

Zestaw funkcji wielu zmiennych na bazie biblioteki
„Benchmark Functions - A Python Library”
do optymalizacji na przedmiocie „obliczenia ewolucyjne”

Link: https://gitlab.com/luca.baronti/python_benchmark_functions

Poniższe opracowanie jest dla funkcji 2 zmiennych.

1) Hypersphere

Wzór w Latex: $f(x) = \sum_{i=1}^N x_i^2$

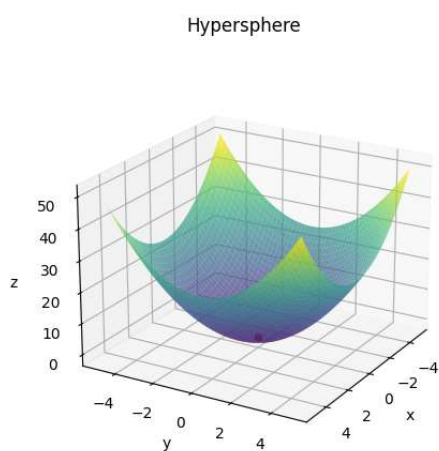
Wzór funkcji:

$$f(x) = \sum_{i=1}^N x_i^2$$

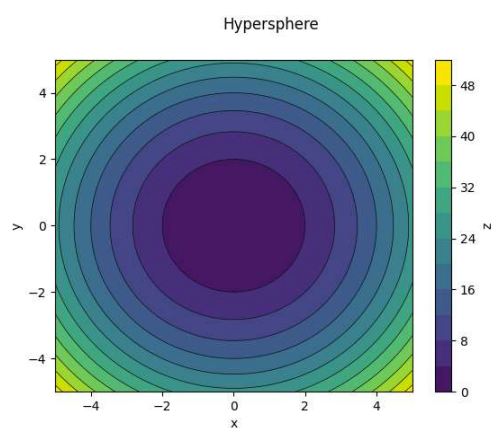
Sugerowany zakres poszukiwań: [-5.0, 5.0]

Globalne minimum równe 0.0 w punkcie [0.0, 0.0]

Wykres:



Wykres typu heatmap:



2) Hyperellipsoid

Wzór w Latex: $f(x)=\sum_{i=0}^{N-1} \sum_{j=0}^i x_j^2$

Wzór funkcji:

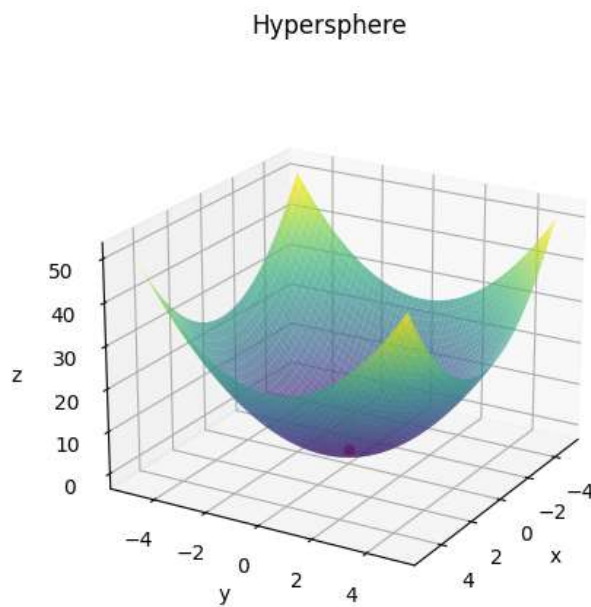
$$f(x) = \sum_{i=0}^{N-1} \sum_{j=0}^i x_j^2$$

Sugerowany zakres poszukiwań: [-65.536, 65.536]

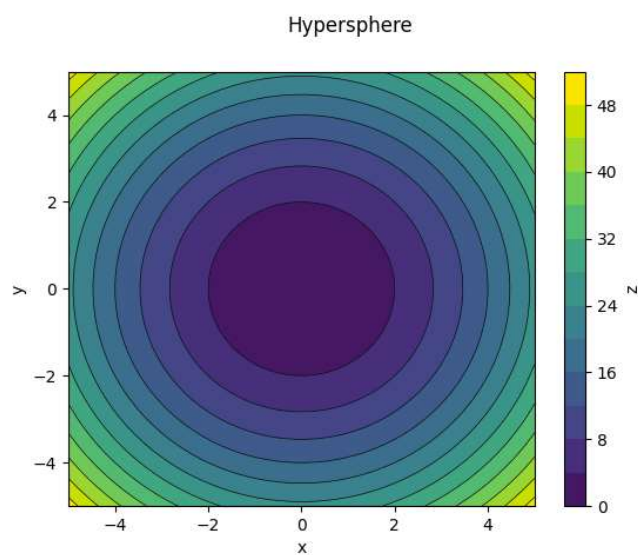
Globalne minima:

(0.0, [0.0, 0.0])

Wykres:



Wykres typu heatmap:



3) Schwefel

Wzór w Latex:

$$f(x) = 418.9829 N \sum_{i=0}^{N-1} x_i \sin(\sqrt{|x_i|})$$

Wzór funkcji:

$$f(x) = 418.9829 N \sum_{i=0}^{N-1} x_i \sin(\sqrt{|x_i|})$$

Sugerowany zakres poszukiwań: [-500.0, 500.0]

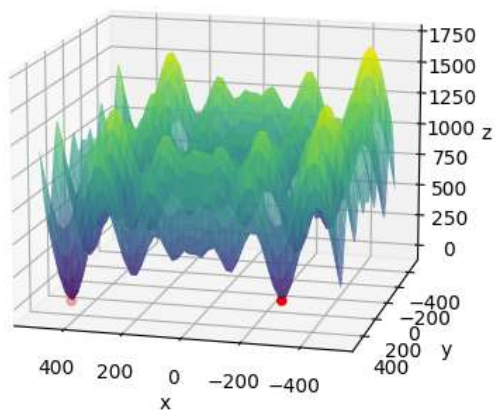
Globalne minima:

(2.545567497236334e-05, [420.9687, 420.9687])

(118.43836006957031, [-302.5249351839932, 420.9687467475071])

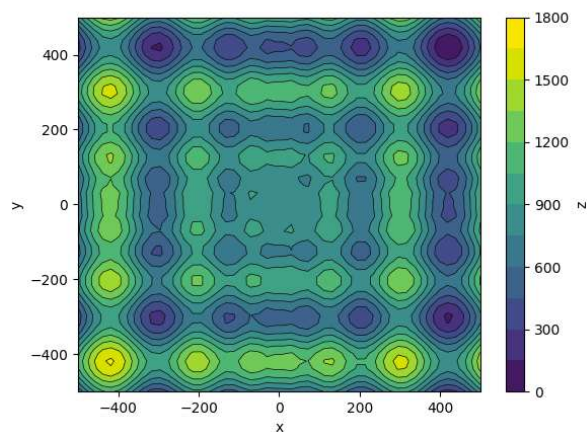
Wykres:

Schwefel



Wykres typu heatmap:

Schwefel



4) Ackley

Wzór w Latex:

$$f_{a,b,c}(x) = -a \exp\left(-b \sqrt{\frac{1}{N} \sum_{i=0}^{N-1} x_i^2}\right) - \exp\left(\frac{1}{N} \sum_{i=1}^N \cos(x_i c)\right) + a + e$$

