FLP

7.03.2025

Seminar 1

$x, y, z \in L$ distincte

1. if $x \leq 7$ then $z := x$ else $z := y$ $\quad \sigma(x) = 0 \quad \sigma(y) = 1, \quad \sigma(z) = 7$

Descrieți formal execuția programelor IMP din stările inițiale indicate, folosind semanticile operaționale big-step și small-step.

$$\frac{\dfrac{x, \sigma \xrightarrow{ID} 0}{x \leq 7, \sigma \to 0 \leq 7} \qquad 0 \leq 7, \sigma \xrightarrow{LEQ\text{-}TRUE} true, \sigma}{\text{if } x \leq 7 \text{ then } z := x \text{ else } z := y, \sigma \to \text{if } 0 \leq 7 \text{ then } z := x \text{ else } z := y, \sigma}$$

$\to$ if true then $z := x$ else $z := y, \sigma \xrightarrow{\text{if true}} z := x, \sigma \to$

$\xrightarrow{ID} z := 0, \sigma \xrightarrow{ASSGN} skip, \sigma_1$

$\sigma_1(x) = 0 \quad \sigma_1(y) = 1, \quad \sigma_1(z) = 0$

2. while $4 \leq X$ do $X := X - y$ din starea σ cu $\sigma(X) = 6$, $\sigma(y) = 3$

$\langle \text{while } 4 \leq X \text{ do } X := X - y, \sigma \rangle \xrightarrow{WHILE}$

$\to \langle \text{if } 4 \leq \boxed{X} \text{ then } (X := X - y; \text{while } 4 \leq X \text{ do } X := X - y) \text{ else skip}, \sigma \rangle \xrightarrow{ID}$

$\to \langle \text{if } 4 \leq 6 \text{ then } (X := X - y; \text{while } 4 \leq X \text{ do } X := X - y) \text{ else skip}, \sigma \rangle$

$\xrightarrow{LEQ\text{-}TRUE} \langle \text{if true then } (X := X - y; \text{while } 4 \leq X \text{ do } X := X - y) \text{ else skip}, \sigma \rangle$

$\xrightarrow{\text{IF-TRUE}} \langle x := \boxed{x}-y \,;\ \text{while } 4 \le x \text{ do } x := x-y,\ \sigma \rangle \longrightarrow$

$\xrightarrow{\text{ID}} \langle x := 6-\boxed{y} \,;\ \text{while } 4 \le x \text{ do } x := x-y,\ \sigma \rangle \longrightarrow$

$\xrightarrow{\text{ID}} \langle x := \underline{6-3} \,;\ \text{while } 4 \le x \text{ do } x := x-y,\ \sigma \rangle \longrightarrow$

$\xrightarrow{\text{SUB}} \langle \underline{x := 3} \,;\ \text{while } 4 \le x \text{ do } x := x-y,\ \sigma \rangle \longrightarrow$

$\xrightarrow{\text{ASGN}} \langle \underline{\text{skip} \,;\ \text{while } 4 \le x \text{ do } x := x-y},\ \sigma_1 \rangle \longrightarrow$ $\sigma_1(x)=3$ $\sigma_1(y)=3$

$\xrightarrow{\text{SEQ}} \langle \underline{\text{while } 4 \le x \text{ do } x := x-y},\ \sigma_1 \rangle \longrightarrow$

$\xrightarrow{\text{WHILE}} \langle \text{if } 4 \le \boxed{x} \text{ then } (x := x-y \,;$
$\text{while } 4 \le x \text{ do } x := x-y)$
$\text{else skip},\ \sigma_1 \rangle \longrightarrow$

$\xrightarrow{\text{ID}} \langle \text{if } \underline{4 \le 3} \text{ then } (x := x-y \,;$
$\text{while } 4 \le x \text{ do } x := x-y)$
$\text{else skip},\ \sigma_1 \rangle \longrightarrow$

$\xrightarrow{\text{LEQ-FALSE}} \langle \underline{\text{if false then } (x := x-y \,;}$
$\underline{\text{while } 4 \le x \text{ do } x := x-y)}$
$\underline{\text{else skip}},\ \sigma_1 \rangle$

$\xrightarrow{\text{IF-FALSE}} \langle \text{skip},\ \sigma_1 \rangle$

3. $\text{while } \neg(y=0) \text{ do } (y := y-1 \,;\ x := 2*x)$

$\sigma(x)=1 \,,\quad \sigma(y)=3$

$W_1 \overset{\text{not.}}{=} \tau$

$\langle \underline{\text{while } \neg(y=0) \text{ do } (y := y-1 \,;\ x := 2*x)},\ \sigma \rangle \xrightarrow{\text{WHILE}}$

$\longrightarrow \langle \text{if } \underline{\neg(y=0)} \text{ then } ((y := y-1 \,;\ x := 2*x) \,;\ W_1),\ \sigma \rangle$
else skip

$\xrightarrow{\text{ID}} \langle \text{if } \neg(\underline{3=0}) \text{ then } ((y := y-1 \,;\ x := 2*x) \,;\ W_1),\ \sigma \rangle$
else skip

$\xrightarrow{\text{EQ-FALSE}} \langle \text{if } \underline{\neg \text{false}} \text{ then } ((y := y-1 \,;\ x := 2*x) \,;\ W_1),\ \sigma \rangle$
else skip

$\xrightarrow{\text{NEQ_FALSE}} \langle \text{if true then } ((y:=y-1;\ x:=2*x);\ W_1),\ \sigma \rangle$
else skip

