CURS 3

Operation algebraica imterna

- Operatie algebrica interna = lege de compositée
 - $\varphi \colon M \times M \longrightarrow M \ \circ \ (\times , y) \longrightarrow \varphi (\times , y)$
 - ex: $S: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ S(x : y) = x + (-y) = x - y peadwise algebries
 - ex: $\mathcal{F}(M) := \mathcal{F}(\mathcal{F}(M)) := \mathcal{F}(M) := \mathcal{F}(M$
 - $\begin{array}{lll} \{x: \mathcal{P}(H) = \{x \mid X \subseteq H\} \\ & (X, X) \longrightarrow X \cup Y : S : X, Y \in \mathcal{J}(H) \\ & (X, X) \longrightarrow X \cap Y : X, Y \in \mathcal{J}(H) \\ & \vdots \\ & (X, X) \longrightarrow X \cap Y : X, Y \in \mathcal{J}(H) \\ \end{array}$
- · 4: M× H → M operate alg. ; P(x,y) = x * y = x ∘ y = x + y
- Societa aditiva: 4(x,y) = x+y
- · Serierea multiplicativa: P(x,y) = xy.
- Asociativitatea: P: M x M → M , M ≠ Ø
 ∀ x,y, 2 ∈ M ⇒ P(x, P(y, z)) = P(P(x,y), z)
- · Comutativitatea: P: M × M → M , M ≠ Ø . Y(x,y) = Y(y,x)
- Element meutre: Ψ: M × M → M → M → M ≠ Ø. ∃ e ∈ M a· ĉ. Ψ(x, e) = Ψ(e,x) = X
 - Element simetrizabil: PH×H→H, M≠Ø ∃ X ∈ H a ? P(x,x') = P(x', x) = e
- Fix M≠¢, φ: M×M→M, ance. n' e=el. meutre => el. sim. unic
- [a] ∈ Zm este imversabil (clase) <=> a e prism eu m
 - $\mathbb{Z}_m = \{ [0], [0], \dots [m-1] \}$

Hamai zi
Manaid = mult nevida, asse, el mutru (+ comutatio)
ex (Fin), 0) marroid, F(n) = 3 f. M-> M3 (P(n), 50) sau (Pm), n) marroin comulation
Morfism de monoini : (malliplicatio)
$H, N = momoire : f: M \rightarrow N = a-2.$ • $f(x,y) = f(x) \cdot f(y) + x, y \in M$ • $f(e) = e^2$, which $e = el m \cdot M \cdot s^2 \cdot e^2 = el \cdot m \cdot N$
(3(H), 0) & (3(H), 0), g (3(H), 0) -> (P(H), 0) (8(X) = CHX (9(X 0Y) = CH(X 0Y) = CHX U CHY = g(X) U g(Y) (9(H)) > CHH = Ø
· Compunisce de mangiti.
M, M, P, momoitis S:M >N, S:N >P moissisme de monoité
(gof) (xy) = g(f(xy)) = g(f(x).f(y)) = g(f(x)).g(f(y)) = (gof)(x).(gof).(y)
$(g \circ g)(e) = g(g(e)) = g(e') = e''$
· Compunizer morf. de moin. « asec.
· Compunizea most de moin. e asac. · M-monoid, Sid 14 a mult. M e moisfism de manoité
Compunized morf de moin, e asse. M-morroid, Sid the a mult. Me morfism de marioité x, y ∈ M => 1 M(xy) = xy = 1 M(x). 1 M(y) or 1 M(e) = e
Compunizea mosf de moin. e asse. M-monoid, Sid 1m. a mult. M e mosfrom de manoité x, y ∈ M => 1m(xy) = xy = 1m(x) · 1m(y) » · 1m(e) = e 1 tamosfrome de mospoité.
Compuniza most de moin. e asse. M-monoid, Sid 14 a mult. Me mostism de monoité x, y ∈ M => 14(xy) = xy = 14(x). 14(y) m 14(e) = e 120 mostisme de mosnoité g: M->N mostism de mosnoité => i20 mossilism de mosnoité
• Compunized most de moin. e asse. • M -imanoid, Sid in a mult. M e marfish de manoité ×, $y \in M = > 1_{M}(xy) = xy = 1_{M}(x) \cdot 1_{M}(y)$ or $1_{M}(e) = e$ • Remarfishe de manoité $g: M \to M$ a.c. $g: g = 1_{M}$ si $g: g = 1_{M}$ $f: g: g: M$ a.c. $f: g: M \to M$ a.c. $f: M \to M$ a.c.
Compuniza most de moin. e asse. M-monoid, Sid 14 a mult. Me mostism de monoité x, y ∈ M => 14(xy) = xy = 14(x). 14(y) m 14(e) = e 120 mostisme de mosnoité g: M->N mostism de mosnoité => i20 mossilism de mosnoité

·HH-momory ? HON <=> HOW

· M RN > N Rb ⇒ WRb

• 1: M → N marfism de manaîze → J. izo. <=? g bij

• Produs direct al momoisilor M, si M2 momoiri comutativi. =)

=> M = manaid comutatio, unde M= M1 × M2

(x, 2xx) (y, 42) = (x, 4, 3 x242)

• M = TT Mi siel ; (xi)iel (yi)iel = (xiyi)iel.

· Couvaint de elemente = son sist. finit din A = 9,029. az

. Multimea L(A) a cur de el, → (multiplicativ) d, β:

de = a, az ... ax b, bz bs (asoc) over el. m. eurântel , vid"

format din submult vida a lui A => L(A) manaid liber

. Daca A= Sag => manoidul liber L(A) ≅ IV. (adibiv.)

Tie A cu L(A) mon liber =) unic mary. $J:L(A). \rightarrow M$ a: $L(A) \rightarrow M$ a: $L(A) \rightarrow M$ a: $L(A) \rightarrow M$ a: $L(A) \rightarrow M$ for L(A) = 0

(propr. de arriverente de morn. liber).

T) Cordar.

A, A' multimer => g: A > A' & by. => 3 unie izamarfism de

momoiti $\overline{J}: L(A) \rightarrow L(A')$ a.î. $\overline{J} \circ i_A = i_A \circ J$, winde $i_A : A \rightarrow Z(A)$

e o induraime comanica a lui A in L(A), jour ix: A'→L(A').

e incluriumes canonica a lui A' in L(A').