Temă

① În sp. vect. euclidian $(\mathbb{R}^3/\mathbb{R}, \langle , \rangle)$ să se construiască o bază orton. pornind de la baza arbitrară:

$B = \{ f_1 = (1,1,1), f_2 = (1,1,-1), f_3 = (1,-1,-1) \}$

P.O.G-S $\longrightarrow B''$ orton.

$e'_1 = f_1 = (1,1,1)$

$e_1 = \dfrac{f_1}{\|f_1\|} = \dfrac{1}{\sqrt{3}}(1,1,1)$

$\|f_1\| = \sqrt{\langle f_1, f_1 \rangle} = \sqrt{1\cdot1 + 1\cdot1 + 1\cdot1} = \sqrt{3}$

$e'_2 = f_2 - \langle f_2, e_1 \rangle e_1 = (1,1,-1) - \dfrac{1}{\sqrt{3}} \cdot \dfrac{1}{\sqrt{3}}(1,1,1) =$

$\langle f_2, e_1 \rangle = \dfrac{1}{\sqrt{3}}(1\cdot1 + 1\cdot1 + (-1)\cdot1) = \dfrac{1}{\sqrt{3}}$

$= (1,1,-1) - \left(\dfrac{1}{3}, \dfrac{1}{3}, \dfrac{1}{3}\right) = \left(\dfrac{2}{3}, \dfrac{2}{3}, -\dfrac{4}{3}\right)$

$e_2 = \dfrac{e'_2}{\|e'_2\|} = \dfrac{3}{\sqrt{24}}\left(\dfrac{2}{3}, \dfrac{2}{3}, -\dfrac{4}{3}\right)$

$\|e'_2\| = \sqrt{\langle e'_2, e'_2 \rangle} = \sqrt{\dfrac{4}{9} + \dfrac{4}{9} + \dfrac{16}{9}} = \dfrac{\sqrt{24}}{3}$

$e'_3 = f_3 - \langle f_3, e_2 \rangle e_2 - \langle f_3, e_1 \rangle e_1 =$

$= (1,-1,-1) - \dfrac{4}{\sqrt{24}} \cdot \dfrac{3}{\sqrt{24}}\left(\dfrac{2}{3}, \dfrac{2}{3}, -\dfrac{4}{3}\right) -$

$\langle f_3, e_2 \rangle = \dfrac{3}{\sqrt{24}}\left(\dfrac{2}{3}\cdot 1 - 1 \cdot \dfrac{2}{3} - \dfrac{4}{3}\cdot(-1)\right)$

$\langle f_3, e_1 \rangle = \dfrac{1}{\sqrt{3}}(1\cdot1 + 1\cdot(-1) + 1\cdot(-1)) = -\dfrac{1}{\sqrt{3}}$

$- \left(-\dfrac{1}{\sqrt{3}}\right)\dfrac{1}{\sqrt{3}}(1,1,1) =$

$= (1,-1,-1) - \dfrac{12}{24}\left(\dfrac{2}{3}, \dfrac{2}{3}, -\dfrac{4}{3}\right) + \dfrac{1}{3}(1,1,1) =$

$$= \left(\frac{3}{3}, -\frac{3}{3}, -\frac{3}{3}\right) - \left(\frac{1}{2}\cdot\frac{2}{3}, \frac{1}{2}\cdot\frac{2}{3}, \frac{1}{2}\cdot\frac{-4}{3}\right) + \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) =$$

$$= \left(\frac{4}{3}, \frac{-2}{3}, \frac{-2}{3}\right) - \left(\frac{1}{3}, \frac{1}{3}, \frac{-2}{3}\right) = (1, -1, 0)$$

$$e_3 = \frac{e'_3}{\|e'_3\|} = \frac{1}{\sqrt{2}}(1, -1, 0)$$

$$\|e'_3\| = \sqrt{\langle e'_3, e'_3\rangle} = \sqrt{1\cdot 1 + (-1)(-1) + 0} = \sqrt{2}$$

$$B'' = \left\{ e_1 = \frac{1}{\sqrt{3}}(1,1,1) ;\ e_2 = \left(\frac{2}{3}, \frac{2}{3}, -\frac{4}{3}\right)\frac{3}{\sqrt{24}} ;\ e_3 = \frac{1}{\sqrt{2}}(1,-1,0) \right\}$$

