In [1]:
```
#Let f(x) = 2x^3 - 4x + 1 and let g(x) = e^x - 1/x
#(a) Use Python code to evaluate the function f at appropriate values to calculate lim

#use sp? np? no.
import sympy as sp

#x = variable
#f_x = function
x = sp.Symbol('x')
f_x = 2*x**3 - 4*x + 1

#limit  as x gets closer to 3
limit_value = sp.limit(f_x, x, 3)
print("lim(x -> 3) f(x) =", limit_value)
```

```
lim(x -> 3) f(x) = 43
```

In [2]:
```
#Use Python code to evaluate the function g at appropriate values to calculate limx→0g


#lod sp, and np
import sympy as sp
import numpy as np

#x= variable
#g_x = function
x = sp.Symbol('x')
g_x = sp.exp(x) - 1/x

#limit as x gets to 0
limit_value = sp.limit(g_x, x, 0)

#calculate g(x) as we get to 0
values_close_to_0 = [g_x.subs(x, 1/n) for n in np.arange(1, 11)]
print("lim(x -> 0) g(x) =", limit_value)
print("Values of g(x) for x getting to 0:", values_close_to_0)
```

```
lim(x -> 0) g(x) = -oo
Values of g(x) for x getting to 0: [1.71828182845905, -0.351278729299872, -1.60438757
491391, -2.71597458331226, -3.77859724183983, -4.81863958713435, -5.84643500510489, -
6.86685154693317, -7.88248093125814, -8.89482908192435]
```

could the limits in parts (a) and (b) be calculated by plugging in the value where the limit is taken into the function? Explain. What does this mean in terms of continuity?

-In part (a) when calculating the limit of the function, as x approaches 3, you can indeed calculate the limit by directly plugging in the value x = 3 into the function. Because f(x) is a polynomial function, it continous over the entire domain, including specific points. In part (b) whn calculating the limit of the function , as x approaches 0, we cant directlyplug in x = 0 to calculate the limit, since the function has a singularity at x=0 due to the division by zero. In terms of continuity, for function (a) it is a polynomial, and they are continous everywhere in their domain. But for function (b) is has a discontinuity at x = 0, due to the singularity created by he divisio of 0, and it discontinues at that point.

In [15]:
```
#The formula for the average rate of change of a function f between two values a and b
# Create a Python function that takes in a mathematical function, e.g., f(x) = 3x^2 ar

#Average rate of change func
def average_rate_of_change(func, a, b):
#calculate fa and fb func
    fa = func(a)
    fb = func(b)

#ARC
    average_rate = (fb - fa) / (b - a)

    return average_rate

#lambda func
f = lambda x: 3 * x**2

#interval a,b= b
a = 2
b = 5

#ARC between a and b
avg_rate = average_rate_of_change(f, a, b)
print("average rate of change:", avg_rate)
```

```
average rate of change: 21.0
```

A baseball is dropped from a tall cliff. Neglecting air resistance, the distance traveled by the baseball inmeters after t seconds is given by the function f(t) = 4.9t^2 (a) Find the average speed (average rate of change of distance) of the baseball between 5 and 6 seconds.

```
ARC = (f(6) - f(5)) / (6 - 5)
f(t) = 4.9t^2
f(6) and f(5)

f(6) = 4.9 * (6^2) = 4.9 * 36 = 176.4
f(5) = 4.9 * (5^2) = 4.9 * 25 = 122.5
   ARC = (176.4 - 122.5) / (6 - 5) = 53.9 meters per second
average speed of the baseball between 5 and 6 seconds is 53.9 meters
per second.
```

b) Find the average speed (average rate of change of distance) of the baseball between 5 and 5.5 seconds.

```
f(5) and f(5.5)

f(5) = 4.9 * (5^2) = 4.9 * 25 = 122.5
f(5.5) = 4.9 * (5.5^2) = 4.9 * 30.25 = 148.225
ARC = (148.225 - 122.5) / (5.5 - 5) = 52.725 meters per second

average speed of the baseball between 5 and 6 seconds is 53.9 meters
per second.
```

(C) Find the average speed (average rate of change of distance) of the baseball between 5 and 5.1 seconds f(5.1) f(5) f(5.1) = 4.9 *(5.1^2) = 4.9* 26.01 = 127.449 f(5) = 4.9 *(5^2) = 4.9* 25 = 122.5 ARC = (127.449 - 122.5)/ (51.5 - 5) = 4.949 meters per second

```
 average speed of the baseball between 5 and 5.1 seconds is 4.949
meters per second.
```

(d) What is the instantaneous speed of the baseball at t = 5 seconds? Hint: You might want to do a few more calculations similar to those done in parts (a) - (c).

```
 f(t) = 4.9t^2

 f(t) = d/dt(4.9t^2) = 2 * 4.9t = 9.8t
 v(5) = 9.8 * 5 = 49 meters per second

 Instantaenous speed of the baseball at t = 5 seconds is 49 meters
per second.
```

(e) find the derivative of f. Hint: The derivative of a quadratic function f(x) = ax^2 + bx + c is given by f'(x) = 2ax + b

```
f(t) = 4.9t^2
f'(t) = 2 * 4.9t^2-1 = 9.8t
the derivative of f(t) is f'(t) = 9.8t
```

(f) Evaluate the derivative of f at t = 5. How does this value compare to the value found in part (d)? Explain what is happening.

```
The derivative of f(t) = 4.9t^2 at t = 5, we substitute t = 5 into
the expressin we did in qustion (e), f'(5) = 9.8 * 5 = 49 which is
the instanatenaeus velocity of the baseball at t = 5 that we got in
part (d), and the values are the same. The same conclusion fro the
instantatneus speed from the derivitate and the value calculated
directly in part (d) is expected. It is showing that the ARC between
two points  gets closer to the instantaneous rate of change as the
time between the two points gets closer to 0.  Basically the  more we
calculate the average speed over samller and smaller time intervals
around t = 5, we get closer to the instantaneous speed at t = 5 , and
the values will match.
```

A multivariate linear model is trained to predict the selling price of a particular used car model. The linear model predicts the selling price (P) in US dollars, using its current condition (C) on a scale of 1 − 10 and the age of the car in years (Y). The model is given by P = 16, 000 + 2, 400C − 1, 800Y .

P = is US dollars c = condition on a scale of 1 - 10 Y = age of car in years

(a) What is this model's predicted selling price of a 5-year old car with a condition rating 8?

```
P = 16,000 + 2,400C - 1,800Y
C = 8
Y = 5
P = 16,000 + 2,400(8) - 1,800(5) = 16,200

Models predicted selling price is $16,200
```

(b)Find ∂P/∂C and interpret this value P = 16,000 + 2,400C - 1,800Y for C: ∂P/∂C = 2,400 ∂P/∂C = 2,400 shows the rate of change of the predicted selling price P, with the condition rating C and holding the age of Y constant. For this example, every one unit increase in the condition rating, the predicted selling price is expected to increase by 42,400 if Y stays the same.

(c) Find ∂P/∂Y and interpret this value. P = 16,000 + 2,400C - 1,800Y ∂P/∂Y = -1,800 ∂P/∂Y = -1,800 represents the rate of change with P, assuming C stays constant. In this example if Y increases, the predicted P is expected to decrease by $1,800 if C stays the same.