Tema 2

① Fie spațiul eucl. $E_3 = (\mathbb{R}^3/\mathbb{R}, <,>)$, cu $B_0 = \{e_1, e_2, e_3\} \subset E_3$ bază canonică

Stabiliți dacă urm. apl. lin. sunt transformări ortogonale

b) $T: E_3 \to E_3$

$$\begin{cases} T(e_1) = e_1 + e_2 \\ T(e_2) = e_2 + e_3 \\ T(e_3) = e_3 + e_1 \end{cases} \Rightarrow \mathcal{T} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$ mat asoc. lui T în rap cu B_0

Avem B_0 bază ortog.

$$\mathcal{T} \cdot {}^t\mathcal{T} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \neq I_3 \Rightarrow$$

$\Rightarrow \mathcal{T} \notin O(3) \Rightarrow T$ nu e transf. ortog.

c) $T: E_3 \to E_3$

$$\begin{cases} T(e_1) = \frac{2}{3} e_1 + \frac{2}{3} e_2 - \frac{1}{3} e_3 \\ T(e_2) = \frac{2}{3} e_1 - \frac{1}{3} e_2 + \frac{2}{3} e_3 \\ T(e_3) = -\frac{1}{3} e_1 + \frac{2}{3} e_2 + \frac{2}{3} e_3 \end{cases}$$

Avem B_0 b. ortog.

$$\mathcal{T} = \begin{pmatrix} 2/3 & 2/3 & -1/3 \\ 2/3 & -1/3 & 2/3 \\ -1/3 & 2/3 & 2/3 \end{pmatrix}$$ mat. asoc. lui T în rap. cu B_0

$$\mathcal{T} \cdot {}^t\mathcal{T} = \begin{pmatrix} \frac{2}{3} & 2/3 & -1/3 \\ \frac{2}{3} & -1/3 & 2/3 \\ -1/3 & 2/3 & 2/3 \end{pmatrix} \begin{pmatrix} 2/3 & 2/3 & -1/3 \\ 2/3 & -1/3 & 2/3 \\ -1/3 & 2/3 & 2/3 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{8}{9}+\frac{1}{9} & \frac{4}{9}-\frac{2}{9}-\frac{2}{9} & -\frac{2}{9}+\frac{4}{9}-\frac{2}{9} \\ \frac{4}{9}-\frac{2}{9}-\frac{2}{9} & \frac{8}{9}+\frac{1}{9} & -\frac{2}{9}-\frac{2}{9}+\frac{4}{9} \\ -\frac{2}{9}+\frac{4}{9}-\frac{2}{9} & -\frac{2}{9}\cdot 2+\frac{4}{9} & \frac{8}{9}+\frac{1}{9} \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I_3 \Rightarrow \sigma \in O_3 \Rightarrow T \text{ e transf. ortog.}$$