$\xrightarrow{\text{IF_TRUE}} \langle (y:=y-1;\ x:=2*x);\ W_1,\ \sigma \rangle$

$\xrightarrow{\text{ID}} \langle (y:=3-1;\ x:=2*x);\ W_1,\ \sigma \rangle$

$\xrightarrow{\text{SUB}} \langle (y:=2;\ x:=2*x);\ W_1,\ \sigma \rangle$

$\xrightarrow{\text{ASSGN}} \langle (\text{skip};\ x:=2*x);\ W_1,\ \sigma_1(y)=2,\ \sigma_1(x)=1 \rangle$

$\xrightarrow{\text{SEQ}} \langle (x:=2*x);\ W_1,\ \sigma_1 \rangle$

$\xrightarrow{\text{ID}} \langle (x:=2*1);\ W_1,\ \sigma_1 \rangle$

$\xrightarrow{\text{MUL}} \langle (x:=2);\ W_1,\ \sigma_1 \rangle$

$\xrightarrow{\text{ASSGN}} \langle (\text{skip});\ W_1,\ \sigma_2(y)=2,\ \sigma_2(x)=2 \rangle$

$\xrightarrow{\text{BLOCK}} \langle \text{skip};\ W_1,\ \sigma_2 \rangle$

$\xrightarrow{\text{SEQ}} \langle W_1,\ \sigma_2 \rangle$

(ar mai trebui făcute încă două iterații pentru while, după același tipar).

• Folosind semantica big-step:

1. if $x \le 7$ then $z := x$ else $z := y$, dintr-o stare σ cu $\sigma(x)=0$, $\sigma(y)=1$, $\sigma(z)=7$

(???)

$$\dfrac{\dfrac{\langle x,\sigma\rangle \Downarrow \langle 0\rangle \quad \langle 7,\sigma\rangle \Downarrow \langle 7\rangle}{\langle x \le 7, \sigma\rangle \Downarrow \langle true\rangle} \quad \dfrac{\langle x,\sigma\rangle \Downarrow \langle 0\rangle}{\langle z := x, \sigma\rangle \Downarrow \langle \sigma_1(x)=1, \sigma_1(y)=1, \sigma_1(z)=0\rangle}}{\langle \text{if } x \le 7 \text{ then } z := x \text{ else } z := y, \sigma\rangle \Downarrow}$$

• ~~Înapoi la small-step~~. (tot big-step)

4. while $\neg(x=y)$ do (if $x \le y$ then $y := y-x$ else $x := x-y$)

din starea σ cu $\sigma(x)=9$, $\sigma(y)=12$

$\sigma_2(\sigma(x)=6, \sigma(y)=3)$

$$\dfrac{\dfrac{\dfrac{*}{\langle \neg(x=y), \sigma_1\rangle \Downarrow \langle true\rangle}\ \text{ID NOT TRUE} \quad \dfrac{\dfrac{\dfrac{\dfrac{*}{\langle x \le y, \sigma\rangle \Downarrow \langle true\rangle}\ \text{ID LEQ TRUE} \quad \dfrac{\dfrac{*}{\langle y-x, \sigma\rangle \Downarrow \langle 3\rangle}}{\langle y := y-x, \sigma\rangle \Downarrow \langle \sigma_1(x)=9, \sigma_1(y)=3\rangle}}{\langle \text{if } x \le y \text{ then } y := y-x \text{ else } x := x-y, \sigma\rangle \Downarrow \langle \sigma_1\rangle}\ \text{IF TRUE} \quad **}{\langle \text{while } \neg(x=y) \text{ do (if } x \le y \text{ then } y := y-x \text{ else } x := x-y), \sigma_1\rangle \Downarrow \langle ??\rangle}\ ②}{\langle \neg(x=y), \sigma\rangle \Downarrow \langle true\rangle}\ \text{ID NOT TRUE} \quad \langle \text{if } x \le y \text{ then } y := y-x \text{ else } x := x-y, \sigma\rangle \Downarrow \langle \sigma_1\rangle\ ①}{\langle \underbrace{\text{while } \neg(x=y) \text{ do (if } x \le y \text{ then } y := y-x \text{ else } x := x-y)}_{\alpha}, \sigma\rangle \Downarrow \langle ?\rangle}$$

$$\text{IF FALSE}\quad \frac{\langle \neg(x=y), \sigma_3\rangle \Downarrow \langle \text{False}\rangle}{\langle \alpha, \sigma_3(\sigma(x)=3, \sigma(y)=3)\rangle \Downarrow \langle \sigma_3\rangle}$$

$$\frac{\dots \quad \dots \text{if} \Downarrow \sigma_2 \quad \dots \langle \text{if}, \sigma_2\rangle \Downarrow \langle \sigma_3\rangle \quad \langle \alpha, \sigma_3\rangle \Downarrow \langle \sigma_3\rangle}{\langle \alpha, \sigma_1\rangle \Downarrow \sigma_3 \quad \langle \alpha, \sigma_2\rangle \Downarrow \langle \sigma_3\rangle}$$

