FLP - consultatie

Care sunt următorii λ-termeni (presupunem u, v , w , x , y, z ∈ V , distincte două câte două)?

$$(\lambda y.(x\ (\lambda w.((v\ w\ )x\ ))))[x := u\ v\ ] = (\lambda y.((u\ v)\ (\lambda w.((v\ w\ )\ (u\ v)\ ))))$$

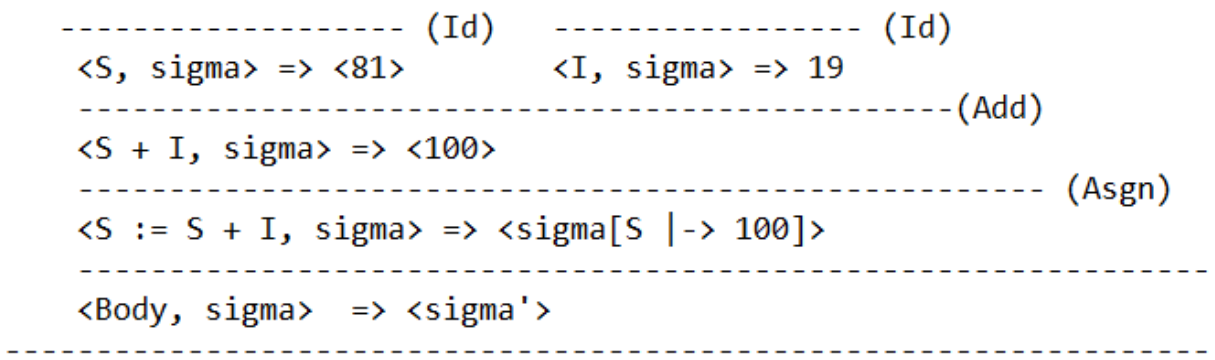
$$(\lambda y.(x\ (\lambda x.x\ )))[x := \lambda y.(xy\ )] = (\lambda y.((\lambda y.(xy\ ))\ (\lambda x.x\ )))$$

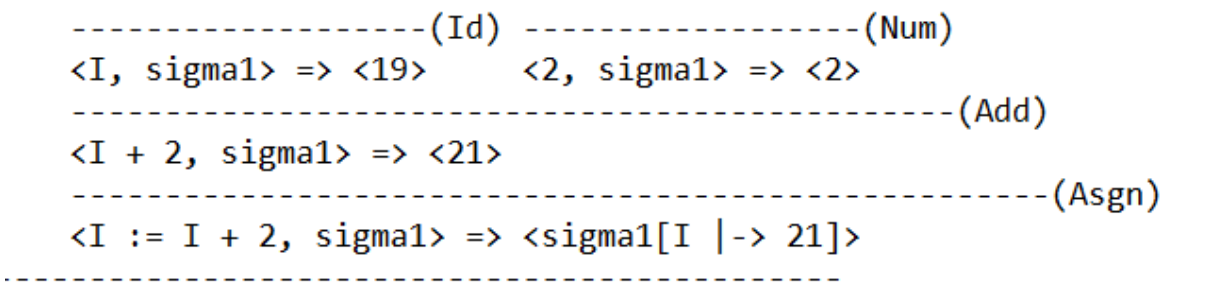
$$(\lambda y.(x\ (\lambda x.x\ )))[x := \lambda x.(xy\ )] = (\lambda z.(x\ (\lambda x.x\ )))[x := \lambda x.(x\ y)] = (\lambda z.((\lambda x.(x\ y))\ (\lambda x.x\ )))$$

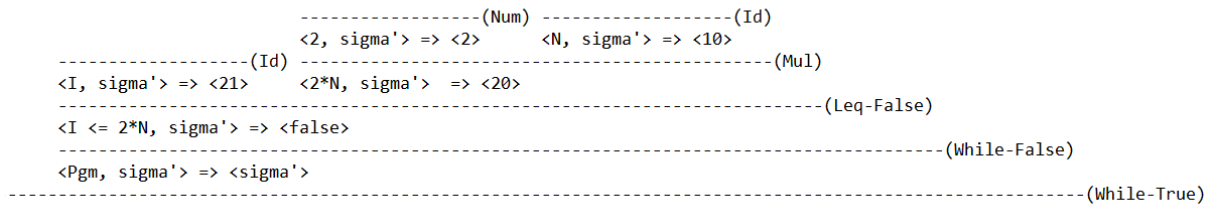
1. Fie I. N. SE L. distincte două câte două. Notam while I <= 2 * N do (S:= S + I; I := I + 2)
   cu Pgm.
   (a) (2 puncte) Să se descrie formal execuţia lui Pgm, dintr-o stare iniţială cu sigma(N) = 10 sigma(I) = 19 sigma(S) = 81 folosind semantica operaţionala big-step SAU cea small-step

```
Body ::= (S:= S + I; I := I + 2)
```

```
                            ------------------ (Num)  ------------------- (Id)
                            <2, sigma> => <2>          <N, sigma> => <10>
------------------ (Id)      ------------------------------------------------ (Mul)
<I, sigma > =>  <19>        <2*N, sigma> => <20>
-------------------------------------------------------------------------------------(Leq-True)
< I <= 2 * N, sigma >  => <true>
-----------------------------------------------------------------------------------------------
```







