1. a) while $I*I \le N$ do((if $I*I=N$ then $P:=1$ else skip); $I:=I+1$))

$\sigma(N)=30$
$\sigma(I)=5$
$\sigma(P)=0$

w = while....
i = if...

<u>Big step</u>

$$\frac{e_\sigma(I*I\le N)=1 \quad (i;\, I:=I+1, \sigma) \Downarrow \sigma_2 \quad (w, \sigma_2)\Downarrow \sigma_1}{(w,\sigma)\Downarrow \sigma_1}$$

$$\frac{(i,\sigma)\Downarrow \sigma_3 \quad (I:=I+1, \sigma_3)\Downarrow \sigma_2}{(i;\, I:=I+1, \sigma)\Downarrow \sigma_2}$$

$\Rightarrow \sigma_3 = \sigma$

$$\frac{e_\sigma(I*I=N)=0,\ (skip, \sigma)\Downarrow \sigma}{(i,\sigma)\Downarrow \sigma}$$

$$\frac{}{(I:=I+1, \sigma_3)\Downarrow \sigma_3{}_{I \to e_{\sigma_3}(I+1)}} = \sigma_2$$

$\sigma_2(N)=30$
$\sigma_2(I)=6$
$\sigma_2(P)=0$

$$\frac{e_{\sigma_2}(I*I\le N)=0}{(w,\sigma_2)\Downarrow \sigma_2}$$

$\Rightarrow \sigma_1 = \sigma_2$ = stare finală $\square$

Small step

$$\frac{}{(w,\sigma) \to (\underbrace{\text{if } I*I \le N \text{ then } ((i; I:=I+1); w) \text{ else skip}}_{i'}, \sigma)}$$

$$\frac{e_\sigma(I*I \le N) = 1}{(i',\sigma) \to ((i; I:=I+1); w, \sigma)}$$

$$\frac{e_\sigma(I*I = N) = 0}{(i,\sigma) \to (\text{skip},\sigma)}$$

$$\frac{(i,\sigma) \to (\text{skip},\sigma)}{(i; I:=I+1, \sigma) \to (\text{skip}; I:=I+1, \sigma)}$$

$$\downarrow$$

$$\frac{}{(I:=I+1,\sigma) \to (\text{skip}, \underbrace{\sigma_{I \to e_\sigma(I+1)}}_{=\ \sigma_1})}$$

$\sigma_1(N) = 30$
$\sigma_1(I) = 6$
$\sigma_1(P) = 0$

$$\frac{(i; I:=I+1, \sigma) \to (\text{skip}, \sigma_1)}{((i; I:=I+1); w, \sigma) \to (\text{skip}, w, \sigma_1)}$$

$$\downarrow$$

$$(w, \sigma_1)$$

$$\frac{}{(w,\sigma_1) \to (i', \sigma_1)}$$

$$\frac{e_{\sigma_1}(I*I \le N) = 0}{(i',\sigma_1) \to (\text{skip},\sigma_1)} \quad \square$$

b) $\{I = 0 \land \exists k(N = k*k)\}$ Pgm $\{P = 1\}$

$\{A\}$ w $\{A \land \neg(I*I \leq N)\}$ $\models \{I = 0 \land \exists k(N = k*k)\} \to \{A\}$
$\models \{A \land \neg(I*I \leq N)\} \to \{P = 1\}$

$\{A \land (I*I \leq N\}$ (i; $I := I+1$) $\{A\}$

$\Rightarrow \{A \land (I*I \leq N\}$ i $\{B\}$ $\Rightarrow$ $\{A \land (I*I \leq N) \land (I*I = N)\}$ ($P := 1$) $\{B\}$ (a)
$\{A \land (I*I \leq N) \land \neg(I*I = N)\}$ (skip) $\{B\}$ (b)

$\{B\}$ ($I := I+1$) $\{A\}$

$B = A[I := I+1]$

(a)
$\models \{A \land (I*I \leq N) \land (I*I = N)\} \to \{B[P := 1]\}$
$\models \{B\} \to \{B\}$

