

Quadrice

Quadricile sunt suprafețe algebrice de gradul al doilea.

Quadrice nedegenerate	Ecuatii
Elipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
Sferoid	$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1$
Sferă	$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{a^2} = 1$
Paraboloid eliptic	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - z = 0$
Paraboloid de rotație	$\frac{x^2}{a^2} + \frac{y^2}{a^2} - z = 0$
Paraboloid hiperbolic	$\frac{x^2}{a^2} - \frac{y^2}{b^2} - z = 0$
Hiperboloid cu o pânză	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
Hiperboloid cu 2 pânze	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$

Quadrice degenerate	Ecuatii
Con	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$
Con de rotație	$\frac{x^2}{a^2} + \frac{y^2}{a^2} - \frac{z^2}{a^2} = 0$
Cilindru eliptic	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
Cilindru de rotație	$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$
Cilindru hiperbolic	$\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$
Cilindru parabolic	$x^2 + 2ay = 0$

Observații: La quadricile degenerate:

- perechi de plane $(ax_1 + bx_2 + cx_3 + d)(a'x_1 + b'x_2 + c'x_3 + d') = 0$
 $a^2 + b^2 + c^2 > 0$, $a'^2 + b'^2 + c'^2 > 0$
- dreapta dublă $x_1^2 + x_2^2 = 0$
- punct dublu $x_1^2 + x_2^2 + x_3^2 = 0$
- quadrică \emptyset $x_1^2 + x_2^2 + x_3^2 + 1 = 0$

① Sferă

$$S(A(a, b, c), R): (x_1 - a)^2 + (x_2 - b)^2 + (x_3 - c)^2 = R^2$$

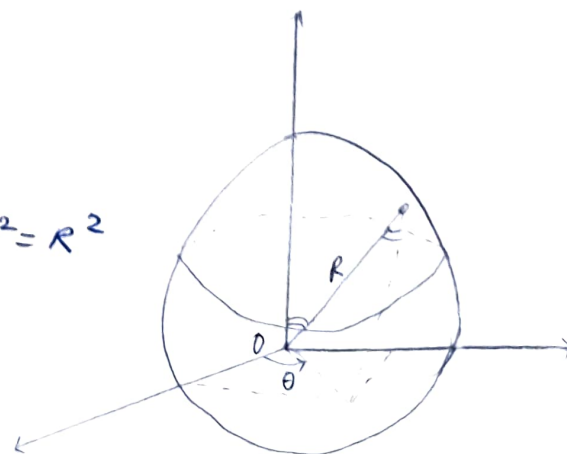
$$S(O(0, 0, 0), R)$$

$$x_1 = R \sin \varphi \cos \theta$$

$$x_2 = R \sin \varphi \sin \theta$$

$$x_3 = R \cos \varphi$$

$$\theta \in [0, 2\pi), \varphi \in [0, \pi]$$

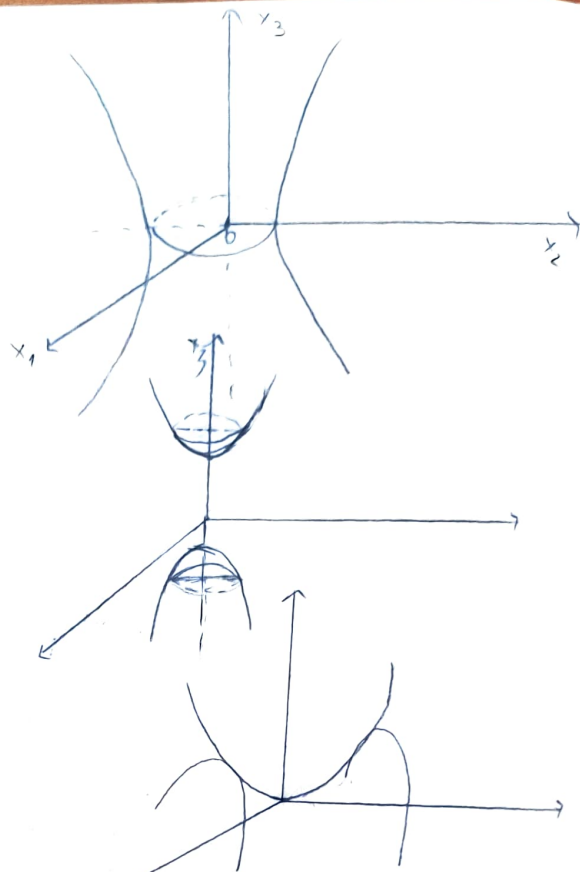


② Hiperboloid cu o pânză

$$\mathcal{H}_{1p} : x_3 = \delta \in \mathbb{R} \Rightarrow \text{elipsă}$$

$$x_1 = \alpha, \alpha \neq \pm a$$

$$x_2 = \beta, \beta = \pm a \mid \text{hiperbolă}$$



③ Hiperboloid cu 2 pânze:

$$\mathcal{H}_{2p} : x_3 = \delta \in (-\infty, -c) \cup (c, \infty) \text{ elipsă}$$

$$x_1 = \alpha$$

$$x_2 = \delta \mid \Rightarrow \text{hiperbolă}$$

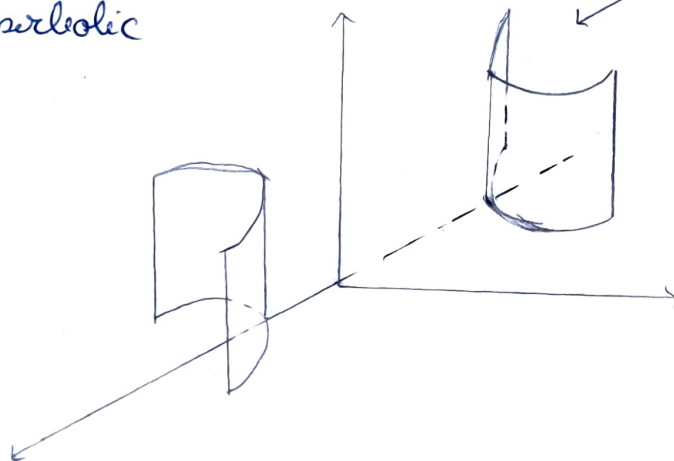
④ Paraboloid hiperbolic

$$x_3 = \delta \neq 0 \text{ hiperbolă}$$

$$x_1 = \alpha$$

$$x_2 = \beta \mid \Rightarrow \text{parabolă}$$

⑤ Cilindru hiperbolic



Δ	δ	Cuadratică
$= 0$	> 0	Con
	< 0	Con
	$= 0$	Cilindru eliptic, hiperbolic, parabolic
$\neq 0$	> 0	Elipsoid, Hiperboloid cu 2 pânze
	< 0	Hiperboloid cu 1 pânză
	$= 0$	Paraboloid eliptic și hiperbolic