# CURS 4

- **Model** = o evaluare $e: V \to \{0,1\}$ dacă $e^+(\varphi) = 1$ (mod. $e \models \varphi$)
- **Satisfiabilă** = $\varphi$ admite model.
- **Contradictorie** = $\varphi$ nu e satisfiabilă
- **Tautologie** = $\forall$ val. a lui $\varphi$ e model
- **Consecință semantică**:
  - $\varphi$ cons. sem. a lui $\psi$ (2 formule) $\Rightarrow$ $Mod(\psi) \subseteq Mod(\varphi)$

    unde $Mod(\varphi)$ = mulț. tuturor modelelor lui $\varphi$
  - notație: $\psi \models \varphi$
- **Echivalență**: $\varphi \sim \psi$ dacă $Mod(\varphi) = Mod(\psi)$
- $\psi \models \varphi \Leftrightarrow \models \psi \to \varphi$
- $\psi \sim \varphi \Leftrightarrow (\psi \models \varphi)$ și $(\varphi \models \psi) \Leftrightarrow \models \psi \leftrightarrow \varphi$
- $\varphi, \chi, \chi'$ formule; instanță de substituție:

  $\varphi_\chi(\chi')$ = expr. $\varphi$ obț. prin înlocuirea tuturor $\chi$ cu $\chi'$
- $\forall\ \varphi, \chi, \chi'$ formule, $\chi \sim \chi'$ implică $\varphi \sim \varphi_\chi(\chi')$
- $\varphi_1 \wedge \varphi_2 \wedge \dots \wedge \varphi_n = \bigwedge_{i=1}^{n} \varphi_i = \bigwedge\limits_{i=1}^{n} \varphi_i$
- $\varphi_1 \vee \varphi_2 \vee \dots \vee \varphi_n = \bigvee_{i=1}^{n} \varphi_i = \bigvee_{i=1}^{n} \varphi_i$

*Pentru orice formule $\varphi, \psi, \chi$,*

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|---|---|
| *terțul exclus* | $\models \varphi \vee \neg\varphi$ |
| *modus ponens* | $\varphi \wedge (\varphi \to \psi) \models \psi$ |
| *afirmarea concluziei* | $\psi \models \varphi \to \psi$ |
| *contradicția* | $\models \neg(\varphi \wedge \neg\varphi)$ |
| *dubla negație* | $\varphi \sim \neg\neg\varphi$ |
| *contrapoziția* | $\varphi \to \psi \sim \neg\psi \to \neg\varphi$ |
| *negarea premizei* | $\neg\varphi \models \varphi \to \psi$ |
| *modus tollens* | $\neg\psi \wedge (\varphi \to \psi) \models \neg\varphi$ |
| *tranzitivitatea implicației* | $(\varphi \to \psi) \wedge (\psi \to \chi) \models \varphi \to \chi$ |

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|---|---|---|
| legile lui de Morgan | $\varphi \vee \psi \sim \neg(\neg\varphi \wedge \neg\psi)$ | |
| | $\varphi \wedge \psi \sim \neg(\neg\varphi \vee \neg\psi)$ | |
| exportarea și importarea | $\varphi \to (\psi \to \chi) \sim \varphi \wedge \psi \to \chi$ | |
| idempotența | $\varphi \sim \varphi \wedge \varphi \sim \varphi \vee \varphi$ | |
| slăbirea | $\models \varphi \wedge \psi \to \varphi$ | $\models \varphi \to \varphi \vee \psi$ |
| comutativitatea | $\varphi \wedge \psi \sim \psi \wedge \varphi$ | $\varphi \vee \psi \sim \psi \vee \varphi$ |
| asociativitatea | $\varphi \wedge (\psi \wedge \chi) \sim (\varphi \wedge \psi) \wedge \chi$ | |
| | $\varphi \vee (\psi \vee \chi) \sim (\varphi \vee \psi) \vee \chi$ | |
| absorbția | $\varphi \vee (\varphi \wedge \psi) \sim \varphi$ | |
| | $\varphi \wedge (\varphi \vee \psi) \sim \varphi$ | |
| distributivitatea | $\varphi \wedge (\psi \vee \chi) \sim (\varphi \wedge \psi) \vee (\varphi \wedge \chi)$ | |
| | $\varphi \vee (\psi \wedge \chi) \sim (\varphi \vee \psi) \wedge (\varphi \vee \chi)$ | |

$$\varphi \to \psi \wedge \chi \sim (\varphi \to \psi) \wedge (\varphi \to \chi)$$
$$\varphi \to \psi \vee \chi \sim (\varphi \to \psi) \vee (\varphi \to \chi)$$
$$\varphi \wedge \psi \to \chi \sim (\varphi \to \chi) \vee (\psi \to \chi)$$
$$\varphi \vee \psi \to \chi \sim (\varphi \to \chi) \wedge (\psi \to \chi)$$
$$\varphi \to (\psi \to \chi) \sim \psi \to (\varphi \to \chi) \sim (\varphi \to \psi) \to (\varphi \to \chi)$$
$$\neg\varphi \sim \varphi \to \neg\varphi \sim (\varphi \to \psi) \wedge (\varphi \to \neg\psi)$$
$$\varphi \to \psi \sim \neg\varphi \vee \psi \sim \neg(\varphi \wedge \neg\psi)$$
$$\varphi \vee \psi \sim \varphi \vee (\neg\varphi \wedge \psi) \sim (\varphi \to \psi) \to \psi$$
$$\varphi \leftrightarrow (\psi \leftrightarrow \chi) \sim (\varphi \leftrightarrow \psi) \leftrightarrow \chi$$
$$\models (\varphi \to \psi) \vee (\neg\varphi \to \psi)$$
$$\models (\varphi \to \psi) \vee (\varphi \to \neg\psi)$$
$$\models \neg\varphi \to (\neg\psi \leftrightarrow (\psi \to \varphi))$$
$$\models (\varphi \to \psi) \to (((\varphi \to \chi) \to \psi) \to \psi)$$