(b)
$\models \{A \land (I*I \leq N) \land \neg(I*I = N)\} \to \{B\}$

$A = \{((I-1)*(I-1) = N)\}$

2. $\{h(x,y,f(g(x),g(y))) = h(y,x,f(y,x))\}$

$\{\cancel{x=y},\ y=x,\ f(g(x),g(y)) = f(y,x)\}$

$\Rightarrow \{\cancel{y=y},\ f(g(y),g(y)) = f(y,y)\}$

$\Rightarrow \{\cancel{g(y)=y},\ g(y)=y\}$

$\Rightarrow \{y = g(y),\ y = g(y)\}$

$\Rightarrow$ "Eșec" (y este in mulțimea var. lui g(y))

3. $\text{shuffle}(\underset{\text{arb}(A,B,C)}{\overset{}{X}} ,\ \text{arb}(\underbrace{\text{arb}(\text{lit}(t), \text{lit}(c), \text{lit}(u))}_{T},\ \underbrace{\text{arb}(\text{lit}(i), \text{lit}(a), \text{lit}(t))}_{U},\ \underbrace{\text{arb}(\text{lit}(e), \text{lit}(t), \text{lit}(r))}_{W}))$

▷ shuffle(A, arb(lit(i), lit(a), lit(t))),
shuffle(B, arb(lit(e), lit(t), lit(r))),
shuffle(C, arb(lit(t), lit(c), lit(u)))

$A = \text{arb}(A_1, A_2, A_3)$
B = ...
C = ...

▷ shuffle(A_1, lit(a)), shuffle(A_2, lit(t)), shuffle(A_3, lit(i)),
shuffle(B_1, lit(t)), ...
....

▷ □

A_1 = lit(a), A_2 = lit(t), ...

X = arb(A, B, C) = arb(arb(lit(a), lit(t), lit(i)),
arb(lit(t), lit(r), lit(e)),
arb(lit(c), lit(u), lit(t)))

4. $t := \lambda r.(\lambda e.((\lambda d.d)(er)))$.

$M = \lambda r{:}R.(\underbrace{\lambda e{:}E.(\overbrace{(\lambda d{:}D.d)(er)}^{M_3})}_{M_2})$

$r_M := \{x : X \mid x \in FV(M)\}$

$c(M, r_M, A) = c(M_2, r_{M_1} = r_M \cup \{r:R\}, A_1) \cup \overbrace{\{A = R \to A_1\}}^{k_1} =$

$= c(M_3, r_{M_2} = r_{M_1} \cup \{e:E\}, A_2) \cup \underbrace{k_1 \cup \{A_1 = E \to A_2\}}_{k_2} =$

$= c(\lambda d{:}D.d, r_{M_2}, B_1) \cup c(er, r_{M_2}, B_2) \underbrace{\cup k_2 \cup \{B_1 = B_2 \to A_2\}}_{k_3} =$

$= c(d, r_{M_2} \cup \{d:D\}, C_1) \cup \{B_1 = D \to C_1\}$

$\cup\, c(e, r_{M_2}, C_2) \cup c(r, r_{M_2}, C_3) \underbrace{\cup \{C_2 = C_3 \to B_2\} \cup k_3}_{k_4} =$

$= \{D = C_1\} \cup \{E = C_2\} \cup \{R = C_3\}$

$\cup \{A = R \to A_1\} \cup \{A_1 = E \to A_2\} \cup \{B_1 = B_2 \to A_2\} \cup \{B_1 = D \to C_1\}$
$\cup \{C_2 = C_3 \to B_2\}$

$= \{A = C_3 \to A_1\} \cup \{A_1 = C_2 \to A_2\} \cup \{B_1 = B_2 \to A_2\}$

$\cup \{B_1 = C_1 \to C_1\} \cup \{C_2 = C_3 \to B_2\}$

$C_1 = B_2 = A_2$

$\Rightarrow \{A = C_3 \to (C_2 \to C_1)\} \cup \{B_1 = C_1 \to C_1\} \cup \{C_2 = C_3 \to C_1\} =$

$= \{A = C_3 \to ((C_3 \to C_1) \to C_1)\} \cup \{B_1 = C_1 \to C_1\}$

5. Dem. in curs 3