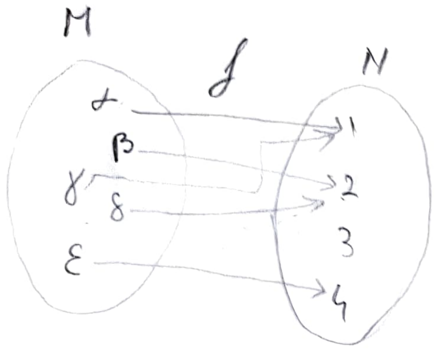
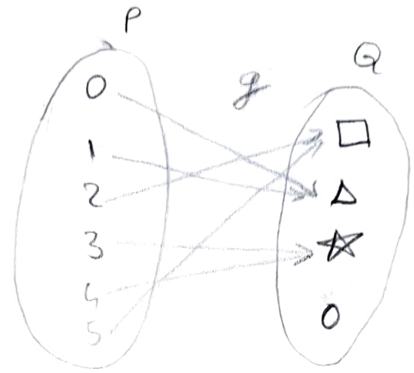
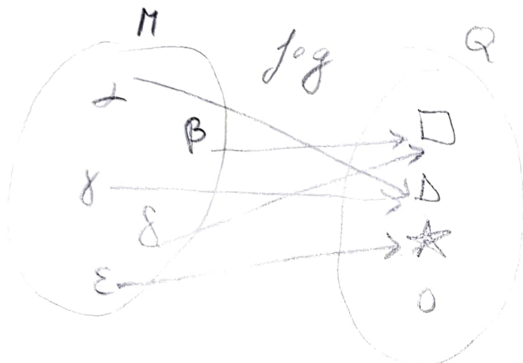
Str. Alg. în Info

10.10.2023

Seminar 2







$f\circ g : M \to Q$

$(f\circ g)(\alpha) = g(f(\alpha)) = \triangle$

$(f\circ g)(\beta) = g(f(\beta)) = \square$

• Cele de mai sus se caracterizează așa:

$f: A\to B$, $g: C \to D$, $B \subset C$

$g\circ f: A \to D$, $(g\circ f)(x) = g(f(x))\ \forall x\in A$ compusa

$\underbrace{A \xrightarrow{f} B \subset C \xrightarrow{g} D}_{g\circ f}$ condiția să se poată face $g\circ f$

ex: $f: \mathbb{N} \to \mathbb{Z}$, $f(x) = 5-3x$

$g: \mathbb{Q}\to\mathbb{Q}$, $g(x) = \frac{x}{x^2+1}$

$g\circ f \Rightarrow \mathbb{N} \xrightarrow{f} \mathbb{Z} \subset \mathbb{Q} \xrightarrow{g} \mathbb{Q}$

dar $f\circ g \Rightarrow \mathbb{Q}\to\mathbb{Q} \not\subset \mathbb{N}\to\mathbb{Z}$ nu se poate

Îm plus, $(g\circ f)(h)$ → citește „h” var. preferată $= g(f(x)) =$

$= \frac{f(x)}{f(x)^2+1} = \frac{5-3x}{(5-3x)^2+1} = \frac{5-3x}{9x^2-30x+26}$

① $f: \mathbb{R}\to\mathbb{R}$, $f(x) = 7-3x$

$g: \mathbb{R}\to\mathbb{R}$, $g(x) = \begin{cases} x^2+1, & x>3 \\ 2-x, & x\le 3 \end{cases}$

$h: \mathbb{R} \to \mathbb{R}, \ h(x) = \begin{cases} 5x-2, & x > -1 \\ x+1, & x \le -1 \end{cases}$

a) $g \circ f : \mathbb{R} \to \mathbb{R}$

$$(g \circ f)(x) = g(f(x)) = \begin{cases} f^2(x) + 1, & f(x) > 3 \\ 2 - f(x), & f(x) \le 3 \end{cases} = \begin{cases} 9x^2 - 42x + 50, & 7 - 3x > 3 \\ 3x - 5, & 7 - 3x \le 3 \end{cases} =$$

$$= \begin{cases} 9x^2 - 42x + 50, & x < \frac{4}{3} \\ 3x - 5, & x \ge \frac{4}{3} \end{cases}$$

b) $f \circ g : \mathbb{R} \to \mathbb{R}$

$$(f \circ g)(x) = f(g(x)) = 7 - 3 \cdot g(x) = \begin{cases} 7 - 3x^2 - 3, & x > 3 \\ 7 + 3x - 6, & x \le 3 \end{cases} = \begin{cases} 4 - 3x^2, & x > 3 \\ 1 + 3x, & x \le 3 \end{cases}$$

