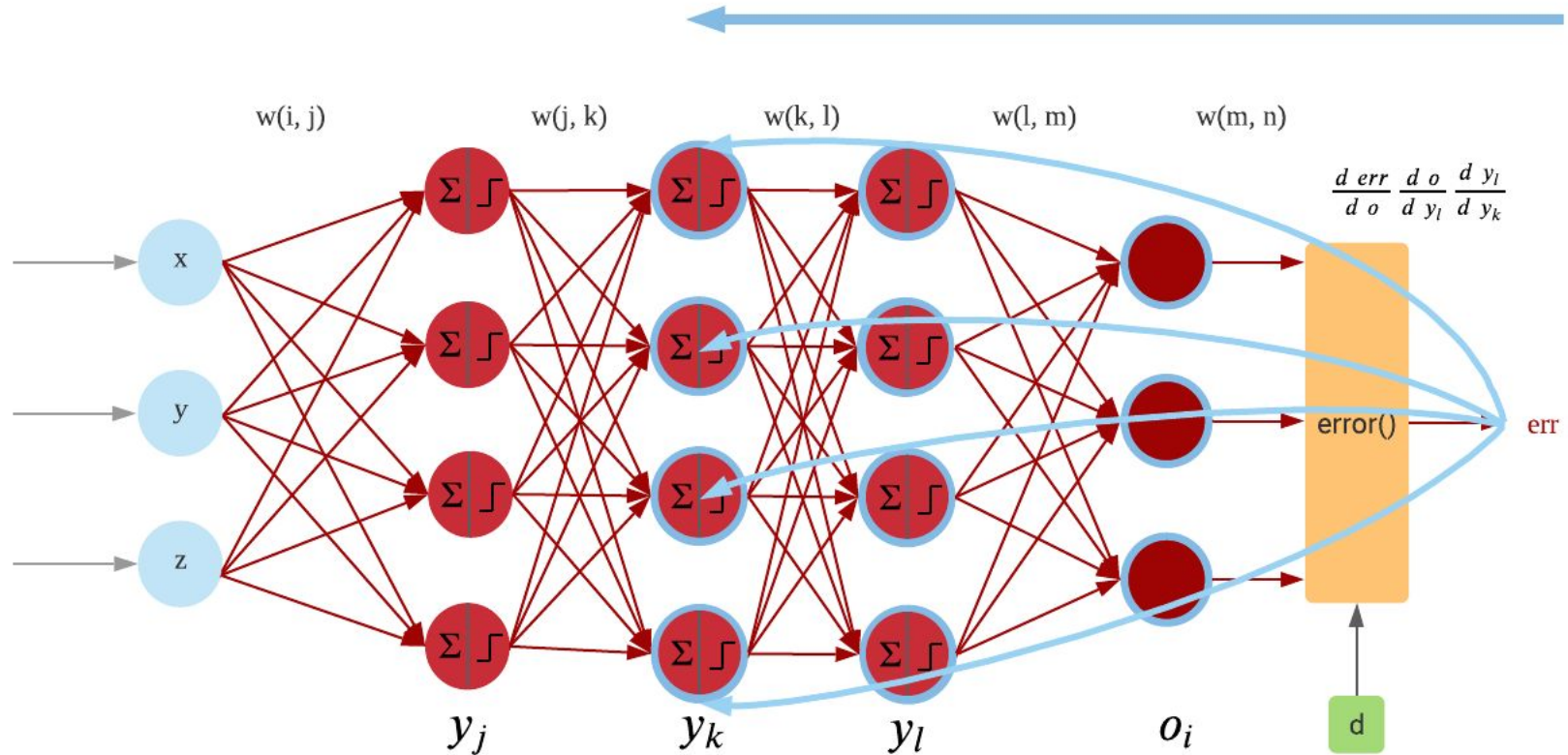
Backpropagation

Backpropagation

Backpropagation

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Backpropagation

All gradients of weights w.r.t error are calculated!