---

2. while $4 \le x$ do $x := x - y$ $\quad \sigma(x) = 6, \ \sigma(y) = 3$

$\boxed{\sigma_1 : x \mapsto 3, \ y \mapsto 3}$

$$① \quad \frac{\overset{\text{ID}}{\langle x, \sigma_1\rangle \Downarrow \langle 3\rangle} \quad \overset{\text{NUM}}{\langle 4, \sigma_1\rangle \Downarrow \langle 4\rangle}}{\langle 4 \le x, \sigma_1\rangle \Downarrow \langle \text{false}\rangle}\ \text{LEQ FALSE}$$

$$\text{(ID)}\ \langle x, \sigma\rangle \Downarrow \langle 6\rangle \quad \overset{\text{NUM}}{\langle 4, \sigma\rangle \Downarrow \langle 4\rangle} \quad \overset{\text{ID}}{\langle x, \sigma\rangle \Downarrow \langle 6\rangle} \quad \overset{\text{ID}}{\langle y, \sigma\rangle \Downarrow \langle 3\rangle}$$

$$\text{LEQ TRUE}\ \frac{\langle 4 \le x, \sigma\rangle \Downarrow \langle \text{true}\rangle}{\langle \text{while } 4 \le x \text{ do } x := x - y, \sigma\rangle \Downarrow \langle \sigma_1\rangle} \quad \frac{\overset{\text{SUB}}{\langle x - y, \sigma\rangle \Downarrow \langle 3\rangle}}{\langle x := x - y, \sigma\rangle \Downarrow \langle \sigma_1\rangle}\ \text{ASSGN} \quad \frac{①}{\langle \text{while} \dots, \sigma_1\rangle \Downarrow \langle \sigma_1\rangle}$$

FLP

Săptămâna 4

21.03.2025

Seminar 02

Semantica axiomatică

Fie $x, y, M, N, C, S, K, I, P \in L$, distincte două câte două, și $n, m \in \mathbb{N}$.

Să se arate că următoarele enunțuri Hoare sunt demonstrabile:

1. $\{x=n \underset{1}{\wedge} y=m\}\ (x:=x+y;\ y:=x-y);\ x:=x-y\ \{x=m \underset{4}{\wedge} y=n\}$

$\{x-y=m \wedge y=n\}\ x:=x-y\ \{x=m \underset{4}{\wedge} y=n\}$ ← Atr

$x:=x+y;\ \underline{y:=x-y}\ \{x-y=m \wedge y=n\}$

$\{x-(x-y)=m \wedge x-y=n\}\ y:=x-y\ \{x-y=m \wedge y=n\}$ Atr

$\{y=m \wedge x-y=n\}\ y:=x-y\ \{x-y=m \underset{3}{\wedge} y=n\}$ Int.pre

$(y=m \wedge x-y=n \rightarrow (x-(x-y)=m \wedge x-y=n))$

$\{y=m \wedge (x+y)-y=n\}\ x:=x+y\ \{y=m \wedge x-y=n\}$ Atr

$\{y=m \underset{1}{\wedge} x=n\}\ x:=x+y\ \{y=m \underset{2}{\wedge} x-y=n\}$ Int.pre

$(y=m \wedge x=n \rightarrow (y=m \wedge (x+y)-y=n))$

Atribuire $\{P(e)\}\ x:=e\ \{P(x)\}$

Secvență $\dfrac{\{P\}\ S_1\ \{R\} \quad \{R\}\ S_2\ \{Q\}}{\{P\}\ S_1;S_2\ \{Q\}}$

Întărirea precondiției: $\dfrac{\{P_1\}\ S\ \{Q\}}{\{P_2\}\ S\ \{Q\}}$ dacă $P_2 \rightarrow P_1$

① $x := x+y$ ②
② $y := x-y$ ③

Secvențiere

① $x := x+y;\ y := x-y$ ③
③ $x := x-y$ ④

Secvențiere

① Program ④

(general: precondiția instrucțiunii k devine postcondiția instrucțiunii k+1)

2. $\{ \overset{①}{I=1} \wedge 1 \le N \wedge S=0 \}$ $\overset{②}{\text{while } I \le 2*N \text{ do}}$ $(S := S+I;\ I := I+2)$ $\{ \underset{③}{S=N*N} \}$

$f(I) = \left(\frac{(I-1)}{2}\right)^2$

Luăm $P := \{ S = \left(\frac{(I-1)}{2}\right)^2 \wedge \neg I \le 2 \cdot N \wedge I \le 2 \cdot N+1 \wedge \exists K.\ I = 2k+1 \}$

$S = S+I;\ \underline{I := I+2}\ \{P\}$

$\{ S = \left(\frac{I+2-1}{2}\right)^2 \wedge \neg(I+2) \le 2N \wedge I+2 \le 2 \cdot N+1 \}$ $I := I+2\ \{P\}$

$\{ S = \left(\frac{I+1}{2}\right)^2 \wedge \neg(I+2) \le 2 \cdot N \wedge I+2 \le 2 \cdot N+1 \}$ $I := I+2\ \{P\}$

While:

$$\frac{\{P \wedge b\}\ S\ \{P\}}{\{P\}\ \text{while } (b)\ S\ \{P \wedge \neg b\}}$$

Slăbirea postcondiției

$$\frac{\{P\}\ S\ \{Q_1\}}{\{P\}\ S\ \{Q_2\}}$$ dacă $Q_1 \longrightarrow Q_2$

$\{S+I=\left(\frac{I+1}{2}\right)^2 \wedge I>2N-2 \wedge I\leq 2N-1\}$ $S:=S+I$ $\{S=\left(\frac{I+1}{2}\right)^2 \wedge I>2N-2 \wedge I\leq 2N-1\}$ 5

$\{S=\left(\frac{I-1}{2}\right)^2 \wedge I>2N \wedge I\leq 2N+1 \wedge I\leq 2N\} \rightarrow$

$\rightarrow \{S+I=\left(\frac{I+1}{2}\right)^2 \wedge I>2N-2 \wedge I\leq 2N-1\}$

3. $\{1\leq N\}$ $(\overset{①}{P:=0}; \overset{②}{C:=1}); \overset{③}{\text{while } \overbrace{C\leq N}^{b} \text{ do } \overbrace{(P:=P+M; C:=C+1)}^{S}}$

$\{P=M*N\}$

Luăm invariantul (postconditia)