c) $h \circ g : \mathbb{R} \to \mathbb{R}$

$$(h \circ g)(x) = h(g(x)) = \begin{cases} 5 \cdot g(x) - 2, & g(x) > -1 \\ g(x) + 1, & g(x) \le -1 \end{cases} = \begin{cases} 5(x^2+1) - 2, & \begin{cases} x^2 + 1 > -1 \\ x > 3 \end{cases} \\ 5(2-x) - 2, & \begin{cases} 2 - x > -1 \\ x \le 3 \end{cases} \\ (x^2 + 1) + 1, & \begin{cases} x^2 + 1 \le -1 \\ x > 3 \end{cases} \\ (2-x) + 1, & \begin{cases} 2 - x \le -1 \\ x \le 3 \end{cases} \end{cases}$$

$$= \begin{cases} 5(x^2+1) - 2, & \begin{cases} x^2 > -2 \\ x > 3 \end{cases} \Rightarrow x > 3 \\ 5(2-x) - 2, & \begin{cases} -x > -3 \\ x \le 3 \end{cases} \Rightarrow x < 3 \\ (x^2+1) + 1, & \begin{cases} x^2 \le -2 \\ x > 3 \end{cases} \Rightarrow \emptyset \\ (2-x) + 1, & \begin{cases} -x \le -3 \\ x \le 3 \end{cases} \Rightarrow x = 3 \end{cases} = \begin{cases} 5x^2 + 3, & x > 3 \\ 8 - 5x, & x < 3 \\ 0, & x = 3 \end{cases}$$

$\boxed{\text{TEMĂ:}}$ $g \circ h$

- O funcție s.n. inj. dacă duce elem. dif. în elem. dif. (? asa ne-a zis)
- $f: A \to B$ s.n. inj. dacă $\forall a_1, a_2 \in A$; $a_1 \ne a_2 \Rightarrow f(a_1) \ne f(a_2)$

Prop: $f: A \to B$ e inj. $\Leftrightarrow \forall a_1, a_2 \in A$; $f(a_1) = f(a_2) \Rightarrow a_1 = a_2$

- O funcție s.n. surj. dacă ea își umple codomeniul.
- $f: A \to B$ s.n. surj. dacă $\forall\, b \in B$, $\exists\, a \in A$ a.î. $f(a) = b$

- surj + inj $\Leftrightarrow$ bij. $\Leftrightarrow$ inv.

① $f, g: \mathbb{R} \to \mathbb{R}$, $f(x) = \begin{cases} x^2+1, & x<0 \\ 1-x, & x \geq 0 \end{cases}$, $g(x) = \begin{cases} x^2+1, & x \geq 0 \\ 1-x, & x<0 \end{cases}$

Fie $x_1, x_2 \in \mathbb{R}$, $x_1 \neq x_2$

Considerăm $x_1 < x_2$

I $x_1 < x_2 < 0$

$-x_1 > -x_2 > 0 \Leftrightarrow x_1^2 > x_2^2 > 0 \Leftrightarrow$

$\Leftrightarrow x_1^2+1 > x_2^2+1 > 1 \Leftrightarrow f(x_1) > f(x_2) > 1 \Rightarrow$

$\Rightarrow f(x_1) \neq f(x_2) \Rightarrow$ inj. (1)

II $x_1 < 0 \leq x_2$

$\left.\begin{array}{l} f(x_1) = x_1^2+1 > 1 \\ f(x_2) = 1-x_2 \leq 1 \end{array}\right| \Rightarrow f(x_1) > f(x_2) > 1 \Rightarrow f(x_1) \neq f(x_2) \Rightarrow$ inj. (2)

III $0 \leq x_1 < x_2$

$f(x_1) = 1-x \geq 1 > 1-x_2 = f(x_2) \Rightarrow f(x_1) \neq f(x_2) \Rightarrow$ inj (3)

~~$f(x_2) =$~~

din (1), (2), (3) $\Rightarrow f$ inj.

Fie $y \in \mathbb{R}$. Vreau $\exists\, x \in \mathbb{R}$, $f(x) = y$

Dacă $y > 1$, luăm $x = -\sqrt{y-1} < 0$. Atunci $f(x) = x^2+1 = (-\sqrt{y-1})^2 + 1 = y$

Dacă $y \leq 1$, luăm $x = 1-y \geq 0$. Atunci $f(x) = 1-x = 1-(1-y) = y$ $\Rightarrow$

$\Rightarrow f$ surj.

$g(2) = g(-1) = 1 \Rightarrow g$ nu e inj.

