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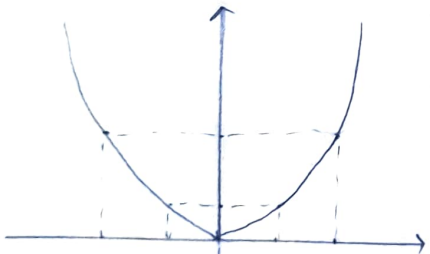
grupa 132

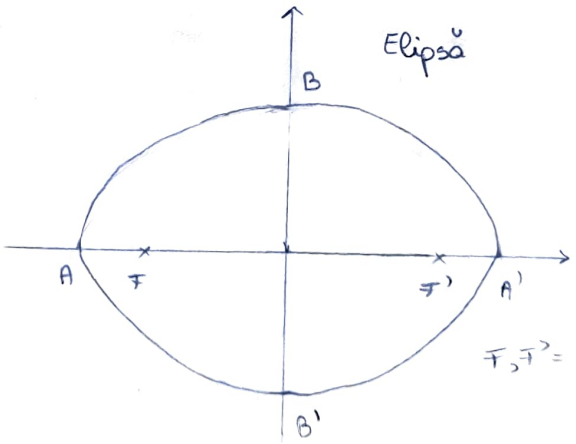
Conice

O conică este curba care se obține prin intersectarea unui plan cu un con.

Secțiune conică	Ecuație
cerc	$x^2 + y^2 = r^2$
elipsă	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
parabolă	$y^2 = 4ax$
hiperbolă	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



Parabolă: $y = ax^2 + bx + c$

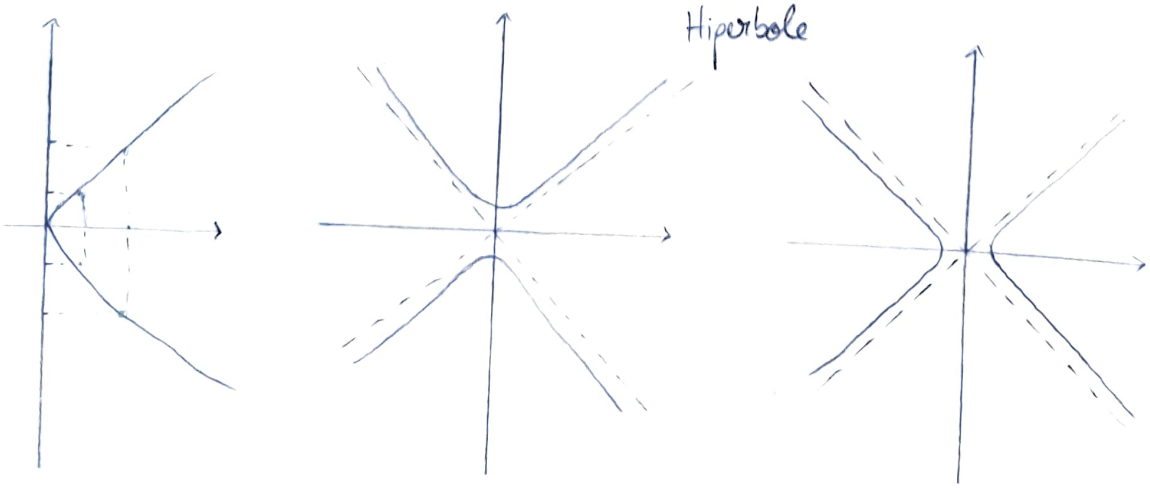


F, F' = pct focalizare

Hiperbole

Parabolă

$y^2 = 2ax$,
$a > 0$



Caz general: $\Gamma = \{(x_1, \dots, x_m) \in E_m \mid f(x_1, \dots x_m) = 0\}$

unde $f : E_m \to \mathbb{R}$, $f(x_1, \dots x_m) = \sum_{i,j=1}^{m} a_{ij} x_i x_j +$

$+ 2 \sum_{i=1}^{m} b_i x_i + c$

$a = (a_{ij})_{i,j=\overline{1,m}}$; $a_{ij} = a_{ji}$ $\forall i,j = \overline{1,m}$ (mat. sim.)

