



# On Bana-Comon Logic

## Seminar-Formal Methods in IT-Security

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## Outline

1. Backgrounds
2. The complexity analysis
3. A Case study of Squirrel
4. Summary

An overview of 2 kinds of models for communication protocols:

Model	Dolev-Yao	Computation
Primitives	Blackbox functions	Maps from bitstrings to bitstrings
Adversary	Restricted behaviour	Probabilistic Turing Machines
Range	Small	Large
Formalisation and Verification	Easy	Hard

How can we take advantage of the both models?

What is an interactive theorem prover(ITP)?

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A software tool that assists with the development of formal proofs by human-machine collaboration.

How are they important in terms of protocol security?

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Automation, easying modelling, universal.

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Our study is based on the 2 ITPs, Isabelle and Squirrel.

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- ▶ Efficiency  $\implies$  Complexity analysis
- ▶ Usefulness  $\implies$  Squirrel

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To summarize: Automation of the proof of protocols under the computational model

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- ▶ The folding process of the protocols
- ▶ Examples in Isabelle

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How should we show that a variable is in certain complexity classes?

$$\exists a \, b. f(n) = a \cdot n + b \implies f \in \mathcal{O}(n)$$

## Our definitions

$$\begin{aligned} msg &:= \mathbf{EMPTY} \mid \mathbf{ITE} \, bl \, msg \, msg \\ &\quad \mid \mathbf{CONS} \, msg \, msg \mid \mathbf{ATOM} \, string \\ bl &:= \mathbf{EQ} \, msg \, msg \mid \mathbf{EQL} \, msg \, msg \mid \mathbf{T} \mid \mathbf{F} \\ functions &: \mathbf{T\_eval} \, \mathbf{T\_bval} \, msg\_len \, bl\_len \end{aligned}$$

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## Our goals

$$\begin{aligned} \exists a \, b. \mathbf{T\_eval} \, msg &\leq a \cdot (\mathbf{msg\_len} \, msg) + b \\ \exists a \, b. \mathbf{T\_bval} \, bl &\leq a \cdot (\mathbf{bl\_len} \, bl) + b \end{aligned}$$

Time for Examples...

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- ▶ Logical operators, e.g. **AND XOR**
- ▶ Length of messages, bit-streams or characters?

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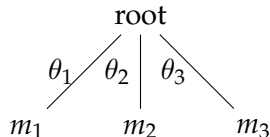
Our choice: as a tree!

Consider the following transition rules:

$$\begin{array}{lll} \text{root}, a \xrightarrow{\theta_1} m_1, b & \text{root}, a \xrightarrow{\theta_2} m_2, b & \text{root}, a \xrightarrow{\theta_3} m_3, b \\ m_2, b \xrightarrow{\eta_1} n_1, c & m_3, b \xrightarrow{\eta_2} n_2, c & m_3, b \xrightarrow{\eta_3} n_3, c \end{array}$$

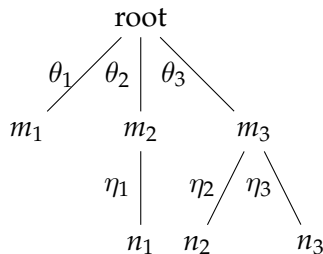
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How do we profit from this tree structure?

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Folding!

root  $\xrightarrow{\text{true}}$

```
if  $\theta_1$  then  $m_1$ 
else if  $\theta_2$  then ( $m_2$ ;
    if  $\eta_1$  then  $n_1$ 
    else empty)
else ( $m_3$ ;
    if  $\eta_2$  then  $n_2$ 
    else  $n_3$ )
```

Our definitions:

$$\begin{aligned} \textit{proc\_trie} &:= \mathbf{Rt} \ (bl \times \textit{proc\_trie}) \ \textit{list} \\ &\quad | \ \mathbf{Nd} \ \textit{msg} \ (bl \times \textit{proc\_trie}) \ \textit{list} \\ \mathbf{fun} \ \mathbf{valid} &:: \textit{proc\_trie} \Rightarrow \textit{bool} \end{aligned}$$

Our goals:

$$\begin{aligned} \mathbf{eval} \ (\mathbf{get\_msg} \ \textit{proc}) &= \mathbf{eval} \ (\mathbf{fold} \ \textit{proc}) \\ \exists a \ b. \mathbf{valid} \ \textit{proc} &\implies \mathbf{T\_fold} \ \textit{proc} \leq a \cdot (\mathbf{sz} \ \textit{proc}) + b \end{aligned}$$

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Evaluation:

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Evaluation:

- Protocol subjects are never repeated.
- Configuration of our tries/prefix trees.

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Squirrel in an overview:

- ▶ 3 basic types of object: message, timestamp and index.
- ▶ Various proof tactics available. Among them, **prf(Pseudorandom Function)** and **euf(Existential Unforgeability)** are Important.

Our experiments:

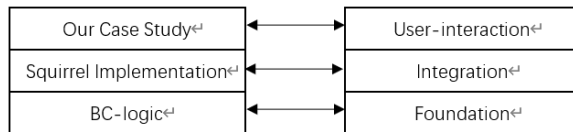
1. The tutorial
2. The simple examples
3. Exploration on the Kerberos protocol

Conclusion: Not easy for starters, incomplete as of beta-version, good future to expect

Expectations:

- ▶ Post-quantum cryptographic schemes
- ▶ A larger library of examples
- ▶ Better automated proof tactics

## Relation of our 2 parts of work



We have answered the 2 questions in the paper of BC-logic:

1. The evaluation and folding of BC-logic is efficient within polynomial time.
2. The BC-logic has its practical usage.

"Rules of logic are to mathematics what those of structure to architecture."

— Bertrand Russell

Thanks!