$R=(C-1)\cdot M) \cancel{\wedge C>N} \wedge (C\leq N+1)$ $\boxed{\neg b = C>N}$

$(P=$

$\{(C-1)M \wedge C\leq N+1 \wedge C\leq N\}$ $(P:=P+M; \underline{C:=C+1})$ $\{P=(C-1)M \wedge C\leq N+1\}$

$\cancel{C:=C+1}$

$\{P=(C+1-1)M \wedge (C+1\leq N+1) \wedge (C+1\leq N)\}$ $C:=C+1$ $\{P=(C-1)M \wedge C\leq N+1\}$

$\{P=C\cdot M \wedge (C+1\leq N+1\}$ $C:=C+1$ $\{P=(C-1)M \wedge C\leq N+1\}$

$\{P+M=CM \wedge (C+1\leq N+1)\}$ $P:=P+M$ $\{P=C\cdot M \wedge (C+1)\leq N+1\}$

$\{P=(C-1)M \wedge C\leq N+1 \wedge C\leq N\}$ $(P:=P+M; C:=C+1)$

$\{P=(C-1)M \wedge C\leq N+1\}$

while

$\{R\}$ while b do S $\{R \wedge \neg b\}$

$\{R \wedge C>N\}$

d

$\{P=(0\cdot M) \wedge (1 \le N+1)\}$ $\quad C:=1$ ② $\quad \{P=(C-1)M \wedge C \le N+1\}$ ⓒ

$\{P=0 \wedge (0 \le N)\}$ $\quad P:=0$ $\quad \{P=0 \wedge 0 \le N\}$ ⓑ

$\{(0=0) \wedge (0 \le N)\}$ $\quad P:=0$ ① $\quad \{P=0 \wedge 0 \le N\}$

$\Updownarrow$

$\{0 \le N\} \leftarrow \{1 \le N\}$ ⓐ

ⓐ ① ⓑ

ⓑ ② ⓒ

ⓒ ③ ⓓ

$$\frac{\{a\}\ 1\ \{b\} \qquad \{b\}\ 2\ \{c\}}{\{a\}\ 1;2\ \{c\}}\ \text{seq.}$$

$$\frac{\{a\}\ 1;2\ \{c\} \qquad \{c\}\ 3\ \{d\}}{\{a\}\ (1;2);3\ \{d\}}\ \text{seq}$$

if:
$$\frac{\{P \wedge C\}\ a\ \{Q\} \qquad \{P \wedge \neg C\}\ b\ \{Q\}}{\{P\}\ \text{if } c \text{ then } a \text{ else } b\ \{Q\}}$$

$$\left.\begin{array}{l}\{R_1\}\ a\ \{Q\} \\ \{R_2\}\ b\ \{Q\}\end{array}\right\} \Rightarrow P=(R_1 \wedge C) \vee (R_2 \wedge \neg C)$$

$P \wedge C = \ldots = R_1 \wedge C$

$P \wedge \neg C = \ldots = R_2 \wedge \neg C$

4. ex: $\{I=0 \wedge \exists K.\ N=K\cdot K\}$ while $I\cdot I \le N$ do
((if $I\cdot I=N$ then $P=1$ else skip); $I:=I+1$) $\{P=1\}$

$\boxed{\text{FLP}}$ 04.04.2025

Seminar 03
Unificare

Scoate: elimină $t \doteq t$
Descompune: $f(t_1,\dots,t_n) \doteq f(t'_1,\dots,t'_n) \Rightarrow t_1 \doteq t'_1; \dots; t_n \doteq t'_n$
Rezolvă: $x \doteq t$, $t \doteq x$
Eșec: - conflict $f(s_1,\dots,s_n) \doteq g(t_1,\dots,t_m)$, $f \neq g$
- ciclu $x \doteq t$ $t \doteq x$, $t \neq x$ și $x \in Var(t)$

$u-z$: variabile
a,b,c: constante ("funcții" de aritate 0)
f,g,h,p: "funcții" (constructori)

Găsiți un unificator pentru termenii:

(2). $f(x, f(x, g(y)))$; $f(u,z)$; $f(g(y), y)$

Descomp $[\underline{f(x, f(x, g(y))) = f(u,z)}, \; f(u,z) = f(g(y), y)]$

Rez $[\underline{x = u}, \; f(x, g(y)) = z, \; f(u,z) = f(g(y), y)]$

$x \mapsto u$ $[\underline{f(u, g(y)) = z}, \; f(u,z) = f(g(y), y)]$
Descomp Rez.

$x \mapsto u$
$z \mapsto f(u, g(y))$ $[\underline{f(u, f(u, g(y))) = f(g(y), y)}]$
Descomp

$x \mapsto u$
$z \mapsto f(u, g(y))$ $[\underline{u = g(y)}, \; f(u, g(y)) = y]$
Rez

$u \mapsto g(y)$,
$x \mapsto g(y)$,
$z \mapsto f(g(y), g(y))$ $[f(g(y), g(y)) = y]$ ciclu!

$\Rightarrow$ Nu putem unifica.

