LMC

22.11.2023

Seminar 3

① $C := \{\neg v_0, v_2\}$

$\varphi = ((v_0 \wedge v_1) \to v_2) \wedge (v_0 \to v_4)$

$\nu(\neg(v_0 \wedge v_1) \vee v_2) \wedge (\neg v_0 \vee v_4)$ (imloc. imp.)

$\nu(\neg v_0 \vee \neg v_1 \vee v_2) \wedge (\neg v_0 \vee v_4)$ (De Morgan) e FNC

$S = \{C_1 = \{\neg v_0, \neg v_1, v_2\}, C_2 = \{\neg v_0, v_4\}\}$

$L = v_1$, rezolventul dintre C_1 și $C_2 \Rightarrow C_3 = \{\neg v_0, v_2\} = C$

② $S = \{\{\neg v_0, \neg v_1, v_2\}, \{\neg v_3, v_1, v_4\}, \{\neg v_0, \neg v_4, v_5\}, \{\neg v_2, v_6\}, \{\neg v_5, v_6\}, \{\neg v_0, v_3\}, \{v_6\}, \{\neg v_6\}\}$

Cu alg. Davis-Putnam

$i = 1$, $S_1 = S$

$P_{1.1}$. $x_1 = v_0 \Rightarrow T^1_{p_1} = \{\{v_0\}\}$

$T^0_{p_1} = \{\{\neg v_0, \neg v_1, v_2\}, \{\neg v_0, \neg v_4, v_5\}, \{\neg v_0, v_3\}\}$

$P_{1.2}$. $U_1 = \{\{\neg v_1, v_2\}, \{\neg v_4, v_5\}, \{v_3\}\}$

$P_{1.3}$. $S'_2 = \{\{\neg v_3, v_1, v_4\}, \{\neg v_2, v_6\}, \{\neg v_5, v_6\}, \{\neg v_6\}, \{\neg v_1, v_2\}, \{\neg v_4, v_5\}, \{v_3\}\}$

$S_2 = S'_2$

$P_{1.4}$. $i = 2$

$P_{2.1}$. $x_2 = v_1 \Rightarrow T^1_{p_2} = \{\{\neg v_3, v_1, v_4\}\}$

$T^0_{p_2} = \{\{\neg v_1, v_2\}\}$

$P_{2.2}$ $U_2 = \{\{\neg v_3, v_4, v_2\}\}$

$P_{2.3}$. $S'_3 = \{\{\neg v_3, v_1, v_4\}, \{\neg v_2, v_6\}, \{\neg v_5, v_6\}, \{\neg v_6\}, \{v_3\}, \{\neg v_4, v_5\}\}$

$S_3 = S'_3$

$P_{2.4}$ $i = 3$

$P_{3.1.}$ $x_3 = v_2$, $T'_{p3} = \{\{v_4, v_2, \neg v_3\}\}$

$T^{\circ}_{p3} = \{\{\neg v_2, v_6\}\}$

$P_{3.2.}$ $U_3 = \{\{\neg v_3, v_4, v_6\}\}$

$P_{3.3.}$ $S'_4 = \{\{\neg v_5, v_6\}, \{\neg v_6\}, \{\neg v_3, v_4, v_6\}, \{\neg v_4, v_5\}, \{v_3\}\}$

$S_4 = S'_4$

$P_{3.4.}$ $i = 4$

$P_{4.1.}$ $x_4 = v_3$, $T'_{p4} = \{\{v_3\}\}$

$T^{\circ}_{p4} = \{\{\neg v_3, v_4, v_6\}\}$

$P_{4.2.}$ $U_4 = \{\{v_4, v_6\}\}$

$P_{4.3.}$ $S'_5 = \{\{\neg v_5, v_6\}, \{\neg v_4, v_5\}, \{\neg v_6\}, \{v_4, v_6\}\}$

$P_{4.4.}$ $S_5 = S'_5$

$i = 5$

$P_{5.1.}$ $x_5 = v_4$ $T'_{p5} = \{\{\neg v_4, v_5\}, \{\neg v_6\}, \{v_4, v_6\}\}$

$T^{\circ}_{p5} = \{\{\neg v_4, v_5\}\}$

$P_{5.2.}$ $U_5 = \{\{v_5, v_6\}\}$

$P_{5.3.}$ $S'_6 = \{\{\neg v_5, v_6\}, \{\neg v_6\}, \{v_5, v_6\}\}$

$S_6 = S'_6$

$P_{5.4.}$ $i = 6$

$P_{6.1.}$ $x_6 = v_5$ $T'_{p6} = \{\{v_5, v_6\}\}$

$T^{\circ}_{p6} = \{\{\neg v_5, v_6\}\}$

$P_{6.2.}$ $U_6 = \{\{v_6\}\}$

$P_{6.3.}$ $S'_7 = \{\{\neg v_6\}, \{v_6\}\}$ $S''_7 = S'_7$

$P_{6.4.}$ $i = 7$

$P_{7.1.}$ $x_7 = v_6$ $T'_{p7} = \{\{v_6\}\}$

$T^{\circ}_{p7} = \{\{\neg v_6\}\}$

$P_{7.2.}$ $U_7 = \{\square\}$

$P_{7.3.}$ $S'_8 = \{\square\} = S_8$

$S_{7.4.}$ S nesatisfiabilă

③ A_1: $a \to (b \to a)$

A_2: $(a \to (b \to c)) \to ((a \to b) \to (a \to c))$

A_3: $(\neg b \to \neg a) \to (a \to b)$

MP: $\frac{a,\ a \to b}{b}$ $\quad$ a, $a \to b$, $\therefore b$

Th. ded: $\Gamma \cup \{a\} \vdash b \Leftrightarrow \Gamma \vdash a \to b$

· $\Gamma \cup \{\neg \varphi\} \vdash \neg(\varphi \to \varphi) \Rightarrow \Gamma \vdash \varphi$

(1) $\Gamma \cup \{\neg \varphi\} \vdash \neg(\varphi \to \varphi)$ (ipoteză) $\quad \vdash$ = „se deduce"

(2) $\Gamma \vdash \neg\varphi \to \neg(\varphi \to \varphi)$ (Th. ded.)

(3) $\Gamma \vdash (\underbrace{\neg \varphi}_{b} \to \neg \underbrace{(\varphi \to \varphi)}_{a}) \to ((\varphi \to \varphi) \to \varphi)$ (A_3 + P2.54(i))

(4) $\Gamma \vdash (\varphi \to \varphi) \to \varphi$ (MP: (2) și (3))

(5) $\Gamma \vdash \varphi \to \varphi$ (P2.61 + P2.55(iii))

(6) $\Gamma \vdash \varphi$ (MP: (5) și (4))

④ a) $\{\varphi, \neg\varphi\} \vdash \varphi$

(1) $\vdash \neg\varphi \to (\neg\varphi \to \neg\varphi)$ (A_1)

(2) $\{\neg\varphi\} \vdash \neg\varphi \to \neg\varphi$ (Th. ded.)

(3) $\{\neg\varphi\} \vdash (\neg\varphi \to \neg\varphi) \to (\varphi \to \varphi)$ (A_3 + P2.54.(i))

(4) $\{\neg\varphi\} \vdash \varphi \to \varphi$ (MP: (2) și (3))

(5) $\{\neg\varphi, \varphi\} \vdash \varphi$ (Th. ded.)

b) $\vdash \neg\varphi \to (\varphi \to \varphi)$

(1) $\{\neg\varphi\} \vdash \varphi \to \varphi$ (Th. ded.)

(2) $\vdash \neg\varphi \to (\varphi \to \varphi)$ (Th. ded.)

c) $\Gamma \cup \{\neg\varphi\} \vdash \psi$ și $\Gamma \cup \{\neg\varphi\} \vdash \neg\psi$ implică $\Gamma \vdash \varphi$

(1) $\Gamma \cup \{\neg\varphi\} \vdash \psi$ (ip.)

(2) $\Gamma \cup \{\neg\varphi\} \vdash \neg\psi$ (ip)

(3) $\Gamma \cup \{\neg\varphi\} \vdash \neg\psi \to (\psi \to \bot)$ (~~3a 4(ii)~~ b) + P2.55.(iii))

(4) $\Gamma \cup \{\neg\varphi\} \vdash \psi \to \bot$ (MP (4) și (1))

(5) $\Gamma \cup \{\neg\varphi\} \vdash \bot$ (MP (4) și (1))

(6) $\Gamma \vdash \varphi$ (R.A. = red. la abs.)

d) $\vdash \neg\neg\varphi \to \varphi$

(1) $\{\neg\neg\varphi, \neg\varphi\} \vdash \bot$ (cuipet a)

(2) $\{\neg\neg\varphi\} \vdash \varphi$ (red. la abs)

(3) $\vdash \neg\neg\varphi \to \varphi$ (Th. ded.)

e) $\vdash \varphi \to \neg\neg\varphi$

(1) $\vdash \neg\neg\neg\varphi \to \neg\varphi$ (d)

(2) $(\vdash \neg\neg\neg\varphi \to \neg\varphi) \to (\varphi \to \neg\neg\varphi)$ (A_3)

(3) $\vdash \varphi \to \neg\neg\varphi$ MP din (1) și (2)

⑤ $\vdash (\neg\varphi \to \varphi) \to \varphi$

(1) $\{\neg\varphi \to \varphi, \neg\varphi\} \vdash \neg\varphi \to \varphi$ P2.54 (ii)

(2) $\{\neg\varphi \to \varphi, \neg\varphi\} \vdash \neg\varphi$ P2.54.(ii)

(3) $\{\neg\varphi \to \varphi, \neg\varphi\} \vdash \varphi$ MP (1) și (2)

(4) $\{\neg\varphi \to \varphi\} \vdash \varphi$ (exa cuipet c.)

(5) $\vdash (\neg\varphi \to \varphi) \to \varphi$ (Th. ded.)