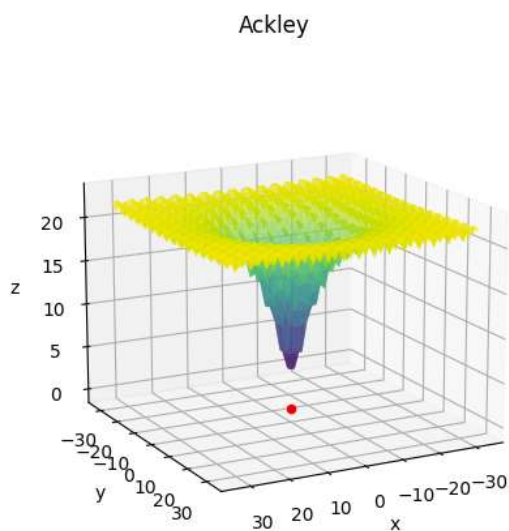
Wzór funkcji:

$$f_{a,b,c}(x) = -a \exp\left(-b \sqrt{\frac{1}{N} \sum_{i=0}^{N-1} x_i^2}\right) - \exp\left(\frac{1}{N} \sum_{i=1}^N \cos(x_i c)\right) + a + e$$

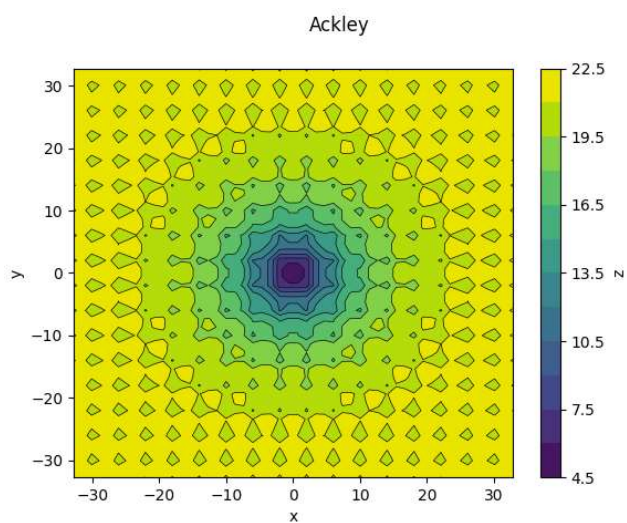
Sugerowany zakres poszukiwań: [-32.768, 32.768]

Globalne minima: 4.440892098500626e-16, [0.0, 0.0])

Wykres:



Wykres typu heatmap:



5) Michalewicz

Wzór w Latex:

$$f_m(x) = -\sum_{i=0}^{N-1} \sin(x_i) \sin^{2m}\left(\frac{x_i^2(i+1)}{\pi}\right)$$

Wzór funkcji:

$$f_m(x) = -\sum_{i=0}^{N-1} \sin(x_i) \sin^{2m}\left(\frac{x_i^2(i+1)}{\pi}\right)$$

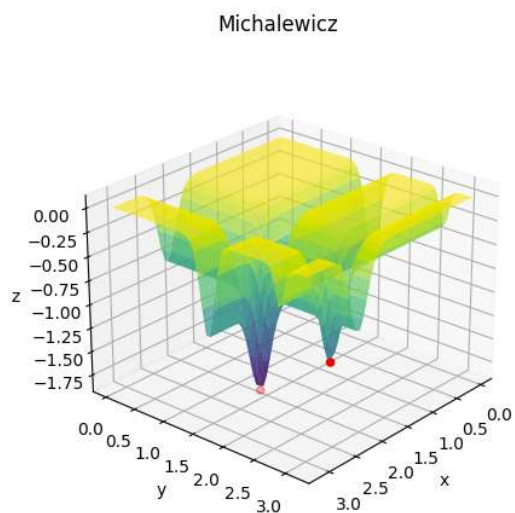
Sugerowany zakres poszukiwań: [0.0, 3.141592653589793]

Globalne minima:

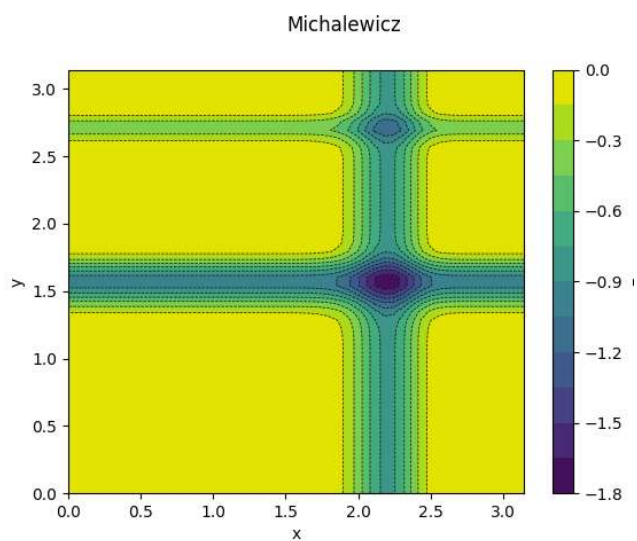
(-1.8013034100904854, [2.202906, 1.570796])

(-1.214059515219654, [2.202900552, 2.71157148384])

Wykres:



Wykres typu heatmap:



6) Rastrigin

Wzór w Latex:

$$f(x) = 10N + \sum_{i=0}^{N-1} (x_i^2 - 10 \cos(2\pi x_i))$$

Wzór funkcji:

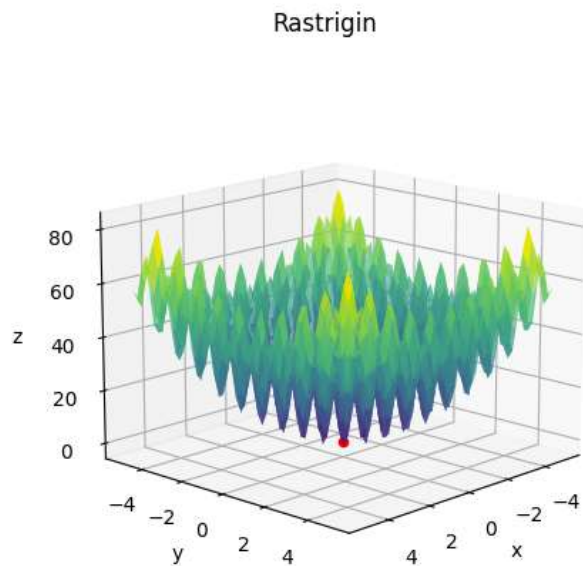
$$f(x) = 10N + \sum_{i=0}^{N-1} (x_i^2 - 10 \cos(2\pi x_i))$$

Sugerowany zakres poszukiwań: [-5.12, 5.12]

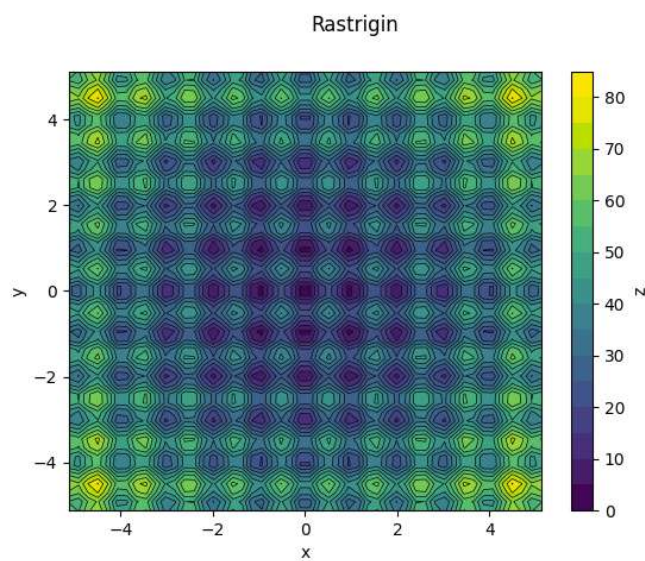
Globalne minima:

$$(0.0, [0.0, 0.0])$$

Wykres:



Wykres typu heatmap:



7) Rosenbrock

Wzór w Latex:

$$f(x) = \sum_{i=0}^{N-2} (100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2)$$

Wzór funkcji:

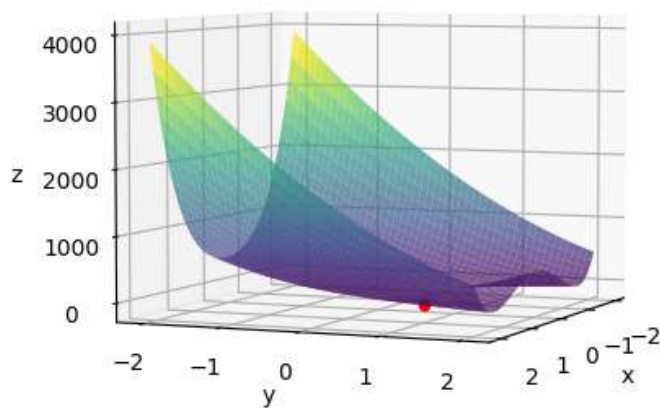
$$f(x) = \sum_{i=0}^{N-2} (100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2)$$

Sugerowany zakres poszukiwań: [-2.048, 2.048]

Globalne minima: (0.0, [1.0, 1.0])

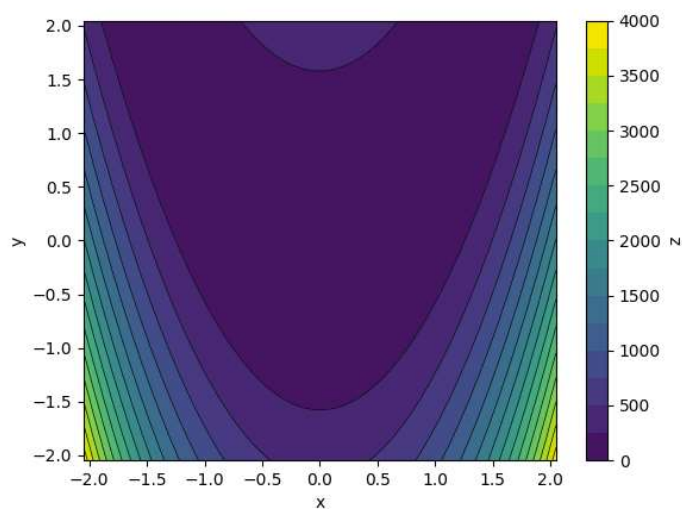
Wykres:

Rosenbrock



Wykres typu heatmap:

Rosenbrock



8) De Jong 3

Wzór w Latex:

$$f(x) = \sum_{i=0}^{N-1} \lfloor x_i \rfloor$$

Wzór funkcji:

$$f(x) = \sum_{i=0}^{N-1} \lfloor x_i \rfloor$$

Sugerowany zakres poszukiwań:

[3.8, 3.8]

Globalne minima:

(-8, [-3.5, -3.5])

(-7, [-2.5, -3.5])

(-6, [-1.5, -3.5])

(-5, [-0.5, -3.5])

(-4, [0.5, -3.5])

(-3, [1.5, -3.5])

(-2, [2.5, -3.5])

(-1, [3.5, -3.5])

(-7, [-3.5, -2.5])

(-6, [-2.5, -2.5])

(-5, [-1.5, -2.5])

(-4, [-0.5, -2.5])

(-3, [0.5, -2.5])

(-2, [1.5, -2.5])

(-1, [2.5, -2.5])

(0, [3.5, -2.5])

(-6, [-3.5, -1.5])

(-5, [-2.5, -1.5])

(-4, [-1.5, -1.5])

(-3, [-0.5, -1.5])

(-2, [0.5, -1.5])

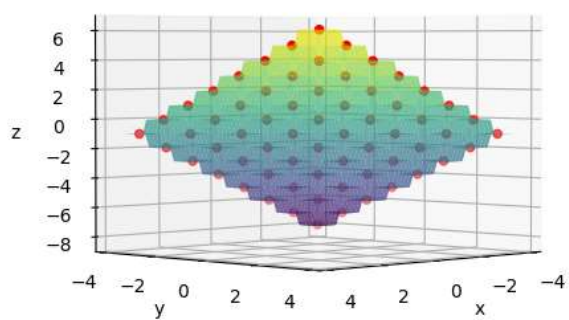
(-1, [1.5, -1.5])

(0, [2.5, -1.5])
(1, [3.5, -1.5])
(-5, [-3.5, -0.5])
(-4, [-2.5, -0.5])
(-3, [-1.5, -0.5])
(-2, [-0.5, -0.5])
(-1, [0.5, -0.5])
(0, [1.5, -0.5])
(1, [2.5, -0.5])
(2, [3.5, -0.5])
(-4, [-3.5, 0.5])
(-3, [-2.5, 0.5])
(-2, [-1.5, 0.5])
(-1, [-0.5, 0.5])
(0, [0.5, 0.5])
(1, [1.5, 0.5])
(2, [2.5, 0.5])
(3, [3.5, 0.5])
(-3, [-3.5, 1.5])
(-2, [-2.5, 1.5])
(-1, [-1.5, 1.5])
(0, [-0.5, 1.5])
(1, [0.5, 1.5])
(2, [1.5, 1.5])
(3, [2.5, 1.5])
(4, [3.5, 1.5])
(-2, [-3.5, 2.5])
(-1, [-2.5, 2.5])
(0, [-1.5, 2.5])
(1, [-0.5, 2.5])
(2, [0.5, 2.5])

$(3, [1.5, 2.5])$
 $(4, [2.5, 2.5])$
 $(5, [3.5, 2.5])$
 $(-1, [-3.5, 3.5])$
 $(0, [-2.5, 3.5])$
 $(1, [-1.5, 3.5])$
 $(2, [-0.5, 3.5])$
 $(3, [0.5, 3.5])$
 $(4, [1.5, 3.5])$
 $(5, [2.5, 3.5])$
 $(6, [3.5, 3.5])$

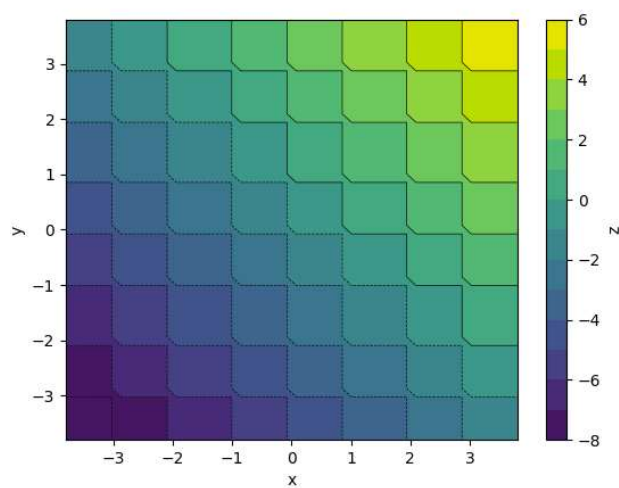
Wykres:

De Jong 3



Wykres typu heatmap:

De Jong 3



9) De Jong 5

Wzór w Latex:

$$f_A(x) = (0.002 + \sum_{i=1}^{25} (i + (x_1 - A_{1i})^6 + (x_2 - A_{2i})^6)^{-1})$$

Uwaga – funkcja tylko dwóch zmiennych

Wzór funkcji:

$$f_A(x) = (0.002 + \sum_{i=1}^{25} (i + (x_1 - A_{1i})^6 + (x_2 - A_{2i})^6)^{-1})$$

Sugerowany zakres poszukiwań: [-65.536, 65.536]

Globalne minima:

(0.9980038377944496, [-31.978333625355454, -31.978335021953196])

(5.928845125400146, [-31.95392172244739, -15.977945021467589])

(10.76318066677915, [-31.941211094929535, -0.02534391546372584])

(15.50381680261233, [-31.9316873909911, 15.96563556596947])

(20.153486960471188, [-31.926446068622827, 31.92644528509213])

(1.9920309002207153, [-15.986389356043315, -31.970336166329645])

(6.903335694220136, [-15.975328836306295, -15.97532332849018])

(11.718699562063055, [-15.968395098335876, -0.02670712890851633])

(16.440907314171774, [-15.96335433919068, 15.963342496424557])

(21.072687508315763, [-15.960229772439092, 31.92235890074413])

