Tema 2 geometrie - Andruță Andra Mihaela - grupa 132

• Fie spațiul vectorial euclidian $E_3=(\mathbb{R}^3/\mathbb{R}, <,>)$, $B_0=\{e_1,e_2,e_3\}\subset E_3$ b. canonică.

Stabiliți dacă urm. aplicații liniare sunt transformări ortogonale

b) $T: E_3 \to E_3$ a.î. $\begin{cases} T(e_1)=e_1+e_2 \\ T(e_2)=e_2+e_3 \\ T(e_3)=e_3+e_1 \end{cases}$

Avem B_0 = b ortonormată

$$J=\begin{array}{c} \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \\ \begin{matrix} T(e_1) & T(e_2) & T(e_3) \end{matrix} \end{array} \to \text{mat. asociată lui } T \text{ în rap cu } B_0$$

Stabilim dacă J e m ortog sau nu. Calculam $J\cdot J^t$

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}=\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}\neq I_3$$

$\Rightarrow J\notin O(3) \Rightarrow T$ nu e transformare ortogonală.

c) $T: E_3\to E_3$ a.î. $\begin{cases} T(e_1)=\frac{2}{3}e_1+\frac{2}{3}e_2-\frac{1}{3}e_3 \\ T(e_2)=\frac{2}{3}e_1-\frac{1}{3}e_2+\frac{2}{3}e_3 \\ T(e_3)=-\frac{1}{3}e_1+\frac{2}{3}e_2+\frac{2}{3}e_3 \end{cases}$

Avem B_0 = b ortonormată

$$J=\begin{array}{c} \begin{pmatrix} \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{pmatrix} \\ \begin{matrix} T(e_1) & T(e_2) & T(e_3) \end{matrix} \end{array} \to \text{mat asoc lui } T \text{ în rap cu } B_0$$

Stabilim dacă J e m. ortog sau nu. Calculam $J\cdot J^t$

$$\begin{pmatrix} \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{pmatrix}\begin{pmatrix} \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}=I_3$$

$\Rightarrow J\in O(3) \Rightarrow T$ e transformare ortogonală.

• $\Gamma: \ x_1^2 + 2x_1x_2 - 5x_2^2 + 4x_1 - 8x_2 - 17 = 0$

Să se aducă la o f. canonică conica Γ prin izometrii

$$A' = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -5 & -4 \\ 2 & -4 & -17 \end{pmatrix} ; \ \Delta = \det A' = 90 \neq 0$$

$$A = \begin{pmatrix} 1 & 1 \\ 1 & -5 \end{pmatrix} : \ \delta = \det A = -6 \neq 0$$

$$\left.\begin{matrix} \Delta \neq 0 \\ \delta < 0 \end{matrix}\right| \Rightarrow \Gamma \text{ e hiperbolă}$$

Centrul conicei Γ e $P_0(x_1^0, x_2^0)$ unde (x_1^0, x_2^0) se det ca sol unică a sistemului

$$\begin{cases} \dfrac{\partial f}{\partial x_1} = 0 \\ \dfrac{\partial f}{\partial x_2} = 0 \end{cases} \Leftrightarrow \begin{cases} 2x_1^0 + 2x_2^0 + 4 = 0 \\ 2x_1^0 - 10x_2^0 - 8 = 0 \end{cases} \Leftrightarrow \begin{cases} x_1^0 + x_2^0 + 2 = 0 \\ x_1^0 - 5x_2^0 - 4 = 0 \end{cases} \Rightarrow x_1^0 = x_2^0 = -1$$

$$\Rightarrow P_0(-1, -1) \text{ centrul conicei } \Gamma$$

Efectuăm translația t.