< Pgm , sigma > => <sigma'>

|

sigma1 ::= sigma[S |-> 100] = {N |-> 10, I |-> 19, S |-> 100 }
sigma' ::= sigma1[I |-> 21] = {N |-> 10, I |-> 21, S |-> 100 }

1. <2, sigma> => <2> (Num)
2. <N, sigma> => <10> (Id)
3. <2*N, sigma> => <20> din (1) și (2) folosind (Mul)
4. <I, sigma > => <19> (Id)
5. < I <= 2 * N, sigma > => <true> din (4) și (3) folosind (Leq-True)

```
< Pgm , sigma >
  ---
=WHILE=> <if I <= 2*N then (Body ; Pgm) else skip, sigma>
                -
=Id=> <if 19 <= 2*N then (Body ; Pgm) else skip, sigma>
                   -
=Id=> <if 19 <= 2*10 then (Body ; Pgm) else skip, sigma>
                  ----
=Mul=> <if 19 <= 20 then (Body ; Pgm) else skip, sigma>
              --------
=Leq-True=> <if true then (Body ; Pgm) else skip, sigma>
               ----------------------------------
=If-True=> < (S := S + I; I := I + 2) ; Pgm, sigma>
                        -
=If-True=> < (S := 81 + I; I := I + 2) ; Pgm, sigma>
```

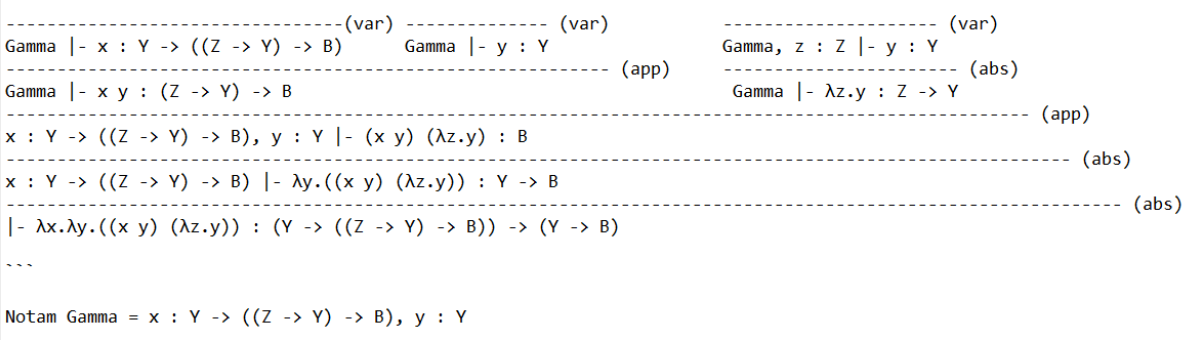
```
Gamma = emptyset

(λx y z.z x y) (λa b c.b) (λd e.e)

Am redenumit variabilele astfel incat sa fie toate distincte.
Presupun ca fiecarei variariabile x îi asociez un tip X
```

```
c(((λx.λ y.λ z.z x y) (λa.λ b.λ c.b)) (λd.λ e.e), F)
= c(((λx y z.z x y) (λa b c.b)), G) U c((λd e.e), H) U {G = H -> F}
=  c((λx y z.z x y), I) U c((λa b c.b), J) U {I = J -> G} U c((λe.e), K) U {H = D -> K}  U {G = H -> F}
= {I = J -> G} U {H = D -> K}  U {G = H -> F} U
  c((λy z.z x y), L) U {I = X -> L} U
  c((λb c.b), M) U {J = A -> M} U
  c(e, N) U {K = E -> N}
= {I = J -> G, H = D -> K, G = H -> F, I = X -> L,
   J = A -> M, K = E -> N} U
  c((λz.z x y), O) U {L = Y -> O} U
  c((λc.b), P) U {M = B -> P} U
  { E = N }
= {I = J -> G, H = D -> K, G = H -> F, I = X -> L,
   J = A -> M, K = E -> N, L = Y -> O, M = B -> P,
   E = N } U
  c((z x y), Q) U { O = Z -> Q} U
  c(b, R) U {P = C -> R}
```

```
= {I = J -> G, H = D -> K, G = H -> F, I = X -> L,
   J = A -> M, K = E -> N, L = Y -> O, M = B -> P,
   E = N, O = Z -> Q, P = C -> R } U
  c((z x), S) U c(y, T) U {S = T -> Q}
  { B = R }
= {I = J -> G, H = D -> K, G = H -> F, I = X -> L,
   J = A -> M, K = E -> N, L = Y -> O, M = B -> P,
   E = N, O = Z -> Q, P = C -> R, S = T -> Q,
   B = R } U
   c(z, V) U c(x, W) U {V = W -> S}
   { Y = T }
= {I = J -> G, H = D -> K, G = H -> F, I = X -> L,
   J = A -> M, K = E -> N, L = Y -> O, M = B -> P,
   E = N, O = Z -> Q, P = C -> R, S = T -> Q,
   B = R, V = W -> S, Y = T, Z = V, X = W }
```

```
Rezolvat
I = (A -> (R -> (C -> R))) -> ((D -> (N -> N)) -> F)
H = D -> (N -> N)
X = ((A -> (R -> (C -> R))))
Z = ((A -> (R -> (C -> R)))) -> (T -> Q)
Y = T
B = R
E = N
V = ((A -> (R -> (C -> R)))) -> (T -> Q)
S = T -> Q
P = C -> R
L = T -> ((((A -> (R -> (C -> R)))) -> (T -> Q)) -> Q)
K = N -> N
J = A -> M
M = R -> (C -> R)
O = (((A -> (R -> (C -> R)))) -> (T -> Q)) -> Q,
G = (D -> (N -> N)) -> F
W = (A -> (R -> (C -> R)))
T = (D -> (N -> N))

Ecuatii
F = (((A -> (R -> (C -> R)))) -> (((D -> (N -> N))) -> Q)) -> Q

|- (λx y z.z x y) (λa b c.b) (λd e.e) : (((A -> (R -> (C -> R)))) -> (((D -> (N -> N))) -> Q)) -> Q
```

```
Gamma = {x : X}

c(x x, A)
= c(x, B) U c(x, C) U {B = C -> A}
= { X = B, X = C, B = C -> A}

Rezolva
X = B
B = C

Ecuatii
C = C -> A
esec

deci termenul x x nu poate avea tip
```

```
c(x y, A) = X = Y -> A

λy.((x y) (λz.y))