(2.9821051568166737, [-0.013229493456781437, -31.965087972948652])

(7.873992977132174, [-0.0218098742844023, -15.973596669242324])

(12.670505811135552, [-0.027787932587848684, -0.027797944168176606])

(17.37440650048896, [-0.032703474009876526, 15.962108272381098])

(21.988407545965707, [-0.036660125242367544, 31.92093500491233])

(3.9682501055681576, [15.981584031018555, -31.96083565740954])

(8.84083596275423, [15.972426685744868, -15.972425383219354])

(13.618608923861016, [15.96614171511398, -0.028838760181507092])

(18.304309518897774, [15.961463078501533, 15.961463740132906])

(22.90063408158465, [15.958600479994116, 31.91959475292217])

(4.950491231786251, [31.958687161100432, -31.958688627118857])

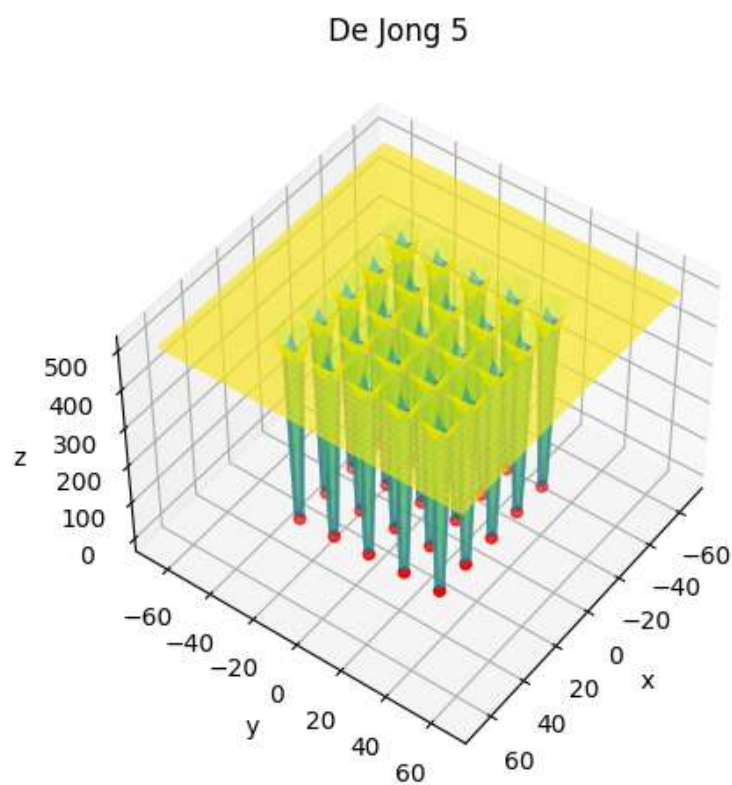
(9.803897942835262, [31.94343414149186, -15.97224527640958])

(14.563054157362322, [31.933413893470714, -0.029596115689988226])

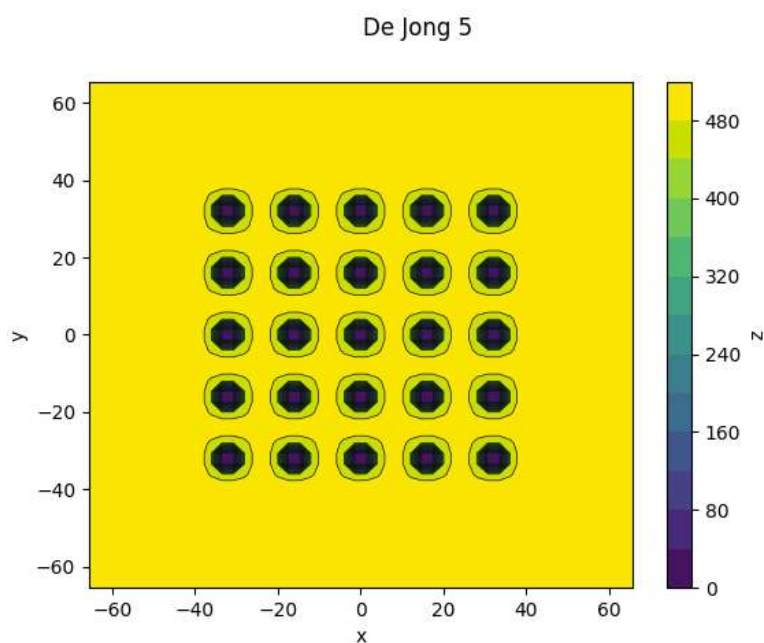
(19.23067813041172, [31.925278492792405, 15.961931498387099])

(23.809434471296303, [31.92109373551219, 31.92109554434111])

Wykres:



Wykres typu heatmap:



10) Martin and Gaddy

Wzór w Latex: $f(x)=(x_1-x_2)^2+\left(\frac{x_1+x_2-10}{3}\right)^2$

Wzór funkcji:

$$f(x) = (x_1 - x_2)^2 + \left(\frac{x_1 + x_2 - 10}{3}\right)^2$$

Uwaga – funkcja tylko dwóch zmiennych

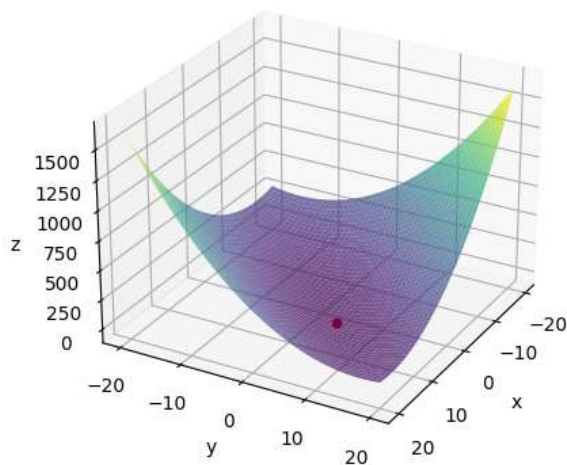
Sugerowany zakres poszukiwań: [- 20, 20]

Globalne minima:

(0.0, [5.0, 5.0])

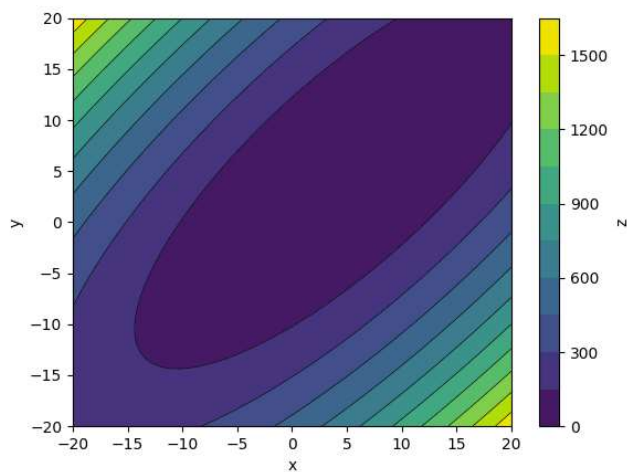
Wykres:

Martin and Gaddy



Wykres typu heatmap:

Martin and Gaddy



11) Griewank

Wzór w Latex:

$$f(x) = \sum_{i=0}^{N-1} \frac{x_i^2}{4000} - \prod_{i=0}^{N-1} \cos\left(\frac{x_i}{\sqrt{i+1}}\right) + 1$$

Wzór funkcji:

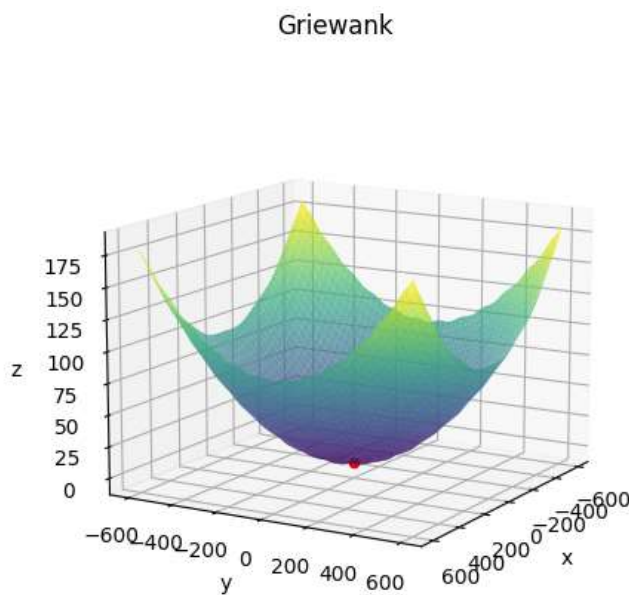
$$f(x) = \sum_{i=0}^{N-1} \frac{x_i^2}{4000} - \prod_{i=0}^{N-1} \cos\left(\frac{x_i}{\sqrt{i+1}}\right) + 1$$

Sugerowany zakres poszukiwań: [-600.0, 600.0]

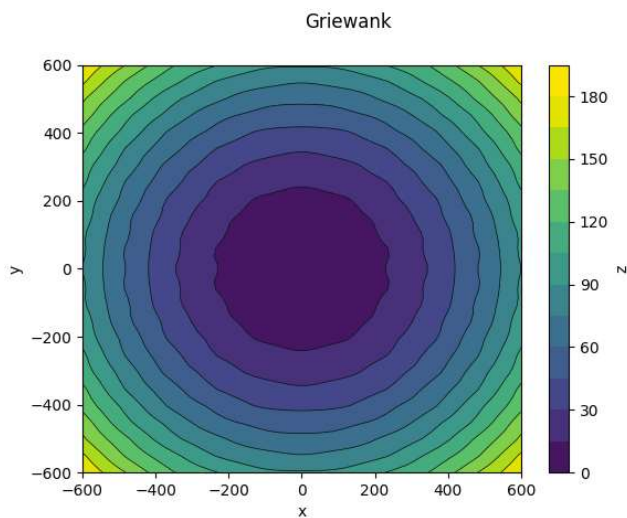
Globalne minima:

(0.0, [0.0, 0.0])

Wykres:



Wykres typu heatmap:



12) Easom

Wzór w Latex:

$$f(x) = -\cos(x_1)\cos(x_2)e^{-(x_1-\pi)^2-(x_2-\pi)^2}$$

Wzór funkcji:

$$f(x) = -\cos(x_1)\cos(x_2)e^{-(x_1-\pi)^2-(x_2-\pi)^2}$$

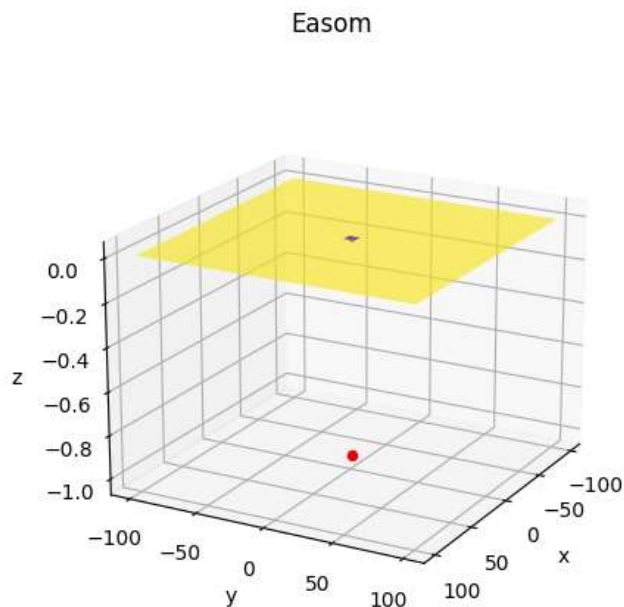
Uwaga – funkcja tylko dwóch zmiennych

Sugerowany zakres poszukiwań: [-100, 100]

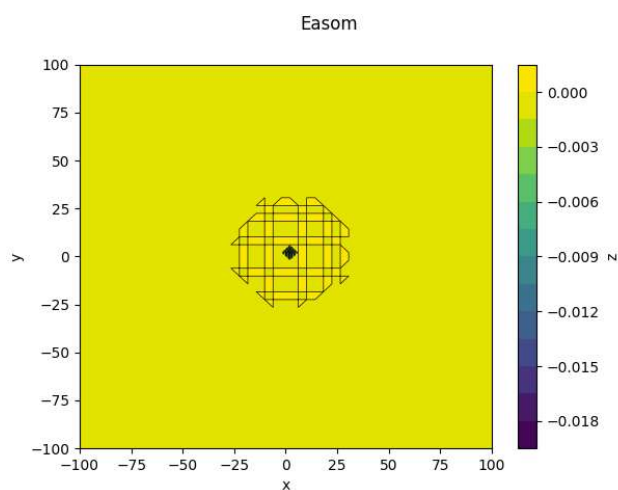
Globalne minima:

(-1.0, [3.141592653589793, 3.141592653589793])

Wykres:



Wykres typu heatmap:



13) Goldstein and Price

Wzór w Latex:

$$f(x) = (1 + (x_0 + x_1 + 1)^2(19 - 14x_0 + 3x_0^2 - 14x_1 + 6x_0x_1 + 3x_1^2)) \cdot (30 + (2x_0 - 3x_1)^2(18 - 32x_0 + 12x_0^2 + 48x_1 - 36x_0x_1 + 27x_1^2))$$

Wzór funkcji:

$$f(x) = (1 + (x_0 + x_1 + 1)^2(19 - 14x_0 + 3x_0^2 - 14x_1 + 6x_0x_1 + 3x_1^2)) \cdot (30 + (2x_0 - 3x_1)^2(18 - 32x_0 + 12x_0^2 + 48x_1 - 36x_0x_1 + 27x_1^2))$$

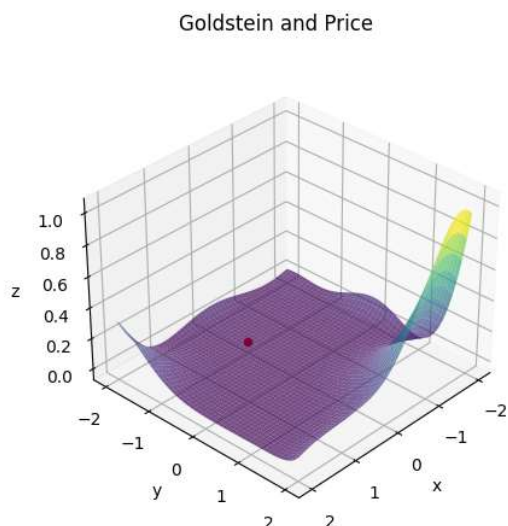
Uwaga – funkcja tylko dwóch zmiennych

Sugerowany zakres poszukiwań: [-2, 2]

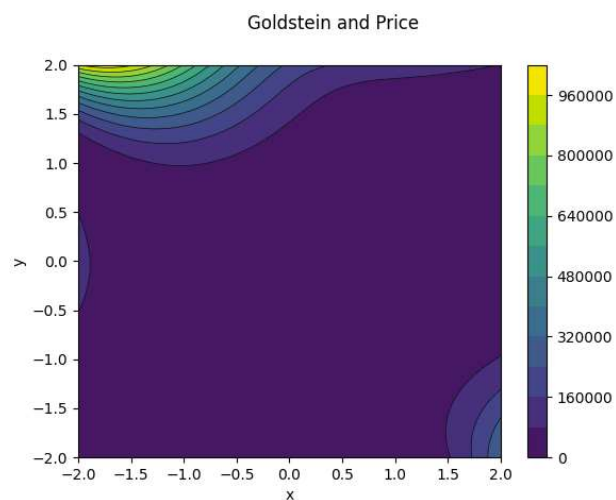
Globalne minima:

(3.0, [0.0, -1.0])