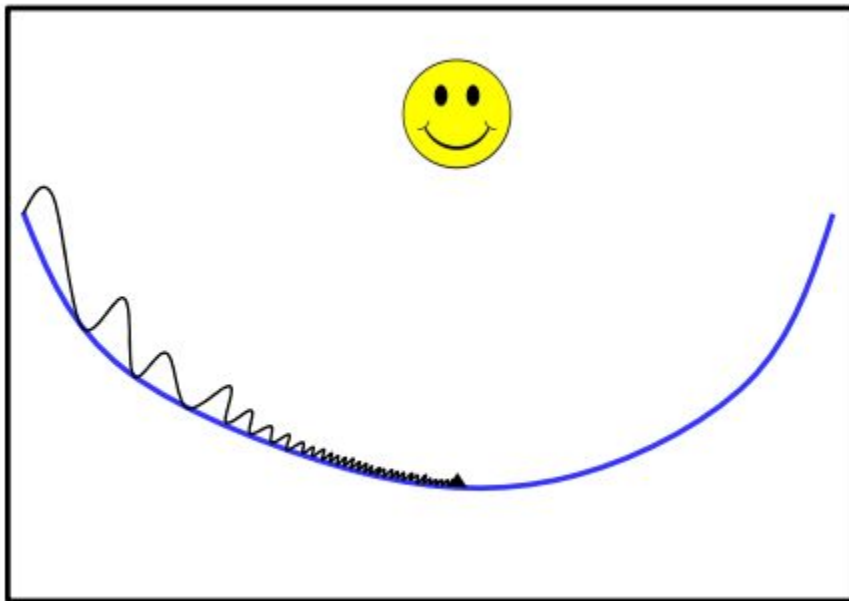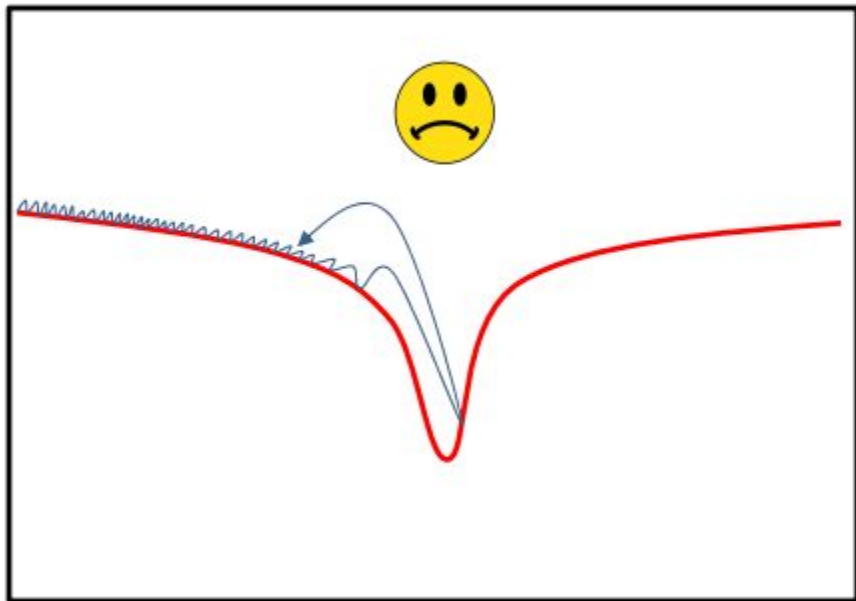# Update Weights

$$W \leftarrow W - \eta \cdot \nabla_W Loss(W)$$

learning
rate

gradient

What should be the learning rate?

https://deeplearning.cs.cmu.edu/F21/document/slides/lec8.optimizersandregularizers.pdf

Optimizers

Gradient Descent:

$$\theta_{t+1} = \theta_t - \alpha \cdot \nabla_\theta J(\theta)$$

Momentum (http://proceedings.mlr.press/v28/sutskever13.pdf):

$$m_{t+1} = \mu \cdot m_t + \alpha \cdot \nabla_\theta J(\theta)$$
$$\theta_{t+1} = \theta_t - m_{t+1}$$