2. Pornind de la baza

$B = \{ f_1 = (0,1,1) ;\ f_2 = (1,0,1) ;\ f_3 = (1,1,0) \} \subset E_3$, unde $E_3 = (\mathbb{R}^3/\mathbb{R}, \langle , \rangle)$, determinați o bază ortan. prin utilizarea procedeului de ortonormalizare Gram-Schmidt

$$e'_1 = f_1 = (0,1,1)$$

$$e_1 = \frac{f_1}{\|f_1\|} = \frac{1}{\sqrt{2}}(0,1,1)$$

$$\|f_1\| = \sqrt{\langle f_1, f_1\rangle} = \sqrt{0\cdot 0 + 1\cdot 1 + 1\cdot 1} = \sqrt{2}$$

$$e'_2 = f_2 - \langle f_2, e_1\rangle e_1 =$$

$$= (1,0,1) - \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}(0,1,1) = (1,0,1) - \left(0, \frac{1}{2}, \frac{1}{2}\right) =$$

$$\langle f_2, e_1\rangle = \frac{1}{\sqrt{2}}\cdot(0\cdot 1 + 0\cdot 1 + 1\cdot 1) = \frac{1}{\sqrt{2}}$$

$$= \left(1, -\frac{1}{2}, \frac{1}{2}\right)$$

$$e_2 = \frac{e'_2}{\|e'_2\|} = \sqrt{\frac{2}{3}}\left(1, -\frac{1}{2}, \frac{1}{2}\right)$$

$$\|e'_2\| = \sqrt{\langle e'_2, e'_2\rangle} = \sqrt{1 + \frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{3}{2}}$$

$$e'_3 = f_3 - \langle f_3, e_2\rangle e_2 - \langle f_3, e_1\rangle e_1 =$$

$$= (1,1,0) - \frac{1}{2}\sqrt{\frac{2}{3}}\sqrt{\frac{2}{3}}\left(1,-\frac{1}{2},\frac{1}{2}\right) - \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}(0,1,1) =$$

$$\langle f_3, e_2\rangle = \sqrt{\frac{2}{3}}\left(1\cdot 1 + \left(-\frac{1}{2}\right)\cdot 1 + \cancel{0\cdot \frac{1}{2}}\right) = \frac{1}{2}\cdot\frac{\sqrt{2}}{\sqrt{3}}$$

$$\langle f_3, e_1\rangle = \frac{1}{\sqrt{2}}(0\cdot 1 + 1\cdot 1 + 0\cdot 1) = \frac{1}{\sqrt{2}}$$

$$= (1,1,0) - \frac{1}{3}\left(1,-\frac{1}{2},\frac{1}{2}\right) - \frac{1}{2}(0,1,1) =$$

$$= (1,1,0) - \left(\frac{1}{3}, -\frac{1}{6}, \frac{1}{6}\right) - \left(0, \frac{1}{2}, \frac{1}{2}\right) =$$

$$= \left(0, \frac{1}{2}, -\frac{1}{2}\right) - \left(\frac{1}{3}, -\frac{1}{6}, \frac{1}{6}\right) =$$

$$= \left(-\frac{1}{3}, \frac{4}{6}, -\frac{4}{6}\right) = \left(-\frac{1}{3}, \frac{2}{3}, \frac{-2}{3}\right)$$

$$e_3 = \frac{e'_3}{\|e'_3\|} = \left(-\frac{1}{3}, \frac{2}{3}, \frac{-2}{3}\right)$$

$$\|e'_3\| = \sqrt{\langle e'_3, e'_3\rangle} = \sqrt{\frac{1}{9} + \frac{4}{9} + \frac{4}{9}} = 1$$

$$B'' \text{ ortom} = \{e_1; e_2; e_3\}$$