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Examen

① while $\underbrace{\neg(B=0)}_{b}$ do if $B \leq A$ then $A := A-B$ else $\{C := A;\ A := B;\ B := C\}$

a) $\sigma(A)=0$, $\sigma(B)=6$ big step sau small-step

$\nabla(\neg(B=0)) = \neg(6=0) = 1$

$\sigma(b)=1$ ($\overbrace{\text{if } B \leq A \text{ then } A := A-B \text{ else } \{C := A;\ A := B;\ B := C\}}^{c}$) $\Downarrow$

$\Downarrow \sigma_3$ (while b do c, σ_3)

(while b do c, σ) $\Downarrow \sigma_2$

$\sigma_2 = A \overset{c=0}{=} 6$

$B \to C$

$\sigma(b)=1$ (if $B \leq A$ then $A := A-B$ else $\{C := A;\ A := B;\ B := C\}$) $\Downarrow$

$\Downarrow \sigma_3$ (while b do c, σ_3)

(while b do c, σ) $\Downarrow \sigma_1$

$\sigma_1 \Rightarrow A = 6-C$

$\sigma(b)=1$ (if $B \leq A$ then $A := A-B$ else $\{C := A;\ A := B;\ B := C\}$) $\Downarrow \sigma_3$ while b do c

(while b do c, σ) $\Downarrow \sigma_4$

$\sigma(b) = \neg$ (if $B \leq A$ then $A := A-B$ else $\{C := A;\ A := B;\ B := C\}$) $\Downarrow \sigma_3$ while b do c, σ_3

(while b do c, σ) $\Downarrow \sigma_2$

$A \Rightarrow C$
$B \rightarrow 6$
$C \rightarrow 6-C$

~~while~~ $\sigma(b)=1$ (if $B \leq A$ then $A := A$ ~~Bdc~~ $\{C := A; B := B; B := C\}$) ⇓ σ_4 while b do c, σ_5

(while b do c, σ) ⇓ σ_2

$A \rightarrow C-6$

$\sigma(b)=1$ (if $B \leq A$ ....) ⇓ σ_5 (while b do c, σ_6)

while b. do c, σ ⇓ σ_2

$C \rightarrow C-6$
$A \rightarrow 6$
$B \rightarrow 6-C$

$\sigma(b)=1$ (if $B \leq A$ ....) ⇓ σ_5 (..... σ_6)

while b do c, σ ⇓ σ_2

$A \rightarrow 6-6+C$
$A \rightarrow C$

$\sigma(b)=1$ (if $B \leq A$ ....) ⇓ σ_6 (..... , σ_6)

while b do c, σ ⇓ σ_2

$C \Rightarrow C$
$A \rightarrow 6-C$
$B \rightarrow C-6$

(nu vei găsi $B=0$)
o iau iar de la capăt

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① a) $\sigma(A)=0$, $\sigma(B)=6$ ss

~~while b do~~

$\sigma(b)=1$ (if $B \le A$ then $A:= \dots$), σ , $\sigma = \{A \to 0, B \to 6\}$

while <if ~~$\neq$~~ $\sigma(b)=1$) then (if $B \le A$ then $A:=A-B$ else $\{\dots\}$;
while $\sigma(b)=1$ do if $B \le A$ then $A:=A-B$ else $\{C:=A \dots\}$ else skip σ
id → <if $\sigma(b)=1$ then (if $\dots\{\dots\}\dots$); while ... , σ >
leg true → <if $\sigma(b)=1$ then (if ...); while ... , σ >
if true → <if $B \le A$ then $A:=A-B$ else $\{\dots\}$; while ... , σ >
id → <if $B \le A$ then ——————————————>
id → <if ~~$6 \le 0$~~ $6 \le 0$ then $A:=A-B$ else $\{\dots\}$, while ... , σ >
leg false → <if false then $A:=A-B$ else $\{\dots\}$ while ... , σ >
if false → < else $\{C:=A; A:=B; B:=C\}$; while ... , σ >
~~$\sigma[C \to 0]$~~ $C \to 0 ; A \to 6 ; B \to 0 \Rightarrow \sigma'$ >
<while $\sigma(b)=1$ do if $B \le A$ then $A:=A-B$ else $\{\dots\}$, else skip, σ >
id → <while $\neg(0=0)$ ... else skip , σ' >
leg false → <while false ... else skip , σ' >
if false → <skip , σ' >

d) conform subpct-ului a) am arătat pentru 2 nr nat A și B
$\forall A, B \in \mathbb{N} \Rightarrow$ execuția programului se va termina

② $\{ f(x, g(y)) = f(h(z), g(h(a))),\ y = h(z) \}$,
$\{ f(x, a) = f(g(y), y),\ y = f(x, x) \}$

1) $f(x, g(y)) = f(h(z), g(h(a))) \xrightarrow{\text{descomp.}} x = h(z),\ g(y) = g(h(a))$

$\ell = \{ x = h(z),\ g(y) = g(h(a)),\ y = h(z) \}$

$g(y) = g(h(a)) \xrightarrow{\text{descomp}} y = h(a)$

$\ell = \{ x = h(z),\ y = h(a),\ y = h(z) \}$

$\ell = \{ x = y,\ y = h(a) \}$ (rezolvă)

$\ell = \{ x = h(a) \}$ (rezolvă)

$\ell = \{ x \mapsto h(a),\ y \mapsto x,\ h(x) \mapsto x \}$ soluție.