În caz de ciclu:
$y = f(y) = f(f(y)) = \dots = f(f(\dots f(y)))$

① $f(x,y),\ f(h(x),x),\ f(x,b)$

$[\underbrace{f(x,y) = f(h(x),x)}_{\text{Descompunere}}\ ;\ f(h(x),x) = f(x,b)]$

$[\underbrace{x = h(x)}_{\text{Ciclu!}},\ y = x,\ f(h(x),x) = f(x,b)]$

* ⑧ $[\underbrace{p(x,x,z) = p(f(a,a),y,y)}_{\text{Descompune}},\ p(f(a,a),y,y) = p(x,b,z)]$

$[\underbrace{x = f(a,a)}_{\text{Rezolvă}},\ x = y,\ z = y,\ p(f(a,a),y,y) = p(x,b,z)]$

$x \leftrightarrow f(a,a)$ $\quad [x = y,\ \underbrace{z = y}_{\text{Rezolvă}},\ p(x,y,y) = p(x,b,z)]$

$f(a,a) \mapsto x$
$y \mapsto z$ $\quad [\underbrace{x = z}_{\text{Rezolvă}},\ p(x,z,z) = p(x,b,z)]$

$z \mapsto x$
$f(a,a) \mapsto x$
$y \mapsto x$ $\quad [\underbrace{p(x,x,x) = p(x,b,z)}_{\text{Descompunere}}]$

$z \mapsto x,\ y \mapsto x,$
$f(a,a) \mapsto x$ $\quad [\underbrace{x = x}_{\text{Elimină}},\ \underbrace{x = b}_{\text{Rezolvă}},\ \underbrace{x = x}_{\text{Elimină}}]$

$x \mapsto b,\ z \mapsto b,$
$y \mapsto b,$
$\underbrace{f(a,a) \mapsto b}_{\text{CONFLICT!!}}$ $\quad \emptyset$

(10) $p(f(x,a), g(y), z)$, $p(f(a,a), z, u)$, $p(v, u, z)$

(6) $f(f(g(x), h(y)), h(z))$, $f(f(u, h(h(x))), h(y))$, $f(v, w)$

S	R
$\emptyset$	$[f(f(g(x), h(y)), h(z)) = f(f(u, h(h(x))), h(y))$; $f(f(u, h(h(x))), h(y)) = f(v, w)]$ Descompun
$\emptyset$	$[f(g(x), h(y)) = f(u, h(h(x)))$; $h(z) = h(y)$; $f(f(u, h(h(x))), h(y)) = f(v, w)]$ Descompun
$\emptyset$	$[g(x) = u$, $h(y) = h(h(x))$, $h(z) = h(y)$, $f(f(u, h(h(x))), h(y)) = f(v, w)]$ Descompun
$\emptyset$	$[u = g(x)$, $y = h(x)$, $h(z) = h(y)$, $f(f(u, h(h(x))), h(y)) = f(v, w)]$ Descompun x2
$\emptyset$	$[u = g(x)$, $y = h(x)$, $z = y$, $f(u, h(h(x))) = v$, $w = h(y)]$ Rezolv
$u \mapsto g(x)$	$[y = h(x)$, $z = y$, $f(g(x), h(h(x))) = v$, $w = h(y)]$ Rezolv
$u \mapsto g(x)$, $y \mapsto h(x)$	$[z = h(x)$, $f(g(x), h(h(x))) = v$, $w = h(h(x))]$ Rezolv
$u \mapsto g(x)$, $y \mapsto h(x)$, $z \mapsto h(x)$	$[f(g(x), h(h(x))) = v$, $w = h(h(x))]$ Rezolv
$u \mapsto g(x)$, $y \mapsto h(x)$, $z \mapsto h(x)$, $w \mapsto h(h(x))$	$[f(g(x), h(h(x))) = v]$ Rezolv
	$\emptyset$

$\Rightarrow S = \{u \mapsto g(x), y \mapsto h(x), z \mapsto h(x), w \mapsto h(h(x)), v \mapsto f(g(x), h(h(x)))\}$

S	R
③. ϕ	$[f(f(x,y),x) = f(g(y),z);$ $\underline{f(g(y),z) = f(u,h(z))}]$
	Descomprime
ϕ	$[\underline{f(f(x,y),x) = f(g(y),z)},$ $g(y) = u,\ z = h(z)]$
ϕ	$[\underline{f(x,y) = g(y)},\ x = z,\ g(y) = u,$ $\underline{z = h(z)}]$

Exec(conflict)

Exec(ciclu).

④ $f(f(x,y),x)$, $f(v,u)$, $f(u,h(z))$

FLP

28.05.2025

Seminar 4
SLD - Respingeri

- x, y, z, u, v, w } variabile (orice altceva: simbol de ~~predicat~~? "funcție")

1) ① $r :- p, q.$
② $s :- p, q.$
③ $v :- t, u.$
④ $w :- v, s.$
⑤ $t.$
⑥ $q.$
⑦ $u.$
⑧ $p.$

$?- w$

① $r \vee \neg p \vee \neg q$ ⑤ t
② $s \vee \neg p \vee \neg q$ ⑥ q
③ $v \vee \neg t \vee \neg u$ ⑦ u
④ $w \vee \neg v \vee \neg s$ ⑧ p

$\neg w$ (4)
$\neg v \vee \neg s$ (3)
$\neg t \vee \neg u \vee \neg s$ (5)
$\neg u \vee \neg s$ (7)
$\neg s$ (2)
$\neg p \vee \neg q$ (8)
$\neg q$ (6)
$\square$

Există o SLD-respingere pt. $\neg w \Rightarrow$ Există răspuns la query-ul "w".

2) $q(X, Y) :- q(Y, X), q(Y, f(f(Y)))$.
$q(a, f(f(X)))$.

$?- q(f(Z), a)$.