```

```
Notam Gamma = x : Y -> ((Z -> Y) -> B), y : Y
```

---

2. Verificați dacă următorii termeni se pot unifica:
f(f(g(x), h(y)), h(z)),
f(f(u, h(h(x))), h(y)),
f(v, w)

R = {
ț = f(f(g(x), h(y)), h(z)),
ț = f(f(u, h(h(x))), h(y)),
ț = f(v, w)

}

S = {}

- ----------REZOLVA

R = {
f(v, w) = f(f(g(x), h(y)), h(z)),
----
f(v, w) = f(f(u, h(h(x))), h(y)),
}

S = {
ț = f(v, w)
}

- ----------DESCOMPUNE

R = {
v = f(g(x), h(y)),
w = h(z),
----
f(v, w) = f(f(u, h(h(x))), h(y)),
}

S = {
ț = f(v, w)
}

- ----------REZOLVA

R = {
v = f(g(x), h(y)),
----
f(v, h(z)) = f(f(u, h(h(x))), h(y)),
}

S = {
ț = f(v, h(z))

w = h(z),
}

- ----------REZOLVA

R = {
f(f(g(x), h(y)), h(z)) = f(f(u, h(h(x))), h(y)),

}

S = {
ț = f(f(g(x), h(y)), h(z))
w = h(z),
v = f(g(x), h(y)),
}

- ----------DESCOMPUNE

R = {
f(g(x), h(y)) = f(u, h(h(x))),
h(z) = h(y),

}

S = {
ț = f(f(g(x), h(y)), h(z))
w = h(z),
v = f(g(x), h(y)),
}

- ----------DESCOMPUNE

R = {
f(g(x), h(y)) = f(u, h(h(x))),
z = y,

}

S = {
ț = f(f(g(x), h(y)), h(z))
w = h(z),
v = f(g(x), h(y)),
}

- ----------REZOLVA

R = {
f(g(x), h(y)) = f(u, h(h(x))),

}

S = {
ț = f(f(g(x), h(y)), h(y))
w = h(y),
v = f(g(x), h(y)),
z = y,
}

- ----------DESCOMPUNE

R = {
g(x) = u,
----
h(y) = h(h(x)),
}

S = {
ț = f(f(g(x), h(y)), h(y))
w = h(y),
v = f(g(x), h(y)),
z = y,
}

- ----------REZOLVA

R = {
h(y) = h(h(x)),

}

S = {
ț = f(f(g(x), h(y)), h(y))
w = h(y),
v = f(g(x), h(y)),
z = y,
u = g(x),
}

- ----------DESCOMPUNE

R = {
y = h(x),

}

S = {
ț = f(f(g(x), h(y)), h(y))
w = h(y),
v = f(g(x), h(y)),
z = y,
u = g(x),
}

- ----------REZOLVA

R = {}

S = {
ț = f(f(g(x), h(h(x))), h(h(x)))
w = h(h(x)),
v = f(g(x), h(h(x))),
z = h(x),
u = g(x),
y = h(x),
}

Deci problema are solutie cu GCU dat de S