Temă

① $f: \mathbb{R}^2 \to \mathbb{R}^3$

$f(x,y) = (x+y, x, -y)$

a) apl. lin ?

$f(X) = AX$ unde $X = \begin{pmatrix} x \\ y \end{pmatrix}$ și $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & -1 \end{pmatrix} \in \mathcal{M}_{3,2}(\mathbb{R})$

Fie $X_1, X_2 \in \mathbb{R}^2$, $\alpha_1, \alpha_2 \in \mathbb{R}$ $\Rightarrow f(\alpha_1 X_1 + \alpha_2 X_2) = A(\alpha_1 X_1 + \alpha_2 X_2) =$

$= \alpha_1 (AX_1) + \alpha_2 (AX_2) = \alpha_1 f(X_1) + \alpha_2 f(X_2) \Rightarrow f$. apl. lin.

d) Teorema rangului defect ?

Știm din subpct b) $\Rightarrow \dim \operatorname{Ker} f = 0$

Știm că $\dim_{\mathbb{R}} \mathbb{R}^2 = 2$

$\operatorname{Im} f = \{ w \in \mathbb{R}^3 \mid \exists!\ v \in \mathbb{R}^2 \text{ a.î. } f(v) = w \}$

Dem. că $\dim_{\mathbb{R}} \operatorname{Im} f = 2$. $\operatorname{Im} f \ni (x', y', z') = (x+y, x, -y) =$

$= (x, x, 0) + (y, 0, -y) = x(1,1,0) + y(1,0,-1) = x v_1 + y v_2$

$S = \{v_1, v_2\} \subset \operatorname{Im} f$ sist de gener. $\Rightarrow$ SLi ?

$\operatorname{rg} \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & -1 \end{pmatrix} = 2 \Rightarrow$ S ebă și SLi $\Rightarrow \dim_{\mathbb{R}} \operatorname{Im} f = 2 \Rightarrow$

(v_1, v_2)

$\Rightarrow \dim_{\mathbb{R}} \operatorname{Im} f + \dim_{\mathbb{R}} \operatorname{Ker} f = \dim_{\mathbb{R}} \mathbb{R}^2$ adevărat