① $\neg q(X, Y) \vee \neg q(Y, X) \vee \neg q(Y, f(f(Y)))$
② $q(a, f(f(X)))$

$\neg q(f(Z), a)$ (1) $\left[\begin{array}{l} X := f(Z) \\ Y := a \end{array}\right]$
$\neg q(Y, X) \vee \neg q(Y, f(f(Y)))$
$\neg q(a, f(Z)) \vee \neg q(a, f(f(a)))$

(2) $[X := a]$ $\neg g(a, f(Z))$

(2) $[Z := f(X)]$

$\neg g(a, f(f(X)))$ $\square$

3) $p(X) :- g(X, f(Y)), r(a).$
$p(X) :- r(X).$
$g(X,Y) :- p(Y).$
$r(X) :- q(X,Y).$
$r(f(b)).$

$?- p(X), g(Y,Z).$

① $p(X) \vee \neg g(X, f(Y)) \vee \neg r(a)$
② $p(X) \vee \neg r(X)$
③ $g(X,Y) \vee \neg p(Y)$
④ $r(X) \vee \neg q(X,Y)$
⑤ $r(f(b))$

$\neg p(X) \vee \neg g(Y,Z)$

$\neg p(X) \vee \neg g(Y,Z)$

(2) $\neg r(X) \vee \neg g(Y,Z)$

(5) $[X := f(b)]$

$\neg g(Y,Z)$

(3)* $\neg p(Z)$

(2)* $\neg r(Z)$

(5) $[Z := f(b)]$

$\underbrace{\neg r(f(b))}_{\text{true}}$

$g(X_1, Y_1) \vee \neg p(Y_1)$

(3)* $[Y := X_1, Z := Y_1]$ sau $\boxed{[X_1 := Y, Y_1 := Z]}$

(2)* $p(X_2) \vee \neg r(X_2)$

$[X_2 := Z]$

① $\neg Q_1 \dots \neg Q_n$. Alegem Q_i
② Alegem o regulă
$R := Q_i \vee \neg P_1 \vee \dots \vee \neg P_k$
③ Redenumim var. din R a.î. să nu le găsim în $Q_1 \dots Q_n$
④ Unificăm Q_i cu Q → θ-substituție
⑤ Continuăm cu $\neg Q_1 \vee \dots \vee \neg P_1 \vee \dots \vee \neg P_k \dots$

09.05.2025

Seminar 5
Programare Funcțională

I. Lambda - calcul fără tipuri

① $\underline{(\lambda x.((x\ y)\ x))(\lambda z.z)} \xrightarrow{\beta} ((x\ y)\ x)[x := \lambda z.z]$

$\equiv (x\ y)[x := \lambda z.z]\ x[x := \lambda z.z]$

$\equiv (x[x := \lambda z.z]\ y[x := \lambda z.z])\ \lambda z.z$

$\equiv ((\lambda z.z)\ y)\ z$

$\xrightarrow{\beta} (z[z := y])\ z$

$\equiv y\ z$

$\lambda x.y \Leftrightarrow f(x) = y$
$\lambda x.y\ z \Leftrightarrow f(z) = y$
(substituim pe x cu z)

② $(\underline{(\lambda x.(\lambda y.x))\ y})\ z \xrightarrow{\beta} ((\lambda y.x)[x := y])\ z$

$\equiv ((\lambda z.x[y := z])[x := y])\ z$

$\equiv (\lambda z.x[x := y])\ z$

$\equiv (\lambda z.y)\ z \xrightarrow{\beta} y[z := z] \equiv y.$

- β-reducție: $(\lambda x.\ ceva)\ altceva \xrightarrow{\beta} ceva[x := altceva]$
- $(a\ b)[x := c] = a[x := c]\ b[x := c]$
- $x[x := c] \equiv c$
- $y[x := c] \equiv y$ dacă $y \neq x$
- $\lambda x.\ ceva[x := altceva] \equiv \lambda x.\ ceva$
- $\lambda y.\ ceva[x := altceva] \equiv \lambda y.(ceva[x := altceva])$, dacă $y \notin FV(altceva)$
- $\lambda y.ceva[x := altceva] \equiv \lambda z.ceva[y := z][x := altceva]$, altfel, $z \notin FV(ceva) \cup FV(altceva)$

variabile libere

2/

③ $(\lambda z.(\lambda x.(\lambda y.(zy))))\ (xy)$

$\equiv (\lambda z.(\lambda t.(\lambda y.(zy))[x:=t]))\ (xy)$

$\equiv (\lambda z.(\lambda t.(\lambda y.(zy))))\ (xy)$

$\rightarrow_\beta (\lambda t.(\lambda y.(zy)))[z:=xy]$

$\equiv \lambda t.(\lambda y.(zy)[z:=xy])$

$\equiv \cancel{\lambda t.(\lambda v\ (\lambda y.(zy))}$

$\lambda t.(\lambda v((zy)[y:=v]))[z:=xy]$

$\equiv \lambda t.(\lambda v((zv))[z:=xy]$

$\equiv \lambda t.(\lambda v(x\ y\ v))$

④ $\underline{((\lambda s.(ss))(\lambda q.q))}\ (\lambda q.q)$

$\rightarrow_\beta ((ss)[s:=(\lambda q.q)])\ (\lambda q.q)$

$\equiv (s[s:=(\lambda q.q)])(s[s:=(\lambda q.q)])\quad (\lambda q.q)$

$\equiv ((\lambda q.q)(\lambda q.q))(\lambda q.q)$

$\rightarrow_\beta (q[q:=(\lambda q.q)])\ (\lambda q.q)$

$\equiv ((\lambda q.q))\ (\lambda q.q)$

$\rightarrow_\beta q[q:=(\lambda q.q)]$

$\equiv \lambda q.q$

3/

Numere în programarea funcțională
- definite într-un mod 'aparte'

$$\underset{\sim}{C_n} = \lambda f.\ \lambda x.\ (\underbrace{f\,(f(\ldots(f\,x)\ldots))}_{n \text{ aplicări}})$$

numărul (numeralul Church asociat lui n)

