LFA

Seminar 3

• Lema de pompare pt. REG

Fie L un limbaj regulat. Atunci $\exists p \in \mathbb{N}$ a.î. $\forall \alpha \in L$ cuvânt cu $|\alpha| \geq p$, $\exists$ o descompunere $\alpha = uvw$ cu proprx.:

1) $|uv| \leq p$
2) $|v| \geq 1$
3) ~~$|uvw|$~~ $uv^i w \in L$, $i \geq 0 \in \mathbb{N}$

① $L_1 = \{ a^m b^{3m} c^{m+3} \mid m \geq 5, m \geq 1 \} \notin REG$

P.p.r.a. că $L_1 \in REG \Rightarrow \exists p \in \mathbb{N}^*$ din Lemă

Alegem $\alpha = a^5 b^{3p} c^{p+3} \in L \Rightarrow |\alpha| = 4p+8 \geq p \ \forall p \in \mathbb{N}^*$

Avem $\alpha = uvw$ a.î. $|uv| \leq p$, $|v| \geq 1 \Rightarrow 1 \leq |v| \leq |uv| \leq p$ (*)

Caz I: Fie $v = a^k b^t$, $k \geq 1$, $t \geq 0 \Rightarrow |v| = k+t \overset{(*)}{\Rightarrow} 1 \leq k+t \leq p$

Alegem $i = 0 \Rightarrow \beta = uv^0 w = uw = a^{5-k} b^{3p-t} c^{p+3} \in L \Leftrightarrow$

$\Leftrightarrow |\beta|_a \geq 5 \Leftrightarrow 5-k \geq 5 \Leftrightarrow k \leq 0$, dar $k \geq 1 \Rightarrow \alpha \notin b$ (1)

Caz II: Fie $v = b^t$, $t \geq 1 \Rightarrow |v| = t \overset{(*)}{\Rightarrow} 1 \leq t \leq p$

Alegem $i = 2 \Rightarrow \beta = uv^2 w = a^5 b^{3p+t} c^{p+3} \in L \Rightarrow |\beta|_b = 3(|\beta|_c - 3)$

$\Leftrightarrow 3p+t = 3(p+3-3) \Leftrightarrow t = 0$, dar $t \geq 1 \Rightarrow \alpha \notin b$ (2)

Din (1) și (2) $\Rightarrow L_1 \notin REG$

② $L_2 = \{ a^{m^2} \mid m \geq 1 \} = \{ a, a^4, a^9, a^{16} \ldots \} \notin REG$

P.p.r.a. că $L_2 \in REG \Rightarrow \exists p \in \mathbb{N}$ din Lemă

Alegem $\alpha = a^{p^2} \in L \Rightarrow |\alpha| = p^2 \ \forall p \in \mathbb{N}$

Avem $\alpha = uvw$ a.î. $|uv| \leq p$, $|v| \geq 1 \Rightarrow 1 \leq |v| \leq p$ (*)

Cat I: (ura singur cor pt că primele 3 litere sunt toate „a")

Fie $w = a^k$, $k \geq 1 \Rightarrow |w| = k \overset{(*)}{\Longrightarrow} 1 \leq k \leq p$

Alegem $i = 2 \Rightarrow \beta = uv^2w = a^{p^2+k} \Leftrightarrow |\beta|_a$ = pătrat perf. $\Leftrightarrow$

$\Leftrightarrow p^2 + k = p \cdot p$.

Știm $1 \leq k \leq p \mid +p^2 \Rightarrow p^2+1 \leq \underbrace{p^2+k}_{|\beta|_a} < p^2+p < (p+1)^2$

$\Leftrightarrow p^2 < |\beta| < (p+1)^2 \Leftrightarrow |\beta|$ nu e pp. $\Rightarrow L_2 \notin REG$

③ $L_3 = \{xx^R \mid x \in \{a,b\}\}$ R = oglindit (vreed)

$\alpha_1 = a^p bb\, a^p \Rightarrow$ merge, dar prea lungă oleru. (cel mai bun cor the)

$\alpha_2 = ab^p b^p a \Rightarrow$ nu merge, putem pompa, $\Delta \neq A$

$\alpha_3 = a^p a^p = a^{2p} \Rightarrow$ nu o să meargă, poți face $\Delta \neq A$

$\alpha_4 = (ab)^p (ba)^p \Rightarrow$ merge

$\alpha_5 = ab^p a^p b\, ba^p b^p a \Rightarrow$ merge

④ $L = \{w \mid w \in a^*b^*\}$ oricâte „a"-uri urmate de oricâte „b"-uri

$\Rightarrow$ pt $\bar{L}$

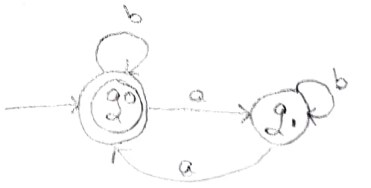
pt $L \Rightarrow F = \{q_0, q_1\}$

pt $\bar{L} \Rightarrow F' = \{q_2\}$

⑤ $L_1 = \{w \mid |w|_a = \text{par}\}$

$L_2 = \{w \mid |w|_b \in \{1,2\}\}$

$\Sigma = \{a,b\}$



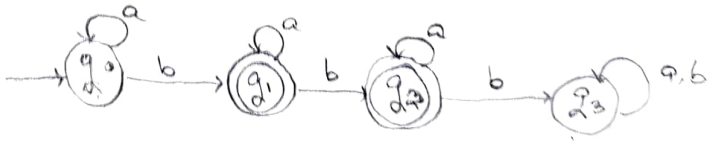
pt L_1

$L = L_1 \cap L_2 \Rightarrow F = \{q_0\pi_1, q_0\pi_2\}$

$L = L_1 \setminus L_2 \Rightarrow F = \{q_0\pi_0, q_0\pi_3\}$

$L = L_2 \setminus L_1 \Rightarrow F = \{q_1\pi_1, q_1\pi_2\}$

$L = L_1 \cup L_2 \Rightarrow F = \{q_0\pi_0, q_0\pi_1, q_0\pi_2, q_0\pi_3, q_1\pi_1, q_1\pi_2\}$



$q = \pi$

pt L_2