Pentru: $(\mathbb{R}^2, \mathbb{R}^2/\mathbb{R}, \varphi)$ ~~$\mathbb{HR}^2$~~

Γ $f(x) = X^T A X + 2BX + c = 0$

$f(x) = a_{11} x_1^2 + 2a_{12} x_1 x_2 + a_{22} x_2^2 + 2b_1 x_1 + 2b_2 x_2 + c = 0$

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = A^T$, $\tilde{A} = \begin{pmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ b_1 & b_2 & c \end{pmatrix}$; $e = \frac{c}{a}$

$B = (b_1 \ b_2)$ $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$\delta = \det A$, $\Delta = \det \tilde{A}$, $r = \operatorname{rg} A \geq 1$, $r' = \operatorname{rg} \tilde{A}$

I. $\delta \neq 0 \Rightarrow$ conică cu centru unic

II. $\delta = 0 \Rightarrow \Gamma$ nu are centru unic

$\mathcal{R} = \{O; e_1, e_2\} \xrightarrow[\text{transf. afine}]{} \mathcal{R}' = \{O; e_1', e_2'\} \xrightarrow[\text{translație}]{} \mathcal{R}'' = \{P; e_1', e_2'\}$

$Q : \mathbb{R}^2 \to \mathbb{R}$, $Q(x) = X^T A X$

$\theta : X = CX'$, $C \in GL(2, \mathbb{R})$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} \Rightarrow \begin{cases} x_1 = c_{11} x_1' + c_{12} x_2' \\ x_2 = c_{21} x_1' + c_{22} x_2' \end{cases}$

$Q(x) = \lambda_1 x_1'^2$, $\lambda_1 \neq 0$ $\begin{pmatrix} \lambda_1 & 0 \\ 0 & 0 \end{pmatrix}$ $\lambda_2 = 0$

$\theta(\Gamma)$: $\lambda_1 x_1'^2 + 2b_1 (c_{11} x_1' + c_{12} x_2') + 2b_2 (c_{21} x_1' + c_{22} x_2') + c = 0$

$\theta(\Gamma)$: $\lambda_1 x_1'^2 + 2 \underbrace{(b_1 c_{11} + b_2 c_{21})}_{b_1'} x_1' + 2 \underbrace{(b_1 c_{12} + b_2 c_{22})}_{b_2'} x_2' + c = 0$

$\lambda_1 x_1'^2 + 2b_1' x_1' + 2b_2' x_2 + c = 0$

$\Delta = \det \begin{pmatrix} \lambda_1 & 0 & b_1' \\ 0 & 0 & b_2' \\ b_1' & b_2' & c \end{pmatrix} = -\lambda_1 b_2'^2$

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a) $\Delta \neq 0 \Rightarrow$ r conică nedegenerată

$b_2' \neq 0$

$\Theta(r): \lambda_1\left(x_1'^2 + \frac{2b_1'}{\lambda_1}x_1' + \frac{b_1'^2}{\lambda_1^2}\right) + 2b_2'x_2' + c' = 0$

$c' = c - \frac{b_1'^2}{\lambda_1}$

$\lambda_1\left(x_1' + \frac{b_1'}{\lambda_1}\right)^2 + 2b_2'\left(x_2' + \frac{c'}{2b_2'}\right) = 0$

Fie $x_1'' = x_1' + \frac{b_1'}{\lambda_1}$ și $x_2'' = x_2' + \frac{c'}{2b_2'}$

Considerăm translația $\mathcal{T}: X' = X'' + X_0$, $X_0 = \begin{pmatrix} -\frac{b_1'}{\lambda_1} \\ -\frac{c'}{2b_2'} \end{pmatrix}$

$\mathcal{T}(\Theta(r)): \lambda_1 x_1''^2 + 2b_2' x_2'' = 0$ parabolă

b) $\Delta = 0 \Rightarrow$ r degenerată, $b_2' = 0$

$\Theta(r) = \lambda_1 x_1'^2 + 2b_1'x_1' + c = 0$

$\lambda_1\left(x_1' + \frac{b_1'}{\lambda_1}\right)^2 + c' = 0$

Fie $x_1'' = x_1' + \frac{b_1'}{\lambda_1}$ și $x_2'' = x_2'$

Considerăm translația $\mathcal{T}: X' = X'' + X_0$, $X_0 = \begin{pmatrix} -\frac{b_1'}{\lambda_1} \\ 0 \end{pmatrix}$

$\left.\begin{array}{l} \Theta: X = RX' \\ \mathcal{T}: X' = X'' + X_0 \end{array}\right| \Rightarrow X = RX'' + RX_0 \Rightarrow RX_0 = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \Rightarrow P(\alpha, \beta)$ în raport cu reperul canonic $\mathcal{R}$

Tabel invarianți Δ și δ

Δ natura	δ genul	Tipul conicii
$\Delta \neq 0$	$\delta > 0$	Elipsă, $\emptyset$
	$\delta < 0$	Hiperbolă
	$\delta = 0$	Parabolă
$\Delta = 0$	$\delta > 0$	Punct dublu
	$\delta < 0$	Drepte concurente
	$\delta = 0$	Drepte confundate, $\parallel$, $\emptyset$