$*$: $add = \lambda n.\ \lambda m.\ \lambda f.\ \lambda x\,((n\,f)((m\,f)\,x))$

$$(m\,f)\,x = \lambda f.\ \lambda x.\ (\underbrace{f(f(\ldots(f\,x))\ldots)}_{m \text{ aplicări}})$$

$$n\,f = \lambda x.\ (f\ldots f(x)))\ )$$

$$(n\,f)((m\,f)\,x) = \underbrace{f\ldots f}_{n \text{ ori}}\ \underbrace{f\ldots f(x)}_{m \text{ ori}} = \underline{\underline{C_{n+m}}}$$

$*$ $mul = \lambda n.\ \lambda m.\ \lambda f.\ (n(m\,f))$

Reduceți termenul (expresia) până la o formă normală:

5) $(mul\ C_2)\,C_2 \equiv ((\lambda n.\ \lambda m.\ \lambda f.\ (n(m\,f)))\,C_2)\ C_2$

$$\xrightarrow{\beta} (\lambda m.\ \lambda f.\ (\underbrace{(n(m\,f))\ [n := C_2]})\ C_2$$

$$\equiv (\lambda m.\ \lambda f.\ (n[n := C_2]\ (m\,f)[n := C_2]))\ C_2$$

$$\equiv (\lambda \underbrace{m}_{x}.\ \underbrace{(\lambda f.\ (C_2\,(m\,f)))}_{a}\ \underbrace{C_2}_{b}$$

$$\xrightarrow{\beta} \lambda f.\ (C_2\,(m\,f))\ [m := C_2]$$

$$\equiv \lambda f.\ ((C_2\,(m\,f))\ [m := C_2])$$

$$\equiv \lambda f.\ (C_2[m := C_2]\ ((m\,f)[m := C_2]))$$

$$\equiv \lambda f.\ (C_2\ (C_2\ f))$$

$$\equiv \lambda f.\ ((\lambda \underbrace{f}_{x}.\ \underbrace{\lambda x\ (f\,(f(x)))}_{a})\ (\underbrace{C_2\,f}_{b}))$$

$$\xrightarrow[\beta]{} \lambda f.\left((\lambda x.(f(f\,x)))[f := c_2\,f]\right)$$

$$\equiv \lambda f.(\lambda x.(f(f\,x))[f := c_2\,f])$$

$$\equiv \lambda f.(\lambda x.(f[f := c_2\,f]\,(f\,x)\,[f := c_2\,f]))$$

$$\equiv \lambda f.(\lambda x.((c_2\,f)(c_2\,f\,x)))$$

$$\equiv \lambda f.(\lambda x.((\lambda f.\lambda x.(f(f\,x)))\,f)\,(c_2\,f\,x)))$$

$$\equiv \lambda f.(\lambda x.(\lambda x.(f(f\,x))[f := f])\,(c_2\,f\,x)))$$

FLP

Seminar 6 Programare Funcțională 27.05.2025

Inferență de tipuri

$\Gamma = \{x : X \mid x \in FV(M)\}$

$M' = M$ în care $\lambda x.A \longrightarrow \lambda x : X.A$

$c(M', \Gamma, w)$ + unificare $\Rightarrow \boxed{W}$

// W este tipul lui M', date fiind tipurile var. în Γ)

Alg:

① $c(x, \Gamma \oplus \{x : \sigma\}, X) = \{X = \sigma\}$ $\quad \sigma = X$

② $c(\lambda x : X.A, \Gamma, W) = c(A, \Gamma \oplus \{x : X\}, V) \cup \{W : X \to W\}$

③ $c(AB, \Gamma, W) = c(A, \Gamma, V) \cup c(B, \Gamma, U) \cup \{V = U \to W\}$

Ex ① $\lambda xyz.(x(yz))$

Găsiți tipul expresiei (fol. alg.)

$\Gamma = \{\}$

$\lambda x : X \; y : Y \; z : Z : (x(yz))$

$\lambda x : X. \lambda y : Y. \lambda z : Z. (x(yz))$

$\underbrace{c(\lambda x : X. \lambda y : Y. \lambda z : Z. (x(yz)), \{\}, W)}$ (0)

$= \underbrace{c(\lambda y : Y. \lambda z : Z. (x(yz)), \{x : X\}, W_1)}_{(1)} \cup \{W = X \to W_1\}$

$(1) = \underbrace{c(\lambda z : Z. (x(yz)), \{x : X, y : Y\}, W_2)}_{(2)} \cup \{W_1 = Y \to W_2\}$

$(2) = \underbrace{c(x(yz), \{x : X, y : Y, z : Z\}, W_3)}_{(3)} \cup \{W_2 = Z \to W_3\}$

α

$(3) = \underbrace{c(x\alpha, W_4)}_{4} \cup \underbrace{c(yz, \alpha, W_5)}_{5} \cup \{W_4 = W_5 \to W_3\}$

$(4) = c(x, \alpha \oplus \{x:X\}, W_4) = \{X = W_4\}$

$(5) = c(y^{(6)}, \alpha, W_6) \cup c(z^{(7)}, \alpha, W_7) \cup \{W_6 = W_7 \to W_5\}$

$(6) = c(y, \alpha \oplus \{y:Y\}, W_6) = \{Y = W_6\}$

$(7) = c(z, \alpha \oplus \{z:Z\}, W_7) = \{Z = W_7\}$

$\Rightarrow c(\lambda x:X.\ \lambda y:Y.\ \lambda z:Z.(x(yz)),\ \{\},\ W) =$

$= \{\underline{W = X \to W_1,\ W_1 = Y \to W_2},\ \underline{W_2 = Z \to W_3},\ \underline{W_4 = W_5 \to W_3},$
$X = W_4,\ W_6 = W_7 \to W_5,\ Y = W_6,\ Z = W_7\}$