<u>Non inj.:</u> $\exists\, a_1, a_2$, $a_1 \neq a_2 \wedge f(a_1) = f(a_2)$

Luăm $y = 0$. Fie $x \in \mathbb{R}$. Dacă $x < 0$, $g(x) = 1-x > 1 > 0 = y \Rightarrow g(x) \neq y$

Dacă $x \geq 0$, $g(x) = x^2+1 \geq 1 > 0 = y \Rightarrow g(x) \neq y$ $\Rightarrow$

$\Rightarrow g$ nu e surj.

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Temă - seminar 2

① $g, h : \mathbb{R} \to \mathbb{R}$ $\quad g(x) = \begin{cases} x^2+1, & x>3 \\ 2-x, & x \le 3 \end{cases}$ $\quad h(x) = \begin{cases} 5x-2, & x>-1 \\ x+1, & x \le -1 \end{cases}$

$$(g \circ h)(x) = g(h(x)) = \begin{cases} h^2(x)+1, & h(x)>3 \\ 2-h(x), & h(x) \le 3 \end{cases} = \begin{cases} (5x-2)^2+1, & \begin{cases} x>-1 \\ 5x-2>3 \end{cases} \\ (x+1)^2+1, & \begin{cases} x \le -1 \\ x+1>3 \end{cases} \\ 2-(x+1), & \begin{cases} x+1 \le 3 \\ x \le -1 \end{cases} \\ 2-(5x-2), & \begin{cases} 5x-2 \le 3 \\ x>-1 \end{cases} \end{cases} =$$

$$= \begin{cases} 25x^2-20x+5, & \begin{cases} x>-1 \\ x>1 \end{cases} \Rightarrow x>1 \\ x^2+2x+2, & \begin{cases} x \le -1 \\ x>2 \end{cases} \Rightarrow \emptyset \\ 1-x, & \begin{cases} x \le 2 \\ x \le -1 \end{cases} \Rightarrow x \le -1 \\ 4-5x, & \begin{cases} x \le 1 \\ x>-1 \end{cases} \Rightarrow x \in (-1,1] \end{cases} = \begin{cases} 25x^2-20x+5, & x>1 \\ 4-5x, & x \in (-1,1] \\ 1-x, & x \le -1 \end{cases}$$

② Studiați injectivitatea și surj. : $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \begin{cases} x^2+3x+1, & x \ge -1 \\ \frac{x-7}{4}, & x<-1 \end{cases}$

Fie $x_1, x_2 \in \mathbb{R}$; $x_1 \ne x_2$.

I $\quad x_1 < x_2 < -1 \;|-7 \iff x_1-7 < x_2-7 < -8 \;|:4 \iff \frac{x_1-7}{4} < \frac{x_2-7}{4} < -2 \iff$

$\iff f(x_1) < f(x_2) < -1 \iff f(x_1) \ne f(x_2)$

II $\quad x_1 < -1 \le x_2$

$\left.\begin{array}{l} f(x_1) = \frac{x_1-7}{4} < -1 \\ f(x_2) = x_2^2+3x_2+1 \end{array}\right| \Rightarrow \frac{x_1-7}{4} < -1 \le \underbrace{x_2^2+3x_2+1}_{\ge -1}$ (A) $\iff f(x_1) < f(x_2) \iff f(x_1) \ne f(x_2)$

$\cancel{x_1-7<-4}$

$x_1<3$

III $\quad -1 \le x_1 < x_2 \;|+\frac{3}{2} \iff \frac{1}{2} \le x_1+\frac{3}{2} < x_2+\frac{3}{2} \;|()^2 \iff \frac{1}{4} \le (x_1+\frac{3}{2})^2 < (x_2+\frac{3}{2})^2 \;|-\frac{5}{4}$

$\iff \frac{1}{4} - \frac{5}{4} \le (x_1+\frac{3}{2})^2 - \frac{5}{4} < (x_2+\frac{3}{2})^2 - \frac{5}{4} \iff -1 \le f(x_1) < f(x_2) \iff f(x_1) \ne f(x_2)$

din I, II, III $\Rightarrow$ f inj.

Fie $y \in \mathbb{R}$, vreau $\exists\, x \in \mathbb{R}$, $f(x) = y$

Dacă $y \ge -1$, luăm $x = \sqrt{y+\frac{5}{4}} - \frac{3}{2}$. Atunci $f(x) = ((y+\frac{5}{4}) - \frac{3}{2} + \frac{3}{2})^2 - \frac{5}{4} = y$

Dacă $y < -2$, luăm $x = 4y+7$. Atunci $f(x) = y$