Adagrad (https://jmlr.org/papers/volume12/duchi11a/duchi11a.pdf):

$$g \leftarrow \nabla_\theta J(\theta)$$
$$r \leftarrow r + g^2$$
$$\triangle\theta \leftarrow \frac{\delta}{\sqrt{r + \epsilon}} \cdot g$$
$$\theta \leftarrow \theta - \triangle\theta$$

Optimizers (Cont')

Adam (https://arxiv.org/pdf/1412.6980.pdf):

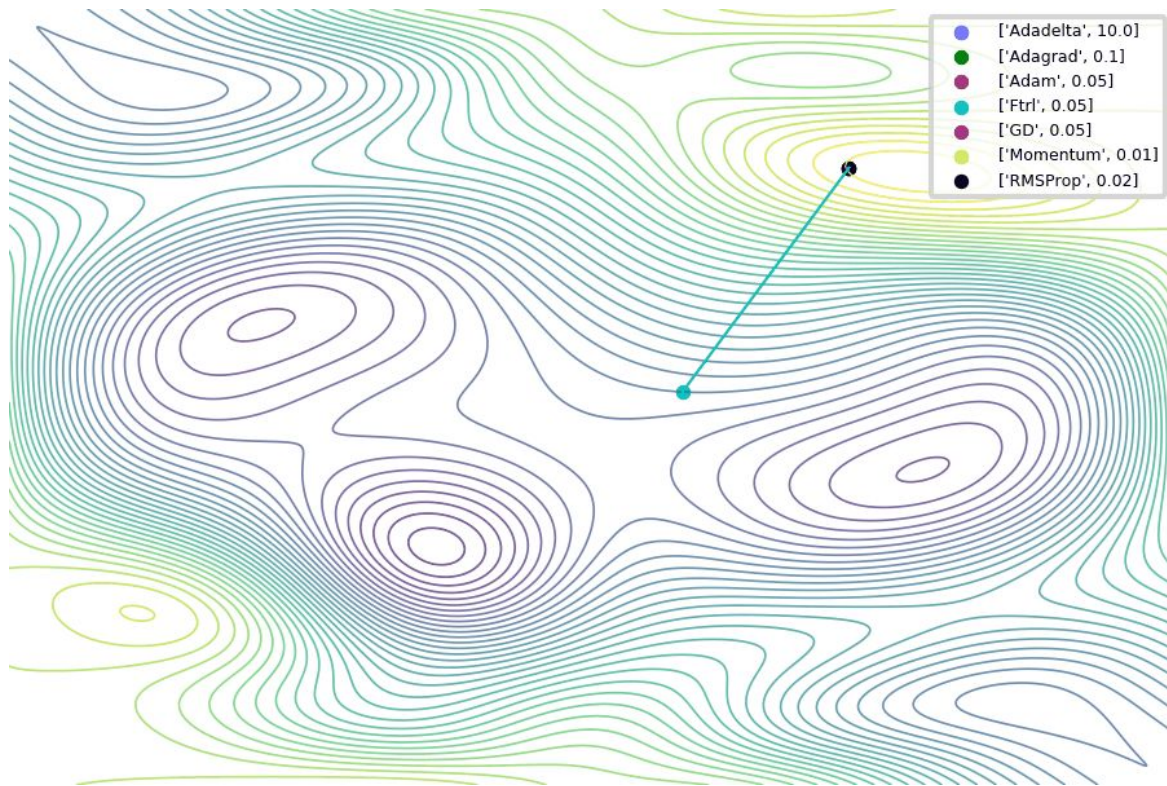$$m_t = \beta_1 \cdot m_{t-1} + (1 - \beta_1) \cdot g_t$$

$$v_t = \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2$$

$$\hat{m}_t = m_t / (1 - \beta_1^t)$$

$$\hat{v}_t = v_t / (1 - \beta_2^t)$$

$$\theta_t = \theta_{t-1} - \alpha \cdot \hat{m}_t / (\sqrt{\hat{v}_t + \epsilon})$$

Visualization

https://github.com/Jaewan-Yun
/optimizer-visualization

Some fun with TF Playground