2) $\ell = \{ f(x, a) = f(g(y), y),\ y = f(x, x) \}$

$f(x, a) = f(g(y), y),\ y = f(x, x)$ (descomp)

$x = g(y),\ y = a,\ y = f(x, x)$ (rezolvă)

$y = a,\ \underbrace{y = f(g(y), g(y))}_{A}$ loop pt că pentru A nu se unifică

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③

$p(X, f(Y)) :- g(X), g(Y)$ ținta $(?-) p(Y, X)$

$g(f(X)) :- r(X)$

$r(a)$

$C_1 \Rightarrow p(X, f(Y)) \vee \neg g(X) \vee \neg g(Y)$

$C_2 \Rightarrow g(f(x)) \vee \neg r(x)$

$C_3 \Rightarrow r(a)$

$G_0: \neg p(Y, X)$ $\quad C_1: p(X_0, f(Y_0)) \vee \neg g(X_0) \vee \neg g(Y_0)$

$\theta_0 = \{Y \mapsto X_0 ; X \to f(Y_0)\}$

$G_1: \underline{\neg g(X_0)} \vee \neg g(Y_0)$ $\quad C_2: g(f(X_1)) \vee \neg r(X_1)$

$\theta_1 = \{X_0 \mapsto f(X_1)\}$

$G_2: \underline{\neg r(X_1)} \vee \neg g(Y_0)$ $\quad C_3: r(a)$

$\theta_2 = \{X_1 \mapsto a\}$

$G_3: \underline{\neg g(Y_0)}$ $\quad C_2: g(f(X_3)) \vee \neg r(X_3)$

$\theta_3 = \{Y_0 \mapsto f(X_3)\}$

$G_4: \underline{\neg r(X_3)}$ $\quad C_3: r(a)$

$\theta_4 = \{X_3 \mapsto a\}$

$G_5: \square$

$\theta = \theta_0 \cdot \theta_1 \circ \theta_2 \cdot \theta_3 \circ \theta_4$

$X \mapsto f(Y_0)$

$Y \mapsto f(X_1)$

④ $M \equiv x(\lambda z.(y\,z))$ ~~$\rho = \lambda z.(y\,z)$~~

$N \equiv a(x\,y)(x\,(y\,z))$

$M \equiv \lambda z\,x.((y\,z)\,x)$

$M' \Rightarrow \lambda z : Z.\ \lambda x : X\,(x\,(y\,z))$ Γ_4

$\mathcal{C}(\lambda z : Z.\ \lambda z : X.\ (x\,(y\,z)),\ \Theta,\ T_1) =$

$= \mathcal{C}(\lambda x.\ x.(x(y\,z)),\ \{z : z\},\ T_2) \cup \{T_1 = z \to T_2,\ T_2 \to X \to T_3\}$

$= \mathcal{C}(\lambda,\ \{z : z, x : X\},\ T_4) \cup \mathcal{C}(y\,z;\ \{z : z, x : X\},\ T_5) \cup$

$\{T_1 = z \mapsto T_2,\ T_2 = X \mapsto T_3,\ T_4 = T_5 \to T_3\}$

$= \mathcal{C}(x,\ \{z : z, x = X\},\ T_4) \cup \mathcal{C}(y\,\{z : z, x : X\},\ T_6) \cup$

$\mathcal{C}(z,\ \{z : z, x : X\},\ T_5) \cup \{T_1 = z \to T_2,\ T_2 = X \mapsto T_3,$

$T_4 = T_5 \to T_3,\ T_6 = T_4 \to T_4\}$

$= \mathcal{C}\{y\}\{z : z, x : X\},\ T_6) \cup \mathcal{C}(z,\ \{z : z\ x : X\},\ T_7) \cup$

$\mathcal{C}(z,\ \{z : z, x : X\},\ T_5) \cup \{T_1 = z \to T_2,\ T_2 = X \to T_3,\ T_5 =$

$T_8 \to T_3,\ T_6 = T_4 \to T_4,\ X = T_4,\ y = T_6$

$\{T_4 = z \to T_2,\ T_2 = X \to T_3,\ T_4 = T_5 \to T_3,\ T_6 = T_7 \to T_5,\ x = T_4,$

$y = T_6$

$z = T_7\}$

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① b) $X, N \in V$

$\{B = X\}\ P_{gcm}\ \{\exists N\ (N * A = X)\}$

$B = 6 \Rightarrow B = X = 0,\ A = 0$

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după

$C \to 0$
$A \to 6$
$B \to 0$

$A \to 6,\ B \to 0,\ C \to 0$

$X \to 0,\ A \to 6 \to$ ~~$\exists N\ (N \cdot A) = X\ (1 \cdot 6$~~ $\exists N\ (N \cdot 6 = 0) \Rightarrow$

$\Rightarrow \{B = x\}\ P_{gcm}\ \{\exists N\ (N * A = X)\}$

↳ dacă e adv. înainte de execuția instr. Pgcm, atunci $\{\exists N\ (N * A = X)\}$ e adv. după

Triplet Hoare