

Temă

①

$$a) A = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 3 & 2 \\ 4 & 3 & -2 \end{pmatrix}$$

$$b) A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$$

$$c) B = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

$$d) C = \begin{pmatrix} 1 & 1 & 3 \\ 2 & 1 & -2 \\ 1 & -1 & 4 \end{pmatrix}$$

$$a) \bar{A} = (A | I_3) = \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ -1 & 3 & 2 & 0 & 1 & 0 \\ 4 & 3 & -2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{L_2 \leftarrow L_2 + L_1}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 4 & 4 & 0 & 2 & 0 \\ 4 & 3 & -2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{L_3 \leftarrow L_3 - 4L_1} \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 4 & 4 & 0 & 2 & 0 \\ 0 & -1 & -10 & -4 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{L_1 \leftarrow L_1 - L_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & -2 & 0 \\ 0 & 4 & 4 & 0 & 2 & 0 \\ 0 & -1 & -10 & -4 & 0 & 1 \end{array} \right) \xrightarrow{L_2 \leftarrow L_2 - 4L_1} \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & -2 & 0 \\ 0 & 4 & 4 & 0 & 2 & 0 \\ 0 & -1 & -10 & -4 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{L_2 \leftrightarrow L_3} \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & -2 & 0 \\ 0 & -1 & -10 & -4 & 0 & 1 \\ 0 & 4 & 4 & 0 & 2 & 0 \end{array} \right) \xrightarrow{L_3 \leftarrow L_3 + 4L_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & -2 & 0 \\ 0 & -1 & -10 & -4 & 0 & 1 \\ 0 & 0 & -36 & -16 & 2 & 4 \end{array} \right)$$

$$\xrightarrow{L_3 \leftarrow \frac{1}{-36} L_3} \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & -2 & 0 \\ 0 & -1 & -10 & -4 & 0 & 1 \\ 0 & 0 & 1 & \frac{4}{9} & -\frac{1}{9} & -\frac{2}{9} \end{array} \right) \xrightarrow{L_2 \leftarrow L_2 + 10L_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{13}{9} & -\frac{11}{9} & -\frac{2}{9} \\ 0 & -1 & 0 & -\frac{20}{9} & \frac{10}{9} & \frac{16}{9} \\ 0 & 0 & 1 & \frac{4}{9} & -\frac{1}{9} & -\frac{2}{9} \end{array} \right)$$

$$b) \bar{A} = (A | I_2) = \left(\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array} \right) \xrightarrow{L_1 \leftrightarrow L_2} \left(\begin{array}{cc|cc} 1 & 3 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{array} \right)$$

$$\sim \left(\begin{array}{cc|cc} 1 & 3 & 0 & 1 \\ 0 & -5 & 1 & -2 \end{array} \right) \sim \left(\begin{array}{cc|cc} 1 & 3 & 0 & 1 \\ 0 & 1 & -1/5 & 2/5 \end{array} \right) \sim \left(\begin{array}{cc|cc} 1 & 0 & -3/5 & 7/5 \\ 0 & 1 & -1/5 & 2/5 \end{array} \right)$$

$$c) \quad \bar{A} = (A \mid I_3) = \left(\begin{array}{ccc|ccc} 2 & -1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array} \right) \sim$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & -2 & 0 & 0 & 0 \\ 1 & 1 & -2 & 0 & 0 & 0 \\ 1 & 1 & -2 & 0 & 0 & 0 \end{array} \right) \xrightarrow{R_2 - R_1, R_3 - R_1} \left(\begin{array}{ccc|ccc} 1 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{2} \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$\left(\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & -1 \\ 0 & 3 & 1 & -1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & -1 \\ 0 & 3 & 0 & -1 & -1 & 3 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array} \right) \sim$$

$$\left(\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1/3 & -1/3 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & -2/3 & 1 \\ 0 & 1 & 0 & -1/3 & -1/3 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array} \right)$$

$$d) \bar{A} = (A | \mathbf{b}) = \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 2 & 1 & -2 & 0 & 1 & 0 \\ 1 & -1 & 4 & 0 & 0 & 1 \end{array} \right) \xrightarrow{L_1 - L_3 \rightarrow L_1}$$

$$\left(\begin{array}{ccc|ccc} 0 & 2 & -1 & 1 & 0 & -1 \\ 2 & 1 & -2 & 0 & 1 & 0 \\ 1 & -1 & 4 & 0 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 0 & 2 & -1 & 1 & 0 & -1 \\ 1 & 2 & -1 & 0 & 1 & 0 \\ 1 & -1 & 4 & 0 & 0 & 1 \end{array} \right) \sim$$

$$\left(\begin{array}{ccc|ccc} 0 & 2 & -1 & 1 & 0 & -1 \\ 1 & 0 & 3 & -1 & 1 & 0 \\ 1 & -1 & 4 & 0 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 0 & 2 & -1 & 1 & 0 & -1 \\ 1 & 0 & 3 & -1 & 1 & 0 \\ 1 & -1 & 4 & 0 & 0 & 1 \end{array} \right) \sim$$

$$\left(\begin{array}{ccc|ccc} 0 & 0 & -15 & 3 & -2 & 1 \\ 0 & 1 & 4 & -1 & 1 & -1 \\ 1 & -1 & 4 & 0 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 0 & 0 & 1 & -7/5 & 2/5 & -1/5 \\ 0 & 1 & 4 & -1 & 1 & -1 \\ 1 & -1 & 4 & 0 & 0 & 1 \end{array} \right)$$

$$2 \left(\begin{array}{ccc|ccc} 0 & 0 & 1 & -1/5 & 2/15 & -1/15 \\ 0 & 1 & 7 & -1 & 1 & -1 \\ 1 & 0 & 11 & -1 & 1 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 0 & 0 & 1 & -1/5 & 2/15 & -1/15 \\ 0 & 1 & 7 & -1 & 1 & -1 \\ 1 & 0 & 11 & -1 & 1 & 0 \end{array} \right)$$

$$2 \left(\begin{array}{ccc|ccc} 0 & 0 & 1 & -1/5 & 2/15 & -1/15 \\ 0 & 1 & 0 & -2/15 & 1/15 & -8/15 \\ 1 & 0 & 1 & -1 & 1 & 0 \end{array} \right) \sim$$

$$2 \left(\begin{array}{ccc|ccc} 1 & 0 & 11 & -1 & 1 & 0 \\ 0 & 1 & 0 & -2/5 & 1/5 & -8/15 \\ 0 & 0 & 1 & -1/5 & 2/5 & -1/15 \end{array} \right) \sim$$

$$2 \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 6/5 & -7/15 & 11/15 \\ 0 & 1 & 0 & -2/5 & 1/15 & -8/15 \\ 0 & 0 & 1 & -1/5 & 2/15 & -1/15 \end{array} \right)$$