LMC

29.11.2023

Seminar 4

① $\{\psi, \neg\varphi\} \vdash \neg(\psi \to \varphi)$ de dem (seminar 3)

$\{\overbrace{\psi, \neg\varphi}^{\Gamma}, \overbrace{\neg\neg(\psi \to \varphi)}^{\neg a}\} \vdash \neg\neg(\psi \to \varphi)$ P2.54(ii)

$\{\psi, \neg\varphi, \neg\neg(\psi\to\varphi)\} \vdash \neg\neg(\psi \to \varphi) \to (\psi \to \varphi)$ S.3.4(iv)

$\{\psi, \neg\varphi, \neg\neg(\psi\to\varphi)\} \vdash \psi \to \varphi$ MP (1) și (2)

$\{\psi, \neg\varphi, \neg\neg(\psi\to\varphi)\} \vdash \psi$ ~~EP~~ P2.54(ii)

—— " —————— $\vdash \varphi$ MP (3) și (4)

—— " —————— $\vdash \neg\varphi$ P2.54(ii)

$\{\psi, \neg\varphi\} \vdash (\psi \to \varphi)$ S3.4.(iii)

② $\vdash (\varphi \to \psi) \to (\neg\psi \to \neg\varphi)$ (seminar 3) Thd $\Gamma \cup \{a\} \vdash b \iff \Gamma \vdash a \to b$

$\{(\varphi \to \psi)\} \vdash \neg\psi \to \neg\varphi$

$\{\neg\psi, \varphi \to \psi\} \vdash \neg\varphi$ | ce am fct eu

$A_3 : (\neg b \to \neg a) \to (a \to b)$

$\{\varphi \to \psi, \neg\psi, \neg\neg\varphi\} \vdash \bot$ (prin reducere la absurd)

(1) $\{\varphi \to \psi, \neg\psi, \neg\neg\varphi\} \vdash \varphi \to \psi$ P2.54(ii)

(2) $\{\varphi \to \psi, \neg\psi, \neg\neg\varphi\} \vdash \neg\neg\varphi \to \varphi$ S.34 (iv) + P2.55(ii)

(3) $\{\varphi \to \psi, \neg\psi, \neg\neg\varphi\} \vdash \neg\neg\varphi$ P.254 (ii)

(4) $\{\varphi \to \psi, \neg\psi, \neg\neg\varphi\} \vdash \varphi$ MP (2) și (3)

(5) $\{\varphi \to \psi, \neg\psi, \neg\neg\varphi\} \vdash \psi$ MP (4) și (1)

(6) $\{\varphi \to \psi, \neg\psi, \neg\neg\varphi\} \vdash \neg\psi$ P 2.54(ii)

(7) $\{\varphi \to \psi, \neg\psi\} \vdash \neg\varphi$ S34. (iii)

(8) $\{\varphi \to \psi\} \vdash \neg\psi \to \neg\varphi$ Th. ded.

(9) $\vdash (\varphi \to \psi) \to (\neg\psi \to \neg\varphi)$ Th. ded.

1) a) $\Gamma = \{v_m \to v_{m+1} \mid m \in \mathbb{N}\}$

Fie $e: V \to \{0,1\}$ a.î. $e \models \Gamma \iff e \models v_m \to v_{m+1}$ pt $\forall m \in \mathbb{N} \iff$

$\iff e^+(v_m \to v_{m+1}) = 1 \ \forall m \in \mathbb{N} \iff e^+(v_m) \to e^+(v_{m+1}) = 1 \ \forall m \in \mathbb{N} \iff$

$\iff e(v_m) \to e(v_{m+1}) = 1 \ \forall m \in \mathbb{N} \iff e(v_m) \leq e(v_{m+1}) \ \forall m \in \mathbb{N}$

Fie $e_k: V \to \{0,1\}$ unde $e_k = \begin{cases} 0, & m < k \\ 1, & m \geq k \end{cases}$ $m \in \mathbb{N}$

	v_0	v_1	v_2	v_3	v_4	...
e_0	1	1	1	1	1	...
e_1	0	1	1	1	1	...
e_2	0	0	1	1	1	...

$e^\infty(v_m) = 0 \ \forall m \in \mathbb{N}$

$Mod(\Gamma) = \{e_k: V \to \{0,1\} \mid k \in \mathbb{N}\} \cup \{e^\infty\}$

b) $\Gamma = \{v_0\} \cup \{v_m \to v_{m+1} \mid 0 \leq m \leq 7\}$

$e(v_0) = 1$

$e(v_m) = 1, \ m \in \overline{0,8}$

$Mod(\Gamma) = \{e: V \to \{0,1\} \mid e(v_m) = 1$ pt $m \in \overline{0,8}\}$

2) Fie $\Gamma = \{v^f \mid v \in V\}$ unde $v^f = \begin{cases} v, & \text{dacă } f(v) = 1 \\ \neg v, & \text{dacă } f(v) = 0 \end{cases}$

$Mod(\Gamma) = \{f\}$

Fie $e: V \to \{0,1\}$ a.î. $e \in Mod(\Gamma) \iff$ pt $\forall v \in V, e \models v^f \iff$

$\iff \forall v \in V, e^+(v^f) = 1 \iff e = f$

„$\Rightarrow$" Știm că pt $\forall v \in V$, $e^+(v^f) = 1$. Vrem să dem că $e = f$.

Fie $v \in V$ → dacă $f(v) = 1 \Rightarrow v = v^f \Rightarrow$ deci $e(v) = e^+(v) = e^+(v^f) = 1 = f(v)$

→ dacă $f(v) = 0 \Rightarrow v^f = \neg v \Rightarrow$ deci $e(v) = e^+(v) = \neg\neg e^+(v) =$
$= \neg e^+(\neg v) \Rightarrow e^+(v^f) = \neg 1 = 0 = f(v)$

$\Rightarrow$

$\Rightarrow$ ~~$e(f) =$~~ $e(v) = f(v) \iff e = f$

„$\Leftarrow$" Știm că $e = f$. Vrem să dem că pt $\forall v \in V$, $e^+(v^f) = 1$

Fie $v \in V$. Atunci $e^+(v^f) = f^+(v^f) \iff e(v) = f(v) = 1$

3) a) Fie Γ mulțime satisfiabilă și finită de formule

Γ - finită $\Rightarrow$ nr finit formule $\Rightarrow$ nr. finit var.: $\bigcup_{\varphi \in \Gamma} Var(\varphi) \subseteq \{v_0, \ldots, v_m\}$

$m \in \mathbb{N}$

Γ - satisfiabilă $\Rightarrow \exists\, e: V \to \{0,1\}$ a.î. $e \vDash \Gamma$

$$e_k: V \to \{0,1\}, \quad e_k(x) = \begin{cases} e(x), & x \in \{v_0, \dots v_m\} \\ 1, & x \in \{v_{m+1}, \dots, v_{m+k}\} \\ 0, & \text{altfel} \end{cases}$$

$$\left.\begin{array}{l} \underbrace{\{e_k \mid k \in \mathbb{N}\}}_{\hookrightarrow \text{ infinită}} \subseteq Mod(\Gamma) \end{array}\right| \Rightarrow Mod(\Gamma) \text{ - infinită}$$

b) $\Gamma = V = \{v_m \mid m \in \mathbb{N}\}$

$e \vDash \Gamma \Leftrightarrow e(v_m) = 1, m \in \mathbb{N}$

$Mod(\Gamma) = \{e: V \to \{0;1\} \mid e(v) = 1 \ \forall\, v \in V\}$

Fie Δ - mulț finită de formule

$$\left.\begin{array}{l} \text{Dacă } \Delta \text{ nesatisfiabilă} \Rightarrow Mod(\Delta) = \emptyset \neq Mod(\Gamma) \\ \text{Dacă } \Delta \text{ satisfiabilă} \Rightarrow Mod(\Delta) \text{ - m. infinită} \end{array}\right| \Rightarrow$$

$\Rightarrow Mod(\Delta) \neq Mod(\Gamma) \Leftrightarrow \Gamma \not\sim \Delta$