Wykres:



Wykres typu heatmap:



14) Picheny, Goldstein and Price

Wzór w Latex:

$$f(x) = 2.427^{-1} (\log[(1 + (x_0 + x_1 + 1)^2 (19 - 14x_0 + 3x_0^2 - 14x_1 + 6x_0x_1 + 3x_1^2)) \cdot (30 + (2x_0 - 3x_1)^2 (18 - 32x_0 + 12x_0^2 + 48x_1 - 36x_0x_1 + 27x_1^2))] - 8.693)$$

Wzór funkcji:

$$f(x) = 2.427^{-1} (\log[(1 + (x_0 + x_1 + 1)^2 (19 - 14x_0 + 3x_0^2 - 14x_1 + 6x_0x_1 + 3x_1^2)) \cdot (30 + (2x_0 - 3x_1)^2 (18 - 32x_0 + 12x_0^2 + 48x_1 - 36x_0x_1 + 27x_1^2))] - 8.693)$$

Uwaga – funkcja tylko dwóch zmiennych

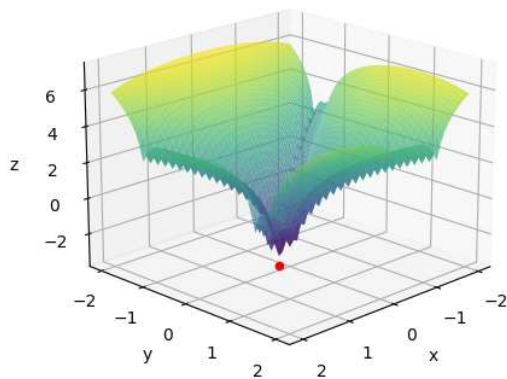
Sugerowany zakres poszukiwań: [-2, 2]

Globalne minima:

(-3.129125550610585, [0.5, 0.25])

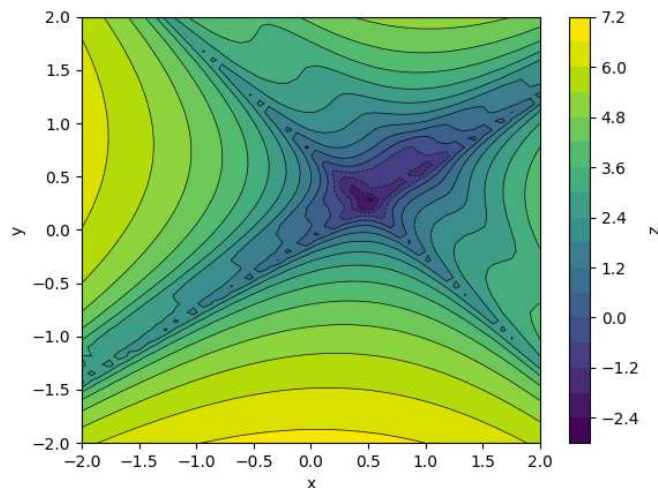
Wykres:

Picheny, Goldstein and Price



Wykres typu heatmap:

Picheny, Goldstein and Price



15) Styblinski and Tang

Wzór w Latex:

$$f(x) = 0.5 \sum_{i=0}^{N-1} (x_i^4 - 16x_i^2 + x_i)$$

Wzór funkcji:

$$f(x) = 0.5 \sum_{i=0}^{N-1} (x_i^4 - 16x_i^2 + x_i)$$

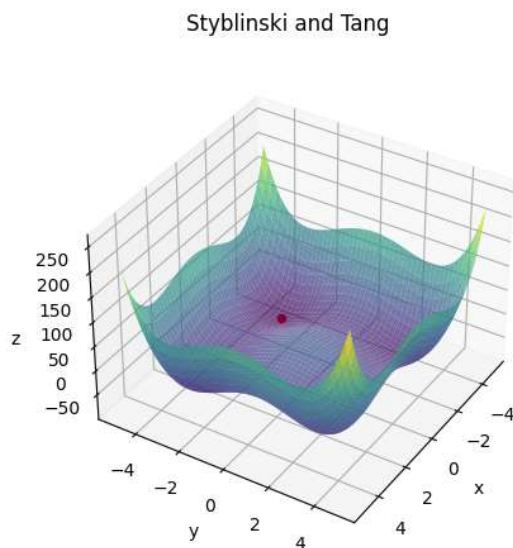
Sugerowany zakres poszukiwań:

[-5, 5]

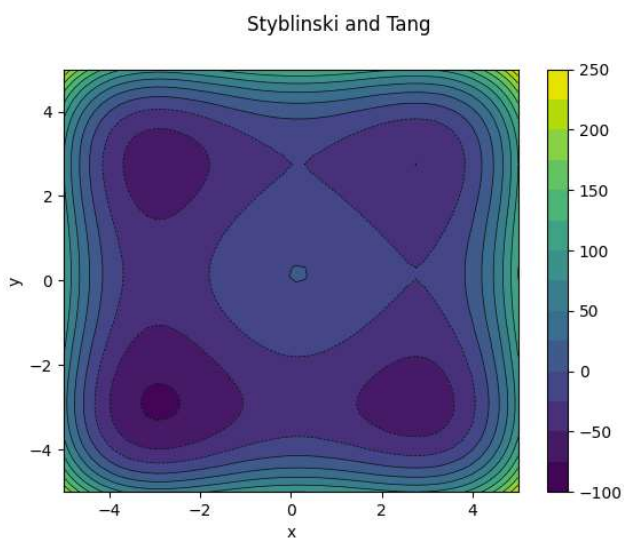
Globalne minima:

(-78.3323314075428, [-2.903534, -2.903534])

Wykres:



Wykres typu heatmap:



16) Mc Cormick

Wzór w Latex:

$$f(x) = \sin(x_0 + x_1) + (x_0 - x_1)^2 - 1.5x_0 + 2.5x_1 + 1$$

Wzór funkcji:

$$f(x) = \sin(x_0 + x_1) + (x_0 - x_1)^2 - 1.5x_0 + 2.5x_1 + 1$$

Uwaga – funkcja tylko dwóch zmiennych

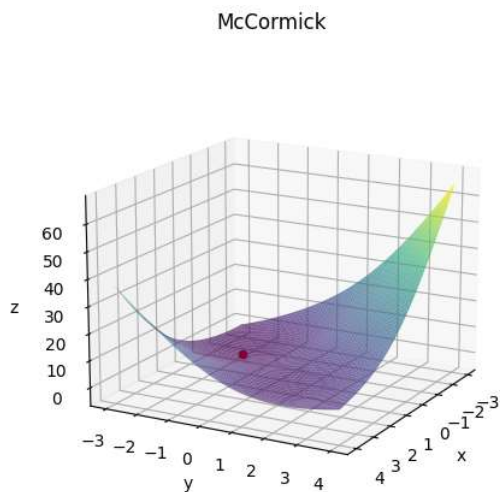
Sugerowany zakres poszukiwań:

$[-3, 4]$

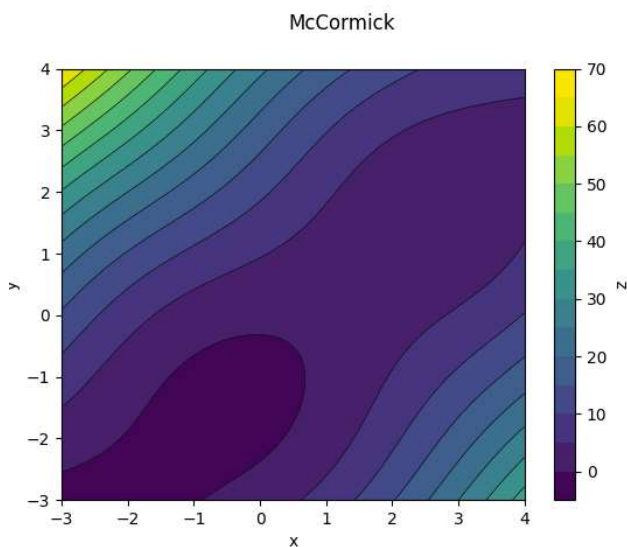
Globalne minima:

$(-1.913222954882274, [-0.54719, -1.54719])$

Wykres:



Wykres typu heatmap:



17) Rana

Wzór w Latex:

$$f(x) = \sum_{i=0}^{N-2} x_i \cos \sqrt{|x_{i+1} + x_{i+1}|} \sin \sqrt{|x_{i+1} - x_{i+1}|} - x_{i+1} \sin \sqrt{|x_{i+1} + x_{i+1}|} \cos \sqrt{|x_{i+1} - x_{i+1}|}$$

Wzór funkcji:

$$f(x) = \sum_{i=0}^{N-2} x_i \cos \sqrt{|x_{i+1} + x_{i+1}|} \sin \sqrt{|x_{i+1} - x_{i+1}|} + (1 + x_{i+1}) \sin \sqrt{|x_{i+1} + x_{i+1}|} \cos \sqrt{|x_{i+1} - x_{i+1}|}$$

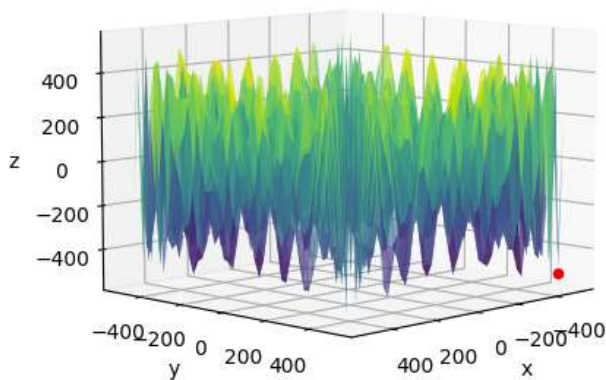
Sugerowany zakres poszukiwań: [-512.0, 512.0]

Globalne minima:

(-511.73288188661934, [-488.632577, 512])

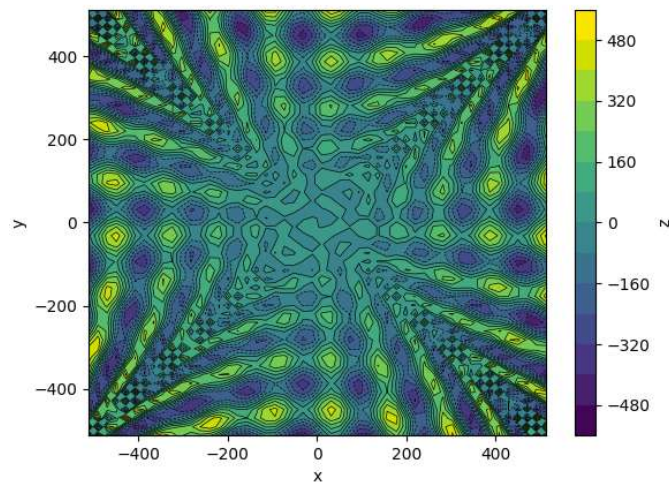
Wykres:

Rana



Wykres typu heatmap:

Rana



18) Egg Holder

Wzór w LaTeX:

$$f(x) = -\sum_{i=0}^{N-2} (x_{i+1} + 47) \sin \sqrt{|x_{i+1} + 47 + 0.5x_i|} + x_i \sin \sqrt{|x_i - (x_{i+1} + 47)|}$$

Wzór funkcji:

$$f(x) = -\sum_{i=0}^{N-2} (x_{i+1} + 47) \sin \sqrt{|x_{i+1} + 47 + 0.5x_i|} + x_i \sin \sqrt{|x_i - (x_{i+1} + 47)|}$$

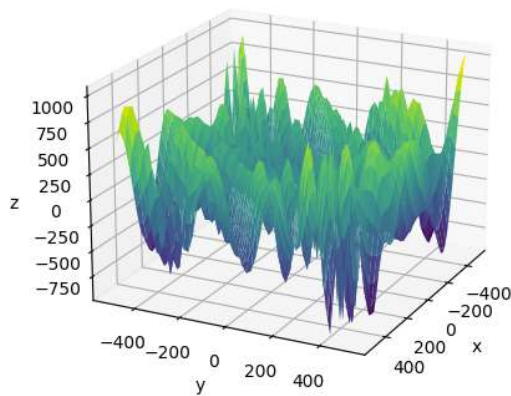
Sugerowany zakres poszukiwań: [-512, 512]

Globalne minima:

(-959.7133283100513, [512.0214856276842, 404.2510470803529])

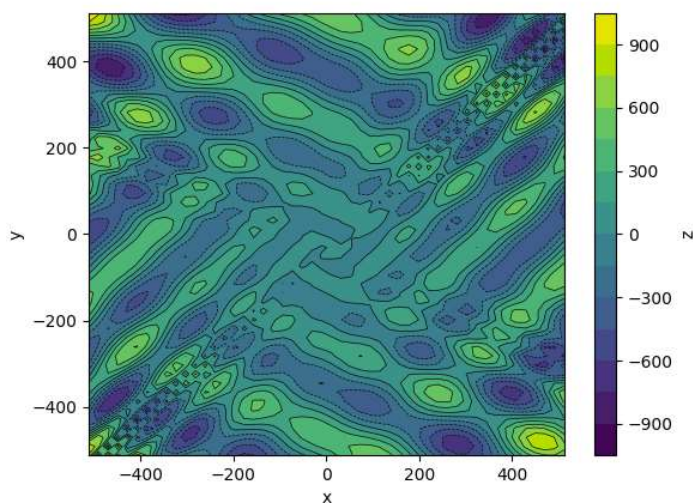
Wykres:

Egg Holder



Wykres typu heatmap:

Egg Holder



19) Keane

Wzór w Latex:

$$f(x) = -\left(\left|\sum_{i=0}^{N-1} \cos^4 x_i - \prod_{i=0}^{N-1} \cos^2 x_i\right|\right) \left(\sum_{i=0}^{N-1} x_i^2 (i+1)\right)^{-\frac{1}{2}}$$

Wzór funkcji:

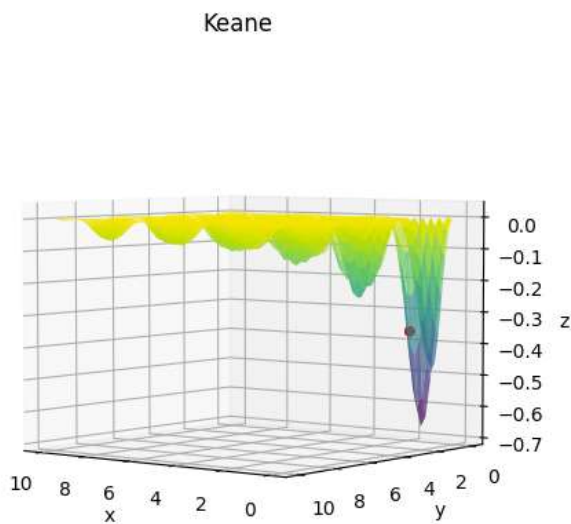
$$f(x) = -\left(\left|\sum_{i=0}^{N-1} \cos^4 x_i - \prod_{i=0}^{N-1} \cos^2 x_i\right|\right) \left(\sum_{i=0}^{N-1} x_i^2 (i+1)\right)^{-\frac{1}{2}}$$

Sugerowany zakres poszukiwań: [0.0, 10.0]