$[\ W := X \to W_1\ ]$

$[\ W_1 := Y \to W_2,\ W: X \to Y \to W_2]$

$[W_2 := Z \to W_3,$
$W_1 := Y \to Z \to W_3$
$W := X \to Y \to Z \to W_3]$

$[W_4 := W_5 \to W_3,\ W_2 := Z \to W_3,\ W_1 := Y \to Z \to W_3,$
$W := X \to Y \to Z \to W_3]$

$[\cancel{X := W_5 \to W_3}$

S

$[W_4 := W_5 \to W_3,$
$W_2 := Z \to W_3,$
$W_1 := Y \to Z \to W_3,$
$W := X \to Y \to Z \to W_3]$

T

$[\underline{X := W_5 \to W_3},\ W_6 := W_7 \to W_5,$
$Y := W_6,\ Z := W_7\ ]$

$[W_4 := W_5 \to W_3,$
$W_2 := Z \to W_3,$
$W_1 := Y \to Z \to W_3,$
$W := W_5 \to W_3 \to Y \to Z \to W_3]$
$X := W_5 \to W_3$

$[\underline{W_6 := W_7 \to W_5},$
$Y := W_6,$
$Z := W_7\ ]$

S

$[W_4 := W_5 \to W_3,$
$W_2 := Z \to W_3,$
$W_1 := Y \to Z \to W_3,$
$W := W_5 \to W_3 \to Y \to Z \to W_3,$
$X := W_5 \to W_3$
$W_6 := W_7 \to W_5\ ]$

T

$[\ Y := W_7 \to W_5,$
$Z := W_7\ ]$

$[W_4 := W_5 \to W_3,$
$Z := W_7,$
$W_2 := W_7 \to W_3,$
$W_1 := Y \to W_7 \to W_3,$
$W := W_5 \to W_3 \to Y \to W_7 \to W_3,$
$X := W_5 \to W_3,$
$W_6 := W_7, W_5\ ]$

$[Y := W_7 \to W_5]$

$[\ W_4 := W_5 \to W_3,$ $\quad Y := W_7 \to W_5$
$Z := W_7,$
$W_2 := W_7 \to W_3,$
$W_1 := (W_7 \to W_5) \to W_7 \to W_3,$
$W := (W_5 \to W_3) \to (W_7 \to W_5) \to W_7 \to W_3,$
$X := W_5 \to W_3,$
$W_6 := W_7 \to W_5\ ]$

$\emptyset$

Demonstrație - corectitudinea tipului

$(\lambda x{:}X.\lambda y{:}Y.\lambda z{:}Z.(x(yz)))$ $\quad (W_5 \to W_3) \to (W_7 \to W_5) \to (W_7 \to W_3)$

$\{X = W_5 \to W_3,\ Y = W_7 \to W_5,\ Z = W_7\}$

Reguli de deducție:

1) $$\frac{}{\Gamma \cup \{x:\sigma\} \vdash x:\sigma}$$

2) $$\frac{\Gamma \cup \{x:\sigma\} \vdash A:\tau}{\Gamma \vdash \lambda x:\sigma.A:\sigma \to \tau}$$

3) $$\frac{\Gamma \vdash A:\sigma \to \tau,\ \Gamma \vdash B:\sigma}{\Gamma \vdash AB:\tau}$$

$$\frac{\{\}\cup\{x:W_5\to W_3\}\vdash \lambda y:W_7\to W_5.\ \lambda z:W_7.(x(yz))}{\{\}\vdash \lambda x:W_5\to W_3.\ \lambda y:W_7\to W_5.\ \lambda z:W_7.(x(yz)) : (W_5\to W_3)\to(W_7\to W_5)\to(W_7\to W_3)}$$

a) $\dfrac{}{\Gamma\vdash y:W_7\to W_5}$

b) $\dfrac{}{\Gamma\vdash z:W_7}$

c) $\dfrac{a \quad b}{\Gamma\vdash yz:W_5}$

d) $\dfrac{}{\Gamma\vdash x:W_5\to W_3}$

e) $\dfrac{d \quad c}{\Gamma\vdash x(yz):W_3}$

⋮

Ex. ② $\lambda xy.(xy(\lambda x.y))$

$\lambda x:X_1 . \lambda y:Y . (xy(\lambda x:X_2 . y))$

P1: $\underbrace{c(\lambda x:X_1 . \lambda y:Y . (xy(\lambda x:X_2 . y)), \{\}, W)}$

P2: (0) $= \underbrace{c(\lambda y:Y . (xy(\lambda x:X_2 . y)), \{x:X_1\}, W_1)}_{1} \cup \{W = X_1 \to W_1\}$

P3: (1) $= \underbrace{c(xy(\lambda x:X_2 . y)), \{x:X_1, y:Y\}, W_2)}_{2} \cup \{W_1 = Y \to W_2\}$

P4: (2) $= \underbrace{c(xy, \{x:X_1, y:Y\}, W_3)}_{3} \cup \underbrace{c(\lambda x:X_2 . y, \{x:X_1, y:Y\}, W_4)}_{4}$
$\cup \{W_3 = W_4 \to W_2\}$

P5: (3) $= \underbrace{c(x, \{x:X_1, y:Y\}, W_5)}_{5} \cup \underbrace{c(y, \{x:X_1, y:Y\}, W_6)}_{6}$
$\cup \{W_5 = W_6 \to W_3\}$

(5) $= \{X_1 = W_5\}$

(6) $= \{Y = W_6\}$

este "dictionar" (cheie-valoare)
$\Rightarrow$ se suprascrie valoarea!

(4) $= c(y, \{x:X_2, y:Y\}, W_4) \cup \{W_2 = X_2 \to W_4$