$$t \begin{cases} x_1' = x_1 - x_1^0 \\ x_2' = x_2 - x_2^0 \end{cases} \Leftrightarrow \begin{cases} x_1' = x_1 + 1 \\ x_2' = x_2 + 1 \end{cases} \Rightarrow \begin{cases} x_1 = x_1' - 1 \\ x_2 = x_2' - 1 \end{cases}$$

$$t(\Gamma): (x_1')^2 + 2x_1'x_2' - 5x_2'^2 + \frac{\Delta}{\delta} = 0 \Leftrightarrow t(\Gamma): x_1'^2 + 2x_1'x_2' - 5x_2'^2 - 15 = 0$$

$$\det(\lambda I_2 - A) = 0 \Leftrightarrow \begin{vmatrix} \lambda - 1 & -1 \\ -1 & \lambda + 5 \end{vmatrix} = 0 \Leftrightarrow \lambda^2 + 4\lambda - 6 = 0$$

$$\Rightarrow \begin{cases} \lambda_1 = -2 - \sqrt{10} \\ \lambda_2 = -2 + \sqrt{10} \end{cases} \text{ val. proprii}$$

$$V_{\lambda_1} = \{ v \in \mathbb{R}^2 \mid Av = \lambda_1 v \} \Rightarrow \begin{cases} (3+\sqrt{10})v_1 + v_2 = 0 \\ v_1 + v_2(-3+\sqrt{10}) = 0 \end{cases} \Rightarrow V_{\lambda_1} = \langle (1, -3+\sqrt{10}) \rangle = \langle f_1 \rangle$$

$$v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$V_{\lambda_2} = \{ v \in \mathbb{R}^2 \mid Av = \lambda_2 v \} \Rightarrow \begin{cases} (3-\sqrt{10})v_1 + v_2 = 0 \\ v_1 + v_2(-3-\sqrt{10}) = 0 \end{cases} \Rightarrow V_{\lambda_2} = \langle (1, -3-\sqrt{10}) \rangle = \langle f_2 \rangle$$

$$\langle f_1, f_2 \rangle = 0 \Rightarrow f_1 \perp f_2$$

Normăm vectorii f_1, f_2 și obț un reper ortonormat.

$$e_1 = \frac{f_1}{\|f_1\|} = \frac{\sqrt{50+15\sqrt{10}}}{10}(1, -3+\sqrt{10}) = \left(\frac{\sqrt{50+15\sqrt{10}}}{10}, -\frac{\sqrt{50+15\sqrt{10}}(3-\sqrt{10})}{10} \right) = \left(+\frac{\sqrt{50+15\sqrt{10}}}{10}, \frac{\sqrt{50-15\sqrt{10}}}{10} \right)$$

$$e_2 = \frac{f_2}{\|f_2\|} = \frac{\sqrt{50-15\sqrt{10}}}{10}(1, -3-\sqrt{10}) = \left(\frac{\sqrt{50-15\sqrt{10}}}{10}, \frac{\sqrt{50+15\sqrt{10}}}{10} \right)$$

$$R = (e_1 | e_2) = \begin{pmatrix} \frac{\sqrt{50-15\sqrt{10}}}{10} & \frac{\sqrt{50+15\sqrt{10}}}{10} \\ -\frac{\sqrt{50+15\sqrt{10}}}{10} & \frac{\sqrt{50-15\sqrt{10}}}{10} \end{pmatrix}.$$

Efectuăm rotația r. $\quad r\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = R\begin{pmatrix} x_1'' \\ x_2'' \end{pmatrix}$

$$r \begin{cases} x_1' = \frac{\sqrt{50-15\sqrt{10}}}{10} x_1'' + \frac{\sqrt{50+15\sqrt{10}}}{10} x_2'' \\ x_2' = -\frac{\sqrt{50+15\sqrt{10}}}{10} x_1'' + \frac{\sqrt{50-15\sqrt{10}}}{10} x_2'' \end{cases}$$

$(r\circ t)(\Gamma): \lambda_1 x_2''^2 + \lambda_2 x_1''^2 + \frac{\Delta}{\delta} = 0$

$(r\circ t)(\Gamma): (-2-\sqrt{10})x_2''^2 + (-2+\sqrt{10})x_1''^2 - 15 = 0 \;\Big|\cdot \frac{1}{15}$

$$\Rightarrow \quad -\frac{x_2''^2}{\left(\sqrt{\frac{30+15\sqrt{10}}{15}}\right)^2} + \frac{x_1''^2}{\left(\sqrt{\frac{-30+15\sqrt{10}}{15}}\right)^2} - 1 = 0$$