Monte Carlo TP 1 Exo 4

$$f(x) = \frac{\lambda}{2} e^{-\lambda|x|} \quad \forall x \in \mathbb{R}$$

(1) $F(t) = \int_{-\infty}^{t} \frac{\lambda}{2} e^{-\lambda|x|} dx$

if $t > 0$: $F(t) = \int_{-\infty}^{0} \frac{\lambda}{2} e^{+\lambda x} dx + \int_{0}^{t} \frac{\lambda}{2} e^{-\lambda x} dx$

$\Leftrightarrow F(t) = \frac{1}{2}\left[e^{\lambda x}\right]_{-\infty}^{0} + \left(-\frac{1}{2}\right)\left[e^{-\lambda x}\right]_{0}^{t}$

$\Leftrightarrow F(t) = \frac{1}{2} + \left(-\frac{1}{2}\right)\left(e^{-\lambda t} - 1\right) = 1 - \frac{e^{-\lambda t}}{2} = \frac{2 - e^{-\lambda t}}{2}$

if $t \leq 0$: $F(t) = \int_{-\infty}^{t} \frac{\lambda}{2} e^{+\lambda x} dx \Leftrightarrow F(t) = \frac{1}{2}\left[e^{\lambda x}\right]_{-\infty}^{t}$

$\Leftrightarrow F(t) = \frac{1}{2}\left(e^{\lambda t} - 0\right) \Leftrightarrow F(t) = \frac{e^{\lambda t}}{2}$

$$F : \begin{cases} \frac{2 - e^{-\lambda t}}{2} & \text{if } t > 0 \\ \frac{e^{\lambda t}}{2} & \text{if } t \leq 0 \end{cases}$$

(2) For $t \geq 0$: $u = \frac{2 - e^{-\lambda t}}{2} \Leftrightarrow u = 2u - 2 = -e^{-\lambda t}$

$\Leftrightarrow u = 2(1-u) = e^{-\lambda t} \Leftrightarrow \ln(2(1-u)) = -\lambda t$

$\Leftrightarrow t = -\frac{\ln(2(1-u))}{\lambda}$ $\quad u > 1$

For $t \leq 0$: $u = \frac{e^{\lambda t}}{2} \Leftrightarrow 2u = e^{\lambda t} \Leftrightarrow \ln(2u) = \lambda t$

$\Leftrightarrow t = \frac{\ln(2u)}{\lambda}$ $\quad u > 0$

$$F^{-1} : \begin{cases} -\frac{\ln(2(1-u))}{\lambda} & \text{if } t > 0 \\ \frac{\ln(2u)}{\lambda} & \text{if } t \leq 0 \end{cases}$$