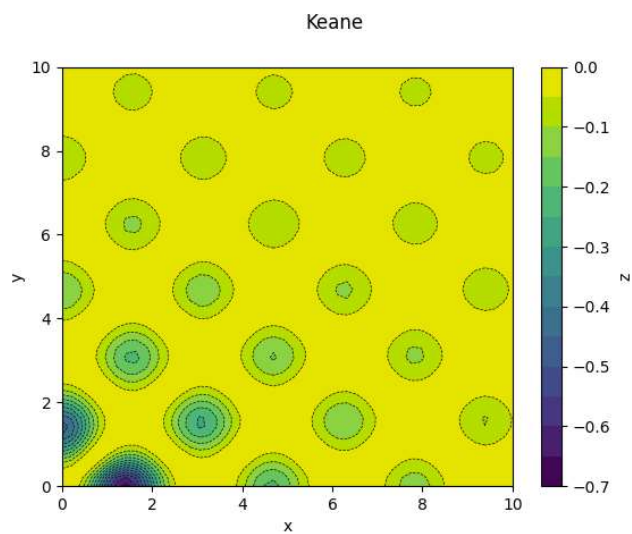
Globalne minima:

(-0.3649799014197145, [1.60086, 0.468498])

Wykres:



Wykres typu heatmap:



20) Schaffer 2

Wzór w Latex:

$$f(x) = 0.5 + \frac{\sin(\sqrt{(x_0^2 + x_1^2)})^2 - 0.5}{(1.0 + 0.001(x_0^2 + x_1^2))^2}$$

Wzór funkcji:

$$f(x) = 0.5 + \frac{\sin(\sqrt{(x_0^2 + x_1^2)})^2 - 0.5}{(1.0 + 0.001(x_0^2 + x_1^2))^2}$$

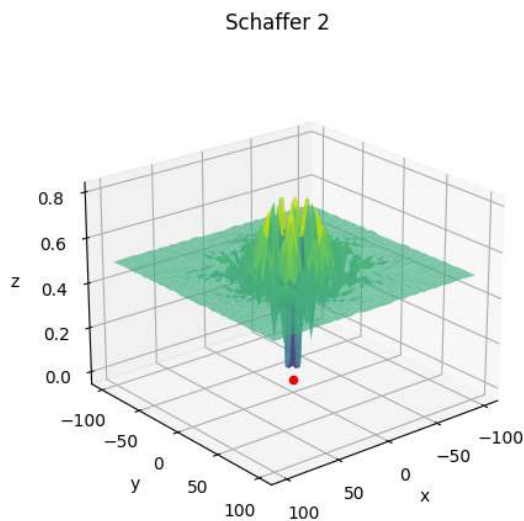
Uwaga – funkcja tylko dwóch zmiennych

Sugerowany zakres poszukiwań: [-100, 100]

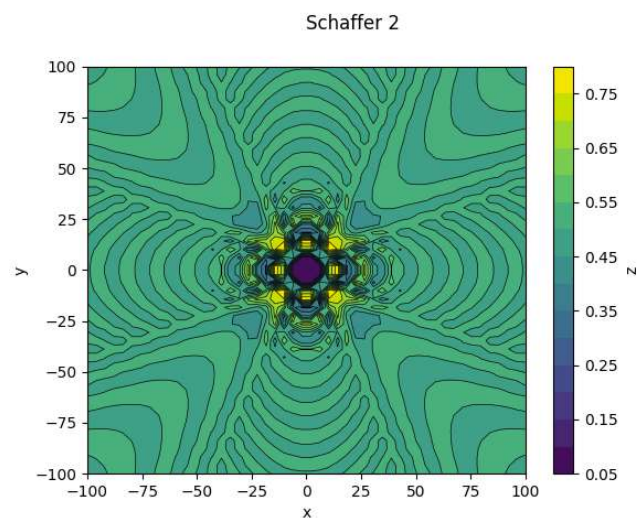
Globalne minima:

(0.0, [0.0, 0.0])

Wykres:



Wykres typu heatmap:



21) Himmelblau

Wzór w Latex:

$$f(x) = (x_0^2 + x_1 - 11)^2 + (x_0 + x_1^2 - 7)^2$$

Wzór funkcji:

$$f(x) = (x_0^2 + x_1 - 11)^2 + (x_0 + x_1^2 - 7)^2$$

Uwaga – funkcja tylko dwóch zmiennych

Sugerowany zakres poszukiwań: [-5, 5]

Globalne minima:

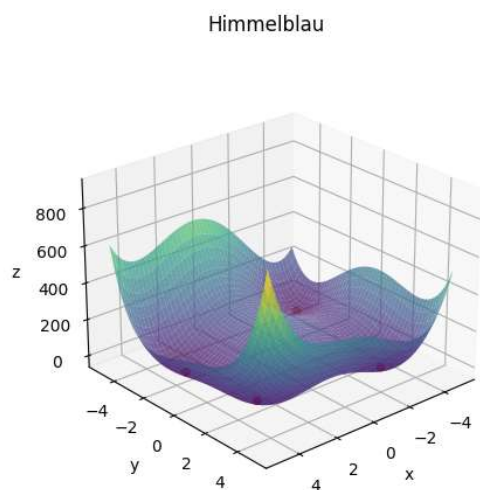
(0.0, [3.0, 2.0])

(1.0989296656869089e-11, [-2.805118, 3.131312])

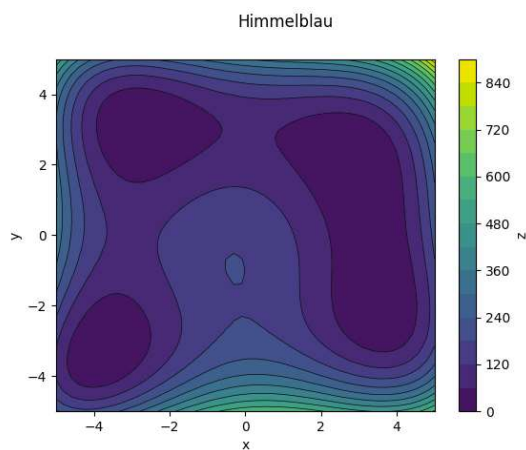
(3.797861082863832e-12, [-3.77931, -3.283186])

(8.894376497582423e-12, [3.584428, -1.848126])

Wykres:



Wykres typu heatmap:



22) Pits and Holes

Wzór w Latex:

$$f_{\mu, C, v}(x) = -\sum_{i=0}^{N-1} N_{PDF}(x, \mu_i, C_i) v_i$$

Wzór funkcji:

$$f_{\mu, C, v}(x) = -\sum_{i=0}^{N-1} N_{PDF}(x, \mu_i, C_i) v_i$$

Uwaga – funkcja tylko dwóch zmiennych

Sugerowany zakres poszukiwań: [20.0, 20.0]

Globalne minima:

(-0.030315347027905098, [0, 0])

(-0.028420525714448686, [20, 0])

(-0.026857396808720072, [0, 20])

(-0.033157279973136014, [-20, 0])

(-0.040672930063224866, [0, -20])

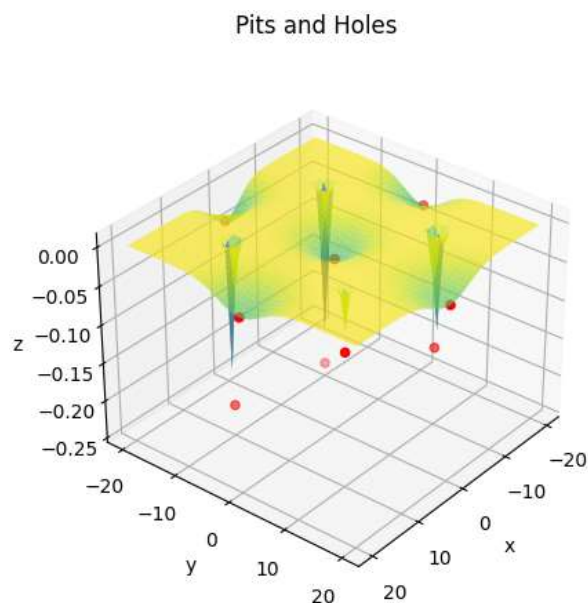
(-0.07965400042977547, [10, 10])

(-0.23874320826749335, [-10, -10])

(-0.152850778099608, [-10, 10])

(-0.21535256572994807, [10, -10])

Wykres:



Wykres typu heatmap:

