# Gauss Jordan

Avem sistem de m ecuații și n necunoscute

A = mat cu coef și B = rezultatele

Se creează mat (A|B) extinsă și se aduce la forma eșalon (1 pe diag. lui A și în rest 0 la A). Soluțiile sistemului sunt rezultatele la B ↳ în principiu

Sist nedet <=>

Sist det <=> pivoți pe fiecare col mai puțin B

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A inversabil dacă det $A \neq 0$ (rg. max) => inversa cu $(A | I_n)$ se încearcă $A = I_n$

rg A = maxim => S.L.I

rg A < max => S.L.D.

rg A = rg $\bar{A}$ = nr. nec. => sol. unică, sist ~~determ~~ compat. det.

rg A = rg $\bar{A}$ < nr. nec => sol ++ => sist. compat. nedet

rg A ≠ rg $\bar{A}$ => sist. incomp., nu are soluții

$\det(A \cdot B) = \det A \cdot \det B$

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$V_1 \cap V_2,\ V_1 + V_2 \subset V$ }}}} $V_1 \cup V_2 \subset V$ dacă $V_1 \subseteq V_2$ sau $V_2 \subseteq V_1$

$V_1 + V_2 = \{x + y \mid x \in V_1, y \in V_2\} \Rightarrow v_1 = x_1 + y_1 \quad v_2 = x_2 + y_2 \Rightarrow v_1 + v_2$

$V_1 \cap V_2 \Rightarrow \alpha x + \beta y \in V_1 \cap V_2 \quad \forall x, y \in V_2$ și $\alpha x + \beta y \in V_{i=1,2}$

$v_1 + v_2 \to (\alpha x_1 + \beta x_2) + (\alpha y_1 + \beta y_2) \in V_1 + V_2$

$V_1 + V_2 = V \Rightarrow V_1 \cap V_2 = \{0_V\}$ și că $\forall x = x_1 + x_2\ \exists! \Rightarrow V_1 \oplus V_2$

$\langle v_1, v_2, \ldots v_n \rangle = \alpha_1 v_1 + \alpha_2 v_2 + \ldots \alpha_n v_n$

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$V_1 \subset V$ dacă $\alpha u + \beta v \in V_1 \Rightarrow \in V$

B = bază dacă B = SLI și $V = \langle B \rangle$

$\dim_K V$ = nr. de elem ale unei baze

$\dim_{\mathbb{R}} \mathbb{R}^n = n$

$\dim_{\mathbb{R}} \mathbb{C} = 2$, $B = \{1, i\}$

$\dim_{\mathbb{C}} \mathbb{C} = 1$

$\dim_K(V_1 + V_2) = \dim_K V_1 + \dim_K V_2 - \dim_K(V_1 \cap V_2)$

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Fie $T : V \to W$

$\operatorname{Ker} T = \{x \in V \mid T(x) = 0_W\}$

$\operatorname{Im} T = \{y \in W \mid \exists x \in V \text{ a.î. } y = T(x)\}$

$\operatorname{Ker} T = \{0_V\} \Rightarrow$ inj

$\operatorname{Im} T = W \Rightarrow$ surj

|=> bij => izo. lin.

$B_1 = \{f_1, f_2, f_3\}$

$A' = C^T A C$

$C = \begin{pmatrix} \vdots & \vdots & \vdots \\ f_1 & f_2 & f_3 \end{pmatrix}$

$\dim_K(\operatorname{Ker} T) + \dim_K(\operatorname{Im} T) = \dim_K V$

Ca să dem. că $B$ e bază a lui $V \Rightarrow$ mat asoc lui $B = A$ și $\det A \neq 0$
(fiecare vector = o coloană)

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$f: V \to V$, $B \subset V$, $f \to A_f$ mat asoc. lui lui $f$

$P(\lambda) = 0 = \det(A_f - \lambda I_m) \Rightarrow$ scoatem $\lambda_i$ rădăcinile ecuației

$V_\lambda = \{ \cancel{A} \mid f(v) = \lambda v \}$ spațiu vect. propriu

vector propriu dacă $A_f v = \lambda_i v$ și $v \in V_{\lambda_i}$ ; $(\lambda I_m - A_f) v = 0$

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$$\Delta = \begin{pmatrix} \lambda_1 & 0 & 0 & 0 & 0 \\ & \lambda_2 & \ldots & & \\ 0 & & & \lambda_m & \\ 0 & 0 & 0 & & \end{pmatrix}$$

Orice end. real cu dim $= m$ cu $m$ val. proprii are $\Delta$

$m_a(\lambda) \leq m_g(\lambda)$ unde $m_a(\lambda)$ de câte ori e răd $\lambda$

$$\Delta \begin{cases} m_a(\lambda_m) + \ldots + m_a(\lambda_p) = m = \dim_K V \\ m_a(\lambda_i) = m_g(\lambda_i) \end{cases}$$

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$v_1 = (x, y, z)$ ; $v_2 = (a, b, c)$

$\langle v_1, v_2 \rangle = x \cdot a + y \cdot b + z \cdot c$

$\| v_1 \| = \sqrt{\langle v_1, v_1 \rangle} = \sqrt{x^2 + y^2 + z^2}$

$\Rightarrow \cos\theta = \dfrac{\langle v_1, v_2 \rangle}{\|v_1\| \, \|v_2\|}$ $\theta \in \{0, \pi)$

$|\langle v_1, v_2 \rangle| = \|v_1\| \, \|v_2\|$

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P. G-S avem. formulele:

$B = \{f_1, f_2, f_3\}$ b. arbitrară

$B' = \{e'_1, e'_2, e'_3\}$ b ortog

$B'' = \{e_1, e_2, e_3\}$ b. ortom.

$e'_1 = f_1 = (\ldots)$

$e_1 = \dfrac{f_1}{\|f_1\|} = \dfrac{e'_1}{\|e'_1\|}$

$e'_2 = f_2 - \langle f_2, e_1 \rangle e_1$

$e_2 = \dfrac{e'_2}{\|e'_2\|}$

$e'_3 = f_3 - \langle f_3, e_2 \rangle e_2 - \langle f_3, e_1 \rangle e_1$

$e_3 = \dfrac{e'_3}{\|e'_3\|}$

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Transf. ortog. $T: E_3 \to E_3$ cu

$T(e_1) = a e_1 + b e_2 + c e_3$

$T(e_2) = d \quad e \quad f$

$T(e_3) = g \quad h \quad i$

$B_0$ - b. ortog.

$$J = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$ mat asoc. lui $T$ în rap. cu $B_0$

dacă $J \cdot {}^t J = I_3 \Rightarrow$ transf. ortog.

Exerciții

① $V = \{(x,y,z) \in \mathbb{R}^3 \mid x+2y+3z=0\} \leq \mathbb{R}^3$

Fie $\left.\begin{array}{l} u,v \in V \\ \alpha,\beta \in \mathbb{R} \end{array}\right| \Rightarrow \left.\begin{array}{l} u = (x_1, y_1, z_1) \\ v = (x_2, y_2, z_2) \end{array}\right| \Rightarrow \alpha u + \beta v = \alpha(x_1,y_1,z_1) + \beta(x_2,y_2,z_2) =$

$$= \underbrace{\alpha x_1 + \beta x_2}_{x} + \underbrace{\alpha y_1 + \beta y_2}_{y} + \underbrace{\alpha z_1 + \beta z_2}_{z}$$

$$x + 2y + 3z = \alpha x + \beta x_2 + 2\alpha y_1 + 2\beta y_2 + 3\alpha z_1 + 3\beta z_2 =$$

$$= \alpha(x_1 + 2y_1 + 3z_1) + \beta(x_2 + 2y_2 + 3z_2) = \alpha \cdot 0 + \beta \cdot 0 = 0$$

$V_2 \leq \mathbb{R}^3$

② $V_1 = \{(x,y,z) \in \mathbb{R}^3 \mid 2x-y+z=0\} \leq \mathbb{R}^3$

$V_2 = \{(x,y,z) \in \mathbb{R}^3 \mid x+2y+z=0\} \leq \mathbb{R}^3$

$V_1 \cap V_2 = ?$

$\boxed{V_1 \cup V_2 \text{ e } V/\mathbb{R} \text{ dacă } V_1 \subseteq V_2 \text{ sau } V_2 \subseteq V_1}$

$V_1 \cap V_2 = \begin{cases} 2x - y + z = 0 \\ x + 2y + z = 0 \end{cases}$ $A = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$

$= \left.\begin{cases} 2x - y = -\alpha \\ x + 2y = -\alpha \end{cases}\right| \Rightarrow$ $z = \alpha$ $x = -\frac{3}{5}\alpha$ și $y = -1/5\,\alpha$

$V_1 \cap V_2 = \{(-3/5\,\alpha;\ -1/5\,\alpha;\ \alpha) \mid \alpha \in \mathbb{R}\}$

$V_1 \oplus V_2 = ?$

$\forall v \in \mathbb{R}^3\ \exists\ v_1 \in V_1$ și $v_2 \in V_2 \Rightarrow$ a.î. $v = v_1 + v_2$

Fie $\left.\begin{array}{l} v = (x, y, z) \\ v_1 = (x_1, y_1, z_1) \\ v_2 = (x_2, y_2, z_2) \end{array}\right| \Rightarrow (x, y, z) = (x_1 + x_2,\ y_1 + y_2,\ z_1 + z_2)$

$x = x_1 + x_2$

$y = y_1 + y_2$

$z = z_1 + z_2$

dar $\begin{cases} 2x - y + z = 0 \\ x + 2y + z = 0 \end{cases}$ $\Rightarrow \begin{cases} 2x - y + z = 0 \\ x + 2y + z = 0 \\ x = x_1 + x_2 \\ y = y_1 + y_2 \\ z = z_1 + z_2 \end{cases}$ $\Rightarrow \begin{cases} 2(x_1 + x_2) - (y_1 + y_2) + (z_1 + z_2) = 0 \\ (x_1 + x_2) + 2(y_1 + y_2) + (z_1 + z_2) = 0 \end{cases}$

$\Leftrightarrow \begin{cases} 2x_1 + 2x_2 - y_1 - y_2 + z_1 + z_2 = 0 \\ x_1 + x_2 + 2y_1 + 2y_2 + z_1 + z_2 = 0 \end{cases} \Leftrightarrow \begin{cases} v_1 + v_2 = 0 \\ v_1 + v_2 = 0 \end{cases} \Rightarrow v_1 + v_2 = 0 \in \mathbb{R}^3$

③ $L = < v_1 = (1,2,0,3), v_2 = (2,-1,3,0), v_3 = (-1,3,2,1) \}$ ~~x~~ $L \subset \mathbb{R}^4$ și $v = (1,-1,1,-1)$

$v \in L = <S>$ ? $\Rightarrow L = \{ \alpha_1 v_1 + \beta v_2 + \gamma v_3 \}$

$v \in L \Leftrightarrow \exists (\alpha, \beta, \gamma) \in \mathbb{R}$ a.î. $v = \alpha v_1 + \beta v_2 + \gamma v_3$

SL. $(1,-1,1,-1) = \alpha(1,2,0,3) + \beta(2,-1,3,0) + \gamma(-1,3,2,1)$

$\begin{cases} \alpha + 2\beta - \gamma = 1 \\ 2\alpha - \beta + 3\gamma = -1 \\ 3\beta + 2\gamma = 1 \\ 3\alpha + \gamma = -1 \end{cases} \Rightarrow \bar{A} = \begin{pmatrix} 1 & 2 & -1 & 1 \\ 2 & -1 & 3 & -1 \\ 0 & 3 & 2 & 1 \\ 3 & 0 & 1 & -1 \end{pmatrix} \Rightarrow \det \bar{A} \neq 0 \Rightarrow$ ~~SL~~ $rg \bar{A} = 4 \neq 3 \Rightarrow$ sist incompatibil $\Rightarrow v \notin L$

④ a) $B = \{ v_1 = (1,2,3), v_2 = (2,-1,1), v_3 \} \subset \mathbb{R}^3$, $v_3 = ?$

Fie $v_3 = (x, y, z) \in \mathbb{R}^3$

$B \subset \mathbb{R}^3$

$\Leftrightarrow \begin{vmatrix} 1 & 2 & x \\ 2 & -1 & y \\ 3 & 1 & z \\ v_1 & v_2 & v_3 \end{vmatrix} \neq 0 \Leftrightarrow 5x + 5y - 5z \neq 0 \Leftrightarrow$

$\Leftrightarrow x + y - z \neq 0 \Rightarrow$

$\Rightarrow$ Deci $v_3 = (x, y, z)$ unde $z \neq x + y$

Un exemplu: $v_3 = (-1, 3, 1+2)$

b) în rap. cu B cu $v = (2,4,5) \Rightarrow v = \alpha v_1 + \beta v_2 + \gamma v_3$

⑤ $B_1 = \{ f_1 = (..), f_2 = (..), f_3(..) \}$ $B_2 = \{ g_1 = (..), g_2 = (..), g_3 = (..) \}$

Det. coord. vect $f_1, f_2, f_3$ în rap. cu $B_2$, $[f_1]_{B_2} = ?$ .....

$B_2 \subset \mathbb{R}^3 \Rightarrow \forall v \in V^3 \; \exists! \; \alpha, \beta, \gamma \in \mathbb{R}$ a.î. $v = \alpha g_1 + \beta g_2 + \gamma g_3$

$v = f_1 \Rightarrow f_1 = \alpha g_1 + \beta g_2 + \gamma g_3 \Rightarrow (0,1,2) = \alpha_1(1,4,-1) + \beta(\cdots) + \gamma(\cdots)$

$\begin{cases} \alpha_1 + \beta - \gamma = 0 \\ \alpha - \beta + \gamma = 1 \\ -\alpha_2 + \beta + \gamma = 2 \end{cases}$ ........ $[f_1]_{B_2} = (\alpha, \beta, \gamma)$

analog pt $f_2, f_3$

⑥ $f : \mathbb{R}^2 \to \mathbb{R}^3,\ f(x,y) = (x+y, x-y, y)$ f. gd. liniară?

a) $f(X) = AX$ unde $X = \begin{pmatrix} x \\ y \end{pmatrix}$ $A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 1 \end{pmatrix} \in \mathcal{M}_{3,2}(\mathbb{R})$

$$\left.\begin{matrix} \text{Fie } X_1, X_2 \in \mathbb{R}^2 \\ \alpha_1, \alpha_2 \in \mathbb{R} \end{matrix}\right| \Rightarrow f(\alpha_1 X_1 + \alpha_2 Y_2) = A(\alpha_1 X_1 + \alpha_2 X_2) = \alpha_1 (AX) + \alpha_2 (AX_2) = \alpha_1 f(X_1) + \alpha_2 f(X_2)$$

b) $[f]_{B_0, B_0'} = A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 0 \end{pmatrix}$ $\quad f(e_1) = f(1,0)$ $\quad B_0 = \{e_1, e_2] \subset \mathbb{R}^2$

$\qquad\qquad \overset{\shortparallel}{f(e_1)}\ f(e_2) \qquad f(e_2) = f(0,1)$

⑦ $f : \mathbb{R}^2 \to \mathbb{R}^3,\ f(x,y) = (x+y, x, -y)$

Ker $f = ?$ și Im $f = ?$

$\operatorname{Ker} f = \{ v = \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \mid f(x) = 0_{\mathbb{R}^2} \} \subset \mathbb{R}^2 \Rightarrow f(x,y) = (0,0,0) \Rightarrow \begin{cases} x+y=0 \\ -y=0 \\ x=0 \end{cases} \Rightarrow A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\operatorname{rg} A = 2 \Rightarrow x = y = 0$ sol unică $\Rightarrow \operatorname{Ker} f = \{0_{\mathbb{R}^2}\}$

$\operatorname{Im} f = \{ u \in \mathbb{R}^3 \mid \exists!\ v \in \mathbb{R}^2$ a.î. $f(v) = u \}$

$\operatorname{Im} f = \{ (x', y', z') \in \mathbb{R}^3 \mid x' - y' + z' = 0 \} \subset \mathbb{R}^3$

$x' - y' + z' = 0 \Rightarrow y' = x' + z' \Rightarrow (x', x'+z', z') = (x', x', 0) + (0, z', z') =$

$= x' \underbrace{(1,1,0)}_{v_1} + z' \underbrace{(0,1,1)}_{v_2}$

$S = \{v_1, v_2\} \subset \operatorname{Im} f$, $\dim_{\mathbb{R}} \operatorname{Im} f = 2$

⑧ $V_{\lambda_1 = -2} = \{ \underset{\begin{pmatrix} x \\ y \end{pmatrix}}{\overset{}{\lambda}} \in \mathbb{R}^2 \mid f(\lambda) = \lambda v \} \subseteq \mathbb{R}^2$

$(\lambda I_2 - A_f) v = 0 \iff (-2 I_2 - A_f) v = 0 \Rightarrow \begin{pmatrix} -5 & -4 \\ -5 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ .....

$V_{\lambda_1} = \{ \alpha (-\frac{4}{5}, 1) \mid \alpha \in \mathbb{R} \}$

$\rightarrow \det(A_f - \lambda I_n)$

① $f:\mathbb{R}^3 \to \mathbb{R}^3$, ~~$f(x,y,z) = (2x+y+z,\ x-y+z,\ x-y-z)$~~

$f(x,y,z) = (2x-y+2z,\ -x+2y-z,\ x+y+z)$

a) $A_f = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & 1 & 1 \end{pmatrix}$

b) $\det(A_f \cdot \lambda \cdot \cancel{\lambda I_3}) = 0 \rightarrow \begin{vmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & 1 & 1 \end{vmatrix} - \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} =$

$$= \begin{vmatrix} 2-\lambda & -1 & 2 \\ -1 & 2-\lambda & -1 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0$$

$[(2-\lambda)^2(1-\lambda)] + (-1)\cdot 1 \cdot 2 + (-1)(-1)\cdot 1 - 2(2-\lambda) - 1(-1)(2-\lambda) - (-1)(-1)(1-\lambda) =$

$= (2-\lambda)^2(1-\lambda) - 2 + 1 - 2(2-\lambda) + (2-\lambda) - (1-\lambda) =$

$= (2-\lambda)^2(1-\lambda) - 1 - (2-\lambda) - (1-\lambda) =$

$= (2-\lambda)^2(1-\lambda) - 1 - 2 + \lambda - 1 + \lambda =$

$= (1-\lambda)(2-\lambda)^2 - 4 + 2\lambda = (1-\lambda)(2-\lambda)^2 - 2(\cancel{4}2-\lambda) =$

$= (2-\lambda)[(1-\lambda)(2-\lambda) - 2] =$

$= (2-\lambda)\{\cancel{2} - \lambda - 2\lambda + \lambda^2 - \cancel{2}\} = \underbrace{(2-\lambda)}_{\lambda_1 = 2}\underbrace{(\lambda^2 - 3\lambda)}_{\lambda^2 - 3\lambda = 0} = 0 \iff \overset{\lambda = 0}{\lambda}\underbrace{(\lambda - 3)}_{\lambda = 3} = 0$

Deci val. proprii: $\lambda_1 = 2$, $\lambda_2 = 0$, $\lambda_3 = 3$

$V_{\lambda_1} = \left\{ v = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \,\middle|\, f(v) = \lambda_1 v \right\} \Rightarrow (A_f - \lambda_1 I_3)v = 0_{\cancel{\mathbb{R}}} \Rightarrow$

$\iff \underbrace{\begin{pmatrix} 0 & -1 & 2 \\ -1 & 0 & -1 \\ 1 & 1 & -1 \end{pmatrix}}_{rg = 2} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} -y + 2z = 0 \iff 2z = y \\ -x - z = 0 \\ x + y - z = 0 \end{cases} \Rightarrow y = 2\alpha$ și $x = -\alpha$

$rg = 2 \Rightarrow z =$ nec. sec. $= \alpha$

$x, y =$ nec. principale

$V_{\lambda_1} = \{ \alpha \underbrace{(-1, 2, 1)}_{v_1} \mid \alpha \in \mathbb{R} \} = \langle v_1 \rangle \Rightarrow \dim_{\mathbb{R}} V_{\lambda_1} = 1$

$V_{\lambda_2} = \left\{ v = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \,\middle|\, f(v) = \lambda_2 v \right\} \Rightarrow (A_f - \lambda_1 I_3)v = 0_{\mathbb{R}}$

$$\underbrace{\begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & 1 & 1 \end{pmatrix}}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} 2x - y + 2z = 0 \\ -x + 2y - z = 0 \\ x + y + z = 0 \end{cases} \Bigg| \Rightarrow \begin{cases} x + y = -\alpha \\ -x + 2y = \alpha \end{cases} \Bigg| \Rightarrow$$

rg $= 2 \Rightarrow z =$ nec. secundare $= \alpha$

$x, y =$ nec. princip

$\Rightarrow -x - 2\alpha - 2x = \alpha \Leftrightarrow x = -\alpha \Rightarrow y = -\alpha - \alpha = 0$

$V_{\lambda_2} = \{ \alpha \underbrace{(-1, 0, 1)}_{v_2} \mid \alpha \in \mathbb{R} \} \cancel{=} \langle v_2 \rangle \Rightarrow \dim_{\mathbb{R}} V_{\lambda_2} = 1$

$V_{\lambda_3} = \{ v = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid f(v) = \lambda_3 v \}$

$$\underbrace{\begin{pmatrix} -1 & -1 & 2 \\ -1 & -1 & -1 \\ 1 & 1 & -2 \end{pmatrix}}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} -x - y + 2z = 0 \\ -x - y - z = 0 \\ x + y + 2z = 0 \end{cases} \Bigg| \Rightarrow \begin{cases} y + z = -\alpha \\ y - 2z = -\alpha \end{cases} \Bigg| \Rightarrow$$

rg $= 2 \Rightarrow x =$ nec. sec. $= \alpha$

$y, z =$ nec. princip

$\Rightarrow z = 0$ și $y = -\alpha$

$V_{\lambda_3} - \{ \alpha \underbrace{(1, -1, 0)}_{v_3} \mid \alpha \in \mathbb{R} \} = \langle v_3 \rangle \Rightarrow \dim_{\mathbb{R}} V_{\lambda_3} = 1$

c) diag?

$m_a(\lambda_1) + m_a(\lambda_2) + m_a(\lambda_3) = \dim_{\mathbb{R}} \mathbb{R}^3 \Leftrightarrow 1 + 1 + 1 = 3$ (adev) $\Bigg| \Rightarrow$

$m_a(\lambda_1) = m_g(\lambda_1) = 1$ și $m_a(\lambda_2) = m_g(\lambda_2) = 1$ și $m_a(\lambda_3) = m_g(K_3) = 1$

$\Rightarrow f$ diagonalizabilă

d) Forma diag $D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$

② $f: \mathbb{R}^3 \to \mathbb{R}^3$, $f(x, y, z) = (x + 4y, 2y + 3z, y)$

Fie $\left.\begin{array}{l} v_1 = (x_1, y_1, z_1) \\ v_2 = (x_2, y_2, z_2) \end{array}\right| \Rightarrow f(v_1 + v_2) = f(x_1 + x_2 + 4y_1 - 4y_2, 2y_1 + 2y_2 + 3z_1 + 3z_2, y_1 + y_2) =$

$= f(v_1) + f(v_2)$ (1)

Fie $v = (x, y, z)$ și $\alpha \in \mathbb{R} \Rightarrow f(\alpha v) = (\alpha x + \alpha 4y, \alpha 2y + \alpha 3z, \alpha y) = \alpha (x + 4y, 2y + 3z, y) =$

$= \alpha f(v)$ (2)

(1) + (2) $\Rightarrow$ ap. lin.