CURS 1

MULȚIMI

- Relația de incluziune „$\subseteq$"
  - reflexivitate $A \subseteq A$
  - antisimetrie $\left.\begin{array}{l} A \subseteq B \\ B \subseteq A \end{array}\right| \Rightarrow A = B$
  - tranzitivitate $\left.\begin{array}{l} A \subseteq B \\ B \subseteq C \end{array}\right| \Rightarrow A \subseteq C$

- Asociativitate $\begin{cases} A \cup (B \cup C) = (A \cup B) \cup C \\ A \cap (B \cap C) = (A \cap B) \cap C \end{cases}$

- Distributivitate $\begin{cases} A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \\ A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \end{cases}$

- $(A \setminus B) \cup (A \cap B) = A$
  $A \setminus B = \emptyset \Leftrightarrow A \subseteq B$

- $M \setminus A = C_M A$ complementara lui A în M

- DeMorgan's Laws $\begin{cases} C_M(A \cup B) = C_M A \cap C_M B \\ C_M(A \cap B) = C_M A \cup C_M B \end{cases}$

- $\left.\begin{array}{l} P(M) = \{A \mid A \subseteq M\} \\ M = \{a, b, c\} \end{array}\right| \Rightarrow P(M) = \{\{\emptyset\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

- $|P(M)| = 2^{|M|}$

FUNCȚII

Def: $\left.\begin{array}{l} (A, B, G) \\ G \subseteq A \times B \\ f: A \to B \end{array}\right| \Rightarrow \forall a \in A, \exists b \in B$ a.î. $(a, b) \in G$

- $f: A \to B = B^A$

- $p_1: A_1 \times A_2 \longrightarrow A_1$
  $p_1(a_1, a_2) = a_1$ proiecția pe A_1

- Produsul cartezian al lui f_1 cu f_2

$$\left.\begin{array}{l} f_1 : A_1 \to B_1 \\ f_2 : A_2 \to B_2 \end{array}\right| \Rightarrow f_1 \times f_2 : A_1 \times A_2 \to B_1 \times B_2$$

$$(f_1 \times f_2)(a_1, a_2) = (f_1(a_1), f_2(a_2))$$

- $A \subseteq T$; χ = chi funcția carac. a lui A

$$\chi_A : T \to \{0,1\} ; \chi_A(x) = \begin{cases} 1 ; & x \in A \\ 0 ; & x \in T \setminus A \end{cases}$$

- $A, A' \subseteq T \Rightarrow A = A' \Leftrightarrow \chi_A = \chi_{A'}$

- Diferența simetrică : $A \Delta A' = (A \setminus A') \cup (A' \setminus A)$

- $\left.\begin{array}{l} f : A \to B , A' \subseteq A \\ f(A') = \{ f(a) \mid a' \in A' \} \end{array}\right| \Rightarrow f(A') \subseteq B$

ex: $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$

$f([0,2]) = [0,4]$

$f([-1,2]) = [0,4]$

$f([-3,-1]) = [1,9]$

- $B' \subseteq B$, $f^{-1}(B') = \{ a \in A \mid f(a) \in B' \}$

ex: $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$

$f^{-1}([0,1]) = [-1,1]$

$f^{-1}([-1,2]) = [-\sqrt{2}, \sqrt{2}] \Leftrightarrow a^2 \leq 2$